

“Soft bang” instead of “big bang”: model of an inflationary universe without singularities and with eternal physical past time

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Abstract. The solution for an inflationary universe without singularities is derived from the Einstein-Lemaître equations. The present state of the universe evolved from a steady state solution for a tiny, but classical micro-universe with large cosmological constant or large equivalent vacuum energy density and with an equal energy density of radiation and/or some kind of relativistic primordial matter in the infinite past. An instability of this state outside the quantum regime caused a “soft bang” by triggering an expansion that smoothly started with zero expansion rate, continuously increased, culminated in an exponentially inflating phase and ended through a phase transition, the further evolution being a Friedmann-Lemaître evolution as in big bang models. As a necessary implication of the model the universe must be closed. All other parameters of the model are very similar to those of big bang models and comply with observational constraints.

Key words: cosmology: theory – cosmology: early Universe

1. Introduction

In classical big bang models the universe starts at infinite density and infinite temperature with infinite expansion velocity and expansion rate. No reason can be given why it expands so rapidly, the conditions at the big bang enter the models as unexplained initial conditions. In the case of an open universe these conditions must even prevail throughout the infinite space which the universe occupied already at the very beginning. Since for matter densities above the Planck-density ρ_{Pl} quantum-gravitational effects become important, the validity of classical big bang models is restricted to densities below ρ_{Pl} and times after the Planck time t_{Pl} at which this density is reached. Theories describing a quantum-evolution of the universe have been presented by several authors: de Witt (1967), Wheeler (1968), Hartle & Hawking (1983) and others. In general, singularities cannot be avoided in these models either. However, when it is assumed that at very early times there are no particles but only vacuum fields in the universe, then nonsingular solutions also exist, having a similar time evolution as the model considered in this paper (Starobinsky 1980).

A serious problem of former big bang models is the enormously large value of about 10^{30} times the Planck-length l_{Pl} obtained for the cosmic scale factor at the Planck time, spanning a range that is not causally connected but nevertheless must be the source of the fantastically isotropic microwave background radiation observed. This problem has been overcome by incorporating into the big bang models a phase of inflationary expansion (Guth 1981, Linde 1982 and Albrecht & Steinhardt 1982), according to which within an extremely short time a causally connected region with the extension of a Planck-length is inflated to the much larger domain obtained from the classical Friedmann-Lemaître evolution. Exponential expansion of an inflationary phase can be obtained for a cosmic substrate with positive energy density and negative pressure (Gliner 1965), and can be explained by the presence of a scalar Higgs field as assumed in the Grand Unified Theories (Georgi & Glashow 1974), other scalar fields, tensor fields or other mechanisms (see Overduin & Cooperstock 1998 for a list of references). The existence of a field of this kind is equivalent to the existence of a - dynamical - cosmological constant, and in the early stages of the universe, the value of the latter must be extremely high, $\Lambda \approx 10^{53}/\text{m}^2$, if the above mentioned difficulty should be overcome.

Thus, there are strong arguments supporting the existence of a large cosmological constant Λ in the very early universe even in the big bang models, classical or combined quantum/classical, and when speaking about big bang models, it will further be assumed that an inflationary phase due to the presence of a large Λ is included. When this necessity is accepted, then cosmological models become possible that avoid the singularities of classical big bang models and even avoid the necessity to invoke a theory of quantum gravity or rather precursors of such a theory which does not yet exist.

A singularity-free model was suggested by Israelit & Rosen (1989) (called IR-model in the following) according to which the (closed) universe was set into existence as a tiny bubble in a homogeneous and isotropic quantum state with a pre-material vacuum energy density corresponding to $\rho_{\text{v}} = \rho_{\text{Pl}} = 5.2 \cdot 10^{96} \text{ kg/m}^3$, with matter and/or radiation density $\rho_{\text{m}} = 0$ and with the diameter of a Planck length as an initial condition. In this model it is assumed that, because it is in the quantum regime, this state can prevail for some time. At the moment when it traverses the barrier to classical behavior it experiences an ac-

celerated expansion described by the expanding branch of a de Sitter solution. For the later evolution a standard model solution with $\Lambda = 0$, $\Omega_0 = 1.16$ and $H_0 = 46.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is assumed. The transition from de Sitter inflation into this evolution is performed by a phase transition (Kirzhnits 1972, Kirzhnits & Linde 1972 and Albrecht & Steinhardt 1982), transforming vacuum energy density into ordinary matter as in the inflation scenarios of big bang models. In order to get a smooth connection to a reasonable later evolution, this transition must occur at the very early time $t \approx 100 t_{\text{PL}}$ when the density of the standard model solution is $\rho_{\text{m}} \approx \rho_{\text{PI}} \cdot (t_{\text{PL}}/t)^{3/2} \approx 10^{-3} \rho_{\text{PI}}$. This is still far above the density of $\approx 10^{86} \text{ kg/m}^3$ that prevails in this model at the temperature corresponding to 10^{16} GeV at which GUTs with simple groups give rise to magnetic monopoles of mass $m_M \approx 10^{16} \text{ GeV}/c^2$ (’t Hooft 1974, Polyakov 1974 and Zeldovich & Novikov 1983). Since inflation is already over at this stage, no possibility exists in this model to dilute the heavy monopoles to such a low concentration that an expansion is possible until the present, and that the detection of magnetic monopoles is most unlikely as expressed by the fact they have not yet been observed.

Blome & Priester (1991) proposed a “big bounce model” (called BP-model in the following) in which the universe is closed, exists for an infinite time and first contracts from an infinite size. After it has passed through a minimal radius larger than l_{PI} it starts re-expanding. In the contraction phase the universe is in a primordial state of a quantum vacuum with finite energy density and negative pressure p . The “big bounce” at the minimal radius is supposed to trigger a phase transition by which ordinary matter is created, the vacuum energy density is reduced and the transition into a Friedmann-Lemaître evolution is achieved as in the inflation scenarios of big bang models. The parameters of the model are adjusted in a way such that the above mentioned monopole condition is satisfied.

The IR-model was further developed by Starkovich & Cooperstock (1992) and by Bayin et al. (1994). The pre-material vacuum energy density of the IR-model is attributed to a scalar Φ field obeying a covariant Klein-Gordon equation that is extended by an additional coupling term to the gravitational field and a potential of the field. Furthermore, an equation of state of the form $p = (\gamma - 1)\rho_v c^2$ for the pressure p is assumed, for the entropy density the equation of state $s = \gamma\rho/T$ is derived, and an adiabatic evolution according to $sS^3 = \text{const}$ is found to comply with the other equations. Three different epochs in the evolution of the universe are assumed for each of which γ is treated as a constant: first an inflationary era with very small γ , second a radiation era with $\gamma = 4/3$, and third a matter era with $\gamma = 1$. The starting and end points of the different eras are determined by critical or limiting values of physical quantities like the Planck density or the Planck temperature, the idea of taking these as limiting values for cosmological models being adopted from Markov (1982). As in the IR-model the (closed) universe starts at the Planck density ρ_{PI} with $S = l_{\text{PI}}$, and in addition to the IR-model the condition $\dot{S} = 0$ is imposed as initial condition, thus providing a start without singularity. (The model starts at the minimum radius of a big bounce model and,

in principle, could be extended further into the past by coupling it to a contraction phase like in the BP-model.) In the inflationary phase, due to the small value of γ the density ρ_v decreases only very slowly whence $T \sim \rho_v S^3$ increases, and the inflation is ended when T reaches the Planck temperature T_{PI} . This way no fine tuning is needed for the adjustment to the following radiation dominated era, and in addition, no re-heating is needed because, differently from usual inflation models, the universe enters the radiation era with the appropriate temperature.

Like big bang models with inflation, the model presented in this paper requires some primordial relativistic matter and/or radiation in addition to a high energy density of some primordial quantum field. It can even avoid singularities that are still present in the BP-model, i.e. infinite extension of the closed universe and infinite velocity of contraction in the far past, $\dot{S} = -\infty$, and it also avoids the problem of missing monopole dilution.

2. The “soft bang” model

The equations for a homogeneous and isotropic universe to be used are the well known Lemaître equations

$$\dot{S}^2 = \frac{8\pi G}{3} \rho S^2 + \frac{\Lambda c^2}{3} S^2 - kc^2, \quad (1)$$

$$\ddot{S} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} - \frac{\Lambda c^2}{4\pi G} \right) S \quad (2)$$

(G = gravity constant, Λ = cosmological constant, c = speed of light, $k = -1, 0, 1$, S = cosmic scale factor, ρ = mass density and p = pressure) following from Einsteins field equations. Eq. (2) is a consequence of Eq. (1) for $\dot{S} \neq 0$ when the energy equation

$$\frac{d}{dt}(\rho S^3) = -\frac{p}{c^2} \frac{d}{dt} S^3 \quad (3)$$

is employed.

Concerning the cosmological constant, as in the usual scenarios of cosmic inflation it is assumed that it can be explained and assumes a very large value due to the presence of some primordial vacuum field which at early times and very high temperatures was in a ground state of very high energy density. There are two possible ways to incorporate this assumption into Eqs. (1)–(2):

1. One can set $\Lambda = 0$ and replace $\rho \rightarrow \rho_{\text{m}} + \rho_{\text{v}}$, where ρ_{v} is the mass density of a quantum field corresponding to its energy density, and ρ_{m} is the mass density of matter plus radiation. In this case the equations must be supplemented with the ansatz $p_{\text{v}} = -\rho_{\text{v}} c^2$ for a negative pressure of the vacuum (Starobinsky 1980 and Zeldovich 1968), and it must be assumed that p_{v} adds to the pressure p_{m} of radiation and matter, yielding the total pressure $p = p_{\text{m}} + p_{\text{v}}$.

2. Equivalently one can set $\rho = \rho_{\text{m}}$, $p = p_{\text{m}}$ and replace Λ by $(8\pi G/c^2)\rho_{\text{v}}$. If in addition the equation of state for relativistic matter and radiation, $p_{\text{m}} = \rho_{\text{m}} c^2/3$, is employed, then from (3) both representations yield

$$\frac{\dot{\rho}_{\text{m}}}{\rho_{\text{m}}} = \frac{4\dot{S}}{S} \quad \text{or} \quad \rho_{\text{m}} S^4 = \rho_* S_*^4 \quad (4)$$

where ϱ_* and S_* are constant values of ϱ_m and S to be specified later, and from (1)–(2) both representations yield

$$\dot{S}^2(t) = \frac{8\pi G}{3} (\varrho_m + \varrho_v) S^2 - kc^2, \quad (5)$$

$$\ddot{S}(t) = -\frac{8\pi G}{3} (\varrho_m - \varrho_v) S. \quad (6)$$

Now the first step is to look for steady state solutions. For these $\dot{S} = \ddot{S} = 0$ is required, and from $\ddot{S} = 0$ and (6) the condition

$$\varrho_m = \varrho_v =: \varrho_* . \quad (7)$$

is obtained. It implies that in the static initial state from which the universe is supposed to evolve, there must be an equipartition between the energy of the vacuum and the energy of relativistic matter and/or radiation. Using this result, from the second requirement $\dot{S} = 0$ and from (6) the conditions $k = 1 > 0$ and

$$S = S_* := c \sqrt{\frac{3}{16\pi G \varrho_*}} \quad (8)$$

are obtained. It must, of course, be expected that, like Einstein’s static universe, this static solution will be unstable. However, it is just this property which opens the possibility that the present universe has evolved from it through an instability.

The instability of the static solution (8) follows from the existence of a dynamic solution that asymptotically converges towards it as $t \rightarrow -\infty$. To prove this and to derive the unstable solution it suffices to solve Eq. (5) because for $\dot{S} \neq 0$ Eq. (6) will then be satisfied automatically. In order to satisfy the condition imposed on the asymptotic behavior, $\varrho_v = \varrho_*$ and (8) must be inserted in Eqs. (4) and (5). However, it is more convenient to use (8) and replace ϱ_* with

$$\varrho_* = \frac{3c^2}{16\pi G S_*^2},$$

and from (5) one thus obtains

$$\dot{S}^2 = \frac{c^2}{2} \left(\frac{S_*^2}{S^2} + \frac{S^2}{S_*^2} - 2 \right) = \frac{c^2}{2} \left(\frac{S}{S_*} - \frac{S_*}{S} \right)^2 \quad (9)$$

or

$$\dot{S} = \pm \frac{c}{\sqrt{2}} \left(\frac{S}{S_*} - \frac{S_*}{S} \right). \quad (10)$$

For the equation with the plus sign one easily finds the solution

$$S = S_* \left(1 + e^{\sqrt{2} c(t-t_0)/S_*} \right)^{1/2} \quad (11)$$

with t_0 being an integration constant. (The solution for the minus sign, obtained from this solution by replacing $t - t_0 \rightarrow t_0 - t$, has $\ddot{S} < 0$ instead of $\ddot{S} > 0$ and is of no interest here because it describes a contracting universe.) For the solution (11) the universe has already existed for an infinite time and was separated from the static solution (8) through an instability in the infinite past. First it starts expanding very slowly, the expansion

velocity \dot{S} becoming larger and larger with time until the exponential term becomes dominant and the expansion exponentially inflating according to

$$S = S_* e^{c(t-t_0)/(\sqrt{2} S_*)}$$

just as after a big bang. During inflation the matter is getting extremely diluted and, as a consequence of this, cooled down as well. Therefore, in principle it would be necessary to replace (4b) at a certain stage by the equation $\varrho_m S^3 = \tilde{\varrho} \tilde{S}^3$ valid for cold matter. However, this is not necessary because at this stage the matter density ϱ_m is much smaller than the vacuum density ϱ_v and can thus be neglected.

It must be assumed that through the process of inflation a phase transition is triggered, transforming energy of the vacuum field into energy of matter as in the usual inflation models. From the simplifying assumption that this transition occurs instantaneously at the scale-factor $S_1 = S_F(t_1)$ of a Friedmann-Lemaître solution for regular matter extending until today, and from (11), the condition

$$S_1 = S_* \left(1 + e^{\sqrt{2} c(t_1-t_0)/S_*} \right)^{1/2},$$

is obtained, which is satisfied by the choice

$$t_0 = t_1 - \frac{S_*}{\sqrt{2} c} \ln \left[\left(\frac{S_1}{S_*} \right)^2 - 1 \right]$$

of the integration constant t_0 .

Since in the phase of inflation, according to (4), radiation and relativistic matter of the joint density ϱ_m are becoming extremely diluted, it must be assumed that energy of the vacuum field with density $\varrho_v c^2$ is transformed into energy of regular matter from which the present matter of the universe derives. Therefore, the time t_1 must be before the creation of quarks but late enough that no remarkable density of magnetic monopoles was able to develop, which would happen for $T(t_1) \geq 10^{29}$ K. With the choice $\varrho_* = 2 \cdot 10^{79}$ kg/m³ corresponding to $S_* \approx 3 \cdot 10^{-27}$ m, according to (8) the intersection of the solution (11) with a Friedmann-Lemaître solution $S_F(t)$ (obtainable from (12)) occurs at the time $t_1 \approx 10^{-33}$ s with $T(t_1) \approx 4 \cdot 10^{26}$ K, and both conditions are met. The radius $S_* \approx 3 \cdot 10^{-27}$ m from which the universe started according to the present model is well above the Planck length by a factor of $\approx 10^8$. Thus the present model is far out of the regime where quantum gravity has to be employed. Fig. 1 shows the time evolution of $S(t)$ for the present model ($\varrho_* = 2 \cdot 10^{79}$ kg/m³), for a big bang model with inflation ($\varrho_v = 2 \cdot 10^{79}$ kg/m³ and $\varrho_m = \varrho_{\text{Pl}} = 5 \cdot 10^{96}$ kg/m³), for the BP-model ($\varrho_v = 2 \cdot 10^{79}$ kg/m³), and finally for the IR-model ($\varrho_v = \varrho_{\text{Pl}} = 5 \cdot 10^{96}$ kg/m³).

A question of considerable interest is how and why the phase transition from inflation to ordinary Friedmann-Lemaître expansion is triggered. A dynamical evolution, for example some instability of the vacuum field, must be supposed, and it must be assumed that it was not present in the infinite time before inflation. Qualitatively it may be expected that, as in the inflation scenarios of big bang models, extreme temperatures before the

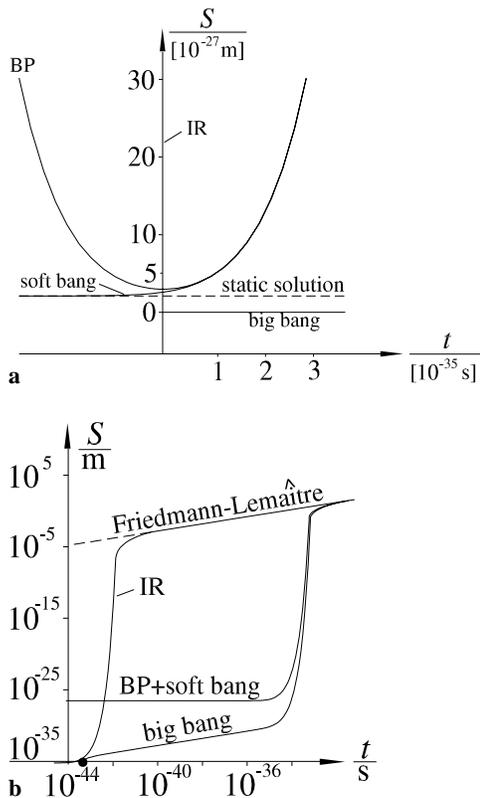


Fig. 1a and b. Evolution of $S(t)$ for the model of this paper in comparison with the IR-model, the BP-model and a big bang model **a** for times before and after the Planck-time in linear time scale, and **b** for $t \geq t_{P1}$ in logarithmic time scale. In the linear time scale according to the IR-model $S(t)$ increases so rapidly that it coincides with the S -axis, while for the big bang model $S(t)$ is still so small that it coincides with the t -axis. For $t \geq t_{P1}$ in logarithmic time scale the present model and the big bounce model become indistinguishable. After inflation all models merge into the same Friedmann-Lemaître evolution.

inflation give rise to thermal fluctuations that change the thermally averaged potential of a quantum field in such a way that it has a large positive minimum value. The phase transition can then be attributed to a decrease of this minimum caused by the rapid cooling through inflationary expansion.

During the phase transition, the extremely low values of density and temperature to which the primordial matter and/or radiation have been brought down through inflation must be restored to the high values that are required as initial values for the Friedmann-Lemaître evolution finally leading to the present state of the universe. For this process several possibilities exist:

1. The primordial matter remains as cold and diluted as it came out of the preceding inflation phase and is cooled down and diluted still further during the following Friedmann-Lemaître evolution. In this case the vacuum energy density ρ_v must be completely converted into the density of hot matter and radiation, and matter of a kind completely different from the primordial matter that existed for $t \rightarrow -\infty$ could be created. As a consequence, in addition to the matter that developed during and after the phase transition there could be a second compo-

nent of quite different matter, although diluted and cooled down extremely.

2. The vacuum energy could be completely used up for re-heating the primordial matter, at the same time increasing its density due to the mass contained in its thermal energy.

3. The third possibility consists in a combination of re-heating of old matter and creation of new matter from vacuum energy.

In this paper, no preference to any of these possibilities is given because the Friedmann-Lemaître evolution following the phase transition is the same for all of them.

In the following differences between the present model and big bang models are searched. Before the phase transition the present model provides much more time than big bang models, and also the ratio ρ_m/ρ_v is quite different, because in the big bang model the density ρ_m has already dropped to $\rho_m \approx 10^{-32} \rho_{P1} \approx 2.5 \cdot 10^{-15} \rho_v$ when it has reached the scale factor S_* , while in the present model $\rho_m = \rho_v$ at this stage. Therefore, the stability behavior with respect to perturbations that locally destroy the high symmetry of the cosmological principle may be quite different. However, it must be expected that differences arising this way are washed out by the exponential inflation. Furthermore, in the process of phase transition, in spite of quite different values of ρ_m , no differences can be expected to arise, because in both models ρ_m is extremely diluted so that it can be neglected in comparison with ρ_v which is the same in both models.

After the period of inflation, both models merge into a Friedmann-Lemaître evolution first described by the Lemaître equation

$$\dot{S}^2 = \frac{8\pi G}{3} \frac{\rho_1 S_1^4}{S^2} + \frac{\Lambda c^2}{3} S^2 - kc^2 \quad (12)$$

and later until today by the Friedmann equation

$$\dot{S}^2 = \frac{8\pi G}{3} \frac{\rho_0 S_0^3}{S} + \frac{\Lambda c^2}{3} S^2 - kc^2. \quad (13)$$

A complete discussion of all possible solutions is given in a survey paper by Felten & Isaacman (1986).

Since during the phase transition most of the vacuum energy is used up for re-heating and/or creation of matter, after it Λ must be either zero or have an extremely small value in comparison with its value before the phase transition. In order to obtain a solution that complies with the matter density presently observed in the universe, Λ must be different from zero.

As long as S is sufficiently small, the Λ - and the k -term on the right hand side of (12) can be neglected and the evolution of the present model and big bang models is essentially identical. Thus the possible appearance of differences is restricted to later times. From the Friedmann equation, for the quantities

$$\Omega_0 = \frac{\rho_m(t_0)}{\rho_{crit}(t_0)}, \quad \lambda_0 = \frac{\rho_\Lambda}{\rho_{crit}(t_0)} \quad (14)$$

with

$$\rho_{crit}(t) = \frac{3H^2(t)}{8\pi G}$$

the condition

$$\Omega_0 + \lambda_0 - 1 = \frac{kc^2}{H_0^2 S_0^2} \quad (15)$$

with H = Hubble parameter can be derived and becomes

$$\Omega_0 + \lambda_0 = 1 + \frac{c^2}{H_0^2 S_0^2} \quad \text{or} \quad \Omega_0 = 1 + \frac{kc^2}{H_0^2 S_0^2} \quad (16)$$

for the present model ($k = 1$) or the standard model ($k = -1, 0, 1$ and $\lambda_0 = 0$) respectively. Furthermore, for the deceleration parameter $q_0 = -\ddot{S}_0/(H_0^2 S_0)$ the result

$$q_0 = \frac{\Omega_0}{2} - \lambda_0 \quad (17)$$

is obtained.

In the present model, as in the standard model, the choice $\lambda_0 = 0$ is possible, and in this case no difference between the two arises if $k = 1$ is chosen in the latter as well. In the standard model $\Omega_0 = 1$ for $k = 0$, and when in the present model λ_0 is zero and Ω_0 is only slightly above 1, then the differences between the two models are again negligible.

The values $\Omega_0 \geq 1$ that have to be chosen in the cases $\lambda_0 = 0$ and $k \geq 0$ considered so far are much larger than the value $\Omega_0 \approx 0.2$ obtained from most observations when luminous matter and the dark matter inferred from galaxy motions are added (see e.g. Riess et al. 1998), and they can be only explained by assuming large amounts of as yet unobserved dark matter. In the standard model the observational value $\Omega_0 \approx 0.2$ can be only obtained for $k = -1$ while in the present model $\lambda_0 > 0.8$ is required. With these choices the differences between the two models are remarkable: In the standard model the expansion is decelerated ($q_0 = 0.1$) while it is accelerated in the present model ($q_0 \leq -0.7$), and in the present model the scale factor S_0 of today and the life time t_0 of the universe after the phase transition, that can be calculated from (16) and (12), are much larger than in the standard model. For convenience the values of S_0 , q_0 and t_0 obtained for the standard model, the present model and other models with $\lambda_0 > 0$ are listed in Table 1 for different choices of the parameters λ_0 , Ω_0 and k . For the evaluation of S_0 and t_0 the value $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ of the Hubble parameter, having a high probability according to latest measurements (see Riess et al. 1998), was used.

The closure of the universe, associated with $k = 1$, could in principle be detected, e.g. by the double observation of a very bright and distant object in opposite directions. So far searches for double observations have not been successful, but if the present model would apply, the result $S_0 = 67.4 \text{ ly} \gg ct_0 = 16.6 \text{ ly}$ would explain why.

It can be concluded in summary that pronounced differences between the present model and standard big bang models with inflation only appear if the universe does not contain large amounts of dark matter, and thus Ω_0 is markedly smaller than 1. The evaluation of observational data some time ago (Liebscher et al. 1992) and just recently (Riess et al. 1998, Perlmutter et al. 1998 and Branch 1998) yields a strong preference for a non-negligible positive cosmological constant of about the magnitude that was employed for the evaluation of the present model

Table 1. Typical parameter values obtained for the standard model, the present model and other models ($\lambda_0 > 0$ and $k < 1$) with $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$. S_0 is given in 10^9 light-years and the life time t_0 of the universe after the phase transition is given in 10^9 years. (For $k = 0$ no value of S_0 follows from (16).)

Model	λ_0	Ω_0	k	S_0	q_0	t_0
standard	0	0.2	-1	16.9	0.1	12.8
standard	0	1	0	—	0.5	8.7
stand./pres.	0	1.1	1	41.3	0.55	8.5
present	0.85	0.2	1	67.4	-0.75	16.6
other	0.75	0.2	-1	67.4	-0.65	15.9
other	0.8	0.2	0	—	-0.7	16.2

in the case $\Omega_0 = 0.2$. The recent observational data also confirm the negative value of the deceleration parameter q_0 connected with it. It can be seen from Table 1 that for comparable values of Ω_0 and λ_0 in an open universe ($k = 0$ and $k = -1$) almost the same results are obtained.

It is illuminating to check the “strong energy condition”

$$\varrho_m + \frac{3p}{c^2} - \frac{\Lambda c^2}{4\pi G} > 0$$

that must be satisfied for all times by solutions starting with a big bang singularity (see e.g. Wald 1984). With the assumptions underlying the present model the condition becomes

$$2(\varrho_m - \varrho_v) > 0.$$

The equilibrium from which the present model evolves through instability has $\varrho_m = \varrho_v$ and thus marks the boundary of the regime in which the strong energy condition is not satisfied; in the later evolution ϱ_m decreases while ϱ_v remains constant so the left hand side of the inequality becomes negative and thus recedes from the boundary.

It is possible to attribute the vacuum energy density of the present model to the same kind of scalar field as introduced by Starkovich & Cooperstock (1992) or by Bayin et al. (1994). In this case the treatment is only slightly modified, and the main results are essentially the same as obtained in this section as will be shown in Sect. 4.

3. Classification of inflationary solutions with $\Lambda = \text{const}$ and $k = 1$

In this section it is shown how the inflationary branch of the soft bang model presented in this paper fits into the framework of general inflationary solutions with $\Lambda = \text{const}$ (or $\varrho_v = \text{const}$ equivalently) for $k = 1$. With $\tau = ct$, $\alpha = 8\pi G/(3c^2)$ and $C = \varrho_m S^4$ Eq. (5) becomes

$$\dot{S}^2(\tau) + V(S) = 0 \quad (18)$$

with

$$V(S) = 1 - \frac{\alpha C}{S^2} - \alpha \varrho_v S^2. \quad (19)$$

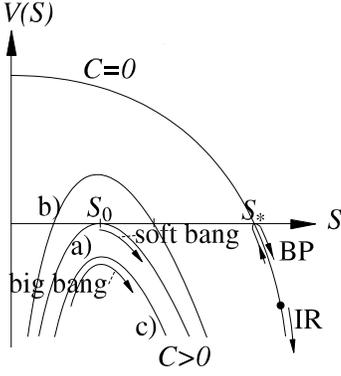


Fig. 2. Potential $V(S)$ for the cases $C = 0$ and $C > 0$. In the case $C > 0$, curve a) applies for $V_{\max} = 0$, b) for $V_{\max} > 0$ and c) for $V_{\max} < 0$.

1. Case $C = 0$

This is the matter free case of the IR-model or the BP-model resp. According to (18), $S(t)$ is restricted to values $S \geq S_*$ with $V(S_*) = 0$, since $V(S) = -\dot{S}^2 \leq 0$. From (19) one obtains

$$S_* = 1/\sqrt{\alpha\varrho_v} \quad (20)$$

with $\dot{S} = 0$ for $S = S_*$ according to (18). In the solution of the BP-model, $S(t)$ comes from ∞ for $t \rightarrow -\infty$, decreases until a minimum value $S_{\min} \gg l_{\text{Pl}}$ is reached, and then turns around to increasing values. This solution is obtained from (18) for $S_* = S_{\min} \gg l_{\text{Pl}}$.

On the other hand, the solution of the IR-model is obtained for $S_* < l_{\text{Pl}}$, starting at $S = l_{\text{Pl}}$ with $\dot{S} > 0$ and $\ddot{S} > 0$.

According to (20) $S_* = l_{\text{Pl}}$ for

$$\varrho_v = \frac{1}{\alpha l_{\text{Pl}}^2} =: \varrho_{v*}, \quad (21)$$

and thus the following classification is obtained:

solution of the BP-model	for	$\varrho_v < \varrho_{v*}$
solution of the IR-model	for	$\varrho_v > \varrho_{v*}$.

2. Case $C > 0$

Fig. 2 shows the potential $V(S)$ with $C > 0$ for three different kinds of solution together with its shape for $C = 0$. For $C > 0$ it has a maximum

$$V_{\max} = 1 - 2\alpha\sqrt{C\varrho_v} \quad \text{at} \quad S_0 = \left(\frac{C}{\varrho_v}\right)^{1/4}. \quad (22)$$

a) For $V_{\max} = 0$ or

$$C = \frac{1}{4\alpha^2\varrho_v}, \quad (23)$$

there is an unstable equilibrium point at $S = S_0$, and the soft bang solution of this paper is obtained. The curve given by (23) is shown in Fig. 3.

b) For $V_{\max} > 0$ or $C < 1/(4\alpha^2\varrho_v)$, big bounce solutions of the same type as in the BP-model are possible as well as solutions of the type used in the IR-model, the difference being

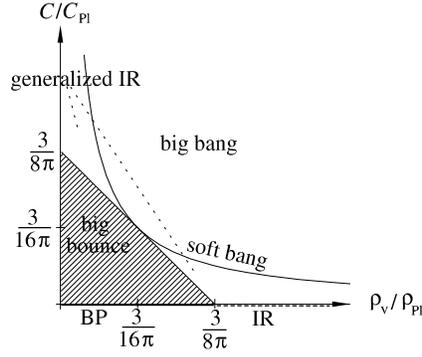


Fig. 3. Diagram C/C_{Pl} versus $\varrho_v/\varrho_{\text{Pl}}$ with location of the different kinds of inflation solutions with $k = 1$. The curve *soft bang* corresponds to (23) and is the location of soft bang solutions, the upper bound of the shaded area corresponds to (25). BP-model solutions are located on the $\varrho_v/\varrho_{\text{Pl}}$ axis in the range from 0 to $3/(8\pi)$, IR-model solutions in the range $\varrho_v/\varrho_{\text{Pl}} > 3/(8\pi)$. In the shaded area generalized big bounce solutions are obtained, in the area above it and below the soft bang curve generalized IR-solutions. The region above the soft bang curve is the location of big bang solutions.

that a matter and/or radiation density $\varrho_m = C/S^4 \neq 0$ coexists with the vacuum energy density ϱ_v . Since for all solutions $S_0 = (C/\varrho_v)^{1/4}$, it follows that $C = \varrho_v S_0^4 = \varrho_m S^4$. Now, $S(t)$ is restricted to $S \geq S_* > S_0$ (see Fig. 2) with

$$S_* = \frac{1}{\sqrt{2\alpha\varrho_v}} \left(1 + \sqrt{1 - 4\alpha^2\varrho_v C}\right)^{1/2} \quad (24)$$

obtained from $V(S_*) = 0$, and therefore

$$\varrho_m = \varrho_v (S_0/S)^4 \leq \varrho_v (S_0/S_*)^4 < \varrho_v.$$

Generalized big bounce solutions of the type considered in the BP-model are obtained for $S_* > l_{\text{Pl}}$, and generalized solutions of the IR-model type for $S_* < l_{\text{Pl}}$. The boundary between the two is given by $S_* = l_{\text{Pl}}$ which with (24) leads to the boundary equation

$$C = l_{\text{Pl}}^2 \left(\frac{1}{\alpha} - \varrho_v l_{\text{Pl}}^2\right) \quad (25)$$

represented in Fig. 3 by the upper boundary of the shaded area. c) For $V_{\max} < 0$ or $C > 1/(4\alpha^2\varrho_v)$ finally, big bang solutions with $k = 1$ are obtained with $\varrho_m = C/S^4 \rightarrow \infty$ for $S \rightarrow 0$.

Fig. 3 shows where the different kinds of inflation solutions with $k = 1$ are located in a C/C_{Pl} versus $\varrho_v/\varrho_{\text{Pl}}$ diagram, the definitions $l_{\text{Pl}} = (\hbar G/c^3)^{1/2}$, $\varrho_{\text{Pl}} = c^5/(\hbar G^2)$ and $C_{\text{Pl}} = \varrho_{\text{Pl}} l_{\text{Pl}}^4$ being used. The boundary curves (21) and (25) don't intersect but only touch at

$$\varrho_v = \frac{1}{2\alpha l_{\text{Pl}}^2} = \frac{3}{16\pi} \varrho_{\text{Pl}}, \quad C = \frac{l_{\text{Pl}}^2}{2\alpha} = \frac{3}{16\pi} C_{\text{Pl}}.$$

4. Attribution of the vacuum energy to a scalar quantum field

In this section it is shown that the vacuum energy can be attributed to a scalar quantum field Φ in essentially the same way

as by Starkovich & Cooperstock (1992) and by Bayin et al. (1994), only some slight modifications being necessary. For clearness the essential steps of their approach is briefly recapitulated.

The scalar field Φ that is responsible for the vacuum energy density is conformally coupled to the Ricci-curvature R and is described by a generalized Klein-Gordon equation

$$g^{\mu\nu} \partial_\mu \partial_\nu \Phi + \xi R \Phi + dV(\Phi)/d\Phi = 0 \quad (26)$$

where $V(\Phi)$ is the scalar field potential and ξ a numerical constant. According to Birrel & Davies (1982) the energy-momentum-tensor of the field is

$$T_{\mu\nu} = (\varrho_\nu + p_\nu/c^2) u_\mu u_\nu - p_\nu g_{\mu\nu} \quad \text{with} \quad (27)$$

$$u_\mu = \frac{\partial_\mu \Phi}{(\partial_\alpha \Phi \partial^\alpha \Phi)^{1/2}}, \quad (28)$$

$$\varrho_\nu c^2 = \frac{\dot{\Phi}^2}{2} + \frac{\Phi^2}{2} \left[\left(\frac{\dot{S}}{S} \right)^2 + \frac{1}{S^2} \right] + \frac{\dot{S}\Phi\dot{\Phi}}{S} + V(\Phi), \quad (29)$$

$$p_\nu = \frac{\dot{\Phi}^2}{6} + \frac{\Phi^2}{6} \left[\left(\frac{\dot{S}}{S} \right)^2 + \frac{1}{S^2} \right] + \frac{\dot{S}\Phi\dot{\Phi}}{3S} - V(\Phi) \quad (30)$$

$$+ \frac{\Phi dV(\Phi)/d\Phi}{3}. \quad (31)$$

With this from Einsteins field equations for $k = 1$ the equations

$$\dot{S}^2 = \frac{8\pi G}{3} \varrho_\nu S^2 - c^2, \quad (32)$$

$$\ddot{S} = \frac{8\pi G}{3} \left(1 - \frac{3\gamma}{2} \right) \varrho_\nu S \quad (33)$$

are obtained where

$$\gamma := 1 + \frac{p_\nu}{\varrho_\nu c^2}. \quad (34)$$

Differentiating (32) with respect to time t yields

$$\frac{d\varrho_\nu}{dS} = \frac{\dot{\varrho}_\nu}{\dot{S}} = \frac{3}{4\pi G S^2} \left(\ddot{S} - \frac{8\pi G}{3} \varrho_\nu S \right),$$

and eliminating \ddot{S} from this equation with (33) yields $d\varrho_\nu/dS = -3\gamma\varrho_\nu/S$ and

$$\varrho_\nu = \frac{\varrho_* S_*^{3\gamma}}{S^{3\gamma}}. \quad (35)$$

From (32) and (33) ϱ_ν can be eliminated yielding

$$\ddot{S} + \left(\frac{3\gamma}{2} - 1 \right) \left(\frac{\dot{S}^2 + c^2}{S} \right) = 0.$$

From this equation the time evolution of S can be determined independently of the evolution of the field Φ if γ is prescribed. Bayin et al. (1994) made the simplifying assumption that during different eras in the evolution of the universe the quantity γ defined in (34) assumed different but constant values, especially a very small one, $\gamma \approx 10^{-3}$, in the inflationary era.

With slight modifications these ideas can be incorporated into the present model. For this purpose it is assumed, that the field Φ can be present in addition to primordial matter of equal density ϱ_m and has the same properties as without matter. (In order to explain the primordial equipartition between energy of the field Φ and energy of matter, some interaction should be present, but this has to be so small that it can be neglected for the dynamical evolution.)

With this assumption, Eqs. (32)–(33) for $k = 1$ must be replaced by the equations

$$\dot{S}^2(t) = \frac{8\pi G}{3} (\varrho_m + \varrho_\nu) S^2 - c^2, \quad (36)$$

$$\ddot{S}(t) = -\frac{8\pi G}{3} \left[\varrho_m - \left(1 - \frac{3\gamma}{2} \right) \varrho_\nu \right] S, \quad (37)$$

and Eq. (35) must be employed instead of the former assumption $\varrho_\nu = \text{const}$ that is recovered from (35) for $\gamma = 0$.

For an equilibrium $\dot{S} \equiv 0$ and $\ddot{S} \equiv 0$ must be satisfied, and in order to obtain the same equilibrium as in Sect. 2 the assumption $\gamma = 0$ must be made. With this, from (36)–(37) one obtains $\varrho = \varrho_\nu =: \varrho_*$ and

$$S = S_* = c \sqrt{\frac{3}{16\pi G \varrho_*}} \quad (38)$$

as equilibrium conditions. The dynamics of deviations from equilibrium is obtained from from (32) with (4) and (35), and similarly as (9) one now obtains the equation

$$\dot{S}^2 = \frac{c^2}{2} \left(\frac{S_*^2}{S^2} + \frac{S^{2-3\gamma}}{S_*^{2-3\gamma}} - 2 \right) \quad (39)$$

for it. With $\gamma = 0$ the same results as in Sect. 2 would be obtained. An evolution similar to that obtained by Bayin et al. (1994) can be achieved by assuming that γ increases as soon as the equilibrium is left, and saturates at the small value $\gamma \approx 10^{-3}$ in order to obtain inflation.

For studying the separation of $S(t)$ from the equilibrium value S_* , the ansatzes

$$S = S_* + s = S_*(1 + \epsilon) \quad \text{with} \quad \epsilon = s/S_*$$

and

$$\gamma = \alpha \epsilon$$

with some constant α are made. Expansion of (39) with respect to ϵ up to terms of order ϵ^2 yields

$$\dot{S}^2 = \frac{c^2}{2} (4 - 3\alpha) \epsilon^2,$$

and for real solutions $\alpha < 4/3$ must be assumed in addition. With these assumptions an expansion evolution of the universe from an initial equilibrium state through instability becomes possible as in Sect. 2.

For $S \gg S_*$ with the assumption $\gamma = \text{const} \approx 10^{-3}$ Eq. (39) can be approximated by

$$\dot{S}^2 = \frac{c^2}{2} \left(\frac{S^{2-3\gamma}}{S_*^{2-3\gamma}} \right), \quad \dot{S} = \frac{c}{\sqrt{2}} \left(\frac{S}{S_*} \right)^{1-3\gamma/2}$$

and yields an evolution

$$S \sim (t - t_0)^{2/(3\gamma)}.$$

Since $2/(3\gamma)$ is very large, this is an inflation-like evolution although algebraic instead of exponential. At some stage and with similar assumptions and consequences as in Sect. 2 a phase transition described by a rapid change of γ must take place in order to enable the transition to a Friedmann-Lemaître evolution.

In the primordial equilibrium state $\gamma = 0$ viz. $p_v = -\rho_v c^2$, $S = S_*$ and $\dot{\Phi} = 0$. With this from (29)–(31) the condition

$$\frac{dV(\Phi)}{d\Phi} = -\frac{2\Phi_*}{S_*^2} \quad (40)$$

is obtained, and with the choice $\Phi_* = 0$ it can be achieved that $V(\Phi)$ has an extremum. $V(\Phi)$ is an arbitrary function and since no condition is obtained for $V(\Phi_*) = V(0)$, it appears that this quantity, which is contained in (29)–(31) and (33), can be chosen such that the extremum becomes a minimum.

5. Observational constraints

Models with a cosmological term Λ (or ρ_v equivalently) must comply with certain observational constraints for which a clear survey was given by Overduin & Cooperstock (1998).

Of course, the flatness constraint $\Omega_0 + \lambda_0 = 1$, where Ω_0 and λ_0 are the present values of $\Omega = \rho_m/\rho_{cr}$ and $\lambda = \rho_v/\rho_{cr}$ with $\rho_{cr} = 3H^2/(8\pi G)$, cannot apply for the present model since a space-time with positive curvature can never become completely flat.

Observations concerning CBM fluctuations, gravitational lens statistics, supernovas etc. restrict λ_0 to a range 0.5–0.8. The value 0.85 used for the calculations in this paper is in fair agreement with this and can be taken even smaller when Ω_0 is raised correspondingly.

The age of the universe is another quantity imposing rather stringent conditions on possible values of λ_0 . The present model has an infinite age of the universe. However, the time t_0 elapsed after the phase transition until today, coinciding with the time that was available for the creation of the elements observed in the universe and the evolution of stars and galaxies, should observe the same conditions as the age of universes with finite past. It was numerically evaluated for the present model, and its value presented in Table 1 is in very good agreement with the latest requirements derived from observational data (see e.g. Riess et al. 1998).

Another constraint is provided by the requirement that, in a closed universe, the antipode must be further away than the most distant object for which gravitational lensing is observed. For the present model, according to Table 1 the distance of our antipode is $\pi S_0 = \pi \cdot 67.4 \cdot 10^9$ ly which is still far beyond our horizon, so the gravitational lensing constraint is well observed.

By nonsingular models the maximum red-shift constraint must be observed which requires that the maximal value of the red-shift

$$z = \frac{S(t_0)}{S(t_{em})} - 1$$

(t_{em} = time of emission), obtained by inserting for $S(t_{em})$ the smallest value that S assumes in the model, must be at least as large as the greatest red-shift observed. In the present model, $S(t_0) = S_0 = 67.4 \cdot 10^9$ ly, the minimal value of S is $S_* \approx 3 \cdot 10^{-27}$ m, and therefore the greatest possible red-shift is much greater than the greatest one observed.

6. Discussion and summary

In the present model all kinds of singularities (e.g. of the expansion velocity, the expansion rate, or $S \rightarrow \infty$ for $t \rightarrow -\infty$ as in the BP-model) are avoided and no necessity arises for employing a theory of quantum gravity. For $t \rightarrow -\infty$ the (closed) universe is a tiny micro-universe in a classical static state. Its expansion is triggered by an instability and starts quite slowly at the velocity $\dot{S} = 0$ and expansion rate $\dot{S}/S = 0$. Only much later it gains appreciably until it becomes exponentially inflating and finally reaches the later expansion rate of an inflationary big bang universe. It is an advantage of the present model that the universe is not “born” with an unexplained and extreme expansion rate like in most big bang models, but that the observed expansion can be explained as the consequence of an instability in its far past.

It should be noted that the dynamical solution describing the departure from the unstable equilibrium obtained in this paper is quite different from the well known Eddington-Lemaître solution of the Einstein-Lemaître equations that describes the departure from Einstein’s static solution. While it has been shown by Börner & Ehlers (1988) and Ehlers & Rindler (1989) that for $\Omega_0 \geq 0.02$ the latter violates the maximum red-shift constraint, the present model is in good agreement with this constraint and other observations, as was shown in Sect. 5.

The coexistence of a quantum field of energy density $\rho_v c^2$ with some sort of primordial relativistic matter and/or radiation is an essential ingredient of the present model that may be critically considered and certainly needs discussion. The fact that a similar coexistence is assumed in big bang models with inflation may be invoked in support, but it may be a weak argument in view of opposing arguments raised by other authors. Priester et al. (1989) emphasize that a quantum vacuum state of the universe is the more natural choice for its primordial stage than a state in which elementary particles already have been present. Usually the origin of the vacuum energy density is assumed to be either the Higgs field that was introduced in elementary particle physics in order to explain the mass of elementary particles through interactions, or some other quantum field or other causes like worm-holes etc. The question is whether a primordial quantum field can exist on its own as a precondition for massive particles to be formed later, or whether it in turn needs these particles for its own existence. When the venerable principle *actio = reactio* is invoked, the second view appears as the more natural one. If this is accepted, then still another feature of the model becomes plausible. At first glance an extreme fine tuning appears to be needed in order that the condition $\rho_m = \rho_v$ is getting satisfied. However, in an equilibrium state equipartition between two interacting ingredients is the only natural

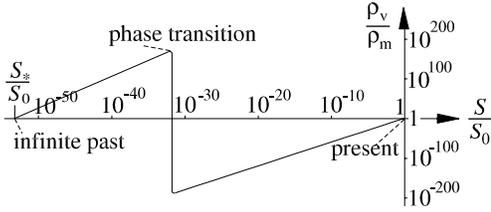


Fig. 4. . Ratio ρ_v/ρ_m as function of S/S_0 for the present model.

constellation, and in this spirit the fact, that equipartition in the equilibrium state results from the the field equations, may even appear as a confirmation.

It is interesting to note that the condition for the existence of an unstable equilibrium state from which the universe evolves through instability is quite contrary for the classical equilibrium of the present model and the quantum equilibrium considered by Starobinsky (1980): in the classical case in addition to the quantum field the presence of particles is necessary while in the quantum case just the absence of particles is required.

In this paper, in agreement with its modern interpretation as the energy density of some quantum field, the cosmological constant is treated as a dynamical variable rather than a geometrical one. However, this treatment is rather crude because Λ is still kept constant for most of the time – at a very large value during early stages of the universe and at its present low value after the phase transition until today. Many models have been proposed coupling the decay of ρ_v to the time evolution of the universe or its scale factor S - a survey is presented in a paper by Overduin & Cooperstock (1998) -, and it should be possible to replace the instant transition assumed in this paper with one of these more refined transitions without major changes in the results. The validity of this assumption was demonstrated in Sect. 4 for one specific model.

A look at the evolution of ρ_v/ρ_m used for the calculations of this paper (Fig. 4) shows that the present model could contribute to a solution of the “coincidence problem” (see Zlatev et al. 1999). This is raised by the latest observations according to which ρ_v lies in the same range as the matter density ρ_m today. The coincidence problem consists in the fact that, according to many models, ρ_m and ρ_v start at very different values in the early universe and require an extreme fine tuning at that time in order to reach almost equal values at present. In the soft bang model of this paper the universe starts with $\rho_m = \rho_v$, during the evolution of the universe there are times at which the two densities temporarily depart from each other, but today they are very close to each other again. It appears that the present model would provide a good starting point for developing a quintessence field with “tracking properties” (see Zlatev et al. 1999) – at least it appears to be in good agreement with the requirements of such a concept.

Conceptually it has been considered as a very satisfying property of big bang models that in them the universe does not have an eternal past but originated from some act of creation. In this sense the eternal past of the present model may appear as a conceptual disadvantage. However, the situation is not as bad as it appears. In physics time is a parameter that is used for ordering changes of states. However, when there are no changes then this order parameter loses its sense. In a very slowly changing situation it may therefore become more useful to consider the changes themselves as the order parameter that represents time instead of using an order parameter ordering no changes. In this sense the lifetime of the universe considered in this paper is not greater than that of big bang models because there has been even a smaller change from the original state to the present.

References

- Albrecht A., Steinhardt P.J., 1982, Phys. Rev. Lett. 48, 1220
 Bayin S.S., Cooperstock F.I., Faraoni V., 1994, ApJ 428, 439
 Birrell N.D., Davies P.C., 1982, In: Quantum Fields in Curved Space. Cambridge Univ. Press, 87
 Blome H.J., Priester W., 1991, A&A 250, 43
 Börner G., Ehlers J., 1988, A&A 204, 1
 Branch D., 1998, Nat 391, 23
 De Witt C.M., 1967, Phys. Rev. 160, 1113
 Ehlers J., Rindler W., 1989, MNRAS 238, 50
 Felten J.E., Isaacman R., 1986, Rev. Mod. Phys. 58, 689
 Georgi H., Glashow S.L., 1974, Phys. Rev. Lett. 32, 438
 Gliner E.B., 1965, Sov. Phys. JETP 22, 378
 Guth A.H., 1981, Phys. Rev. D 23, 347
 Hartle J.B., Hawking S.W., 1983, Phys. Rev. D 28, 2960
 ’t Hooft G., 1974, Nucl. Phys. B 79, 297
 Israelit M., Rosen N., 1989, ApJ 342, 627
 Kirzhnits D.A., 1972, JETP Lett. 15, 529
 Kirzhnits D.A., Linde A.D., 1972, Phys. Lett. 42 B, 471
 Liebscher D.E., Priester W., Hoell J., 1992, Astron. Nachr. 313, 265
 Linde A.D., 1982, Phys. Lett. 108 B, 389
 Markov M.A., 1982, JETP Lett. 36, 265
 Overduin J.M., Cooperstock F.I., 1998, Phys. Rev. D 58, 1
 Perlmutter S., Aldering G., Della Valle M., et al., 1998, Nat 391, 51
 Polyakov A.M., 1974, JETP Lett. B 20, 430
 Priester W., Hoell J., Blome H.J., 1989, Phys. Bl. 45, 51
 Riess A.G., Filippenko A.V., Challis P., et al., 1998, AJ 116, 1009
 Starkovich S.P., Cooperstock F.I., 1992, ApJ 398, 1
 Starobinsky A.A., 1980, Phys. Lett. 91 B, 99
 Wald R.M., 1984, General Relativity. The University of Chicago Press, 220
 Wheeler, J.A., 1968, In: De Witt C.M., Wheeler J.A. (eds.) Battelle Recontres. Benjamin (Publ.), New York
 Zeldovich Y.B., 1968, Sov. Phys. Usp. 11, 381
 Zeldovich Y.B., Novikov I.D., 1983, Relativistic Astrophysics Vol. II, Univ. of Chicago Press
 Zlatev I., Wang L., Steinhardt P.J., 1999, Phys. Rev. Lett. 82, 896