

Do we really see a cosmological constant in the supernovae data?

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Abstract. The magnitude-redshift relation is one of the tools for a direct observational approach to cosmology. The discovery of high redshift Type Ia supernovae (SNIa) and their use as “standard candles” has resurrected interest in this approach. Recently collected data have been used to address the problem of measuring the cosmological parameters of the universe. Analysed in the framework of homogeneous models, they have yielded, as a primary result, a strictly positive cosmological constant. However, a straight reading of the published measurements, conducted with no a priori idea of which model would best describe our universe at least up to redshifts $z \sim 1$, does not exclude the possibility of ruling out the Cosmological Principle - and cosmological constant - hypotheses. It is therefore shown here how the large scale homogeneity of this part of the universe can be tested on our past light cone, using the magnitude-redshift relation, provided sufficiently accurate data from sources at redshifts approaching $z = 1$ would be available. An example of an inhomogeneous model with zero cosmological constant reproducing the current observations is given. The presently published SNIa data can thus be interpreted as implying either a strictly positive cosmological constant in a homogeneous universe or large scale inhomogeneity with no constraint on Λ . An increase in the number and measurement accuracy of the candidate “standard candles” at very high redshift is therefore urgently needed, for progress in both fundamental issues of the Cosmological Principle and of the cosmological constant.

Key words: cosmology: observations – stars: supernovae: general

1. Introduction

The use of astronomical observations to directly determine the space-time geometry of our universe is a long time proposed instrument (Sandage 1961, Kristian & Sachs 1966, Ellis et al. 1985). First presented as an ideal program which could be realised “in principle”, it has of late become a new field in cosmology, due to recent tremendous progress in instrumental technology.

The magnitude-redshift relation is one of the tools for a direct observational approach. The discovery of high redshift

($z \sim 0.5-1$) Type Ia supernovae (SNIa), and of their potential use as “standard candles” (Phillips 1993, Riess et al. 1995, Hamuy et al. 1995, 1996a, 1996b, Perlmutter et al. 1997, Tripp 1998, Jha et al. 1999) has resurrected interest in this approach. Recently collected data, from ongoing systematic searches, have been used to address the problem of measuring the cosmological parameters of the universe. Analysed in the framework of homogeneous models, they have yielded, as a primary result, a strictly positive cosmological constant, many orders of magnitude smaller than the energy of the vacuum expected in standard particle physics models, as proposed by Riess et al. (1998), hereafter referred to as R98, and by Perlmutter et al. (1999), hereafter referred to as P99.

A non zero cosmological constant was long ago proposed as a possible interpretation of magnitude-redshift observations of cluster galaxies and quasars by Solheim (1966). But the accuracy of the data was not then sufficient to probe this hypothesis.

Anything that contributes to the energy density of the vacuum acts just like a cosmological constant. Analysed in the framework of Friedmann models of the universe, this can be viewed as an argument in favor of a zero cosmological constant, since some unknown symmetry of particle physics theory could presumably cancel the vacuum energy density (see e.g., Weinberg 1989). If the value of the cosmological constant was confirmed to be in the range favored by the SNIa announced results, it would be necessary to explain how it is so small, yet non zero. As such a result would have a revolutionary impact on our understanding of the fundamental laws of physics, it is of tremendous importance to check the different available interpretations.

Provided every other source of potential bias or systematic uncertainties has been correctly taken into account, one of the most vulnerable points of the data collecting procedure is the difference in the absolute magnitude of the supernovae due to an evolution of their progenitors. This point is discussed at length in R98 and P99 who conclude that their results cannot be markedly affected by this effect. Other authors (Drell et al. 1999, Dominguez et al. 1999) claim contrary evidence that high and low redshift supernovae observed so far, substantially differ from one another, and thus contest their use as “standard candles”. A progressive dimming of the SNIa by intergalactic dust has also been proposed as a possible systematic

effect that could mimic the behaviour of cosmic acceleration, and thus allow cosmologies without a cosmological constant (Aguirre 1999). This important problem is not treated in the present work, where the working assumption is made that the SNIa data actually measure the magnitude-redshift relation up to the precisions claimed in R98 and P99.

The theoretical interpretation is another matter which has to be dealt with carefully to avoid a priori assertions which would lead to incorrect results.

P98 discuss the possibility that the magnitude-redshift relation they find from the analysis of their data is due, not to a cosmological constant, but to an evolving field of unknown nature that contributes to the total energy density of the universe (see e.g., Steinhardt 1996, Caldwell et al. 1998, Garnavich et al. 1998). This entity, which can have an equation-of-state ratio different from that of the cosmological constant, would lead to a different expansion history.

It is proposed here to focus on a less exotic alternative, namely the possibility of large scale inhomogeneity of the part of the universe which can be probed with SNIa measurements.

Large scale spatial homogeneity of the universe is a belief most commonly shared by current cosmologists. It proceeds from a hypothesis brought to the status of Cosmological Principle by Einstein (1917). Its justification is based on two arguments:

1. The isotropy, or quasi-isotropy, of the temperature of the Cosmic Microwave Background Radiation (CMBR) around us.
2. The Copernican assumption that, as our location must not be special, the isotropy we observe must be observed identically from any point of the universe.

Since a matter distribution which is seen isotropic from everywhere implies homogeneity, the Cosmological Principle follows. However, of the two above arguments, only the first is observation grounded. The second, purely philosophical, has never been verified, and cannot thus constitute acceptable evidence.

As will be shown in Sect. 2, the claim for a strictly positive cosmological constant from SNIa data proceeds from an a priori homogeneity assumption. This, and the central role played in the whole cosmological field by the Cosmological Principle, is sufficient motivation to examine to which extent this Principle can be observationally tested.

As has been stressed by Ellis (1979), the situation is completely different within and outside our past light cone. For models exhibiting particle horizons, there are regions of the universe, outside our past light cone, from which we cannot receive any information at the present time. There is thus no way we can observationally verify the spatial homogeneity of such far out regions.

The possibility of direct verification on our past light cone has been studied by Partovi and Mashhoon (1984). These authors have explored the extent to which it is possible to distinguish between large scale homogeneous and inhomogeneous spherically symmetrical models, using magnitude-redshift data.

Owing to the then state of observational technique, they concluded this distinction could not be performed.

Analyses of galaxy redshift surveys have recently received increased attention. A controversy over whether the universe is smooth on large scale (Guzzo 1997, Cappi et al. 1998, Martinez et al. 1998, Scaramella et al. 1998, Wu et al. 1999) or presents an unbounded fractal hierarchy (Sylos Labini et al. 1996, Sylos Labini et al. 1998) has developed and waits for the next generation of wider and deeper galaxy catalogues which may provide a more conclusive answer (Martinez 1999).

The supernovae data have already been investigated for what they can tell about (in)homogeneity. Kim et al. (1997) used the first seven SNIa discovered by the Supernova Cosmology Project at $0.35 < z < 0.65$ and compared them to a nearby sample at $z \leq 0.1$ to declare the ruling out of the hypothesis of a locally underdense bubble. The nearby sample has also been examined for evidence of a local ‘‘Hubble Bubble’’ by Zehavi et al. (1998). The marginal signal identified by these authors is very sensitive to the cosmological model retained. Its value has been estimated for an Einstein-de Sitter universe, but would be less significant, and could be considered as included in the limits calculated by Kim et al., for the type of models proposed by R98 and P99.

At another redshift extreme, Starkman et al. (1999) have shown that the segment of the universe sampled by the current supernovae data is not large enough to determine the overall properties of the expansion. But these authors only consider the Einstein-de Sitter homogeneous case to complete their calculations.

It will however be shown, in Sect. 2, that in the near future, we would in principle, be able to probe homogeneity on scales up to, at least $z = 1$. In principle here means provided the evolution effect comes under control, as well as any other bias or systematic effect. But, as long as the relevance of the homogeneous models used by R98 and P99 as a framework for their data analyses is not verified, any claim to cosmological parameter measurements remains premature. Furthermore, a straight reading of the presently published results does not exclude a ruling out of the homogeneity hypothesis. It is moreover shown, in Sects. 3 and 4, that large scale inhomogeneity can mimic a cosmological constant in an homogeneous universe up to the precision achieved by current measurements.

2. Magnitude-redshift relation and homogeneous models

The luminosity distance D_L of a source is defined as the distance from which the radiating body, if motionless in an Euclidean space, would produce an energy flux equal to the one measured by the observer. It thus verifies

$$l = \frac{L}{4\pi D_L^2}, \quad (1)$$

L being the absolute luminosity, i.e. the luminosity in the rest frame of the source, and l the measured bolometric flux, i.e. integrated over all frequencies by the observer.

In Friedmann-Lemaître-Robertson-Walker (FLRW) models, the distance measure at redshift z is a function of z and of the parameters of the model (Carroll et al. 1992)

$$D_L = \frac{c(1+z)}{H_0\sqrt{|\kappa|}} \mathcal{S} \left(\sqrt{|\kappa|} \int_0^z [(1+z')^2(1+\Omega_M z') - z'(2+z')\Omega_\Lambda]^{-\frac{1}{2}} dz' \right), \quad (2)$$

H_0 being the current Hubble constant, Ω_M , the mass density parameter, and, Ω_Λ being defined as

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

where Λ is the cosmological constant and with

$$\begin{aligned} \text{for } \Omega_M + \Omega_\Lambda > 1 & \quad \mathcal{S} = \sin & \quad \text{and } \kappa = 1 - \Omega_M - \Omega_\Lambda \\ \text{for } \Omega_M + \Omega_\Lambda < 1 & \quad \mathcal{S} = \sinh & \quad \text{and } \kappa = 1 - \Omega_M - \Omega_\Lambda \\ \text{for } \Omega_M + \Omega_\Lambda = 1 & \quad \mathcal{S} = \text{I} & \quad \text{and } \kappa = 1. \end{aligned}$$

The apparent bolometric magnitude m of a standard candle of absolute bolometric magnitude M , at a given redshift z , is thus also a function of z and of the parameters of the model. Following Perlmutter et al. (1997), hereafter referred to as P97, it can be written, in units of megaparsecs, as

$$\begin{aligned} m &= M + 5 \log D_L(z; \Omega_M, \Omega_\Lambda, H_0) + 25 \\ &\equiv \mathcal{M} + 5 \log \mathcal{D}_L(z; \Omega_M, \Omega_\Lambda). \end{aligned} \quad (3)$$

The magnitude ‘‘zero-point’’ $\mathcal{M} \equiv M - 5 \log H_0 + 25$ can be measured from the apparent magnitude and redshift of low-redshift examples of the standard candles, without knowing H_0 . Furthermore, $\mathcal{D}_L(z; \Omega_M, \Omega_\Lambda) \equiv H_0 D_L(z; \Omega_M, \Omega_\Lambda, H_0)$ depends on Ω_M and Ω_Λ with different functions of redshift. A priori assuming that FLRW models are valid to describe the observed universe, R98 and P99 thus proceed as follows to determine Ω_M and Ω_Λ .

A set of apparent magnitude and redshift measurements for low-redshift ($0.004 \lesssim z \lesssim 0.1$) SNIa is used to calibrate Eq. (3), and another set of such measurements for high-redshift ($0.16 \lesssim z \lesssim 0.97$) is used to determine the best fit values of Ω_M and Ω_Λ ¹. The results of these measurements are plotted in Figs. 4 and 5 of R98 and Figs. 1 and 2 of P99. The magnitude-redshift relation from the data and the theoretical curves obtained for different values of the cosmological parameters are then compared and best-fit confidence regions in the parameter space are plotted, see Figs. 6 and 7 of R98 and Fig. 7 of P99.

It is convenient at this stage to make the following remarks.

If one considers any cosmological model for which the luminosity distance D_L is a function of the redshift z and of the other cosmological parameters of the model, and if this function is Taylor expandable near the observer, i.e. around $z = 0$, the

¹ The methods used by the two survey teams, the stretch factor method for P99 and the multicolor light curve shape and template fitting methods for R98, differ in their conceptions and byproducts: P99 calibrate \mathcal{M} and obtain direct fitting of Ω_M and Ω_Λ ; R98 calibrate M , which yields an estimate for H_0 . However, the following remarks apply to each of them.

analysis of observational data at $z < 1$, in the framework of this model, can legitimately use the Taylor expansion

$$\begin{aligned} D_L(z; cp) &= \left(\frac{dD_L}{dz} \right)_{z=0} z + \frac{1}{2} \left(\frac{d^2D_L}{dz^2} \right)_{z=0} z^2 \\ &+ \frac{1}{6} \left(\frac{d^3D_L}{dz^3} \right)_{z=0} z^3 + \frac{1}{24} \left(\frac{d^4D_L}{dz^4} \right)_{z=0} z^4 + \mathcal{O}(z^5), \end{aligned} \quad (4)$$

as, by definition of luminosity distances, $D_L(z=0) = 0$.

Here, cp denotes the set of cosmological parameters, pertaining to the given model, which can be either constants, as in FLRW models, or functions of z , as in the example presented in Sects. 3 and 4.

Luminosity distance measurements of sources at different redshifts $z < 1$ yield values for the different coefficients in the above expansion. Going to higher redshifts amounts to measuring the coefficients of higher power of z . For very low redshifts, the leading term is first order; for intermediate redshifts, second order. Then third order terms provide significant contributions. For redshifts approaching unity, higher order terms can no longer be neglected.

Therefore, for cosmological models with very high (or infinite) number of free parameters, such data only provide constraints upon the values of the parameters near the observer. But for cosmological models with few parameters, giving independent contributions to each coefficient in the expansion, the method not only provides a way to evaluate the parameters, but, in most cases, to test the validity of the model itself.

For FLRW models, precisely, one obtains from Eq. (2)

$$D_L^{(1)} \equiv \left(\frac{dD_L}{dz} \right)_{z=0} = \frac{c}{H_0}, \quad (5)$$

$$D_L^{(2)} \equiv \frac{1}{2} \left(\frac{d^2D_L}{dz^2} \right)_{z=0} = \frac{c}{4H_0} (2 - \Omega_M + 2\Omega_\Lambda), \quad (6)$$

$$\begin{aligned} D_L^{(3)} \equiv \frac{1}{6} \left(\frac{d^3D_L}{dz^3} \right)_{z=0} &= \frac{c}{8H_0} (-2\Omega_M - 4\Omega_\Lambda - 4\Omega_M\Omega_\Lambda \\ &+ \Omega_M^2 + 4\Omega_\Lambda^2), \end{aligned} \quad (7)$$

$$\begin{aligned} D_L^{(4)} \equiv \frac{1}{24} \left(\frac{d^4D_L}{dz^4} \right)_{z=0} &= \frac{5c}{72H_0} (8\Omega_\Lambda + 4\Omega_M\Omega_\Lambda + 2\Omega_M^2 \\ &- 16\Omega_\Lambda^2 - 12\Omega_M\Omega_\Lambda^2 + 6\Omega_M^2\Omega_\Lambda - \Omega_M^3 + 8\Omega_\Lambda^3). \end{aligned} \quad (8)$$

These coefficients are independent functions of the three parameters of this class of models, H_0 , Ω_M and Ω_Λ . With the method retained by P99 and described above, H_0 is hidden in the magnitude ‘‘zero-point’’ \mathcal{M} . The coefficients $\mathcal{D}_L^{(i)}$ of the expansion of \mathcal{D}_L are thus functions of Ω_M and Ω_Λ alone. However, the same following remarks, with D replacing \mathcal{D} , apply to the analysis and results of R98.

In standard models, i.e. for $\Omega_\Lambda = 0$

$$\mathcal{D}_L^{(2)} = \frac{c}{4} (2 - \Omega_M). \quad (9)$$

If the analysis of the measurements gives $\mathcal{D}_L^{(1)} < 2 \mathcal{D}_L^{(2)}$, it implies $\Omega_M < 0$, which is physically irrelevant. The model

is thus ruled out. This is what happens with the SNIa data, and what induces R98 and P99 to postulate a strictly positive cosmological constant, to counteract the $-\Omega_M$ term.

To test FLRW models with $\Omega_\Lambda \neq 0$, one has to go at least to the third order to have a chance to obtain a result. This seems to be the order currently reached by the SNIa surveys. This last assertion rests on the two following remarks:

1. As stressed in P97, R98 and P99, the well-constrained linear combination of Ω_M and Ω_Λ obtained from the data is not parallel to any contour of constant current ‘‘deceleration’’² parameter $q_0 = \Omega_M/2 - \Omega_\Lambda$. As $\mathcal{D}_L^{(2)} = \mathcal{D}_L^{(1)}(1 - q_0)/2$, this implies that higher order terms effectively contribute.
2. The contribution of the fourth and higher order terms is negligible. One can easily verify that, for the higher redshifts reached by the surveys, the contribution of the fourth order term does not overcome the measurement uncertainties, of the order of 5 to 10% (see Figs. 4 and 5 of R98 and Figs. 1 and 2 of P99).

A ruling out of FLRW models with non zero cosmological constant, at the third order level, i.e. due to a negative value for Ω_M , would occur provided (see Eqs. (5) to (7))

$$1 - \frac{\mathcal{D}_L^{(3)}}{\mathcal{D}_L^{(1)}} - 3 \frac{\mathcal{D}_L^{(2)}}{\mathcal{D}_L^{(1)}} + 2 \left(\frac{\mathcal{D}_L^{(2)}}{\mathcal{D}_L^{(1)}} \right)^2 < 0. \quad (10)$$

This cannot be excluded by the results of R98 and P99. It corresponds to the left part of the truncated best-fit confidence ellipsoidal regions in the $\Omega_M - \Omega_\Lambda$ plane of R98 Figs. 6 and 7 and P99 Fig. 7, and to the upper part of the error bars for the higher redshifts data in R98 Figs. 4 and 5 and P99 Figs. 1 and 2.

P99 propose an approximation of their results, which they write as

$$0.8 \Omega_M - 0.6 \Omega_\Lambda \approx -0.2 \pm 0.1, \quad (11)$$

which corresponds, in fact, to:

$$2 \left(\frac{\mathcal{D}_L^{(2)}}{\mathcal{D}_L^{(1)}} \right)^2 - 4.2 \frac{\mathcal{D}_L^{(2)}}{\mathcal{D}_L^{(1)}} - \frac{\mathcal{D}_L^{(3)}}{\mathcal{D}_L^{(1)}} \approx -1.8 \pm 0.1. \quad (12)$$

It must be here stressed that the results published as primary by P99 and R98, under the form of best-fit confidence regions in the $\Omega_M - \Omega_\Lambda$ plane, proceed from a Bayesian data analysis, for which a prior probability distribution accounting for the physically (in Friedmann cosmology) allowed part of parameter space is assumed. Results as given in the form of Eq. (11) are thus distorted by an a priori homogeneity assumption, which would have to be discarded for the completion of the test here proposed.

It may however occur that future, more accurate measurements yield values for the $\mathcal{D}_L^{(i)}$ s verifying Eq. (10) with the $>$ sign, which would correspond to a physically consistent positive Ω_M . In this case, the FLRW models will have to be tested

² A better name would be ‘‘acceleration’’ parameter, as $q_0 < 0$ implies an accelerated expansion for a homogeneous universe.

to the fourth order, i.e. with sources at redshifts nearer $z = 1$. A final test for the homogeneity hypothesis on our past light cone would be a check of the necessary condition, obtained from Eqs. (5) to (8), and which can be written as

$$\begin{aligned} \mathcal{D}_L^{(4)} = & -\frac{10}{9} \mathcal{D}_L^{(2)} + \frac{10}{3} \frac{\mathcal{D}_L^{(2)2}}{\mathcal{D}_L^{(1)}} - \frac{20}{9} \frac{\mathcal{D}_L^{(2)3}}{\mathcal{D}_L^{(1)2}} - \frac{20}{9} \mathcal{D}_L^{(3)} \\ & + \frac{10}{3} \frac{\mathcal{D}_L^{(2)}\mathcal{D}_L^{(3)}}{\mathcal{D}_L^{(1)}}. \end{aligned} \quad (13)$$

The above described method only applies to data issued from $z < 1$ supernovae. If the ongoing surveys were to discover more distant sources, up to redshifts higher than unity, the Taylor expansion would no longer be valid.

In practice, one will have to consider the Hubble diagram for the largest sample of accurately measured standard candles at every available scale of redshift from the lowest to the highest (e.g. Figs. 4 and 5 of R98 and Figs. 1 and 2 of P99). The FLRW theoretical diagram best fitting the data at intermediate redshift ($z \approx 0.6-0.7$ for measurements with 5% accuracy, $z \approx 0.8$ for 10% accuracy) will be retained as candidate. If it corresponds to a negative value for Ω_M , the assumption of large scale homogeneity for the observed part of the universe will have to be discarded, and no higher redshift data will be needed, at least to deal with this issue. If it corresponds to a positive Ω_M , the diagram will have to be extended to higher redshift data. If these data confirm the best fit of this candidate model up to $z \approx 0.8-0.9$ (for measurements at some 5–10% accuracy), the homogeneity hypothesis (and the corresponding values of the model parameters) would receive a robust support (provided no unlikely fine tuning of another model of universe parameters).

Another practical way of analysing the data is given by the very clever method described by Goobar and Perlmutter (1995), hereafter referred to as GP. An application of this method to probe large scale homogeneity of the observed universe would, in principle, imply the measurement of the apparent magnitude and redshift of only three intermediate and high redshift standard candles. In practice, more would certainly be needed to smooth out observational uncertainties. The method applies to sources with any redshift values and runs as follows.

Using Eqs. (2) and (3), one can predict the apparent magnitude of a source measured at a given redshift, in a peculiar FLRW model (i.e. for a given pair of values for Ω_M and Ω_Λ). Fig. 1 of GP shows the contours of constant apparent magnitude on the Ω_Λ -versus- Ω_M plane for two sufficiently different redshifts. When an actual apparent measurement of a source at a given redshift is made, the candidate FLRW model selected is the one with values for Ω_M and Ω_Λ narrowed to a single contour line. Since one can assume some uncertainty in the measurements, the allowed ranges of Ω_M and Ω_Λ are given by a strip between two contour lines. Two such measurements for sources at different redshift can define two strips that cross in a more narrowly constrained ‘‘allowed’’ region, shown as a dashed rhombus in Fig. 1 of GP. Now, if a third measurement of a source at a redshift sufficiently different from the two others is made, a third strip between two contour lines is selected. If this

strip clearly crosses the previously drawn rhombus (and if the measurement uncertainties are kept within acceptable values), it provides support for the homogeneity hypothesis, with the above pointed out reserve. In case this strip clearly misses the rhombus, this hypothesis is ruled out, at least on our past light cone.

If the latter happens to be the case, we shall need an alternative model to fit the data. An example of a model, able to fulfil this purpose without the help of a cosmological constant is proposed in the following section.

3. Example: Lemaître-Tolman-Bondi model with zero cosmological constant

The complexity of the redshift-distance relation in inhomogeneous models and their deviation from the Friedmann relation have recently been stressed by Kurki-Suonio and Liang (1992) and Mustapha et al. (1998). In a previous very interesting paper, Partovi and Mashhoon (1984) have shown that the luminosity distance-redshift relation, in local models with radial inhomogeneities and the barotropic equation of state for the source of gravitational energy, cannot be distinguished from the FLRW one, at least to second order in z . A special case of the solutions studied by these authors is used here to explore to which extent such inhomogeneous models can mimic homogeneity.

A class of spatially spherically symmetrical solutions of Einstein's equations, with dust (pressureless ideal gas) as a source of gravitational energy, was first proposed by Lemaître (1933). It was later on discussed by Tolman (1934) and Bondi (1947), and became popular as the "Tolman-Bondi" model. In the following, it will be referred to as LTB model.

Where the Cosmological Principle could be ruled out, the LTB solution would appear as a good tool for the study of the observed universe in the matter dominated region (Célérier & Schneider 1998, Schneider & Célérier 1999). It is used here as an example to show that a non vanishing cosmological constant in a FLRW universe can be replaced by inhomogeneity with a zero cosmological constant to fit the SNIa data.

The LTB line-element, in comoving coordinates (r, θ, φ) and proper time t , is

$$ds^2 = -dt^2 + S^2(r, t)dr^2 + R^2(r, t)(d\theta^2 + \sin^2\theta d\varphi^2), \quad (14)$$

in units $c = 1$.

Einstein's equations, with $\Lambda = 0$ and the stress-energy tensor of dust, imply the following constraints upon the metric coefficients:

$$S^2(r, t) = \frac{R'^2(r, t)}{1 + 2E(r)}, \quad (15)$$

$$\frac{1}{2}\dot{R}^2(r, t) - \frac{GM(r)}{R(r, t)} = E(r), \quad (16)$$

$$4\pi\rho(r, t) = \frac{M'(r)}{R'(r, t)R^2(r, t)}, \quad (17)$$

where a dot denotes differentiation with respect to t and a prime with respect to r , and $\rho(r, t)$ is the energy density of the matter.

$E(r)$ and $M(r)$ are arbitrary functions of r . $E(r)$ can be interpreted as the total energy per unit mass and $M(r)$ as the mass within the sphere of comoving radial coordinate r .

One easily verifies that Eq.(16) possesses solutions for $R(r, t)$, which differ owing to the sign of function $E(r)$ and run as follows.

1. with $E(r) > 0$, for all r

$$R = \frac{GM(r)}{2E(r)}(\cosh u - 1), \quad (18)$$

$$t - t_0(r) = \frac{GM(r)}{[2E(r)]^{3/2}}(\sinh u - u).$$

2. with $E(r) = 0$, for all r

$$R(r, t) = \left[\frac{9GM(r)}{2}\right]^{1/3} [t - t_0(r)]^{2/3}. \quad (19)$$

3. with $E(r) < 0$, for all r

$$R = \frac{GM(r)}{-2E(r)}(1 - \cos u), \quad (20)$$

$$t - t_0(r) = \frac{GM(r)}{[-2E(r)]^{3/2}}(u - \sin u).$$

where $t_0(r)$ is another arbitrary function of r , usually interpreted, for cosmological use, as a Big-Bang singularity surface³, and for which $R(r, t) = 0$. One can choose $t_0(r) = 0$ at the symmetry center ($r = 0$) by an appropriate translation of the $t = \text{const.}$ surfaces and describe the universe by the $t > t_0(r)$ part of the (r, t) plane, increasing t corresponding to going from the past to the future.

The physical interpretation of $R(r, t)$ must also be discussed. Bondi (1947) presents it as the radial luminosity distance of a radiating source. But, with the definition used here (above Eq. (1)) and a corrected expression for apparent luminosity, it corresponds in fact to the luminosity distance divided by a factor $(1 + z)^2$, as is shown below.

Assuming that the light wavelength is much smaller than any reasonably defined radius of curvature for the universe (geometrical optics approximation), Kristian & Sachs (1965) established that the apparent intensity of a source, as measured at any point of one of its emitted light rays by an observer with proper velocity u^a , is

$$l = \mu(k_a u^a)^2, \quad (21)$$

where μ is a scalar, corresponding to the magnitude of the electromagnetic tensor components as measured by the observer.

From the definition of redshift, one obtains (Ellis 1971)

$$1 + z = \frac{(k_a u^a)_s}{(k_a u^a)_o}, \quad (22)$$

³ In fact, the physical interpretation of this surface is improper. One can always consider that as the energy density increases while reaching its neighbourhood, radiation becomes the dominant energy component, pressure can no more be neglected, and the LTB model no longer holds.

where k_a is tangent to the null geodesics on which light travels, the subscripts s and o denoting values at the source and at the observer, respectively.

For a measure realised at the source by an observer motionless in the rest frame of the source

$$(k_a u^a)_s^2 = 1, \quad (23)$$

which gives⁴

$$l = \frac{\mu}{(1+z)^2}. \quad (24)$$

Assuming that the electromagnetic tensor for the light emitted by a distant source verifies Maxwell's equations for vacuum (Kristian & Sachs 1966, Bondi 1947), it follows, for a radial measurement realised on any light ray by an observer located at the symmetry center ($r = 0$), that

$$l = \frac{L}{4\pi R^2(1+z)^4}. \quad (25)$$

The definition of luminosity distance given by Eq. (1) yields the expression retained by Partovi & Mashhoon (1984), namely

$$D_L = (1+z)^2 R. \quad (26)$$

One thus sees that the luminosity distance D_L is a function of the redshift z and through R , of the parameters of the model: $M(r)$, $E(r)$ and $t_0(r)$.

A light ray issued from a radiating source with coordinates (t, r, θ, φ) and radially directed towards an observer located at the symmetry center of the model, satisfies, from Eqs. (14) and (15),

$$dt = - \frac{R'(r, t)}{\sqrt{1+2E(r)}} dr. \quad (27)$$

For a given function $R(r, t)$, Eq. (27) possesses an infinite number of solutions of the form $t(r)$, depending on the initial conditions at the source or on the final conditions at the observer. One can thus consider two rays emitted by the same source at slightly different times separated by τ . The equation for the first ray can be written

$$t = T(r). \quad (28)$$

The equation for the second ray is therefore

$$t = T(r) + \tau(r). \quad (29)$$

Assuming: $\tau(r) \ll T(r)$ for all r , Eq. (27) implies

$$\frac{dT(r)}{dr} = - \frac{R'[r, T(r)]}{\sqrt{1+2E(r)}}, \quad (30)$$

$$\frac{d\tau(r)}{dr} = - \frac{\tau(r)}{\sqrt{1+2E(r)}} \frac{\partial R'}{\partial t}[r, T(r)], \quad (31)$$

which gives the equation of a ray (Eq. (30)) and the equation for the variation of $\tau(r)$ along this ray (Eq. (31)).

If one considers $\tau(r)$ as the period of oscillation of some spectral line emitted by the source and $\tau(0)$, its period as measured by the observer at $r = 0$, the definition of the redshift is

$$\tau(r) = \frac{\tau(0)}{1+z}. \quad (32)$$

From this equation, another way of writing the rate of variation of $\tau(r)$ along a ray is

$$\frac{d\tau(r)}{dr} = -\tau(0) \frac{\partial z}{\partial r} \frac{1}{(1+z)^2}. \quad (33)$$

One can always choose z as a parameter along the null geodesics of the rays and obtain from the above equations

$$\frac{dr}{dz} = \frac{\sqrt{1+2E(r)}}{(1+z)\dot{R}'[r, T(r)]}. \quad (34)$$

Eq. (27) therefore becomes

$$\frac{dt}{dz} = - \frac{R'[r, T(r)]}{(1+z)\dot{R}'[r, T(r)]}. \quad (35)$$

Eqs. (34) and (35) form a system of two partial differential equations of which each null geodesic is a solution starting with z at the source and finishing at the observer with $z = 0$.

Successive partial derivatives of R with respect to r and t , and derivatives of $E(r)$ with respect to r , evaluated at the observer, contribute to the expression of the coefficients of the luminosity distance (Eq. (26)) expansion in powers of z . It is therefore interesting to note the behaviour of $R(r, t)$ and $E(r)$ near the symmetry center of the model, i.e. near the observer (Humphreys et al. 1997):

$$R(r, t) = R'(0, t) r + \mathcal{O}(r^2), \quad (36)$$

$$E(r) = \frac{1}{2} E''(0) r^2 + \mathcal{O}(r^3). \quad (37)$$

One thus easily sees that R , \dot{R} , and higher order derivatives of R with respect to t alone, vanish at the observer, as do E and E' .

Expressions for the coefficients of the luminosity distance expansion naturally follow; after some calculations, one obtains

$$D_L^{(1)} = \frac{R'}{\dot{R}'}, \quad (38)$$

$$D_L^{(2)} = \frac{1}{2} \frac{R'}{\dot{R}'} \left(1 + \frac{R' \ddot{R}'}{\dot{R}'^2} + \frac{R''}{R' \dot{R}'} - \frac{\dot{R}''}{\dot{R}'^2} \right), \quad (39)$$

$$D_L^{(3)} = \frac{1}{6} \frac{R'}{\dot{R}'} \left(-1 - \frac{R' \ddot{R}'}{\dot{R}'^2} + 3 \frac{R'^2 \ddot{R}'^2}{\dot{R}'^4} - \frac{R'^2 \ddot{R}''}{\dot{R}'^3} - 6 \frac{R' \ddot{R}' \dot{R}''}{\dot{R}'^4} + 4 \frac{R'' \ddot{R}'}{\dot{R}'^3} + 2 \frac{R' \ddot{R}''}{\dot{R}'^3} - 3 \frac{R'' \dot{R}''}{R' \dot{R}'^3} + 3 \frac{\dot{R}''^2}{\dot{R}'^4} + \frac{R'''}{R' \dot{R}'^2} - \frac{\dot{R}'''}{\dot{R}'^3} + \frac{E''}{\dot{R}'^2} \right), \quad (40)$$

with implicit evaluation of the partial derivatives at the observer.

⁴ Bondi improperly writes: $l = \mu (U = \overline{E^{\mu\nu}}$ in Bondi's notation).

Following Humphreys et al. (1997), one can adopt a covariant definition for the Hubble and deceleration parameters of a spherically symmetric inhomogeneous universe. These authors (see also Partovi & Mashhoon (1984)) derive expressions for H_0 and q_0 at the observer, in units $c = 1$:

$$H_0 = \frac{1}{D_L^{(1)}} = \left(\frac{\dot{R}'}{R'} \right)_0, \quad (41)$$

$$q_0 = -H_0 D_L^{(2)} - 3. \quad (42)$$

Substituting into Eqs. (38) to (40), it is easy to see that the expressions for the FLRW coefficients in the expansion of D_L in powers of z can mimic LTB with $\Lambda = 0$ ones, at least to third order. This is straightforward for $D_L^{(1)}$. The case of $D_L^{(2)}$ is discussed at length in Partovi and Mashhoon (1984). For higher order terms, it implies constraints on the LTB parameters, which will be illustrated below on the peculiar example of flat models. In fact, owing to the appearance of higher order derivatives of the parameter functions in each higher order coefficient, LTB models are completely degenerate with respect to any magnitude-redshift relation, while FLRW ones, of which the parameters are constants, are more rapidly constrained and cannot thus fit any given relation, when tested at sufficiently high redshifts.

4. Illustration: Flat LTB ($\Lambda = 0$) models

To illustrate which kind of constraints can be imposed on LTB parameters by current observational results, the peculiar case of spatially flat LTB ($\Lambda = 0$) models is analysed here.

Spatial flatness is a property of the subclass of LTB models verifying $E(r) = 0$ (Bondi 1947). In this case, the expression for R is given above by Eq. (19) and the calculation of the successive derivatives of R , contributing to the expressions for the expansion coefficients, is straightforward.

As the mass function $M(r)$ remains constant with time, it can be used to define a radial coordinate r : $M(r) \equiv M_0 r^3$, where M_0 is a constant.

With the covariant definition for H_0 above mentioned (Eq. (41)), the $D_L^{(i)}$ s, as derived from Eqs. (38) to (40), can thus be written, in units $c = 1$, in the form

$$D_L^{(1)} = \frac{1}{H_0}, \quad (43)$$

$$D_L^{(2)} = \frac{1}{4H_0} \left(1 - 6 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} \right), \quad (44)$$

$$D_L^{(3)} = \frac{1}{8H_0} \left(-1 + 4 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + 7 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} - 10 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} \right), \quad (45)$$

with the previously indicated choice $t_0(0) = 0$, where t_p is the time-like coordinate at the observer. It is convenient to note that t_p is not a free parameter of the model, since its value proceeds from the currently measured temperature at 2.73 K (Célérier & Schneider 1998).

A comparison with the corresponding FLRW coefficients gives the following relations:

$$\Omega_M \longleftrightarrow 1 + 5 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + \frac{29}{4} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + \frac{5}{2} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}}, \quad (46)$$

$$\Omega_\Lambda \longleftrightarrow -\frac{1}{2} \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + \frac{29}{8} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + \frac{5}{4} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}}. \quad (47)$$

The above Eq. (47) implies that a non vanishing cosmological constant in a FLRW interpretation of data at $z < 1$ corresponds to a mere constraint on the model parameters in a flat LTB ($\Lambda = 0$) interpretation.

Any magnitude-redshift relation, established up to the redshifts and with the precisions achieved by current measurements, i.e. at third order level, can thus be interpreted in either model. For instance, the latest results published in P99, and given under the form of Eq. (11), correspond, in a flat LTB ($\Lambda = 0$) interpretation, to

$$4.3 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + 3.625 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + 1.25 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} \approx -1 \pm 0.1. \quad (48)$$

Such a result would imply a negative value for at least one of the two quantities $t'_0(0)$ or $t_0''(0)$, which would be an interesting constraint on the ‘‘Big-Bang’’ function in the observer’s neighbourhood. For instance, a function $t_0(r)$ decreasing near the observer would imply, for a source at a given $z < 1$, an elapsed time from the initial singularity that is longer in an LTB model than in the corresponding FLRW one, i.e. an ‘‘older’’ universe⁵. A decreasing $t_0(r)$ has thus an analogous effect in a LTB universe to a positive cosmological constant in a FLRW one. They both make the observed universe look ‘‘older’’.

5. Conclusion and discussion

The interpretation of recently published data from high redshift SNIa surveys, devoted to the measurement of the magnitude-redshift relation, has been re-examined with no a priori idea about which model would best describe our universe, at least up to redshifts $z \sim 1$.

It has been shown that a straight reading of these data does not exclude the possibility of ruling out the Cosmological Principle.

A method of testing large scale homogeneity on our past light cone, provided sufficiently accurate data from ‘‘standard candles’’ at redshifts approaching $z = 1$ would be available, has been proposed. This method could be applied using the SNIa

⁵ In fact, see footnote 3, a lower temperature.

data, provided every source of potential bias or systematic uncertainties would be correctly taken into account. An evolution of the progenitors is generally considered as one of the more likely sources of potential systematic errors in the analysis of the supernovae data. Mustapha et al. (1997) have made the point that, given isotropic observations about us, homogeneity cannot be proved without either a model independent theory of source evolution, or distance measures that are independent of source evolution. If a perceptible evolution effect was to be put forward, its impact on the SNIa data would thus have to be evaluated with a model independent method, before using them as a test for homogeneity.

An example of an inhomogeneous model with a zero cosmological constant reproducing the current observations has been given. However, a vanishing cosmological constant is a minimal feature here imposed on the model only to prove that Λ is not necessary to fulfill the observational constraints. A non zero Λ would only add a new free parameter in the LTB equations and would not alter the following primary statement: the presently published SNIa data can be interpreted as implying either a strictly positive cosmological constant in an homogeneous universe or large scale inhomogeneity with no constraint on Λ .

The choice of over-simple inhomogeneous models as examples can, of course, be questioned. The most naive assumptions of spherical symmetry and centered observer, which have been discussed in other works (see e.g. C  l  rier & Schneider 1998, C  l  rier 1999), have been mainly retained for simplification purpose, but do not distort the above conclusions. It could be actually interesting to probe the possibility of an off-center observer (see e.g. Humphreys et al. 1997) with the SNIa data. Spherical symmetry is grounded on the observed quasi-anisotropy of the CMBR temperature, and can thus be considered as a sufficiently good working approximation.

The possibility of testing finer features of the model depends on observational improvements. But before aiming at such ambitious goals, it is of the utmost importance to complete the large scale homogeneity test. A previous increase in the number and measurement accuracy of the candidate ‘‘standard candles’’ at very high redshifts is therefore urgently needed, for progress in both fundamental issues of the Cosmological Principle and of the cosmological constant.

It has been shown however that LTB models are highly degenerate with respect to any magnitude-redshift relation, but this is not the case for FLRW models. The best way to prove large scale inhomogeneity would therefore be to disprove homogeneity. Conversely, the best way to prove a non zero cosmological constant would be to prove large scale homogeneity. But, as the Friedmann distance-redshift relation is a necessary but not a sufficient condition for homogeneity, its observed verification would not, in principle, be enough to support the Cosmological Principle. Even if this would imply a fine tuning of its parameters, the possibility for an inhomogeneous universe to mimic such a relation could not be excluded.

To consolidate the robustness of the future magnitude-redshift tests, it would therefore be worth confronting their re-

sults with the full range of available cosmological data. The most recent attempt to do so is the work by Bahcall et al. (1999) who conclude that the analysed observations can be considered as consistent with the standard Big-Bang picture of the expansion of the universe, but do not discuss the homogeneity hypothesis. A cross-check with a model independent analysis of the CMBR anisotropy would, for instance, likely yield a substantial improvement, as the CMBR data are known to provide orthogonal constraints on the models parameters.

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References

- Aguirre A.N., 1999, ApJ 512, L19
 Bahcall N.A., Ostriker J.P., Perlmutter S., Steinhardt P.J., 1999, in press, astro-ph/9906463
 Bondi H., 1947, MNRAS 107, 410
 Caldwell R.R., Dave R., Steinhardt P.J., 1998, Phys. Rev. Lett. 80, 1582
 Cappi A., Benoist C., da Costa L.N., Maurogordato S., 1998, A&A 335, 779
 Carroll S.M., Press W.H., Turner E.L., 1992, ARA&A 30, 499
 C  l  rier M.N., Schneider J., 1998, Phys. Lett. A249, 37
 C  l  rier M.N., 1999, Class. Quantum Grav., in press
 Dominguez I., H  flich P., Straniero O., Wheeler C., 1999, In: Future Directions of Supernovae Research: Progenitors to Remnants. in press, astro-ph/9905047
 Drell P.S., Loredo T.J., Wasserman I., 1999, ApJ, in press, astro-ph/9905027
 Einstein A., 1917, Sitzungsber. Preuss. Akad. Wiss. Phys.-Math. K1 142, Berlin
 Ellis G.F.R., 1971, In: Sachs R.K. (ed.) General Relativity and Cosmology. Rend. Scuola Int. Fis. Enrico Fermi, XLVII Corso, Academic Press, New York
 Ellis G.F.R., 1979, Gen. Rel. Grav. 11, 281
 Ellis G.F.R., Nel S.D., Maartens R., et al., 1985, Phys. Rep. 124, 315
 Garnavich P.M., Jha S., Challis P., et al., 1998, ApJ 509, 74
 Goobar A., Perlmutter S., 1995, ApJ 450, 14
 Guzzo L., 1997, New Astron. 2, 517
 Hamuy M., Phillips M.M., Maza J., et al., 1995, AJ 109, 1
 Hamuy M., Phillips M.M., Schommer R.A., et al., 1996a, AJ 112, 2391
 Hamuy M., Phillips M.M., Suntzeff N.B., et al., 1996b, AJ 112, 2398
 Humphreys N.P., Maartens R., Matravets D.R., 1997, ApJ 477, 47
 Jha S., Garnavich P.M., Kirshner R.P., et al., 1999, ApJS, in press, astro-ph/9906220
 Kim A.G., Gabi S., Goldhaber G., et al., 1997, ApJ 476, L63
 Kristian J., Sachs R.K., 1966, ApJ 143, 379
 Kurki-Suonio H., Liang E., 1992, ApJ 390, 5
 Lema  tre G., 1933, Ann. Soc. Sci. Bruxelles A53, 51
 Martinez V.J., Pons-Borderia M.J., Moyeed R.A., Graham M.J., 1998, MNRAS 298, 1212
 Martinez V.J., 1999, Sci 284, 445
 Mustapha N., Hellaby C., Ellis G.F.R., 1997, MNRAS 292, 817
 Mustapha N., Bassett B.A.C.C., Hellaby C., Ellis G.F.R., 1998, Class. Quantum Grav. 15, 2363
 Partovi M.H., Mashhoon B., 1984, ApJ 276, 4
 Perlmutter S., Gabi S., Goldhaber G., et al., 1997, ApJ 483, 565
 Perlmutter S., Aldering G., Goldhaber G., et al., 1999, ApJ 517, 565
 Phillips M.M., 1993, ApJ 413, L105
 Riess A.G., Press W.H., Kirshner R.P., 1995, ApJ 438, L17

- Riess A.G., Press W.H., Kirshner R.P., 1996, ApJ 473, 88
Riess A.G., Filippenko A.V., Challis P., et al., 1998, AJ 116, 1009
Sandage A., 1961, ApJ 133, 355
Scaramella R., Guzzo L., Zamorani G., et al., 1998, A&A 334, 404
Schneider J., C  lerier M.N., 1999, A&A 348, 25
Solheim J.E., 1966, MNRAS 133, 321
Starkman G., Trodden M., Vachaspati T., 1999, in press, astro-ph/9901405
Steinhardt P.J., 1996, Nat 382, 768
Sylos Labini F., Gabrielli A., Montuori M., Pietronero L., 1996, Physica A226, 195
Sylos Labini F., Montuori M., Pietronero L., 1998, Phys. Rep. 293, 61
Tolman R.C., 1934, Proc. Nat. Acad. Sci. 20, 169
Tripp R., 1998, A&A 331, 815
Weinberg S., 1989, Rev. Mod. Phys. 61, 1
Wu K.K.S., Lahav O., Rees M.J., 1999, Nat 397, 225
Zehavi I., Riess A.G., Kirshner R.P., Dekel A., 1998, ApJ 503, 483