

*Letter to the Editor***Ion acceleration in connection with a modulated solar wind termination shock: phase space propagation and complete energy spectra**

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**Abstract.** Using the general kinetic transport equation for cosmic ray particles the energization of heliospheric pick-up ions (PUI's) to Anomalous Cosmic Ray particles (ACR's) by means of non-linear wave-particle interactions is described. After creation of PUI's with initial energies of 1 keV/nuc they undergo phase space diffusion by Fermi-1 and Fermi-2 scattering processes in the heliosphere and at the termination shock and eventually appear as ACR's in the 10 MeV/nuc energy regime for which spatial diffusion becomes important. We present ion energy spectra derived from a fully consistent theoretical context and covering the range from 1 keV/nuc to 100 MeV/nuc. Our solutions of the kinetic transport equation reflect the effect of simultaneous operations of convective and diffusive transport processes in heliospheric phase space. As demonstrated in our calculations the kick-up from pick-up's to ACR's in our approach completely works on known physical grounds with no need to prescribe a specific injection of seed ACR's at the shock. The spectra resulting near the termination shock are far from being pure power-laws as is expected in classical shock acceleration theories. Thus the idea to start from measured VOYAGER ACR spectra and to apply a necessary demodulation up to a pure power-law cannot be used to find the shock location.

**Key words:** Sun: solar wind – interplanetary medium – ISM: cosmic rays – acceleration of particles – shock waves – turbulence

**1. The particle-modulated shock scenario**

Pick-up ions are produced in the heliosphere by ionization of interstellar neutral atoms (e.g. Rucinski et al. 1993). After pick-up by solar wind magnetic fields these ions are subject to pitch-angle-scattering, adiabatic deceleration, energization via Fermi-2 processes and transit time damping (Fisk et al. 1974, 1976a; Vasyliunas & Siscoe 1976; Klecker 1977; Möbius et al. 1988; Isenberg 1987; Fahr & Ziemkiewicz 1988; Bogdan et al. 1991; Chalov et al. 1995a, 1997; Fichtner et al. 1996).

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Early results (Pesses et al. 1981; Potgieter & Moraal 1988; Jokipii 1992; Chalov & Fahr 1994, 1995a) suggested that the solar wind termination shock and the region upstream with finite efficiency may convert preenergized pick-up ions into ACR particles by Fermi-1 and shock drift acceleration processes. After ACR's have become sufficiently energized they eventually can diffuse upstream from the shock and thus in addition to pick-up's appear in the inner solar system with typical energies of about 10 MeV/nucleon. The energy density of ACR particles near the termination shock proves to be high enough to enforce a dynamic modification of the shock structure. Two-fluid approaches of cosmic-ray-modified shock waves were already proposed by Donohue & Zank (1993), Zank et al. (1993), Krülls & Achterberg (1994), Chalov & Fahr (1994, 1995a, 1995).

More recently Chalov & Fahr (1997) have presented a three-fluid model of an ACR-modulated shock structure. In this approach pick-up ions are treated as a separate keV-energetic plasma fluid reducing the effective preshock solar wind Mach number and thus the shock strength. This three-fluid model includes a continuous injection of primary pick-up ions into the secondary ACR fluid all over the region of a decelerated plasma flow.

Upstream of the termination shock the energy distribution of pick-up ions has already developed a high-energy tail such that upon arrival at the shock selected particles from this tail can participate in further acceleration near the shock (see Liewer et al. 1993; Kucharek & Scholer 1995; Scholer & Kucharek 1999; Giacalone et al. 1994; Chalov & Fahr 1996; Zank et al. 1996; Lee et al. 1996). Some of these ions are thus subject to consecutive bouncing processes ahead of the shock till eventually they reach ACR energies.

We do not follow these processes explicitly here, but rather want to consider general features of phase space propagation of heliospheric energetic particles. It was soon recognized that this phase space transport allows for a split into two distinct forms: a) Convection and momentum diffusion of low energy particles, and b) spatial diffusion and adiabatic acceleration of high energy particles. The relevant kinetic transport equation allows to derive two different solutions for these different cases both of which

were treated separately in the literature up to now. As bridge between these two cases an artificial injection of ACR particles at the shock had to be introduced describing the percentage of preaccelerated pick-up's injected into the diffusive ACR regime.

In this paper we present a new numerical model to study the joint and simultaneous operation of all relevant phase-space transport processes like pre-acceleration of convected pick-up ions, energetization by the termination shock velocity profile and diffusive acceleration without any need to arbitrarily specify an injection process.

## 2. Theoretical approach and solutions

We look for the solution of the complete kinetic particle transport equation describing all relevant convective and diffusive transport processes in heliospheric phase space for particles with energies between 1 keV/nuc and 100 MeV/nuc without any need to prescribe a specific particle injection at the shock or to explicitly follow reflection processes at the shock-associated electric potential well. The phase-space dynamics rather is due to wave-particle induced couplings to the solar-wind flow. Assuming radial symmetry conditions and a pitch-angle isotropy of the distribution function  $f(r, p)$  ( $r$ : radial distance,  $p$ : particle momentum) which is expected to be maintained by effective pitch-angle scattering processes especially in the region near the termination shock we can formulate the relevant transport equation for the stationary case in its conservation law form valid for the flux of the differential particle density  $N = 4\pi r^2 p^2 f(r, p)$  in the form (see Dworsky 1999 or Dworsky & Fahr 1999):

$$\begin{aligned} & \frac{\partial}{\partial r} \left[ \left( \frac{\partial \kappa}{\partial r} + \frac{2\kappa}{r} + u \right) N \right] \\ & + \frac{\partial}{\partial p} \left[ \left( \frac{\partial D}{\partial p} + \frac{2D}{p} - \frac{p}{3r^2} \frac{\partial (r^2 u)}{\partial r} \right) N \right] \\ & = \frac{1}{2} \frac{\partial^2}{\partial r^2} (2\kappa N) + \frac{1}{2} \frac{\partial^2}{\partial p^2} (2DN) + 4\pi r^2 p^2 S \end{aligned} \quad (1)$$

In the above equation  $\kappa(r, p)$  and  $D(r, p)$  denote the spatial and the momentum diffusion coefficients,  $u(r)$  is the local solar wind velocity, and  $S(r, p)$  is the spectral local pick-up ion production rate. This linear partial differential equation of second order can be solved for very general physical conditions after transcribing it into an equivalent system of Ito-stochastic linear differential equations (see Achterberg & Krülls, 1992, Krülls & Achterberg, 1994, Chalov & Fahr, 1997, 1998, 1999). By use of stochastic transport terms and introduction of a two-dimensional stochastic variable  $W = \{W_r, W_p\}$  we arrive at the following system of first-order differential equations equivalent to Eq. (1):

$$dr = \left( \frac{\partial \kappa}{\partial r} + \frac{2\kappa}{r} + u \right) dt + \sqrt{2\kappa} dW_r, \quad \text{and} \quad (2)$$

$$dp = \left( \frac{\partial D}{\partial p} + \frac{2D}{p} - \frac{p}{3r^2} \frac{\partial (r^2 u)}{\partial r} \right) dt + \sqrt{2D} dW_p. \quad (3)$$

This system is integrated using Runge-Kutta methods for a representative sample of about  $10^6$  stochastic particles allowing to cover in a statistically significant manner the whole heliospheric phase-space. This procedure then permits to synthesize the local distribution function  $f(r, p)$  at all places in the heliosphere.

The energy-dependent momentum diffusion coefficient  $D(r, p)$  is used in the form adopted by Chalov & Fahr (1995a) or Fichtner et al. (1996) and is given by:

$$D = D_0 \frac{u_E^3}{r_E} \left( \frac{p}{u_E} \right)^{\gamma-1} \left( \frac{r}{r_E} \right)^{\gamma-\alpha} \quad (4)$$

where  $D_0$  is a constant factor and where  $\alpha$  and  $\gamma$  are power exponents of the spatial and the wave-number dependence of Alfvénic wave turbulences. The meaning of the unexplained quantities in the above equation is found in Chalov & Fahr (1995a). The spatial diffusion is treated here as diffusion perpendicular to the azimuthal magnetic field in the ecliptic and is described by a coefficient  $\kappa(r, p)$  identical to that used by Le Roux & Fichtner (1997). It is given by:

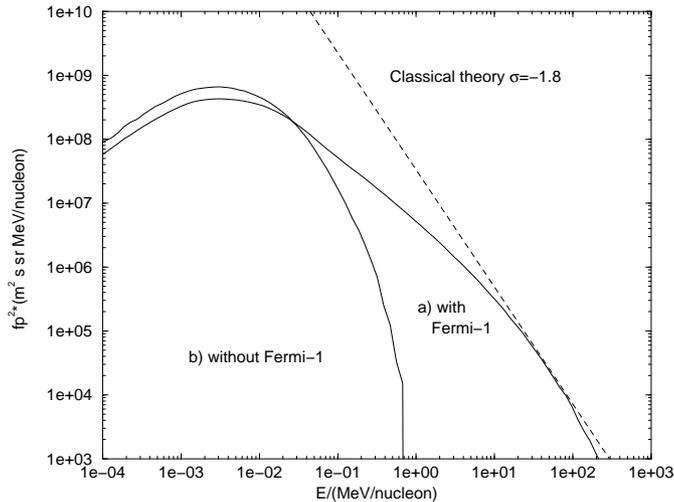
$$\kappa(r, p) = 0.015\kappa_0 \frac{p}{mc} \frac{r}{r_E} \begin{cases} 0.4 & \text{for : } R \leq 0.4\text{GV} \\ \frac{R}{R_0} & \text{for : } R > 0.4\text{GV} \end{cases} \quad (5)$$

with  $R$  being the particle rigidity, and  $\kappa_0 = 3.36 \times 10^{22} \text{ cm}^2 \text{ s}^{-1}$  and  $R_0 = 1 \text{ GV}$  being reference values. The spectral pick-up ion production rate is given by:  $S(r, p) = S(r)\delta(p - p_0)/\pi p_0^2$ , where the total local production rate  $S(r)$  due to charge exchange- and photo- ionisation of interplanetary H-atoms is taken from Rucinski et al. (1993), and where  $p_0$  is defined by:  $p_0^2/2m = 1 \text{ keV}$  (i.e. the kinetic energy of a solar wind proton). The transport equation (1) in principle is dynamically coupled to the solar wind equations, but is nevertheless solved in a test particle approximation here. That means we base our solutions onto the particle-modified solar wind velocity profile  $u = u(r)$  given by Le Roux & Fichtner (1997) with an inherent shock at 79.9 AU and a shock compression factor of  $s = 3.4$ .

## 3. Summary and discussion

The results which we obtain with the above mentioned approach are unique and innovative since they describe the origin and energy spectra of ACR particles as consequence of a continuous energization of pick-up ions up to the 100 MeV/nuc range by cooperative and simultaneous actions of Fermi-1 and Fermi-2 processes. Thus no injection of ACR particles at the shock has to be specified like done in earlier papers e. g. by Drury (1983), Lee (1983), Potgieter & Moraal (1988), Chalov & Fahr (1994), or le Roux & Fichtner (1997). In this respect we present comprehensive energy spectra for the range from keV up to 100 MeV.

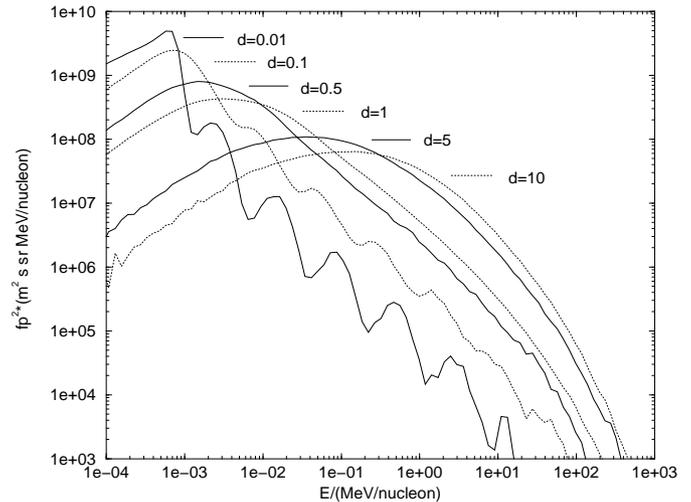
Fig. 1 shows two spectra a) and b) both obtained at the shock (i. e. 79.9 AU). Spectrum a), the complete spectrum, called "reference spectrum", results when all processes are operating the way as formulated in the preceding section of this paper. Contrary, spectrum b) results when diffusive shock acceleration



**Fig. 1.** Shown are two PUI/ACR spectra a) and b) calculated at the position of the heliospheric termination shock (at 79.9 AU): a) Standard spectrum with all relevant processes operating as taken into account in this paper. b) Spectrum obtained for identical conditions, but the diffusive shock acceleration (Fermi-1) is suppressed setting  $\kappa(r, p) = 10^{-3} \cdot \kappa_0(r, p)$ . To guide the eye in addition that power law is shown which is expected from classical theory for  $s = 3.4$  (i. e.  $\sigma = -1.8$ ).

(Fermi-1) is suppressed by arbitrarily reducing the reference value of the spatial diffusion coefficient  $\kappa(r, p)$  by a factor of  $10^{-3}$ . As can clearly be recognized, spectrum a) does show a high energy shoulder extending to energies larger than 100 MeV/nuc, whereas spectrum b) strongly falls off at energies below 1 MeV/nuc. Furthermore by the dashed line we show the power law spectrum with spectral index  $\sigma = (s + 1)/(1 - s) = -1.8$  which is expected by the classical shock acceleration theory by Drury (1983) on the basis of the shock compression value of  $s = 3.4$  underlying the present calculations. As obvious this power law only fits spectrum a) reasonably well at energies of 100 MeV/nuc whereas at energies below 100 MeV/nuc spectrum a) is much less steep.

The reason for this interesting outcome is evident: In the theory by Drury (1983) the shock is idealized by a one-dimensional discontinuous step in the thermodynamical properties of the background plasma, with homogeneous upstream and downstream plasma conditions. Due to the existence of pressure-relevant pick-up's and ACR's the shock, however, in reality is modified and structured into an extended precursor and a subshock as shown in papers by Chalov & Fahr (1994, 1995a, 1997) or le Roux & Fichtner (1997). Since particles with higher energies have larger spatial diffusion coefficients and correspondingly a larger mean free path  $\Lambda(r, p)$  for pitch-angle scattering they may see an extended part of the shock with a net compression factor  $s_{\text{eff}}(p)$  higher than that seen by lower energy particles. In the shock acceleration theory by Drury (1983), however, a monotonic increase of the spectral index  $\sigma$  with increasing  $s$  is expected, i. e.  $d\sigma/ds = 2/(1 - s)^2$ , suggesting a flattening of the spectrum with increasing energies. This outcome would even yield a much poorer fit of spectrum a) in Fig. 1 which is ob-

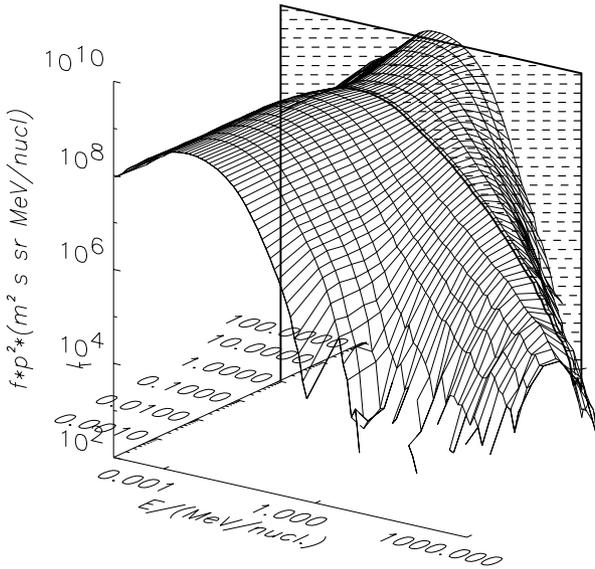


**Fig. 2.** Shown are spectra obtained at the termination shock calculated for conditions identical to those underlying curve a) in Fig. 1, however, with the factor  $d$  the momentum diffusion coefficient  $D(r, p) = d \cdot D_0(r, p)$  is varied with respect to the standard one, i. e.  $D_0(r, p)$ . The standard spectrum a) from Fig. 1 is thus appearing here as the curve for  $d = 1$ .

tained from our consistent calculations and reveals a steepening towards higher energies.

The fact that in no case the results by Drury (1983) or by Potgieter & Moraal (1988) can be reconciled with our results rather is due to the fact that no injection of particles in form of a delta function of the energy, i. e. by  $q \cong \delta(p - p_{\text{inj}})$ , is introduced by us here. The main injection into the Fermi-1 acceleration process in our approach is due to consecutive action of the adiabatic acceleration and the Fermi-2 acceleration term in connection with the spatial diffusion term. This effect is clearly demonstrated in Fig. 2 showing various spectra deviating from the reference spectrum, when momentum diffusion coefficients  $D(r, p) = d \cdot D_0(r, p)$  are applied which compared to the reference function  $D_0(r, p)$  are arbitrarily changed by factors  $d$  between 0.01 and 10.0.

In Fig. 3 in addition we have studied the effect of a variation of the spatial diffusion coefficient  $\kappa(r, p) = k \cdot \kappa_0(r, p)$ . The influence of the spatial diffusion coefficient  $\kappa(r, p)$ , however, turns out to be less univocal. While the peak intensity nearly stays unshifted for a variation in the range  $0.0001 \leq k \leq 100$ , the high energy spectral slope is stronger (steeper spectral decrease) both for larger ( $k \geq 1$ ) and smaller ( $k \leq 1$ ) than standard values for  $\kappa(r, p)$ . For small values of  $\kappa$  the diffusive shock acceleration (Fermi-1) is ineffective, since the probability of particles to reappear after shock passage in the region upstream of the shock is small (i. e. diffusive propagation velocity is small compared to convective motion). On the other hand, for large values of  $\kappa$  and consequently large mean free paths  $\Lambda$  particles can easily escape from the spherical shock structure and thus are lost for further Fermi-1 acceleration processes. This may also give a hint for the most likely fact that at different phases of the solar activity cycle when interplanetary Alfvénic turbulence



**Fig. 3.** Shown are spectra obtained at the termination shock calculated for conditions identical to those underlying curve a) in Fig. 1. except for the value of  $\kappa(r, p) = k \cdot \kappa_0(r, p)$  used. In the three-dimensional plot we have displayed spectra as function of energy in MeV/nuc for various values of  $k$ . The standard spectrum a) from Fig. 1 is thus appearing here as the curve for  $k = 1$ .

levels, and thus diffusion coefficients  $D$  and  $\kappa$ , may differ from their reference values  $D_0$  and  $\kappa_0$ , the particle spectra may look different in an activity-specific way.

Finally we would like to stress again the point that at the solar wind termination shock no power-law ACR-spectrum is obtained by us, while following Drury (1983) a pure power law could be expected at the shock with a power spectral index of  $\sigma = (s + 1)/(1 - s)$ .

In the case treated by us in this paper a compression factor of  $s = 3.4$  applies at the subshock (see le Roux & Fichtner 1997, from where the solar wind velocity profile is taken!) yielding  $\sigma = -1.8$ . In Fig. 1 we have shown this power law spectrum for the purpose of a comparison with our standard spectrum a) in Fig. 1. As evident, even at the shock position the resulting real spectrum a) very much differs from a simple power law putting attempts of Stone et al. (1996) into severe problems to simply derive the actual compression ratio at the solar wind termination shock and its location from needed demodulations of spectra obtained by VOYAGER-1/2. From these recently obtained spectra (i. e. at about 60 to 70 AU) which are already looking similar to our reference spectrum a) in Fig. 1, it thus cannot be excluded that these NASA satellites are already very

close to the termination shock, since no demodulation of the observed spectra for the sake to reach the expected power-law at the shock is required.

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