Astron. Astrophys. 353, 227–238 (2000)

ASTRONOMY AND ASTROPHYSICS

Loss of angular momentum of magnetic Ap stars in the pre-main sequence phase

K. Stępień[1]

Warsaw University Observatory, Al. Ujazdowskie 4, 00-478 Warszawa, Poland (kst@astrouw.edu.pl)

Received 3 August 1999 / Accepted 8 October 1999

Abstract. A model for rotation evolution of an intermediate mass star with the primordial magnetic field in the pre-main sequence (PMS) phase was developed. It takes into account the accretion of matter along the magnetic field lines, the stellar field-disk interaction and a magnetized wind. Variations of stellar moment of inertia were included based on evolutionary models of PMS evolution of such stars. Stellar mass and magnetic moment were assumed constant during the PMS evolution. Values of the parameters describing the strength of the magnetic field, accretion rate and mass loss rate were taken from observations. In addition, the life time of the disk was varied. An equation describing the evolution of the rotation rate of a magnetic PMS star was derived and solved for different stellar masses.

The results indicate that the interaction of the stellar ymagnetic field with circumstellar environment wipes out quickly a memory of the initial rotation period. The ZAMS period depends solely on the details of this interaction. Accretion spins up a star early in its PMS life and if the disk disappears right after that the star may keep its faster rotation until ZAMS and appear there as a Be star.

A wide variety of parameters describing the evolution of stellar AM results in typical ZAMS rotation periods of magnetic stars several times longer than of normal stars. This agrees well with the observations. Under special circumstances a star can reach an exceptionally long rotation period of several years (up to 100 years). This requires a long PMS life time, an existence of a disk for only a part of the PMS phase and the wind in the strong magnetic field existing for the rest of the PMS life. The observations confirm indeed that extremely slowly rotating Ap stars are lower mass stars with strong magnetic fields.

Key words: stars: chemically peculiar – stars: emission-line, Be – stars: magnetic fields – stars: pre-main sequence – stars: rotation

1. Introduction and observational data

Approximately 10–15% of all main-sequence (MS) stars of spectral types between late B and early F show chemical peculiarities in their spectra (Seitter & Duerbeck 1982). The peculiarities are believed to result from an upwards selective diffusion of some elements for which the radiative force exceeds the gravitational attraction and a downwards gravitational settling of other elements. Such a separation produces apparent overabundances of the floating elements and under-abundances of the sinking ones in atmospheres of chemically peculiar stars (Michaud 1970). A necessary condition for the occurrence of this process is a lack of vigorous mixing in a stellar atmosphere. Intense mixing is expected to exist in atmospheres of rapidly rotating stars due to meridional circulation currents, hence no peculiarities should occur in them. Observations show indeed that the two major groups of peculiar stars, Am and Ap stars, rotate on average substantially slower than normal stars of the same spectral types. The presence of a large scale surface magnetic field additionally stabilises a stellar atmosphere. Such fields are not observed in Am stars but a high percentage of binaries has been detected among them. This suggests that their slower rotation rates may be the result of tidal interaction between the components. Ap stars show, as a rule, measurable magnetic fields, in several cases with a kilogauss intensity, which indicates a casual relation between the existence of a strong field and a slow rotation. The present paper deals with this group of stars.

Statistical data about rotation rates of Ap stars were published, among others, by Abt et al. (1972), Wolff (1981) and, more recently, by Abt & Morrell (1995). According to Wolff, the average $v \sin i$ is equal to 54 km s^{-1} for Ap stars with masses equal to 4 $M_\odot$ (data for 63 stars), 36 km s^{-1} for 3 $M_\odot$ (64 stars measured), and 24 km s^{-1} for 2 $M_\odot$ (61 stars measured). She did not give comparable data for normal stars so her observations must be compared with the catalogue data. Assuming that the B7-B8 type stars have 4 $M_\odot$, their $< v \sin i >$ is equal to 165–170 km s^{-1} according to Allen (1973) and about 220 km s^{-1} according to Schmidt-Kaler (1982). Similarly, assuming B9-A0 for 3 $M_\odot$, one finds 145–150 km s^{-1} or 180–190 km s^{-1}, respectively, and if A5 stars have 2 $M_\odot$, their average rotation rates are 115 km s^{-1}, or 170 km s^{-1}, respectively. Comparing the latter numbers with the data for Ap stars one sees that the 4 $M_\odot$ Ap stars rotate on average 3–4 times slower, 3 $M_\odot$ Ap stars 4–5 times slower and 2 $M_\odot$ up to 6–7 times slower than their normal counterparts.

Abt & Morrell (1995) measured rotation of Ap stars as a part of a larger program aimed at all bright A-type stars. Their results show that $< v \sin i >$ of 188 normal A0-A1 stars is equal to 142 km s^{-1}, whereas this value for 43 Ap stars of the same spectral types is equal to 32 km s^{-1}, which gives the ratio of 4.4. For A2-A4 stars the corresponding numbers are 131 km s^{-1} for 242 normal stars and 41 km s^{-1} for 29 Ap stars (ratio 3.2), and for A5-F0 stars they obtained 125 km s^{-1} for 234 normal stars and 30 km s^{-1} for 17 Ap stars (ratio 4.2). Abt & Morrell (1995) determined not only the average values but they also analysed the $v \sin i$ distribution for each group of stars. They reached a conclusion that all these distributions can be well represented by separate Maxwellian distributions. This supports an earlier conclusion by Preston (1970) that rotation rates of peculiar stars do not belong to a slow rotation tail of a single distribution of all MS stars of the respective spectral type but form a separate Maxwellian distribution.

The data of Wolff and Abt & Morrell are in a reasonable agreement: they both show that Ap stars rotate about 4 times slower than normal MS stars of the same spectral types.

We can summarise the above discussion with a conclusion that, under the assumption that internal rotation of both groups of stars is not distinctly dissimilar, Ap stars have on average only one quarter of the average angular momentum (AM) of normal stars from the same spectral range. Had they been formed with similar AMs as normal stars they must have subsequently lost on average at least 75% of their initial AM.

The average observed $v \sin i$ of normal late B and A stars corresponds to the rotation period of the order of 0.5–1 day whereas that of typical Ap stars to the rotation period of about 2.5–4 days. Slow down of these Ap stars will be the main subject of the present paper.

While we can expect to find among stars with known rotation periods a limited number of stars with significantly longer periods, the probability of observing an extremely long period – in the range of years, is extremely low if rotation rates of all Ap stars are described by a single Maxwellian distribution (Preston 1970). Yet there are several Ap stars with known variability periods of the order of many years (Mathys et al. 1997). If, according to the commonly accepted oblique rotator model, these periods are identified with the stellar rotation periods, the extremely slowly rotating Ap stars are overrepresented by many orders of magnitude within a sample of peculiar stars with known rotation rates (Preston 1970).

The presence of so many extremely slowly rotating stars indicates the existence of an additional, very efficient braking mechanism operating for some Ap stars. Such a mechanism will also be discussed.

The reasons for slower than normal rotation of Ap stars may be as follows:

- they are formed from proto-stellar clouds possessing particularly low AM,
- they lose most of the original AM during early phases of contraction,
- they emerge from the intense contraction phase with AM similar to AM of normal stars but they lose most of it in the pre-MS (PMS) phase
- they land on ZAMS with AM similar to normal stars but they lose AM during the subsequent MS phase of evolution.

The first possibility seems unlikely; many Ap stars are members of stellar clusters, including associations. Have they been formed from a low rotation tail of the initial distribution of AM of proto-stellar clouds, they should occupy a low rotation tail of a single distribution of all cluster members (assuming no difference in subsequent rotational history of normal and Ap stars). This is not the case. Instead, as discussed above (Abt & Morrell 1995), both groups belong to two different Maxwellian distributions. The second possibility cannot be excluded off-hand but must wait until we can model these early phases of contraction of a rotating protostar sufficiently accurately. At present it remains purely speculative, hence we will not discuss it in the present paper. The fourth possibility can be rejected on the observational ground. The problem of evolution of the rotation period of an Ap star during its MS evolution has been discussed many times in the past; for references see North (1998). The earlier papers, based on insufficient data, suggested that Ap stars lose a significant fraction of their AM during the MS evolution (Abt 1979, Wolff 1981). But later, more complete data showed that the period distribution of peculiar members of very young clusters is indistinguishable from the distribution of field stars supposed to be much older (Borra et al. 1985, North 1984a, 1987). North (1984b) discussed the dependence of the observed period of field Ap stars on age measured by the gravity. He concluded that the rotation period increases when gravity decreases, just as expected from the conservation of AM during the MS life. Recently, North (1998) confirmed this conclusion for Si stars using their Hipparcos parallaxes. He stresses again that no evidence is found for any loss of AM on the MS. Wolff & Simon (1997) considered rotation evolution of normal MS stars of intermediate mass. They reached a similar conclusion, that stars with masses higher than 1.6 $M_\odot$ do not show any indication of AM loss during the MS life.

After discarding the three discussed processes we are left with the third possibility. It will be considered in the present paper. Sect. 2 summarises briefly properties of PMS stars of the intermediate mass which are progenitors of Ap stars. Sect. 3 discusses possible mechanisms for AM loss of these stars. The equation for AM evolution is derived which includes evolutionary variation of the stellar moment of inertia, AM gain from accretion, AM loss via a magnetized wind and AM loss due to magnetic star-disk link. The equation is numerically solved for stars with masses 1.5, 2, 2.5 and 3 $M_\odot$ in Sect. 4 for the selected values of the free parameters specifying the considered mechanisms. It is shown that for typical, observed values of these parameters an amount of AM lost in the PMS phase is of the required order. It is also shown that the extremely long rotation periods can be reached before ZAMS under special circumstances and only in case of less massive stars. Sect. 5 contains main conclusions of the paper.

2. PMS stars of intermediate mass

Low mass stars spend a large fraction of their PMS life in a phase of full convection (Stahler 1988). Duration of the full convection phase decreases, however, rapidly with an increasing mass of the PMS star: from more than 10^6 years for a 1 $M_\odot$ and 10^5 years for a 1.5 $M_\odot$ down to 10^4 years for 2 $M_\odot$ and zero for masses above 2.4 $M_\odot$ (Palla & Stahler 1993). While a time scale for the Ohmic dissipation of a large scale stellar magnetic field is longer than a MS life time of an intermediate mass star, the time scale for a turbulent dissipation is believed to be substantially shorter. As a result, any primordial magnetic field is expected to be destroyed during the full convection phase of less massive stars and replaced with dynamo generated fields. For stars more massive than about 1.5 $M_\odot$ the full convection phase becomes probably too short to fully dissipate the fossil magnetic field hence such stars are expected to possess primordial fields. Observations support this conjecture: magnetic fields observed in cool MS stars are well correlated with the stellar rotation (Stępień 1991, Saar 1996) whereas those observed in upper MS stars are most likely of primordial origin (Borra et al. 1982). The mass of about 1.5 $M_\odot$ is a limit beyond which no fossil fields are observed (Landstreet 1991).

If the above picture is correct, about 10% of PMS stars of intermediate mass are expected to possess fossil magnetic fields strong enough to alter their rotation rates and produce Ap phenomenon later on the MS. It is not clear whether a shallow and short-living convection zone connected with deuterium burning can produce appreciable magnetic fields in these stars but the observations of activity of Herbig Ae/Be stars - identified as intermediate mass PMS stars, indicate a correlation of the activity level with effective temperature rather than rotation rate, as would be expected for dynamo generated fields (Böhm & Catala 1995). The conclusion about a negligible role of dynamo generated fields in these stars is further supported by a comparison of the observed rotation velocity of Herbig Ae/Be stars with ZAMS stars. The comparison led Böhm & Catala (1995) to the conclusion that unless AM is redistributed inside intermediate mass stars during their approach to ZAMS, it must be conserved. This cannot be true, of course, in case of Ap stars which rotate on ZAMS about four times slower than normal stars.

Detailed models of PMS stars of intermediate mass were computed by Iben (1965), and more recently by Palla & Stahler (1993). The latter models show that the PMS phase of these stars lasts substantially shorter than hitherto assumed: from less than 10^7 years for a 2 $M_\odot$ star down to 2×10^5 years for a 5 $M_\odot$ star. Shorter times scales require more efficient spin down mechanism for a given AM loss.

Regarding the circumstellar environment the observations of Herbig Ae/Be stars indicate many similarities with T Tau stars. Both types of stars show the presence of stellar winds and accretion disks. Accretion rates, determined for Herbig Ae/Be stars are within an interval $10^{-8} - 10^{-6}$ $M_\odot$ /year (Hillenbrand et al. 1992) but there exist several apparently disk-less stars, which indicates that disks may be present only throughout a part of the PMS life (de Winter et al. 1997), just as it is in case of T Tau stars. The estimate of mass loss of Herbig Ae/Be stars due to a stellar wind is of the order of 10^{-8} $M_\odot$ /year (Catala 1989, Palla 1991). An interaction of the stellar magnetic field with the circumstellar matter will influence stellar AM. This influence will now be considered in detail and the resulting equation governing the evolution of AM of a PMS star will be derived.

3. AM loss of magnetic stars

Evolution of angular velocity of an intermediate mass star approaching ZAMS and possessing the primordial magnetic field is influenced by the following processes (Cameron & Campbell 1993, Cameron et al. 1995, Armitage & Clarke 1996)

- interaction of the stellar magnetic field with disk.
- accretion of matter from the disk onto the star,
- magnetized stellar wind,
- change of stellar momentum of inertia due to the approach to ZAMS.

It is assumed that the magnetic moment does not vary during the approach to ZAMS. This is a consequence of the assumption that the magnetic field is of primordial origin and the fact that a time scale of the approach is much shorter than a time scale for Ohmic decay of the large scale field. As detailed models show, an apparent dipolar magnetic field inside a rotating radiative star originates from electric currents strongly concentrated toward a center (Wright 1969, Moss 1974). The present assumption corresponds to a situation when such a central dipole is maintained throughout the approach to ZAMS.[1]

Moment of inertia of a star is given by

$$I = k^2 M R^2 , \tag{1}$$

where k^2 is radius of gyration, M mass of the star and R its radius. It is assumed that mass loss and accretion rates are small enough to neglect any variation of the stellar mass during the considered phase of evolution. In absence of any interaction of a star with its environment AM is conserved: $J = I\omega = \mathrm{const}$, where ω is angular velocity of the star. An interaction can be described as a torque T exerted on the star, such that

$$\frac{\mathrm{d}J}{\mathrm{d}t} = T . \tag{2}$$

It will be assumed below that T consists of three parts, resulting from magnetic disk-star interaction, denoted by $T_{\rm disk}$, accretion of matter onto the star, denoted by $T_{\rm acc}$, and magnetized wind, denoted by $T_{\rm wind}$

$$T = T_{\rm disk} + T_{\rm acc} + T_{\rm wind} . \tag{3}$$

[1] The referee remarked that other authors (Tayler 1987, Cameron & Campbell 1993) make a frozen magnetic flux assumption, more appropriate for a collapse of a perfectly conducting protostellar cloud. With this assumption the magnetic moment decreases as stellar radius ($\mu \sim R$). None of the conclusions of the present paper is altered significantly by such a behavior of the magnetic moment, although the initial values of the magnetic moment should now be increased by a factor of about 1.5 (depending on mass, see Table 1) to reach the same ZAMS periods as in case of constant μ. Detailed shapes of curves describing period evolution also become slightly different, mainly due to a decrease of the value of an equilibrium period, enforced by stellar field-disk interaction, with decreasing μ.

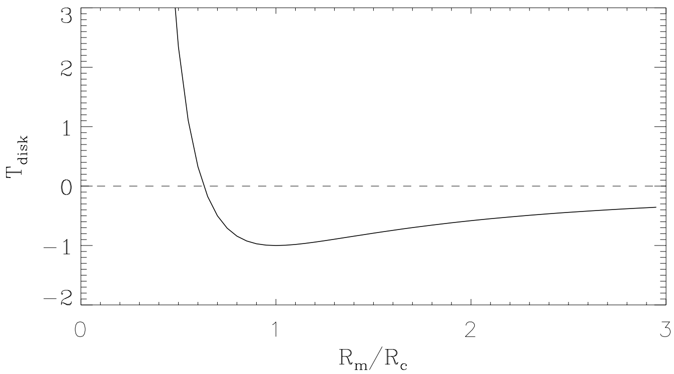

Fig. 1. Dependence of the torque resulting from a disk-star linkage on the ratio of radius of the magnetosphere to corotation radius

The disk torque was discussed by Armitage & Clarke (1996). They derived an expression for $T_{\rm disk}$ based on the disk-star interaction model of Livio & Pringle (1992) in which reconnection of twisted field lines in the magnetosphere limits the growth of the toroidal field produced by the interaction of disk with stellar poloidal field. The field lines are thus twisted until $B_\varphi \sim B_z$, where B_φ and B_z are toroidal and vertical components of the magnetic field. The resulting equation for the magnetic torque is

$$T_{\rm disk} = \frac{\mu^2}{3}\left(\frac{1}{R_{\rm m}^3} - \frac{2}{R_{\rm m}^{3/2}R_{\rm c}^{3/2}}\right) , \quad (4)$$

where $\mu = B_{\rm surf}R^3$ is the magnetic moment of the stellar magnetic field, $R_{\rm m}$ is the radius of the magnetosphere (= inner radius of the disk), and $R_{\rm c}$ the corotation radius

$$R_{\rm c} = \left(\frac{GM}{\omega^2}\right)^{1/3} , \quad (5)$$

with G denoting the gravity constant.

Fig. 1 shows the dependence of $T_{\rm disk}$ on $R_{\rm m}/R_{\rm c}$: for $R_{\rm m} < R_{\rm c}/2^{2/3}$ a disk spins up a star whereas for all values of $R_{\rm m}$ larger than that the disk brakes the star rotation. A maximum efficiency of spin down occurs for $R_{\rm m} = R_{\rm c}$. The equation for $T_{\rm disk}$ simplifies substantially in this case. This condition will be adopted throughout the rest of the paper. Any possible decrease of efficiency of spin down will be modeled by varying the value of the magnetic moment of inertia μ and the life time of the disk.

The disk torque does not depend on mass of the disk as long as the mass exceeds a certain limiting value (Armitage & Clarke 1996, Cameron et al. 1995). It is assumed here that the disk mass fulfils this condition.

The torque due to accretion can be determined assuming that the matter is accreted from the inner edge of the disk along the magnetic field lines so that it deposits onto the star all its AM. For $R << R_{\rm m}$ this can be approximated by

$$T_{\rm acc} = \omega \dot{M}_{\rm acc} R_{\rm c}^2 = \frac{(GM)^{2/3}\dot{M}_{\rm acc}}{\omega^{1/3}} . \quad (6)$$

Loss of AM via a magnetized wind has been considered e. g. by Weber & Davis (1967), Mestel (1984), Kawaler (1988) and Stępień (1995). Assuming that the wind velocity at the Alfvén surface is equal to escape velocity and adopting a dipolar geometry for the magnetic field one obtains from the formula (9) in Stępień (1995)

$$T_{\rm wind} = -\frac{\omega R^{3/5}\mu^{4/5}\dot{M}_{\rm wind}^{3/5}}{3(2GM)^{1/5}} , \quad (7)$$

where $\dot{M}_{\rm wind}$ is the mass loss rate via the magnetized wind. Putting together all the derived terms and remembering that ${\rm d}J/{\rm d}t = \omega {\rm d}I/{\rm d}t + I{\rm d}\omega/{\rm d}t$ we have from Eq. (2)

$$\frac{{\rm d}\omega}{{\rm d}t} = \frac{(GM)^{2/3}\dot{M}_{\rm acc}}{I\omega^{1/3}} - \frac{\mu^2\omega^2}{3IGM} - \frac{\omega R^{3/5}\mu^{4/5}\dot{M}_{\rm wind}^{3/5}}{3I(2GM)^{1/5}} - \omega\frac{{\rm d}I}{I{\rm d}t} . \quad (8)$$

This is the final equation for the rotation evolution of an Ap magnetic star in its PMS phase. It will be solved numerically in the next section and the results discussed.

4. Results

4.1. Input parameters

Eq. (8) is solved for four stellar masses, covering a majority of known magnetic Ap stars: 1.5, 2.0, 2.5 and 3.0 $M_\odot$. A knowledge of the time dependence of their radii and moment of inertia is necessary because these functions appear explicitly in the equation for rotation evolution, and both, R and I vary considerably during the PMS phase (Palla & Stahler 1993). Values of $R(t)$ were taken directly from the paper by Palla & Stahler (1993), either from tables or read off from the figures. The authors do not publish, however, any data on I. For the comparison of the rotation period of PMS stars with young MS stars Böhm & Catala (1995) used very few values of I for 2, 3 and 5 solar masses, made available to them by Palla & Stahler. These sparse values were used by Stępień (1998) in the preliminary discussion of AM loss of Ap stars. Nevertheless, the paper by Palla & Stahler (1993) contains all necessary data to calculate values of I for the considered stars during their PMS evolution. To obtain I as a function of time it was assumed that a fully convective star has a gyration radius $k^2 = 0.20$. This value was obtained from the values of I, R and M, tabulated by James (1964) who performed numerical calculations of an internal structure of rotating polytropes. It was assumed that a $n = 1.5$ polytrope corresponds to a fully convective case. For a fully radiative configuration $k^2 = 0.05$ was adopted and a linear variation of k^2 in time was assumed for phases when a star develops a radiative core. Polynomial relations were fitted to the thus obtained values of $R(t)$ and $I(t)$. It turned out that linear relations were sufficient in case of 2 and 2.5 $M_\odot$, parabolic for $R(t)$ in case of 1.5 and 2 $M_\odot$, whereas the third order polynomials were adopted for $I(t)$ in case of 1.5 and 2 $M_\odot$.

The values of $I(t)$ from Table 1 can be compared with the results of the detailed model calculations given by Böhm & Catala

Table 1. Stellar input parameters

time (years)	R ($R_\odot$)	k^2 adopted	I ($M_\odot R_\odot^2$)
	1.5 $M_\odot$		
10^6	3	0.18	2.4
9.5×10^6	2	0.05	0.3
1.2×10^7	1.7	0.05	0.22
	2 $M_\odot$		
10^6	3.2	0.17	3.5
3.4×10^6	3	0.05	0.9
8.4×10^6	1.8	0.05	0.31
	2.5 $M_\odot$		
9×10^5	5	0.05	3.1
3.9×10^6	2.1	0.05	0.55
	3 $M_\odot$		
4×10^5	6.5	0.05	6.3
2×10^6	2.3	0.05	0.8

(1995). Their initial value for the 2 $M_\odot$ star is equal to 3.67 in our units – about 5% more than given here. For $t = 2.1 \times 10^6$ years they give 2.45 whereas the present calculations (not listed in Table 1) give 2.3, and their ZAMS value is equal to 0.31, the same as here. In case of 3 $M_\odot$ star Böhm & Catala take as an initial time the value of 10^6 years, appreciably later than adopted here. At that time the radius of the star is already substantially smaller (4 $R_\odot$, as opposite to 6.5 $R_\odot$ given in Table 1) as also is I. Their ZAMS value is equal to 0.87 - about 10% more than the present one. One can conclude that the values of the stellar moment of inertia calculated with the above assumptions are satisfactorily close to the values obtained from model calculations. The accuracy of the polynomial fits is also of the order of 10%.

Considering accretion rates, Hartmann et al. (1998) determined recently accretion rates for several T Tauri stars. The average accretion rate is equal to $10^{-8} M_\odot/\mathrm{year}$ with values for individual stars deviating by about an order of magnitude in either direction. Hillenbrand et al. (1992) obtained considerably higher accretion rates for Herbig Ae/Be stars – of the order of $10^{-6} - 10^{-5} M_\odot/\mathrm{year}$, but the derived accretion rate shows a clear dependence on stellar mass. For masses $\leq 3M_\odot$ it is of the order of $10^{-7} - 10^{-6} M_\odot/\mathrm{year}$. The average life time of a disk depends, again, on the stellar mass but seems to be in many cases considerably shorter than the duration of the PMS phase of stars with intermediate masses (Hillenbrand et al. 1992, de Winter et al. 1997). Because $M = \mathrm{const}$ is assumed throughout the computations, values of accretion rate have to be adjusted to the life time of a disk to avoid a violation of this assumption, but generally, values from the range $10^{-8} - 10^{-6} M_\odot/\mathrm{year}$ were considered.

Regarding mass loss via a wind, Catala & Kunasz (1987) found a value of $10^{-8} M_\odot/\mathrm{year}$ for the mass loss rate of AB Aur, a Herbig Ae/Be star with mass about 3 $M_\odot$. Nisini et al. (1995) determined $\dot{M}_{\rm wind}$ for 14 Herbig Ae/Be stars. All but one values are in the range $2 \times 10^{-8} - 4 \times 10^{-7} M_\odot/\mathrm{year}$ and a good correlation with $L_{\rm bol}$ of the star, hence its mass, was found. A value of $10^{-8} M_\odot/\mathrm{year}$ will be adopted here as the most probable value of the mass loss rate, but different values will also be considered. Contrary to what is observed in case of T Tauri stars where mass loss seems to be powered by a disk and mass loss rate in these stars is usually assumed to be a given fraction of an accretion rate, Nisini et al. (1995) concluded that winds of Herbig Ae/Be stars seem to be powered by stars themselves. Hence, $\dot{M}_{\rm wind}$ can be considered as independent of $\dot{M}_{\rm acc}$ and one can assume that even when a disk disappears a wind can still go on.

A third important input parameter is the intensity of the stellar magnetic field. Here the basic value $\mu = 2.7 \times 10^{36}$ in cgs units is adopted. It corresponds to the surface magnetic field of 1 kG on a 2 $R_\odot$ star (this is equivalent to the effective magnetic field of about 330 G, Preston 1971). Such a field is rather moderate by standards of Ap magnetic stars; many of them have kilogauss effective magnetic fields. Stronger and weaker fields will also be considered. Note that while a variation of $\dot{M}_{\rm acc}$ or $\dot{M}_{\rm wind}$ affects only one term in Eq. (8), a variation of μ affects two terms: that connected with a star-disk interaction, and another describing a magnetized wind.

To allow for changes of all the above discussed parameters in Eq. (8), efficiency factors were introduced, defined in the following way

$$\begin{aligned} \dot{M}_{\rm acc} &= k_{\rm acc} \times 10^{-8} M_\odot/\mathrm{year}, \\ \mu &= k_{\rm mag} \times 2.7 \times 10^{36}, \\ \dot{M}_{\rm wind} &= k_{\rm wind} \times 10^{-8} M_\odot/\mathrm{year}\,. \end{aligned} \quad (9)$$

In terms of Eq. (8) the coefficients can be regarded as as a direct measure of the respective parameters, i. e. $k_{\rm acc}$ = 10 corresponds to the accretion rate $\dot{M}_{\rm acc} = 10^{-7}$, or $k_{\rm mag}$ = 5 to the surface magnetic field of 5 kG. In more general terms these coefficients describe also any other possible variation of efficiency of the respective mechanism, e. g. if the radius of the magnetosphere is not much larger than the stellar radius or when a deviation from a condition $R_{\rm m} = R_{\rm c}$ occurs. The coefficients $k_{\rm acc}, k_{\rm mag}$ and $k_{\rm wind}$ will be used when discussing different cases of the period evolution.

The last input parameter is the initial rotation period. Very few is known about the rotation periods of Herbig Ae/Be stars in early phases of evolution. Neuhäser et al. (1998) has recently determined a rotation period of a 3 $M_\odot$, PMS star, Par 1724 in Orion. Judging from its spectral type K0 the star lies close to the birth line. Its period is equal to 5.7 days. The measurements of $v \sin i$ of Herbig Ae/Be stars refer mostly to later stages (e. g. Böhm & Catala 1995), so they cannot be used for calculating initial rotation periods. More is known about rotation periods of young T Tauri stars. Their typical values are of the order of days (Bouvier et al. 1995, Adams et al. 1998). In the present calculations three values will be considered: 3, 5 and 7 days. As it will be shown below, the value of the initial rotation period is essentially unimportant because rotation periods merge very quickly to a single value dependent only on the parameters describing AM loss/gain – the result known from modeling the rotation evolution of T Tauri stars (Armitage & Clarke 1996, Cameron et al. 1995).

Equation (8) was solved using the 4th order Runge-Kutta method (Press et al. 1986). A rigid rotation of a star was assumed during the whole PMS phase, following the fact that the primordial magnetic field should be able to enforce it. The results of computations will be compared with periods resulting from conservation of AM, henceforth called "normal". These periods do not correspond exactly to periods of non-magnetic stars undergoing the same PMS evolution because a disk accretion also spins up a non-magnetic star. The spin up via a boundary layer is, however, usually considerably less efficient than in case of magneto-spheric accretion and is not described correctly by the accretion term in Eq. (8). The comparison of the observed $v \sin i$ of Herbig Ae/Be stars with their young counterparts on MS showed that, depending on the degree of AM mixing inside the stars, the both compared groups have identical or not much different AM (Böhm & Catala 1995). Errors connected with neglecting any AM change of non-magnetic stars should therefore be much less than a difference between ZAMS rotation period of a normal and an Ap star.

4.2. Evolution of rotation of PMS stars

4.2.1. Stars with masses of 3 and 2.5 $M_{\odot}$

The two masses will be discussed together because they share a couple of common properties: the time scale of their PMS evolution is only a few million years, and they remain in a fully radiative configuration throughout all, or nearly all of this time. The latter property results in only a moderate change of moment of inertia during the approach to ZAMS. The ratio $I_{\rm init}/I_{\rm ZAMS}$ = 6 and 8 for the 2.5 and 3 $M_{\odot}$, respectively (Table 1) but the relatively high ratios adopted here are the result of a special choice of a phase of maximum expansion as the initial moment of time. With a slightly different choice of initial conditions adopted e. g. by Böhm & Catala 1995 the ratio drops to about 1.5 in case of 3 $M_{\odot}$. Note that in absence of any mechanism altering AM of a star its ZAMS rotation period scales directly as moment of inertia, i. e. $P_{\rm ZAMS} = P_{\rm init} I_{\rm ZAMS}/I_{\rm init}$. For fully radiative stars this holds independently of whether a rigid rotation or AM conserved in shells is adopted.

Before we discuss the joint role of all three mechanisms altering stellar AM, it may be instructive to see how each of them separately operates. To achieve this, only one of the first three terms in Eq. (8) was retained in succession, together with the last term, whereas the other two were put identically to zero. Such a situation is not fully physically consistent because in reality the mechanisms are not independent of one another (e. g. disk accretion can not go on without a disk). Notwithstanding, the results can be used to estimate a relative importance of the considered mechanisms in different phases of the PMS evolution.

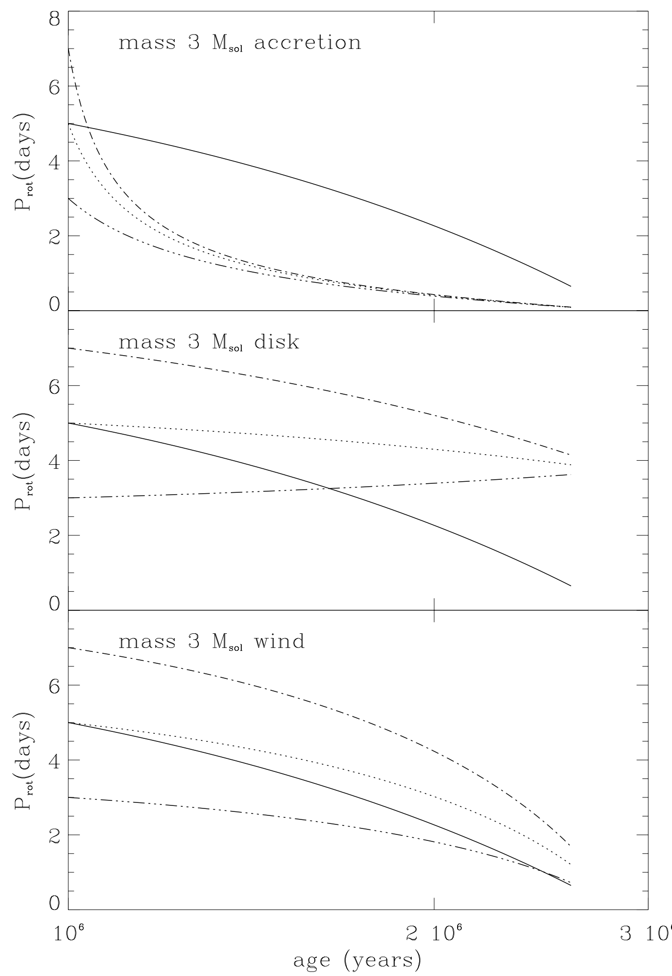

Fig. 2. The influence of individual mechanisms altering stellar AM

Fig. 2 shows the rotation period evolution of a 3 $M_{\odot}$ star when only magneto-spheric accretion is operating (top), only the interaction with a disk exists (middle) and only a wind acts on the star (bottom). The k-parameters took the values 10, 3 and 10, respectively (see Eq. (9)). In the "wind only" case also a nonzero value for μ is necessary, so $k_{\rm mag}$ = 1 was adopted. The rather high values of the efficiency coefficients were adopted just to enhance the effect of each mechanism. The variation of the rotation period with constant AM is shown (for clarity only one) as a solid line. For the sake of convenience the starting point was shifted in all figures to 10^6 years.

The following conclusion can be drawn from the figure: all mechanisms tend to produce a uniform rotation, independent of the value of initial period (a wind does not change the ratios of the periods but decreases differences). This is, of course, a consequence of their dependence on ω (see Eq. (8)) and it will be even better visible in case of less massive stars when the mechanisms operate over longer time scales. The accretion is very efficient in spinning up the star in early phases of the considered time interval. Very effective is also star-disk linkage. It tends to lock stellar rotation to some sort of equilibrium value but the time scale of the locking is longer than the time scale

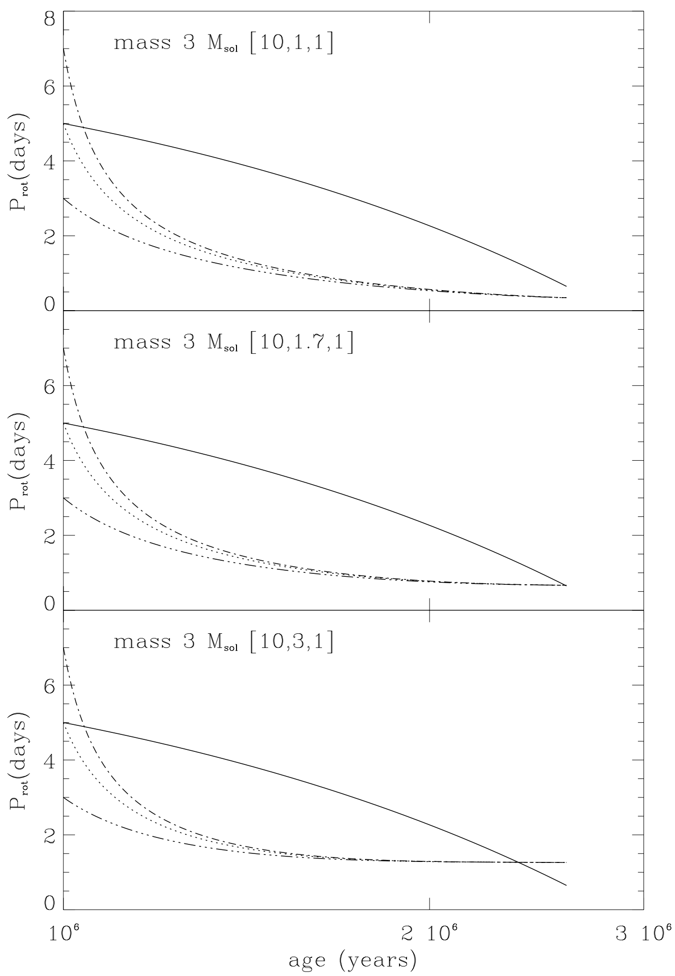

Fig. 3. Evolution of a rotation period of a star with 3 $M_\odot$ during the PMS phase. The vector gives the adopted values of the efficiency coefficients k, see definition (9). The weakest magnetic field, $k_{\rm mag}$ = 1 (top), results in $P_{\rm ZAMS}$ = 0.35 day, for a limiting value of $k_{\rm mag}$ = 1.7 (middle) the ZAMS period is, by definition, the same as normal (0.65 day) whereas a strong magnetic field, $k_{\rm mag}$ = 3 (bottom), slows down a star to a period of 1.26 day

for spin up by accretion. The wind seems less important in this case even with $k_{\rm wind}$ = 10. Its role is, however, crucial in long time scales, particularly when the disk disappears (see below).

Fig. 3 shows the results of the computations when all three mechanisms act simultaneously over the whole PMS phase of evolution. The efficiency coefficients describing accretion and wind are set at their most probable values, as discussed in the previous section: $k_{\rm acc}$ = 10 and $k_{\rm wind}$ = 1 whereas $k_{\rm mag}$ varies from 1 (top) through 1.7 (middle) to 3 (bottom). As it is seen from Fig. 3, disk accretion always dominates the period variations in the early phases of the PMS phase. This is due to a fact that the corresponding term in Eq. (8) is inversely proportional to (some power of) ω while the two other terms are proportional to positive powers of ω. For low rotation rates, close to the initial values, the accretion term is substantially larger than the others. As period decreases and ω increases, the two spin down terms take over. Nevertheless, for the moderate magnetic field, the star is spun up when it lands on the ZAMS (top). For strong enough magnetic field the disk and wind cancel closely the influence of accretion and the star lands on ZAMS with a rotation period close to normal (middle). Such a value of the field will be called limiting. Still stronger magnetic fields produce slow rotators (bottom) – here the ZAMS period is equal to 1.2 days, as compared to 0.65 day for a normal star with the initial rotation period of 5 days (solid line). For still stronger magnetic field the ZAMS rotation period becomes even longer, e. g. $P_{\rm ZAMS}$ = 2.1 days when $k_{\rm mag}$ = 5.

The calculations were also performed for a life time of a disk equal to 8×10^5 years, i. e. a half of the total considered PMS phase (see Table 1). Switching off the disk corresponds to putting the two first terms in Eq. (8) identically to zero with the magnetized wind still operating. The results changed little compared to the previous situation except for the case when a very massive wind ($k_{\rm wind}$ = 10) in the presence of a strong magnetic field ($k_{\rm mag}$ = 5) operated over the whole disk-less phase. In such a case $P_{\rm ZAMS}$ = 2.3 day. Much stronger fields (with $k_{\rm mag} \geq 30$) in the presence of massive winds ($k_{\rm wind} \geq 10$) are necessary to reach periods of the order of one year and even more extreme values are needed to obtain periods longer than 10 years. We draw a conclusion that massive stars with rather strong magnetic fields can reach rotation periods within the typical range observed for Ap stars but the existence of massive stars with extremely long periods of the order of several years is improbable. The mechanism which can lead to such large values is a magnetized wind in the absence of a disk, but its spin down time scale is of the same order as the duration of the PMS phase of a massive star, at least for values of the mass loss rate and surface magnetic field from within the observed range. On the other hand, the short duration of the PMS phase of massive stars favors spin up of stars with weak and moderate magnetic fields. The magneto-spheric disk accretion is more effective than the disk accretion in the absence of the magnetic field, hence stars with weak but nonzero magnetic fields may rotate faster on ZAMS than non-magnetic stars. This may explain the origin of Be stars as stars with fossil magnetic fields of the order of one, or a few hundred Gauss which were spun up by a disk accretion in the PMS phase. Calculations obtained for a 5 $M_\odot$ star having still shorter PMS life time showed that a spin up occurs for $k_{\rm mag} \leq 6$ which corresponds to the surface magnetic field of the order of one kilogauss or less. For magnetic fields equal to about a half of this limiting value the ZAMS rotation periods two times shorter than normal were obtained. Several authors invoke an existence of moderate magnetic fields in Be stars to explain their observed properties (e. g. Porter 1997 and references therein).

The results for a 2.5 $M_\odot$ star are not much different from the above discussed case although the total PMS phase considered here is longer than in a case of a 3 $M_\odot$ which results in an extension of time when the merged curves corresponding to the different initial periods evolve. Fig. 4 shows the period evolution for the values of the efficiency coefficients indicated and the disk existing the whole PMS time (top and middle). The limiting value of $k_{\rm mag}$, for which the ZAMS period is identical with normal, is equal now to 1.5, only slightly less than in the 3 $M_\odot$

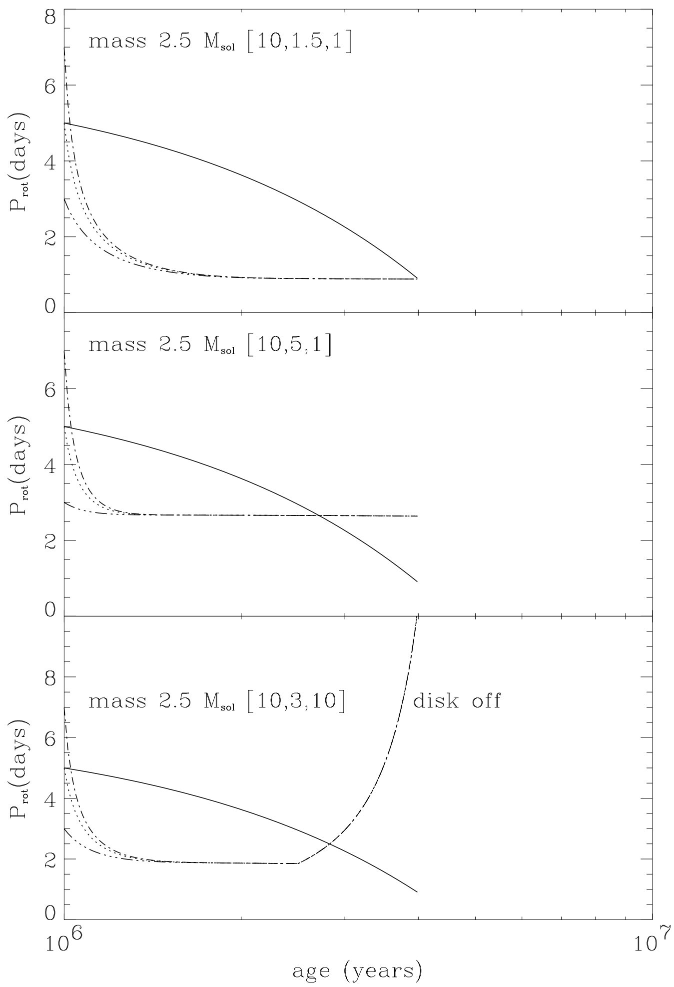

Fig. 4. Evolution of a rotation period of a star with 2.5 $M_{\odot}$ during the PMS phase. The vector gives the adopted values of the efficiency coefficients k, see definition (9). Top and middle figures present results for two different strengths of the magnetic field and the disk existing the whole PMS time.Values of the ZAMS period much larger than normal can be reached in case when the disk exists for a fraction of the PMS life and later only the magnetized wind operates (bottom)

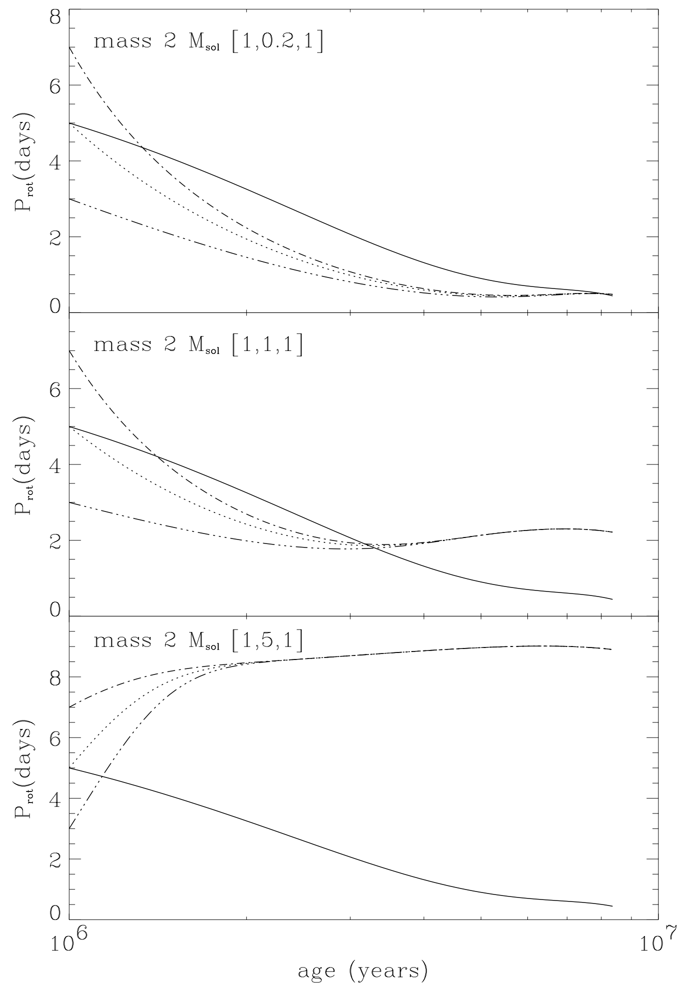

Fig. 5. Evolution of a rotation period of a star with 2 $M_{\odot}$ during the PMS phase with a disk existing all the time. The vector gives the adopted values of the efficiency coefficients k, see definition (9). The limiting value of $k_{\rm mag}$ for which spin down balances spin up is equal here to only 0.2 (top). For moderate (middle) and strong (bottom) magnetic fields the ZAMS rotation periods are several times longer than normal (2.2 and 9 days, respectively, compared to the normal value of 0.44 day)

case. Significantly stronger magnetic fields result in a larger value of the equilibrium period as the middle figure shows (here $P_{\rm ZAMS}$ = 2.6 days as compared to 0.9 day given by a solid line). The bottom figure presents the results of calculations when the disk was switched off after 1.5 mln years but the wind in a rather strong magnetic field spun down the star to the rotation period of 10 days. Note a sudden turn up of the broken line at the time when the disk disappeared. Similarly as in the previous case of a 3 $M_{\odot}$ star a massive wind in the presence of an extremely strong magnetic field is necessary to obtain periods of the order of several years yet the requirements for the parameters describing it became less stringent as the stellar PMS life time increases.

4.2.2. Stars with masses of 2 and 1.5 $M_{\odot}$

These stars differ from their more massive counterparts in two important aspects. First, their moment of inertia varies by a larger factor between the birth line and ZAMS, due to not only the variation of the stellar radius but also of the gyration radius – they are born as fully convective. As a result, $I_{\rm ZAMS}$ is by a factor of at least 20 lower than the moment of inertia on the birth line. The most rapid phase of decrease of I, which takes place just very close to the birth line, was skipped in the present model but even here the moment of inertia changes by a factor of about 11 within the considered interval of time. Second, the duration of the PMS is substantially longer than in more massive stars (see Table 1). This gives more time for the spin down mechanisms to operate. On the other hand, a long time scale may lead to a violation of the assumption about constant mass if a too large accretion or mass loss rate is adopted. This restricted the considered ranges.

Fig. 5 shows the results of calculations with a disk existing the whole PMS phase when $k_{\rm acc} = k_{\rm wind}$ = 1 and $k_{\rm mag}$ takes

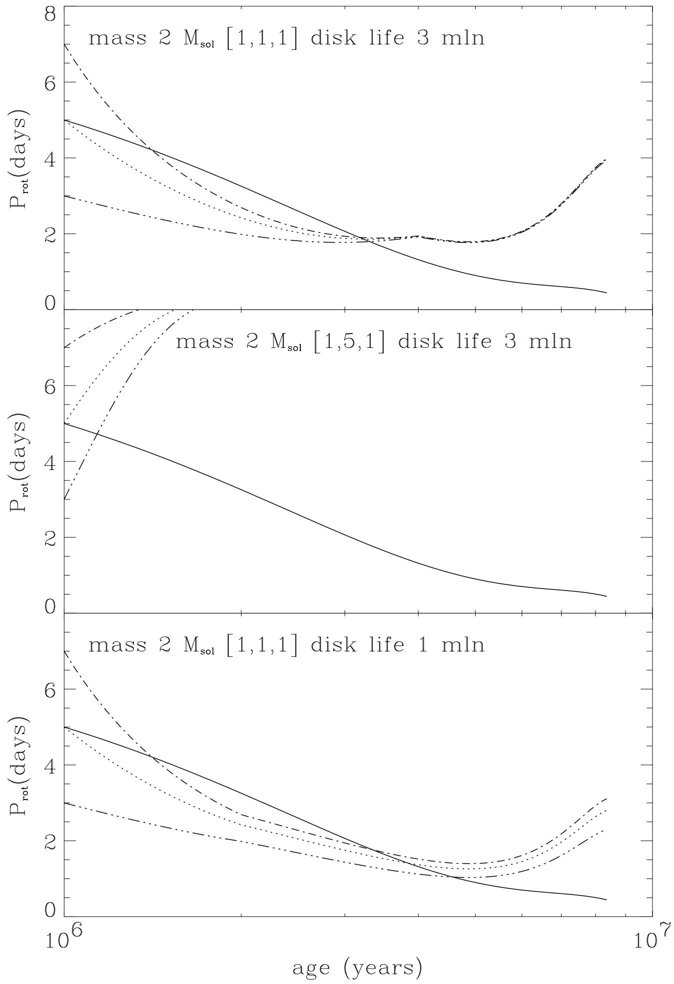

Fig. 6. Evolution of a rotation period of a star with 2 $M_\odot$ during the PMS phase with a life time of a disk shorter than the PMS phase. The vector gives the adopted values of the efficiency coefficients k, see definition (9). For a given life time of the disk the ZAMS period is a sensitive function of the intensity of the magnetic field: 4 days for $k_{\rm mag}$ = 1 (top) and 2066 days for $k_{\rm mag}$ = 5 (middle). A shortening of the life time of the disk without changing k does not result in a longer ZAMS period as the comparison of the top and bottom figures indicate

three values: the limiting value of 0.2 (top), corresponding to a very weak field close to a limit of detection, a moderate value of 1, typical of many Ap stars (middle) and a large value of 5, characteristic of strong field stars (bottom). Regarding the influence of accretion, disk-star interaction and wind, all the trends observed in case of more massive stars are also present here. The evolutionary curves for different periods merge very quickly so that a memory of initial period is lost. Contrary to massive stars, accretion dominates period variations in early phases of the PMS evolution only for relatively weak stellar magnetic fields. In later phases of the PMS life disk locking occurs and the period stays constant. ZAMS rotation periods several times longer than normal are easily reached but if the disk exists the whole PMS time, it is again impossible to obtain very long periods – disk locking is so effective that it prevents a star from any additional spin down.

If the disk is dispersed after a relatively short time but the wind still exists, much longer periods can be reached. Fig. 6 presents such a situation. When a disk is switched off after 3×10^6 years and a typical wind ($k_{\rm wind}$ =1) exists in the presence of a moderate magnetic field ($k_{\rm mag}$ = 1) the resulting ZAMS period is not much different from a case with a disk existing the whole PMS life (compare Fig. 5 middle and Fig. 6 top). The efficiency of the wind increases, however, rapidly with the strength of the magnetic field. For $k_{\rm mag}$ = 5 and all the other parameters unchanged the ZAMS period reaches 5.6 years (Fig. 6 middle). Additionally, an increase of $k_{\rm wind}$ to 3 (not shown here) results in the ZAMS rotation period of the order of 100 years.

The reason for such an enormous slow down by the wind is that its time scale, estimated from the third term in Eq. (8) and being not much different as in case of more massive stars, acts now over a much longer time. The wind has enough time to slow a star's rotation down to periods even of the order of 100 years. What happens when the disk disappears still earlier? Can similarly long periods be reached with lower mass loss rates? Fig. 6 (bottom) presents the results of calculations for a disk life time equal to 1 mln years, i. e. 3 times shorter than before. It is seen that the broken lines do not turn up so rapidly as before – the term describing a decrease of I plays now an important role in early disk-less phases, forcing the curves to bend downwards. Only after some time, when I is already close to its final value and varies rather slowly, the term connected with the wind starts to dominate. It can be concluded that early disappearance of the disk does not necessarily lead to much longer ZAMS periods because a rapid decrease of the moment of inertia can partly offset the influence of the spin down by the wind in the early phases of the PMS life of a star.

The results for a 1.5 $M_\odot$ star are very similar to the 2 $M_\odot$ case. The calculations of the ZAMS periods in case of a disk existing the whole PMS phase indicate again that periods several times longer than normal can easily be obtained with different combinations of the efficiency factors k from within the observed range. The qualitative results for a disk life time shorter than the PMS life time of a 1.5 $M_\odot$ are also very similar as in case of a 2 $M_\odot$ star. Fig. 7 shows the results of calculations for $k_{\rm acc} = k_{\rm mag} = k_{\rm wind}$ = 1 and the life time of disk equal to 5.5 mln years (top) and for the magnetic field five times stronger (middle). In the first case the ZAMS period is equal to 14 days and in the other case it is equal to about $2 \times 10^4 {\rm days} \approx$ 60 years. Similarly as in the case of a 2 $M_\odot$ star the shortening of the disk life does not influence significantly the ZAMS period if all the other parameters are kept constant (compare Fig. 7 top and bottom). A "hook" visible on the curves describing the variation of the rotation period in Fig. 7 is the result of the behavior of I of a 1.5 $M_\odot$ star, which suddenly decreases right before ZAMS (see Palla & Stahler 1993).

5. Discussion and conclusions

Intermediate mass stars in the PMS phase share several properties with T Tauri stars: both groups show the presence of accretion disks and outflows of matter, or winds. The observed pa-

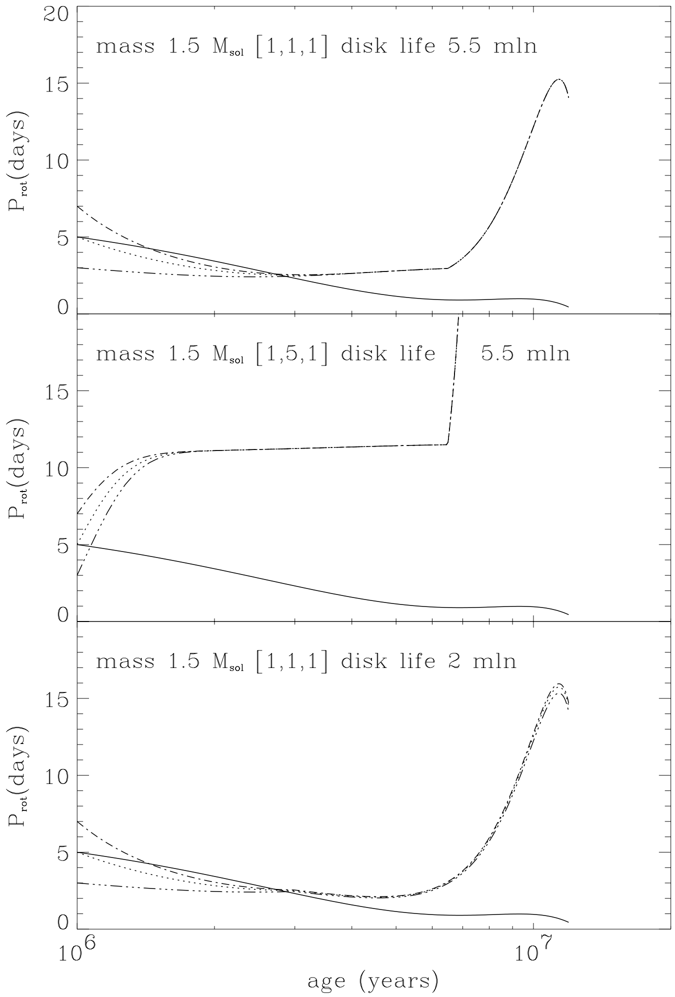

Fig. 7. Evolution of a rotation period of a star with 1.5 $M_\odot$ during the PMS phase with a life time of a disk shorter than the PMS phase. The vector gives the adopted values of the efficiency coefficients k, see definition (9). For a given life time of the disk the ZAMS period is a sensitive function of the intensity of the magnetic field: 14 days for $k_{\rm mag}$ = 1 (top) and 2×10^4 days for $k_{\rm mag}$ = 5 (middle). A shortening of the life time of the disk without changing k does not result in a longer ZAMS period as the comparison of the top and bottom figures indicate

rameters describing these phenomena, i. e. the duration time, accretion rate and wind mass loss rate vary significantly from one star to another. There exist, however, also notable differences between the two groups of stars: low mass stars possess deep convective envelopes over the whole PMS phase of evolution, in which hydromagnetic dynamo operates generating magnetic fields. Interaction of the field with circumstellar matter fully controls the stellar rotation in the PMS phase. The results of modeling the rotation evolution indicate that the ZAMS rotation rates of lower MS stars are completely determined by the PMS environmental conditions (mainly by the life time of the accretion disk) but they do not depend at all on the initial rotation periods achieved by stars leaving the protostellar phase. Intermediate mass stars with masses between 1.5 and 2.5 $M_\odot$ reach a fully radiative configuration long before ZAMS whereas still more massive stars are fully radiative from the beginning of the PMS phase. A convective zone due to deuterium burning is probably too shallow to be able to produce appreciable magnetic field in these stars (Palla & Stahler 1993). It seems therefore that a typical intermediate mass star does not possess any significant magnetic field when it evolves towards ZAMS. A weak or no interaction of the star with its environment should be the consequence of a lack of magnetic field. As a result, little, or no change of stellar AM is expected during the PMS phase. A comparison of the rotation rates of Herbig Ae/Be stars with young MS stars of the same mass supports this view: depending on whether a rigid rotation is assumed during approach to ZAMS, or AM conserved in shells, no, or at most a moderate loss/gain of AM is needed to evolve the observed rotation rates of Herbig Ae/Be stars to ZAMS (Böhm & Catala 1995).

There exists, however, a group of intermediate mass stars possessing measurable fossil magnetic fields which should be able to modify stellar AM in the PMS phase. In particular, when these magnetic stars possess accretion disks, they should accrete matter along the magnetic field lines. In addition, the field should interact with the disk itself producing a torque exerted on the star, and if the star losses mass via a wind, its magnetic field, frozen in the wind matter, should carry away stellar AM. Such stars were considered in the present paper.

The above processes influencing stellar AM are described by four parameters: accretion rate, mass loss rate via a wind, the strength of the stellar magnetic field and the life time of the disk. It was always assumed that the wind goes on after disappearance of the disk. Although the broad parameter space with their values restricted by observations was investigated, the detailed discussion was restricted for clarity to the two most important parameters: magnetic field strength and disk life time. The presence of the disk forces a star to spin up in early phases of the PMS life due to accretion process but later a disk locking occurs with an equilibrium rotation period which practically stays constant as long as the (massive enough) disk exists. The value of the equilibrium period results from a condition $\mathrm{d}\omega/\mathrm{d}t \approx 0$ (see Eq. (8)) but does not depend on the initial rotation period. This condition is most sensitive to the input parameter μ describing the strength of the magnetic fields, because μ appears with the highest power in Eq. (8). This is why the discussion was concentrated on the μ-dependence of the stellar rotation period. Nevertheless, if e. g. the accretion rate in increased, the resulting equilibrium period will be correspondingly shorter as the inspection of Eq. (8) shows. An increase of $\dot{M}_{\rm wind}$ acts similarly. Hence, the final ZAMS rotation period of a star possessing a disk over the whole, or nearly the whole PMS phase is controlled by a combination of the three parameters varying randomly from one star to another. The typical, resulting periods are several times longer than in case of a star with conserved AM. This agrees well with the observed ratio of rotation rates of normal and Ap stars. The disk-locking prevents the stars, however, from reaching extreme values of the rotation periods equal to many years.

All these conclusions apply to situation when both, the disk and the wind disappear earlier and simultaneously. If, however,

Table 2. Extremely slowly rotating Ap stars

HD	Other	Sp. pec.	P(years)	$\log T_e$ (ref.)	$\log L/L_\odot$	$< B_{\rm surf} >$ (kG)
9996	GY And	A0 SiSrCr	21.9	3.987 (1)	1.61	4.4
94660	HR 4263	A0 EuCrSi	7.4	4.002 (4)	1.81	6.2
110066	AX CVn	A1 SrCrEu	13.4 or 26.8	3.959 (2)	1.71	4.1
137949	33 Lib	F0 SrCrEu	≥ 75	3.875 (3)	1.12	4.7
187474	V3961 Sgr	A0 EuCrSi	6.4	4.004 (1)	1.79	5.0
201601	γ Equ	A9 SrEu	77	3.886 (2)	1.10	3.8

(1) North (1998), (2) Adelman et al. (1995), (3) Babel (1994), (4) from $[u-b]$ using the relation given by Stępień (1994); magnetic field from Mathys et al. (1997)

a disk survives only a fraction of the PMS life but the magnetized wind exists for the rest of the PMS phase the situation changes drastically. Because the PMS life time is a very sensitive function of the stellar mass this parameter becomes very important in determining the ZAMS period of a magnetic star. Switching off disk early in the PMS life of a massive star leaves the star spun up and the subsequent spin down by a wind is inefficient because of a too short time left until the star reaches ZAMS. The presence of the magnetic field may result in such a case in a ZAMS star which rotates faster than normal. The magnetic field should not be too strong so that the equilibrium period is not reached before dispersing the disk. It is suggested that Be stars may be formed in this way. For decreasing stellar mass, early disappearance of the disk leaves more and more time for the wind to operate. As a result, very long rotation periods, of the order of years, (or even 100 years in case of the least massive stars) can be reached by stars which possess winds till ZAMS. According to these predictions Be stars should be the stars with masses equal at least 3 $M_\odot$ possessing moderate magnetic fields. Such fields have not yet been detected directly in Be stars although the existence of nonzero fields has been postulated (Fox 1993, Porter 1997). On the other hand, the present results indicate that extremely slowly rotating stars should occur with an increasing probability among low mass Ap stars and they should possess significant magnetic fields. Recently Mathys et al. (1997) discussed magnetic fields of long period Ap stars. The authors give results for 31 stars with rotation periods longer than one month. All of the stars possess relatively strong magnetic fields and they all seem to have moderate or low masses (the last property was also noted by Wolff 1981). Table 2 gives more detailed data for 6 stars with rotation periods longer than 5 years. The consecutive columns give identifications of each stars, spectral type and peculiarity, rotation period in years, the adopted effective temperature with references and luminosity calculated with the Hipparcos parallaxes and bolometric corrections from Stępień (1994). No corrections for interstellar absorption were applied because all the six stars lie closer than 150 pc. The last column gives the strength of surface magnetic fields taken from Mathys et al. (1997). Note that the star with the longest known period of 77 years, γ Equ, possesses less than 2×10^{-5} of the average AM of a normal star!

The positions of the stars from Table 2 in the Hertzsprung-Russell (HR) diagram do not differ appreciably from positions of other magnetic stars (Hubrig et al. 1998). All six lie in the middle of the MS i. e. they are only moderately evolved. If an additional braking mechanism operated efficiently during the *whole* MS phase, we would expect the most slowly rotating stars to lie close to terminal age MS. This is clearly not the case, hence the braking mechanism must be most efficient in phases close to the zero-age MS (ZAMS). As the last column shows, all stars have strong surface magnetic fields with $k_{\rm mag}\approx 5$. Comparing their positions in the HR diagram with evolutionary tracks of Schaller et al. (1992) one sees that two stars with the longest periods, i. e. 33 Lib and γ Equ, have masses close to 1.7 $M_\odot$ whereas the other four stars have masses in the range 2.3–2.6 $M_\odot$. These properties agree well with the predictions of the present model, although the adopted duration of the PMS phase of a 2.5 $M_\odot$ star seems to be too short for an extreme slow down to occur.

Detailed numerical results of the modeling depend sensitively on the adopted duration of the PMS phase of stars with different masses. If, instead of the data taken from Palla & Stahler (1993), significantly longer PMS life times were adopted, the estimates regarding Be stars and extremely long period stars would apply to the correspondingly higher masses than obtained here. This would remove an apparent discrepancy between the predicted upper limit for extremely slowly rotating mass ($\sim 2M_\odot$) and observations ($\sim 2.5M_\odot$).

Because ZAMS periods of magnetic Ap stars depend on several apparently uncorrelated parameters no correlation with any of them is expected. Assuming that the values of these parameters do not change significantly with stellar mass, the present model does predict, however, that a longer time scale of the PMS phase of less massive stars may result in a relatively stronger spin down so that the ratio of the average rotation rate of Ap stars to the average rotation rate of normal stars should increase with decreasing mass. The data of Wolff (1981) suggest indeed the presence of such a trend (from about 3–4 for 4 $M_\odot$ to 6–7 for 2 $M_\odot$) but the results are not conclusive because different data sets on stellar rotation of Ap and normal stars are here compared (see Introduction).

To summarise the following results were obtained.

- Magnetic stars of the upper MS lose quickly memory of the initial rotation period during their PMS evolution. Their ZAMS periods are the result of an interaction of the stellar magnetic field with circumstellar environment, similarly as in case of stars of the lower MS.

- Accretion from a disk along the magnetic field lines spins up a star in early phases of the PMS evolution which may result in faster than normal rotation on ZAMS for favorable conditions (massive stars with short PMS life, moderate magnetic fields and early disappearance of disk). This may produce Be star phenomenon.
- A star-disk interaction in later phases of the PMS life results in a locking of the rotation rate to a sort of equilibrium value which is typically several times longer than a normal star rate. The disappearance of the disk at this stage simultaneously with the stellar wind results in typical observed ZAMS rotation periods of Ap stars, several times longer than normal stars. Because the ZAMS period depends on values of several independent parameters describing the AM evolution no direct correlation of the rotation period of Ap stars with any of these parameters is expected.
- If the magnetized wind exists after the disappearance of the disk, it may slow down a star with a long time of approach to ZAMS to an extremely long rotation period. This can be achieved only in case of a lower mass star with a long PMS life time and a strong magnetic field. Periods up to 100 years can thus be reached.
- It is the highly nonlinear response of the PMS star and its accretion disk to the presence of the magnetic field (little AM change for weak fields and relatively large but limited AM loss for stronger fields, with a rapid transition between the two regimes) which splits a single initial rotation distribution into two ZAMS distributions. Under exceptional conditions (a combination of a strong magnetized wind and an early disappearance of a disk) an additional loss of a very large fraction of AM may occur.

Acknowledgements. This work was finished during my stay in Hamburger Sternwarte as an Alexander von Humboldt Fellow. I thank Jürgen Schmitt for hospitality, the AvH Foundation for financial support and an anonymous referee for valuable remarks and pointing out several typing errors. A partial support of the State Committee for Scientific Research under a grant 2P03D 010 12 is also acknowledged.

References

Abt H.A., 1979, ApJ 230, 485
Abt H.A., Morrell N.J., 1995, ApJS 99, 135
Abt H.A., Chaffee F.H., Suffolk G., 1972, ApJ 175, 778
Adams N.R., Walter F.M., Wolk S.J., 1998, ApJ 116, 237
Adelman S.J., Pyper D.M., Lopez-Garcia Z., Caliskan H., 1995, A&A 296, 467
Allen C.W., 1973, Astrophysical Quantities. The Athlone Press, London
Armitage P.J., Clarke C.J., 1996, MNRAS 280, 458
Babel J., 1994, A&A 283, 189
Böhm T., Catala C., 1995, A&A 301, 155
Borra E.F., Landstreet J.D., Mestel L., 1982, ARA&A 20, 191
Borra E.F., Beaulieu A., Brousseau D., Shelton I., 1985, A&A 149, 266
Bouvier J., Covino E., Kovo O., et al., 1995, A&A 299, 89
Cameron A.C., Campbell C.G., 1993, A&A 274,309
Cameron A.C., Campbell C.G., Quaintrell H., 1995, A&A 298, 133
Catala C., 1989, In: Reipurth B. (ed.) Low mass star formation and pre-Main-Sequence evolution. ESO, München, p. 471
Catala C., Kunasz P.B., 1987, A&A 174, 158
Fox G.K., 1993, MNRAS 260, 525
Hartmann L., Calvet N., Gullbring E., D'Alessio P., 1998, ApJ 495, 385
Hillenbrand L.A., Strom S.E., Vrba F.J., Keene J., 1992, ApJ 397, 613
Hubrig S., North P., Mathys G., 1998, In: North P., Schnell A., Ziznovsky J. (eds.) Proceedings of the 26th meeting and workshop of the European Working Group on CP stars (Vienna, 27–29 October 1997), Contr. Astr. Obs. Skalnate Pleso 27, 249
Iben I., 1965, ApJ 141, 993
James R.A., 1964, ApJ 140, 552
Kawaler S.D., 1988, ApJ 333, 236
Landstreet J.D., 1991, In: Tuominen I., Moss D., Rüdiger G. (eds.) The Sun and Cool Stars: activity, magnetism, dynamos. IAU Coll. No. 130, Springer, Berlin, p. 342
Livio M., Pringle J.E., 1992, MNRAS 259, 23P
Mathys G., Hubrig S., Landstreet J.D., Lanz T., Manfroid J., 1997, A&AS 123, 353
Mestel L., 1984, In: Baliunas S.L., Hartmann L. (eds.) Cool Stars, Stellar Systems, and the Sun. Springer, Berlin, p. 49
Michaud G., 1970, ApJ 160, 641
Moss D.L., 1974, MNRAS 168, 61
Neuhäser R., Wolk S.J., Torres G., et al., 1998, A&A 334, 873
Nisini B., Milillo P., Saraceno P., Vitoli F., 1995, A&A 302, 169
North P., 1984a, A&A 141, 328
North P., 1984b, A&A 148, 165
North P., 1987, A&AS 69, 371
North P., 1998, A&A 334, 181
Palla F., 1991, In: Falgerone K., et al. (eds.) Fragmentation of molecular clouds and star formation. Kluwer, Dordrecht, p. 331
Palla F., Stahler S.W., 1993, ApJ 418, 414
Porter J.M., 1997, A&A 324, 597
Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1986, Numerical Recipes. Cambridge University Press, Cambridge
Preston G.W., 1970, In: Sletteback A. (ed.) Stellar Rotation. Reidel, Dordrecht, p. 254
Preston G.W., 1971, ApJ 164, 309
Saar S.H., 1996, In: Strassmeier K.S., Linsky J.L. (eds.) Stellar Surface Structure. IAU Symp. No. 176, Kluwer, Dordrecht, p. 237
Schaller G., Schaerer D., Meynet G., Maeder A., 1992, A&AS 96, 269
Schmidt-Kaler Th., 1982, In: Schaifers K., Voigt H.H. (eds.) Landolt-Boernstein: Astronomy and Astrophysics. Vol. 2, Springer, Berlin, p. 449
Seitter W.C., Duerbeck H. W., 1982, In: Schaifers K., Voigt H.H. (eds.) Landolt-Boernstein: Astronomy and Astrophysics. Vol. 2, Springer, Berlin, p. 269
Stahler S.W., 1988, ApJ 332, 804
Stępień K., 1991, Acta Astron. 41, 1
Stępień K., 1994, In: Zverko J., Ziznovsky J. (eds.) Chemically peculiar and magnetic stars. Tatranska Lomnica, p. 8
Stępień K., 1995, MNRAS 274, 1019
Stępień K., 1998, In: North P., Schnell A., Ziznovsky J. (eds.) Proceedings of the 26th meeting and workshop of the European Working Group on CP stars (Vienna, 27–29 October 1997), Contr. Astr. Obs. Skalnate Pleso 27, 205
Tayler R.J., 1987, MNRAS 227, 553
Weber E.J., Davis L. Jr., 1967, ApJ 148, 217
de Winter D., Koulis C., Thé P.S., et al., 1997, A&AS 121, 223
Wolff S.C., 1981, ApJ 224, 221
Wolff S.C., Simon T., 1997, PASP 109, 759
Wright G.A.E., 1969, MNRAS 146, 197