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The zoo of dwarf novae: illumination, evaporation and disc radius variation

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Abstract. In the framework of the dwarf nova thermal-viscous disc instability model, we investigate the combined effects on the predicted dwarf nova lightcurves of irradiating the accretion disc and the secondary star and of evaporating the inner parts of the disc. We assume the standard values of viscosity. We confirm the suggestion by Warner (1998) that the large variety of observed outbursts' behaviour may result from the interplay of these three effects. We are able to reproduce light curves reminiscent of those of systems such as RZ LMi or EG Cnc. We can obtain long lasting outbursts, very similar to superoutbursts, without assuming the presence of a tidal instability.

Key words: accretion, accretion disks – instabilities – stars: binaries: close – stars: novae, cataclysmic variables

1. Introduction

Dwarf novae (DN) are cataclysmic variable binary systems which, every few weeks, exhibit 4–6 mag outbursts, which last for a few days (see e.g. Warner 1995a). In several subclasses of DN both the outburst durations and recurrence times can be very different from the values quoted above. It is generally accepted that DN outbursts are due to a "thermal-viscous" instability. This instability occurs in the accretion disc in which the viscosity is given by the α-prescription (Shakura & Sunyaev 1973), at temperatures close to 8000 K. Hydrogen is then partially ionized and opacities are a steep function of temperature (see Cannizzo 1993 for a review and Hameury et al. 1998 for the most recent version of the model). Modeling dwarf nova lightcurves requires a varying Shakura-Sunyaev parameter α (Smak 1984): it must be of the order of 0.1–0.2 (0.2 according to Smak 1999b) in outburst and of the order of 0.01 in quiescence, when the temperature is below the hydrogen ionization temperature. One could therefore expect that the disc instability model (DIM) might offer useful constraints on mechanisms which generate accretion disc viscosity. This assumes, of course, a successful application of the model to the observed DN outburst cycles.

However, despite its success in explaining the overall characteristics of DNs, the DIM in its standard version faces several serious difficulties when one tries to account for the detailed properties of dwarf nova outbursts. Some of these difficulties are the result of an incomplete version of the DIM. For example it was believed that a truncation of the inner parts of the disc is necessary to explain the long delay between the rise of optical light and that of UV and EUV in systems such as SS Cyg. As shown by Smak (1998), however, when correct outer boundary condition are assumed, the standard DIM reproduces the observed delays. On the other hand, observed quiescent X-ray fluxes far exceed the predictions of the model and seem to require an inner 'hole' in the disk. Such a hole can either be due to evaporation of the disc close to the white dwarf (Meyer & Meyer-Hofmeister 1994), or to the presence of a magnetic field strong enough to disrupt the disc. In addition, systems such as WZ Sge, which have long recurrence time and large amplitude, long outbursts require very low values of α ($\alpha < 10^{-4}$) if interpreted in the framework of the standard DIM (Smak 1993, Osaki 1995a, Meyer-Hofmeister et al. 1998). These values, much lower than those of other DNs at similar orbital periods, are, however, left unexplained. On the other hand, WZ Sge systems can be explained with standard values of α, provided that the disc is truncated as in other systems so that it is either stable or marginally unstable (Lasota et al. 1995, Warner et al. 1996) and the mass transfer from the secondary is significantly increased during the outburst under the influence of illumination by radiation from the accreting matter (Hameury et al. 1997).

SU UMa systems are a subclass of dwarf novae which occasionally show long outbursts during which a lightcurve modulation (superhump) is observed at a period slightly longer than the orbital period; these superoutbursts are, usually, separated by several normal outbursts. The superhump is due to a 3:1 resonance in the disc which causes the disc to become eccentric and to precess (Whitehurst 1988). Osaki (1989) proposed that the related tidal instability is also responsible for the long duration and large amplitude of superoutbursts (see Osaki 1996 for a review of the thermal-tidal instability model). In his model,

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the tidal torques which remove angular momentum from the outer parts of the disc are increased by approximately an order of magnitude when the disc reaches the 3:1 resonance radius (typically $0.46a$, where a is the orbital separation) until the disc outer radius has shrunk to typically $0.35a$, i.e. by about 30%. This model accounts for many of the properties of SU UMa systems; it has, however, difficulties in explaining systems with very short superoutburst cycles such as RZ LMi, for which one must assume that the tidal instability stops when the disc has shrunk by less than 10% (Osaki 1995b), i.e. by much less than assumed in the standard case.

Finally, it must be noted that a number of systems exhibit very bizarre lightcurves: we have already mentioned the case of RZ LMi, which has similarities, with systems such as ER UMa, V1159 Ori and DI UMa. The case of the December 1996 superoutburst of EG Cnc which was followed by 6 closely spaced normal outbursts in 1996 has not been reproduced by simulations, except by Osaki (1998) who assumed that the viscosity parameter $\alpha_{\rm c}$ in the cold state was increased to 0.1, almost the value in the hot state, for 70 days after the superoutburst, and then returned to its quiescent value $\alpha_{\rm c} = 0.001$. It is finally worth mentioning that the prototypical classical dwarf nova U Gem exhibited an unusually long outburst in 1985, lasting 45 days, with a shape similar to superoutbursts, but without superhumps (Mattei et al. 1987). Since in this system the radius of the 3:1 resonance is larger than the size of the primary's Roche lobe, one can conclude that while superhumps can be attributed to the 3:1 resonance, the tidal instability is obviously not the sole cause of very long outbursts.

The inability of numerical models to reproduce the large variety of observed light curves may indicate that additional physical effects should be added to the DIM. One such effect is the tidal instability. Another important class of effects is the illumination of the disc and the secondary star. These effects are usually not included in simulations, and it was suggested by Warner (1995c) that irradiation of the secondary star, that gives rise to high and low states of mass transfer, and of the inner disc, that drastically affects the disc instability, account for the light curves of VY Scl stars. He also suggested (Warner 1998) that the wide spectrum of superoutburst behaviours is generated by the interplay of reactions of the disc and the secondary to irradiation. This suggestion is supported by observations since there is evidence that the mass transfer rate from the secondary star increases during outbursts (Smak 1995) in dwarf novae such as Z Cha or U Gem, most probably as a result of illumination of the secondary.

Recently Smak (1999a) argued that properties of outburst cycles of "standard" U Gem–type dwarf novae, which cannot be reproduced by the DIM (e.g. the same maximum brightness of narrow and wide outbursts), are well explained if *all* outbursts are associated with some mass–transfer enhancement due to the secondary's irradiation. (Long outbursts would be due to important mass transfer enhancements, making the mass transfer rate larger than the critical value for stability). If this is the case the 'pure' DIM would find no application in the real world.

It also appears that illumination of the disc itself by the hot white dwarf has strong effects on the stability properties of the disc as soon as the white dwarf temperature exceeds 15,000 K (King 1997, Hameury at al. 1999).

In this paper, we investigate the influence of the combined effects of illumination of the disc, of the secondary star and of evaporation of the inner parts of the disc, on the predictions of the DIM in which standard values of viscosity are assumed. Our free parameters are the white dwarf mass, the quiescent white dwarf temperature, the mass transfer rate in the absence of illumination, and two parameters describing in a crude manner evaporation effects and the influence of illumination of mass transfer from the secondary. We show that a large variety of light curves are predicted by the models, many of which have an observational counterpart. In Sect. 2, we show that the long 1985 outburst of U Gem requires enhanced mass transfer during the outburst. In Sect. 3, we describe the model and our assumptions; in Sect. 4, we give our results and compare them with lightcurves of observed systems, and we discuss briefly possible extensions of this work.

2. The long 1985 outburst of U Gem

U Gem is a prototypical dwarf nova that undergoes outbursts that last for 7–14 days, with an average recurrence time of 120 days (Szkody & Mattei 1984). The orbital period is 4.25 hr, and the primary and secondary masses are respectively 1.1 and 0.5 $M_\odot$ (Ritter & Kolb 1998); for such parameters, the outer disc radius, defined as the radius of the last non-intersecting orbits, is 4.15 10^{10} cm.

The 1985 outburst that lasted for 45 days is therefore exceptional; long outbursts are observed in other systems (SS Cyg for example), and are a natural outcome of models in which the outer disc radius is kept constant (see e.g. Hameury at al. 1998), which may happen when the outer disc radius reaches the tidal truncation radius. However, one does not normally obtain extremely long outbursts; the 1985 outburst of U Gem was in fact so long that more mass was accreted during this outburst than what was contained in the pre-outburst disc. The maximum mass $M_{\rm d,max}$ of a quiescent disc during quiescence is the integral of the maximum surface density $\Sigma_{\rm max}$ on the cool branch; using the fits given by Hameury et al. (1998), one gets

$$M_{\rm d,max} = 2 \times 10^{20} \alpha_{\rm c}^{-0.83} M_1^{-0.38} \left(\frac{r_{\rm out}}{10^{10} {\rm cm}} \right)^{3.14} \quad (1)$$

where $\alpha_{\rm c}$ is the Shakura-Sunyaev viscosity parameter on the cool branch, M_1 is the primary mass in solar units, and $r_{\rm out}$ is the outer disc radius.

On the other hand, during a long outburst, the whole disc is entirely on the hot branch, and the local mass transfer rate in the outer regions of a disc must be large enough to prevent a cooling wave formation; using again the analytical fits of Hameury et al. (1998), this gives

$$\dot{M}_{\rm out} > 8 \times 10^{15} M_1^{-0.89} \left(\frac{r_{\rm out}}{10^{10} {\rm cm}} \right)^{2.67} \quad (2)$$

Here, $\dot{M}_{\rm out}$ is the local mass transfer rate in the outer parts of the disc, which is close to the mass accretion rate onto the white dwarf if the whole disc sits for a long time on the hot branch.

The maximum duration of such an outburst is thus $t_{\rm max} = M_{\rm d,max}/\dot{M}$. For the parameters appropriate for U Gem, one gets

$$t_{\rm max} = 26 \left(\frac{\alpha_{\rm c}}{0.01}\right)^{-0.83} M_1^{0.51} \left(\frac{r_{\rm out}}{4.1\ 10^{10}{\rm cm}}\right)^{0.47}\ {\rm d} \quad (3)$$

As $\alpha_{\rm c}$ is larger than 0.01 for this prototypical dwarf nova (Livio & Spruit 1991 for example find that $\alpha_{\rm c}$ must be equal to 0.044 to account for the timing properties of U Gem), $t_{\rm max}$ can never be as high as 45 days. This means that the total amount of mass accreted during this long outburst is larger than the mass of the disc in quiescence; this is possible only if the mass transfer rate from the secondary has increased to a value close to the mass accretion rate onto the white dwarf, i.e. is close to the critical rate for stable accretion. Such an increase of the mass transfer rate is very likely caused by the illumination of the secondary. This reinforces the conclusion of Smak (1999a) that long outbursts result from large mass–transfer enhancements.

3. The model

3.1. Disc irradiation

We use here the numerical code described in Hameury et al. (1998). This code solves the usual mass, angular momentum and energy conservation equations on an adaptive grid, with a fully implicit scheme. This allows to resolve narrow structures in the accretion disc (Menou et al. 1999), and avoids the Courant condition which would severely limit the time step. To describe disc irradiation we use a version of the code described in Dubus et al. (1999) (see also Hameury et al. 1999). A grid of vertical structures is used to determine the cooling rate of the disc as a function of the vertical gravity, the integrated disc surface density Σ, the central temperature $T_{\rm c}$ and the illumination temperature $T_{\rm ill}$, defined as $T_{\rm ill} = (F_{\rm ill}/\sigma)^{1/4}$ where $F_{\rm ill}$ is the illuminating flux. In what follows, we use $\alpha_{\rm hot} = 0.2$ and $\alpha_{\rm cold} = 0.04$, except where otherwise stated. We also neglect the albedo β of the disc; taking it into account introduces a multiplicative factor $(1-\beta)^{-1/4}$ for the white dwarf temperatures.

It must also be stressed that the white dwarf surface temperature cannot be too large; this is because the intrinsic (quiescent) white dwarf luminosity must be significantly less than the accretion luminosity in outburst; for M_1 = 0.6 M$_\odot$, the quiescent white dwarf temperature has to be smaller than 33,000 K if the outburst amplitude is to be larger than 2 magnitudes.

3.2. Illumination of the secondary

We are interested here in the effect of illumination of the secondary on its mass transfer rate on short time scales (days), and we do not consider any long term effects that may lead to cycles accounting for the observed dispersion of the average mass transfer rate for a given orbital period (McCormick & Frank 1998). Even on short time scales, the response of the secondary to illumination is complex (see e.g. Hameury et al. 1988); we prefer to use here a simpler approach in which we assume a linear relation between the mass transfer rate from the secondary $\dot{M}_{\rm tr}$ and the mass accretion rate onto the white dwarf $\dot{M}_{\rm acc}$, i.e.

$$\dot{M}_{\rm tr} = \max(\dot{M}_0, \gamma \dot{M}_{\rm acc}) \quad (4)$$

where $\dot{M}_0$ is the mass transfer rate in the absence of illumination. This is similar to the formula used by Augusteijn et al. (1993) in the context of soft X-ray transients. Although it is an extremely crude approximation, it has, nevertheless, the advantage of having only one free parameter γ. Its value must be in the range $[0\text{–}1]$ for stability reasons.

Such an approach obviously requires the illumination to have a noticeable effects on the secondary's surface layers. As mentioned earlier, strongly irradiated companion stars are observed in several systems. To describe the effects of irradiation of a Roche-lobe filling star we shall follow the approach of Osaki (1985) and Hameury et al. (1986).

The mass transfer rate from the secondary can be written as (Lubow & Shu 1975):

$$\dot{M}_{\rm tr} = Q \rho_{L1} c_s \quad (5)$$

where Q is the effective cross section of the mass transfer throat at the Lagrangian point L_1, $Q = 1.9 \times 10^{17} T_4 P_{\rm hr}^2$ cm^2, where T_4 is the surface temperature of the secondary and $P_{\rm hr}$ the orbital period in hours; c_s is the sound speed, and ρ_{L1} the density at L_1, which, in the case of an isothermal atmosphere, can be expressed as:

$$\rho_{L1} = \rho_0 e^{(R-R_{L1})/H} \quad (6)$$

where ρ_0 is the density calculated at a reference level, R the secondary radius (defined by this reference position), R_{L1} the Roche lobe radius and H the scale height.

In quiescence, the mass transfer rate is low, of order of $10^{15} - 10^{16}$ g s^{-1}. At short orbital periods and correspondingly low secondary's temperatures, both Q and c_s are small. If one assumed that ρ_0 corresponds to the photospheric density, which for a low mass main sequence star is of the order of 10^{-5} g cm^{-3}, one would find $\dot{M}_{\rm tr} \sim 10^{16} e^{(R-R_{L1})/H}$ g s^{-1}. This in turn would mean that $(R-R_{L1})/H$ is small so that $\dot{M}_{\rm tr}$ varies as $T_4^{3/2}$, and therefore is not very sensitive to illumination. In the irradiated case, however, the reference density ρ_0 does not correspond to the photospheric density but to the density at the base of the isothermal atmosphere, which extends much deeper than the unilluminated photosphere when the atmosphere is illuminated with an irradiation temperature exceeding 10^4 K. For those high illumination fluxes, the outer layers of the star are affected on a thermal time scale (seconds) at least down to a point where the unperturbed temperature equals the irradiation temperature. This point is the base of the isothermal layers in the illuminated case, and from models of very low mass stars (Dorman et al. 1989), one gets $\rho_0 \sim 10^{-3}$ g cm^{-3}, with resulting very high mass transfer rates, exceeding 10^{18} g s^{-1}.

The latter estimate is an upper limit, as it does not take into account the fact that a fraction of the secondary is shielded

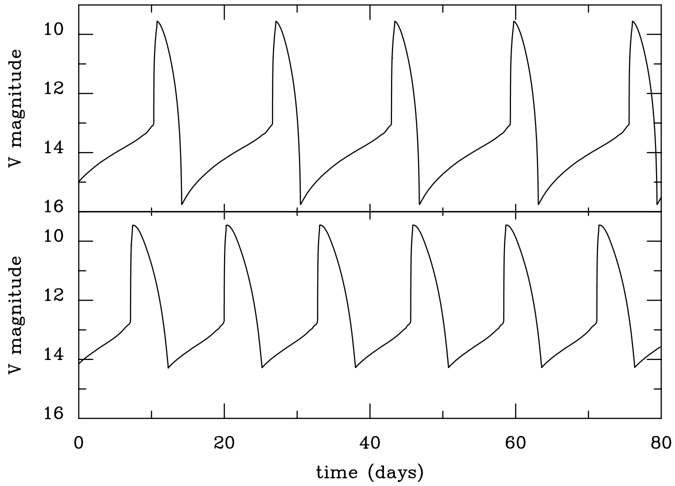

Fig. 1. Predicted light curve in the standard model (no irradiation, fixed inner radius) for an average mass transfer rate of 3×10^{16} g s^{-1}, an average disc radius of 1.9×10^{10} cm, and a 0.6 $M_\odot$ (top panel), or 1.0 $M_\odot$ (bottom panel) primary.

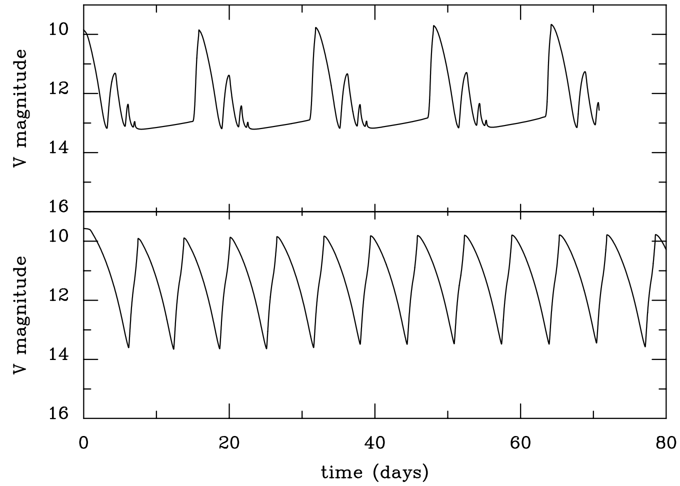

Fig. 2. Predicted light curve when disc illumination is taken into account. The parameters are the same as in Fig. 1. The upper panel shows the case of a 0.6 $M_\odot$ primary in which the radiation from the boundary layer is ignored, the lower panel corresponds to a 1$M_\odot$ primary, and we have taken into account the accretion luminosity. In both cases, the quiescent white dwarf temperature is 30,000 K.

from irradiation by the accretion disc; partial shielding does not suppress the enhancement of mass transfer, since circulation at the surface of the star prevents the existence of large temperature gradients, but reduces it in a complex way which we are not attempting to describe here.

Observations show, however, that the increase can be quite significant; the mass transfer rate rises by a factor 2 in U Gem and Z Cha (Smak 1995), whereas Vogt (1983) finds that in VW Hyi, the bright spot luminosity increases by a factor ~ 15 during maximum and decline of outbursts close to a superoutburst, which he attributed to a corresponding increase in mass transfer under the effect of illumination. Although the evidence is not very strong, there are some indications that the hot spot brightening, and hence the mass transfer increase, is delayed with respect to the eruption by a day or two; this could be either the response time of the secondary (Smak 1995), or the thermal inertia of the white dwarf (only an equatorial belt is instantaneously heated by accretion, so irradiation of the secondary is delayed).

3.3. Inner disc radius

There is evidence that in dwarf novae accretion discs are truncated, as indicated by emission line profiles (Mennikent & Arenas 1998), or the detection of a significant quiescent X-ray and UV flux (Lasota 1996). If this is due to the presence of a magnetic field, the inner disc radius $r_{\rm in}$ is a simple function of the mass accretion rate onto the white dwarf:

$$r_{\rm in} = 9.8 \times 10^8 \dot{M}_{15}^{-2/7} M_1^{-1/7} \mu_{30}^{4/7} \text{ cm} \qquad (7)$$

where $\dot{M}_{15}$ is the mass accretion rate in units of 10^{15} g s^{-1}, M_1 is the white dwarf mass and μ_{30} is the magnetic moment in units of 10^{30} Gcm^3. The value of μ_{30} should be such as to allow $r_{\rm in} = R_1$ in outbursts, where R_1 is the primary radius, as most DNs do not show then coherent pulsations. In quiescence, however, coherent oscillations are observed (Patterson et al. 1998), and for example in WZ Sge $\mu_{30} \approx 50$ (Lasota et al. 1999).

An inner hole in the disc can be also due to evaporation. The physics of evaporation is poorly understood but several models were proposed (Meyer & Meyer-Hofmeister 1994, Liu et al. 1997, Kato & Nakamura 1998, Shaviv et al. 1999). The evaporation rates $\dot{\Sigma}$ in the disc are, however, quite uncertain. Evaporation is normally accounted for by introducing the additional term $\dot{\Sigma}$ in the mass conservation equation. However, because evaporation is expected to increase towards the accreting body, and since the local mass transfer rate in the disc increases sharply with radius during quiescence, the effects of evaporation are important essentially very close to the disc inner edge, and can be treated assuming that the disc inner radius is a function of the accretion rate onto the white dwarf, just as in the case of the formation of a magnetosphere. To first order, the effect of evaporation is to create a hole in quiescence which increases the recurrence time; what matters is thus the inner disc radius in quiescence and not the detailed way $r_{\rm in}$ varies with $\dot{M}_{\rm acc}$.

We shall therefore use Eq. (7) in all cases, using μ_{30} as a free, unconstrained parameter that merely describes the size of the hole generated in the disc by either the presence of as magnetic field, or by evaporation.

4. Results

In the following, we discuss the influence of each individual effect mentioned above; our reference situation is that of a system with a 1.0 $M_\odot$ primary, whose radius is 5×10^8 cm; the mass transfer rate is 3×10^{16} g s^{-1}, and the average outer disc radius is 1.8×10^{10} cm. These parameters are typical of short period dwarf novae with massive primaries. The light curve corresponding to the standard version of the DIM is given in Fig. 1.

For comparison we also show the case of a system with a 0.6 $M_\odot$ primary.

4.1. Influence of the disc irradiation

The effects of irradiation of the disc by both the hot white dwarf and the boundary layer have been described in detail in Hameury et al. (1999), and we summarize here the most important results. For very hot white dwarfs ($T_{\rm eff} > 20\,000$ K), the temperature in the innermost parts of the disc exceeds the hydrogen ionization temperature during quiescence; the viscosity is therefore high in these regions, which are thus partially depleted as first suggested by King (1997). The transition region between the hot inner disc and the outer, cool parts is strongly destabilized by irradiation, and the model predicts several small outbursts between major ones. In particular, many reflares are expected at the end of a large outburst (see Fig. 2). In certain cases, the reflares may dominated the light curve; this depends on whether the heating front can reach the outer edge of the disc or not. The reflares we obtained do not have the observed amplitudes but it is tempting to attribute the succession of several normal outbursts after a superoutburst in EG Cnc to this effect. Playing with parameters would produce a result corresponding better to the observed lightcurve but the merit of such an exercise is rather dubious considering the important uncertainties of the model itself.

4.2. Influence of the secondary irradiation

Irradiation of the secondary enhances mass transfer. Hameury et al. (1997) showed that if one assumes that the effect of irradiation is given by Eq. (4), outbursts having the general characteristics of superoutbursts (long durations, flat top or exponential decay with an abrupt cut-off) are expected. This model was, however, applied to a case in which the quiescent mass transfer was low enough for the disc to be stable on the cool branch; the instability was triggered by an external perturbation of the mass transfer from the secondary. This was required to explain the very long recurrence times of systems such as WZ Sge when standard values of α are assumed. Marginally unstable mass transfer rates as in Warner et al. (1996) can give similar recurrence times but also in this case the amount of mass accreted during the superoutburst requires a substantial enhancement of mass transfer.

Fig. 3 shows the light curve obtained when one includes the secondary irradiation in the model. We neglect here disc irradiation and the disc is not truncated. We have taken γ = 0.5, and all other parameters are as in Fig. 1, for a 1 $M_\odot$ primary (i.e. $\dot M_0 = 3 \times 10^{16}$ g s^{-1}). The light curve is similar to those observed in SU UMa systems; it shows several normal outbursts separated by a large one which is sustained by enhanced mass transfer from the secondary.

Large outbursts occur when the surface density at the outer edge of the disc is large enough that a cooling wave does not start immediately after the heating wave has arrived; equivalently, the disc mass must be larger than some critical value, and one therefore expects that the recurrence time of such large

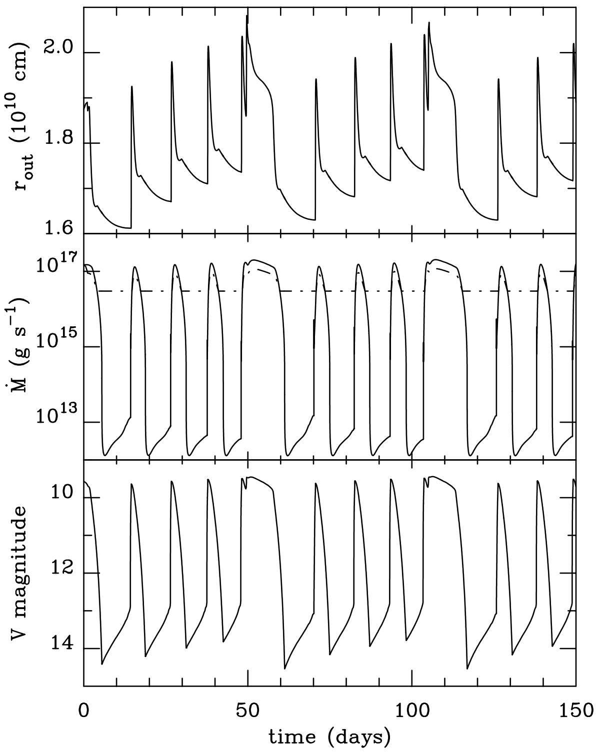

Fig. 3. Effect of secondary illumination on the predicted light curve; the parameters are those of Fig. 1, with M_1 = 0.6 $M_\odot$ and γ = 0.5. The upper panel shows the outer disc radius, the middle panel shows the accretion rate onto the white dwarf (solid line) and the mass transfer rate from the secondary (dashed line); the lower panel shows the visual magnitude of the disc.

outbursts $T_{\rm s}$ varies roughly as $\dot M_0{}^{-1}$. This, however, requires that the disc mass keeps increasing despite the presence of small outbursts, which means that $\dot M_0$ must be large enough to refill the disc with more mass than is lost during such outbursts. For $\dot M_0 = 3 \times 10^{15}$ g s^{-1}, which is more appropriate for short period systems, one does not get long outbursts, at least for the value of α considered here. Short outbursts are (as all our outbursts) of the inside-out type, and their recurrence time $T_{\rm n}$ is the viscous time, and therefore do not depend on the mass transfer rate.

The correlation between $T_{\rm s} \propto T_{\rm n}^{0.5}$ found in SU UMa systems (Warner 1995b) is interpreted, in the framework of the tidal-thermal instability (TTI) model, as resulting from $T_{\rm s}$ varying as $\dot M^{-1}$, as in our case, and from $T_{\rm n} \propto \dot M^{-2}$ for outside-in outbursts (Osaki 1995a). One must however be careful with such a simple interpretation. This explanation is valid only if $\dot M$ is the only parameter determining both $T_{\rm n}$ and $T_{\rm s}$; this is clearly not the case, as quantities such as the viscosity in quiescence (that may vary by orders of magnitude from WZ Sge type systems to "normal" SU UMa's), the disc radius (to the power 5.6), and the orbital period enter together with $\dot M$ in expressions for $T_{\rm n}$ and $T_{\rm s}$ (Osaki 1995a). It must also be stressed out that, for the low mass transfer rates of SU UMa stars, outside-in outbursts are not a natural outcome of the models and are produced by lowering the value of $\alpha_{\rm c}$ or by making it an appropriate function of radius.

The time evolution of the outer disc radius is different from the predictions of the TTI model in several respects: (i) $r_{\rm out}$ varies during a normal outburst, whereas in the TTI model, $r_{\rm out}$ remains roughly constant during an outburst after an initial increase during the rise (ii) $r_{\rm out}$ oscillates at the beginning of a large outburst, (iii) the disk extends to a larger radius during a large outburst than during a short outburst, allowing for the possibility of the development of superhumps if the radius can reach the 3:1 resonance radius, whereas in the TTI model the disk size varies by 30% during a superoutburst, with an average that is smaller than the average size during the previous normal outburst, and (iv) $r_{\rm out}$ remains approximately constant during a superoutburst, showing only a slow decline.

Our results are similar to those of Smak (1991b), who considered the effect of enhanced mass transfer during superoutbursts, and concluded that observed disc radius variations in Z Cha and the length of the cycle in VW Hydri appeared to support the enhanced mass transfer model. The main difference with our work comes from the approximations describing the effect of illumination: whereas we assume a dependence between $\dot{M}_{\rm tr}$ and $\dot{M}_{\rm acc}$ given by Eq. 4, in Smak's model $\dot{M}_{\rm tr}$ is increased by about one order of magnitude after the maximum of a normal outburst, during a fixed period. As a consequence, the disc radius we obtain at the end of a large outburst is much smaller than in Smak's model. In our case, when the cooling wave starts propagating, the accretion rate onto the white dwarf, and hence the mass transfer from the secondary, is unaffected, and the disc contracts as in the unilluminated case, whereas in Smak's model, the disk expands rapidly when mass transfer is reduced by a factor 10; the surface density then drops at the outer edge below the critical value, and a cooling wave starts in quite a large disc.

Ichikawa et al. (1993) also considered a mass transfer outburst as the source of superoutbursts. They compared the resulting lightcurves with those produced by the tidal–thermal model and concluded in this last model gives a much better representation of observed properties of SU UMa's system. One should stress, however, that in Ichikawa et al. (1993) superoutbursts are triggered by a mass transfer 'instability'. In our case, superoutbursts are triggered by the usual thermal-viscous instability and it is only the subsequent evolution of the outburst which is modified by an enhanced mass transfer. In Ichikawa et al. (1993), the mass transfer $\dot{M}_{\rm tr}$ is increased by a factor 100, whereas in our case the peak mass transfer rate from the secondary is 1.2×10^{17} g s^{-1}, i.e. increased only by a factor 4. It is much smaller than the maximum possible rate from an irradiated low-mass star (see Sect. 2.1). Note that the average mass transfer rate, as given by Eq. (4) is larger than $\dot{M}_0$, and thus larger than for Fig. 1; it is in this case 4.8×10^{16} g s^{-1}.

Observations are, for the moment, not of much help in deciding which of the models is right. There are good reasons to believe that a tidal instability is required to account for the superhump phenomena. What is not known, however, is (i) the increase in the tidal torque resulting from this instability, and (ii) the radius at which the instability stops. There is also good evidence for an enhanced mass transfer, due to irradiation, during outbursts but a reliable description of this effect is missing. In both models, however, the observed correlation with superoutburst and normal outburst frequency can be obtained only by playing with the viscosity prescription which, of course, is not very satisfactory.

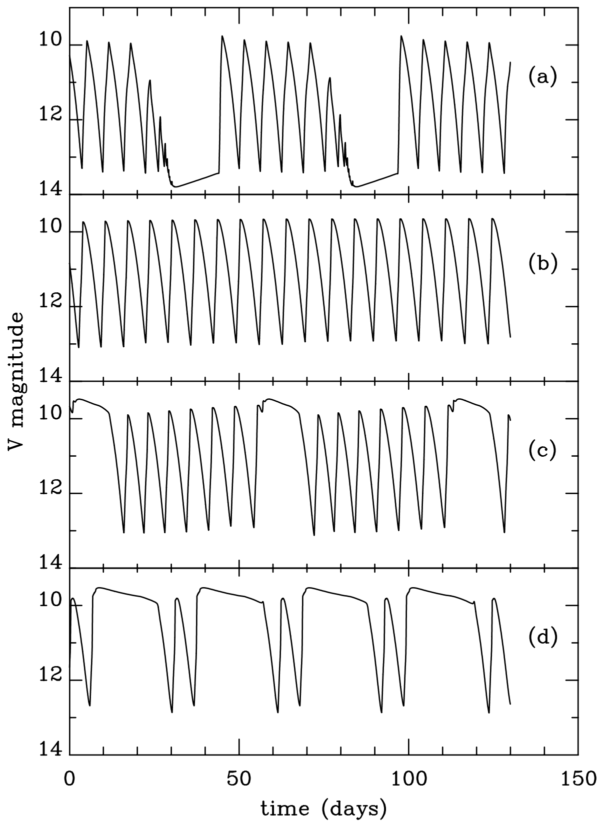

Fig. 4a–d. Visual magnitudes obtained in the case of a system containing a 1 $M_\odot$ primary, whose radius is 5×10^8 cm and surface temperature 35,000 K in quiescence. γ = 0.5 for all four panels, and $\dot{M}_0$ = 10^{16}, 3×10^{16}, 4×10^{16}, and 7×10^{16} g s^{-1} from top to bottom. Note that the average mass transfer from the secondary is larger than $\dot{M}_0$.

The quiescent luminosities and accretion rates are almost identical in the standard and enhanced mass transfer cases, as expected, since after the passage of the cooling wave, the disc has essentially forgotten its initial conditions; differences arise only from the different disk sizes which are smaller in the illuminated case because of the large mass transfer increase.

4.3. Parameter dependence

4.3.1. Mass transfer rate

In this section we shall consider the combined effect of irradiation of both the disc and the secondary and shall determine how the resulting lightcurves depend on the value of rate $\dot{M}_0$ at which mass is transferred from an unilluminated secondary.

Fig. 4 shows various light curves obtained by varying $\dot{M}_0$. We considered a system containing a 1 $M_\odot$ primary, whose radius is 5×10^8 cm and surface temperature 35,000 K in quiescence. We have taken γ = 0.5, and $\dot{M}_0$ ranges from 10^{16} to

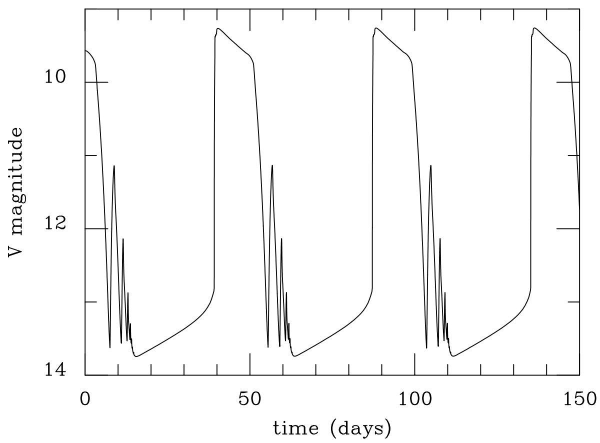

Fig. 5. Light curves obtained with $\alpha_{\rm cold} = 0.02$; all other parameters are the same as for Fig. 4b.

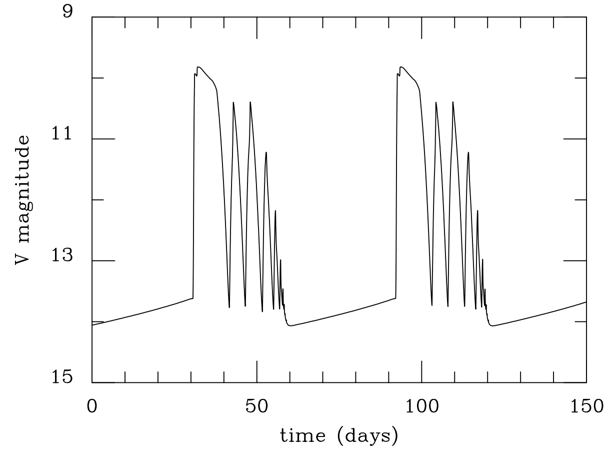

Fig. 6. Example of a lightcurve obtained for a small disk; we consider a 1 $M_\odot$ primary with surface temperature 35,000 K, $\alpha_{\rm cold} = 0.02$, $\alpha_{\rm hot}$ = 0.2, $\gamma = 0.5$, and $\dot{M}_0 = 10^{16}$ g s^{-1}.

7×10^{16} g s^{-1}; this corresponds to average transfer rates in the range 1.5–8.6 $\times 10^{16}$ g s^{-1}. The inner disc radius is equal to the white dwarf radius. The white dwarf temperature has been chosen in the upper range of observed values in order to emphasize illumination effects. The effect is quite dramatic. Light–curves corresponding to cases (a) and (b) do not seem to be observed (as mentioned in the previous section the mass transfer rate is too low for long outburts to be present). It might be that the corresponding systems exist, but have not yet been discovered because they are intrinsically rare – the parameters of Fig. 4 are at the upper range of allowed values, or that some of the curves we obtain are artifacts due to our oversimplified treatment of the secondary response to illumination. Lightcurves (c) and (d), however, compare very well with those of systems having very short supercycles such as RZ LMi. In our model they are obtained for high mass transfer rates, which is natural since these systems spend most of their time in the high state, whereas in the tidal–thermal instability such lightcurves require an ad hoc reduction of the parameter describing the tidal interaction. The agreement with RZ LMi can be improved; in particular a reduction of the duration of the long outburst will be obtained by decreasing γ.

One should note, however, that we have assumed that $\dot{M}_{\rm tr}$ responds immediately to changes in $\dot{M}_{\rm acc}$, whereas one could argue that there is a delay of the order of one or two days between illumination and the increase of mass transfer as discussed above. The introduction of such a delay makes the occurrence of long outbursts more difficult, as these require a near balance between mass transfer and accretion onto the white dwarf that must be established within a short outburst. We checked that if we use in Eq. (4) the average of the mass accretion rate over the past 2.5 days (the duration of short outbursts), long outbursts are suppressed.

Finally, it is worth pointing out that small outbursts in Fig. 4 are intrinsically different from those obtained when the disc illumination is not taken into account. Whereas in a non–irradiated disc small amplitude outbursts appear when the heating front cannot bring the whole disc into a hot state, here the disc never returns to quiescence in its inner parts, so the cooling wave is reflected into a heating wave when it gets close to the stable hot inner part of the disc. The amplitude of these reflares grows as a consequence of the enhanced mass transfer during maximum, until the disc mass has grown up to a point where a self-sustained long outburst is possible.

4.3.2. Viscosity

The reflares properties also depend on the ratio $\alpha_{\rm hot}/\alpha_{\rm cold}$: the smaller this ratio, the more important the reflares (see Menou et al. 1999 for a discussion of this effect in the context of X-ray transients). This is simply due to the fact that, the lower this ratio, the larger $\Sigma/\Sigma_{\rm max}$ after the passage of a cooling front, $\Sigma_{\rm max}$ being the maximum surface density on the cold stable branch; in the limiting case $\alpha_{\rm hot} = \alpha_{\rm cold}$ there are no outbursts (Smak 1984), but a heating/cooling wave that propagates back and forth. Fig. 5 shows the effect of changing $\alpha_{\rm cold}$ to a smaller (by a factor 2) value. Successive reflares no longer reach the outer edge of the disc, and their amplitude therefore decreases from one mini-outburst to the next one. This accounts for the presence of flat top outbursts which were absent in Fig. 4b. The lightcurves are similar for all mass transfer rates, showing the pattern of Fig. 5 with longer recurrence times for smaller $\dot{M}_{\rm tr}$. The only exception is for 7×10^{16} g s^{-1}, which is close to stability, and for which the main outbursts are of the outside-in type.

4.3.3. Outer disc radius

Since small discs favour large reflares, it is not surprising that when one considers discs with average $r_{\rm out} = 1.3 \times 10^{10}$ cm, and one takes $\alpha_{\rm cold}$ = 0.02 and $\alpha_{\rm hot}$ = 0.2, one obtains a combination of the lightcurves shown in the two previous sections; Fig. 6 is a good example of this. It is worth noting that such a light curve is reminiscent of that of EG Cnc, even though the timescales are not quite the same. We do obtain the right pattern

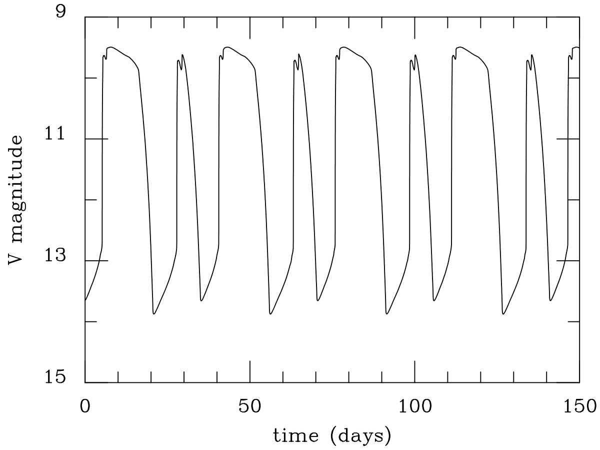

Fig. 7. Lightcurve obtained when the inner disc is truncated at a radius given by Eq. (7), with a magnetic moment of 2×10^{30} G cm^3, the other parameters being that of Fig. 4c.

for the reflares, but we do not reproduce the very long superoutburst of EG Cnc (100 days), that would require γ to be very close to unity, meaning that the linear approximation in Eq. 4 is invalid. For WZ Sge, one already had to assume a relatively large value of γ (0.87) in order to reproduce the observed 25 days duration; since the outburst duration varies as $1/\log(\gamma)$ (Hameury et al. 1997), we would need $\gamma = 0.97$ to obtain 100 days. Another difference with EG Cnc is the amplitude of the minioutbursts: the observed ones have approximately the same amplitude, whereas we get two identical minioutbursts, the others being of decreasing amplitude. We have not been able to reproduce this behaviour with our parameterization; a possible solution is to introduce a time dependent temperature of the white dwarf. This is expected, because the superoutburst lasted long enough to heat up the surface of the white dwarf that will then cool.

If the rebrightenings of EG Cnc are indeed due to illumination effects, this implies that $\alpha_{\rm cold}$ cannot be small as we do not obtain reflares when $\alpha_{\rm cold}$ is significantly less than 0.01; Osaki et al. (1997) reached the same conclusion, but on different grounds; they assumed that $\alpha_{\rm cold}$ was increased to 0.1 during the superoutburst, remained high for 2 months, and then decreased back to small values (0.001), and had therefore to set $\alpha_{\rm cold}$ to be an explicit function of time.

4.3.4. Inner disc radius

Finally we consider the effect of removing the inner disc regions, keeping both the disc and secondary irradiated.

Apart from increasing the delay between the onset of an outburst in the disc and accretion onto the white dwarf, a large inner disc radius has a stabilizing effect on the disc itself, by preventing inside-out outbursts. This effect is quite noticeable in the case where the white dwarf surface temperature is high: if the inner disc radius $r_{\rm in}$ is large enough, the unstable transition region between the stable region heated above hydrogen ionization temperature and the cooler, quiescent external part does not exist. This suppresses the bounces after a longer outburst, as can be seen in Fig. 7.

For a given mass transfer rate, there is a critical value of the inner radius above which the disc is stable; when $r_{\rm in}$ approaches this value, the recurrence rate goes to infinity. Arbitrarily long recurrence rates could therefore be expected, but only at the expense of very fine tuning; in the normal case where the mass accretion rate onto the white dwarf is negligible in quiescence as compared with the mass transfer rate from the secondary, the reasoning used by (Smak 1993) applies. The recurrence time $t_{\rm rec}$ is equal to:

$$t_{\rm rec} = \frac{\Delta M}{\dot{M}_{\rm tr}} = f \frac{M_{\rm crit}}{\dot{M}_{\rm tr}} \qquad (8)$$

where f is the ratio of the amount of mass transferred during an outburst ΔM and the maximum possible disc mass $M_{\rm crit}$, obtained assuming that the surface density is everywhere the critical surface density. Numerical models show that the surface density is not very far from its critical value even at large radii, and that the amount of mass transferred during a normal outburst is typically 10% of the total disc mass. Therefore, f is not a very small parameter that could freely vary, and large changes in $t_{\rm rec}$ cannot result from variations in $r_{\rm in}$ alone. A similar situation is encountered in the case of soft X-ray transients (Menou et al. 1999).

5. Conclusions

We have shown that many types of light curves can be produced by numerical models that include the illumination of both the secondary and the accretion disc, thereby explaining a great variety of observed light curves. These effects account for phenomena such as post-outburst rebrightening (e.g. EG Cnc), long outbursts (U Gem for example), or SU UMa systems with extremely short supercycles. In order to explore further these possibilities, one would need to determine from observations the mass transfer rate from the secondary as a function of the mass accretion rate onto the white dwarf, with a better accuracy than it is available now.

Despite the fact that our approximations are very crude, in particular the one concerning the response of the secondary to illumination, we can nevertheless draw a number of conclusions. First, the illumination of the disc is important only if the white dwarf is relatively massive, so that it can have a high temperature without contributing too much to the light emitted by the system in outburst, and that the efficiency of accretion is high. Rebrightenings also require α not to be too low in quiescence.

The fact that we can reproduce an alternance of normal and long outbursts when the illumination of the secondary is included does not of course imply that the thermal-tidal instability model for SU UMa is incorrect; a tidal instability is most probably required to account for the superhump phenomenon. The question of the precise role of this instability is, however, still open and our results raise some doubt on the validity of the parameters derived when fitting the observations, in particular for systems having a very short supercycle.

This also means that the determination of the viscosity from the modeling of light curves is a far more difficult task than previously estimated. We obviously need some progress in the determination of the tidal torque; we also need to know how the secondary responds to illumination. We finally should include 2D effects in our models. First because the orbits in the outer disc are far from being circular, and second because the presence of a hot spot whose temperature can be of order of 10,000 K could in principle significantly alter the stability properties of the outer disc.

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