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Linear interaction between pulsations and convection, scattering and line profiles of solar p-modes

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Abstract. First we show that when the linear stability analysis is done taking into account the interactions between convection and pulsation, it includes the influence of scattering. However, in practice, the solutions are obtained using a simple time independent solution for the unperturbed convection. Secondly, we investigate the consequences of taking the convection spectrum into account. We find that the terms which fix the value of the peak frequency shifts and of the line widths in the power spectrum are not influenced by mode coupling in opposition to the Goldreich & Murray (1994) prediction. Other terms produce line profiles with are not rigorously Lorentzian. All these terms are present even when the interaction between convection and pulsation is ignored but indeed their value varies with the theory being used. However, new terms appear. They do not modify the eigenvalues but influence the line asymmetry. Most of them lead to mode coupling. Though they are very difficult to compute, an order of magnitude estimate suggests that these terms are very small.

Key words: Sun: oscillations – stars: oscillations

1. Introduction

In convective zones, stars have no radial symmetry and no static state as all variables show fluctuations over small horizontal scales. In the usual approach, convection is given by an underlying velocity field and fluctuations of the thermodynamic variables which vary over short horizontal lengths. These convective fluctuations are superposed to the average model which is assumed time independent and spherically symmetric. Consequently, waves crossing a convective region see moving layers whose properties vary with time and which have no radial symmetry. Fortunately, the convective fluctuations are usually small and can be considered as perturbations. Convection influences the waves in two ways:

1. the convective velocity produces a Doppler shift, this leads to scattering.
2. the waves modify the state of convection and this in turns modifies the waves through terms such as the Reynolds tensor or the convective flux.

Goldreich & Murray (1994) have for the first time discussed the scattering of solar p-modes by convection. They found that scattering contributes significantly to line widths in the 5 min. range. Recently Gruzimov (1998) has also discussed a related problem. Goldreich & Murray consider that convection is the sum of linear modes and they compute the interaction between the unperturbed convection and the p-modes; they obtain the rate of energy pumping out of the p-modes by convection which leads to an increase of the line widths.

However the energy pumped out of the p-modes perturbs the convection and that perturbed convection influences the stability of the p-modes through the perturbation of the Reynolds tensor and the convective flux (and also other terms which are often neglected). Goldreich & Murray disregarded this point. In other words they neglected the linear perturbation of convection and its feedback on pulsations. When this problem is studied, there is no need to introduce the notion of scattering; all the terms are present in the equations. But the difficulty is to obtain a good enough solution to this problem.

An accurate solution of the stability of solar p-modes (that is to say the solution of the non-adiabatic linear equations of stellar pulsation) is necessary to understand their physics since:

1. the damping time gives the line width
2. the dissipations also produce a small frequency shift, of the order of $1\mu Hz$ according to Balmforth (1992), and this increases as the frequency approaches the cut-off.

This study must be made using a time dependent theory of convection since in the outer convection envelope, where the non-adiabatic effects are the most important, the characteristic time of convection is of the same order as the periods. Even when convection is slow compared to pulsations (deep in the envelope or in a convective core), we cannot blindly perturb the static equations for convection as was shown by Boury et al. (1964). For instance they show that, in such a situation, the perturbation of $\Delta T/T$ the average relative temperature difference between rising convective globules and the average medium cannot be obtained from the perturbation of the static equation but is very close to zero.

In Sect. 2, we obtain the equations for pulsation and for its interactions with convection using a method different form, though equivalent to, the usual one. These equations show that the interactions between convection and pulsations include all the scattering terms. However, the equations for these interactions are always linearized for two reasons. Firstly, this leads, when the star does not pulsate, to a time independent solution which is identical to the mixing length theory (this theory is nearly always used in the computation of models and for consistency the equations used to compute the static model and its perturbations must be obtained from the same set of equations). Secondly, this allows us to obtain a solution for the interactions between convection and pulsation without tremendous difficulties. Because, with this approximation, the unperturbed solution for convection is time independent and has no spectrum, only the interactions between convection and the mode considered have to be taken into account. In other words, convection does not lead to mode coupling. One aim of this paper is to consider the consequences of giving up this approximation. In Sects. 3 and 4, we discuss the solution for unperturbed and perturbed convections when the spectrum is taken into account, keeping however the local approximation. Even if we are unable to write down the explicit solution, we can give some general properties. This allows us to give the general form of the perturbation for terms such as the Reynolds tensor and the convective flux which appear in the equations for global oscillations (i.e. for the star). Each of them can be split in two parts. The first one, which has a non-zero statistical mean, does not couple the global modes. These terms are obtained in the usual studies of linear interactions between convection and pulsations. The second one, which has a zero statistical average, couples all the global modes.

In Sect. 5, we use this solution to discuss the influence of the linear interactions between pulsation and convection on the stability of stars. We find that the terms with a non-zero statistical mean, already present when the convection spectrum is neglected, lead in the power spectrum to a shift of the peak frequencies and a modification of the line widths. Also they produce a departure from Lorentz profiles which gives skew lines. The terms with a zero statistical mean do not modify the eigenvalues. They change the total power in the lines and also they produce new departures from Lorentz profile. Unfortunately, we lack a good enough solution for convection to carry out a reliable evaluation of these terms but a rough estimate suggests that they are very small.

2. Equations of the problem

Let us start with a preliminary remark concerning the definition of the radially symmetric part and of the convective fluctuations of the variables in the static model. We may define the spherically symmetric part of only two thermodynamic variables as the average taken over the surface of the sphere of radius r i.e. by $y_0 = 1/S \int y d\Omega$. Then $y = y_0 + \Delta y$ where Δy is the convective fluctuation. Here we will do this for the density and the pressure. For the other thermodynamic variables, two methods are used to define their horizontal average. Let us consider the energy U per unit mass. In practice its horizontal average U_0 is defined by $U_0 = U(p_0, \rho_0)$, but in theoretical studies it is usually defined as the average of ρU over the surface of the sphere of radius r divided by ρ_0. With this second definition, we get to the second order in the convective fluctuations: $(\rho U)_0 = \rho_0 U(p_0, \rho_0) + 1/S \int \Delta\rho \Delta U d\Omega$. This leads to an relative difference between the two definitions of the order of $(\Delta\rho/\rho)^2$ which is very small in the Sun (but not necessarily in giants) and will therefore be neglected (the error is smaller than the uncertainty on the equation of state). The same remark can be made for the temperature, the energy generation rate and all other variables.

For the velocity, we define the spherically symmetric part $\boldsymbol{u}$ through $\rho_0 \boldsymbol{u} = 1/S \int \rho \boldsymbol{v} d\Omega$. Rather than defining a convective velocity $\boldsymbol{V}$, we use the convective momentum $\Delta \boldsymbol{q} = \rho \boldsymbol{V}$ defined as $\Delta \boldsymbol{q} = \rho \boldsymbol{v} - \rho_0 \boldsymbol{u}$. All convective variables Δy and $\Delta \boldsymbol{q}$ are assumed to have projections only over very high degree spherical harmonics, much higher than the pulsations which will be considered. This means that we neglect large scale convection which is expected to have little energy. For the static model, we write:

$$\begin{aligned} y &= y_0 + \Delta y \\ \rho \boldsymbol{v} &= \rho_0 \boldsymbol{u} + \Delta \boldsymbol{q} \end{aligned} \tag{1}$$

In practice $\boldsymbol{u} = 0$ and y_0 is spherically symmetric (but see the discussion below). When pulsations are excited, we decide to split each perturbation in two parts and we write:

$$\begin{aligned} y &= y_0 + \Delta y + y_0' + \Delta y' \\ \rho \boldsymbol{v} &= \rho_0 \boldsymbol{u} + \Delta \boldsymbol{q} + \Delta \boldsymbol{q}' \end{aligned} \tag{2}$$

where y_0' and $\boldsymbol{u}$ which describe the global oscillation of the star, have projections on spherical harmonics of relatively low degrees only while $\Delta y'$ and $\Delta \boldsymbol{q}'$ which describe the induced perturbation of convection have only projections on very high degrees. Notice that we can also keep the notation of Eq. (1) with the convention that y_0 (Δy) of that equation corresponds to $y_0 + y_0'$ ($\Delta y + \Delta y'$) of Eq. (2). Then y_0 is the projection on low degrees of y. This allows us to use the equations obtained below through a projection over low degrees to get the perturbed equations for global oscillations.

We note that if first order terms in the convective fluctuations have a projection on high degree spherical harmonics only, second order terms in the convective fluctuations have a projection on low degree spherical harmonics. (Such terms appear in the global equations for both the static and for the perturbed model; for instance the Reynolds tensor and the convective flux.) The simplest way to see this is to go to plane coordinates. Then the horizontal dependence of the global eigenfunctions is given by $\exp(-\imath \boldsymbol{k}_p.\boldsymbol{r})$ where $\boldsymbol{k}_p$ is a small horizontal vector. It is always possible to find nearly opposite convective wave numbers such that the horizontal component of $\boldsymbol{k'}_c + \boldsymbol{k''}_c$ is small. Consequently, if we consider for instance $\Delta \boldsymbol{q}(\boldsymbol{k'}_c) \Delta \boldsymbol{q}'(\boldsymbol{k''}_c + \boldsymbol{k}_{p,j})$ there exist values of $\boldsymbol{k'}_c, \boldsymbol{k''}_c$ and $\boldsymbol{k}_{p,j}$ such that $(\boldsymbol{k'}_c + \boldsymbol{k''}_c)_H + \boldsymbol{k}_{p,j} = \boldsymbol{k}_{p,i}$ (the subscript "c" and "p" refer respectively to convection and pulsation, $\boldsymbol{k}_{p,j}$ refers to one of the modes which perturb convection (i.e. one of the excited modes) and $\boldsymbol{k}_{p,i}$ to the mode which is directly considered, also $\boldsymbol{k}_c \gg \boldsymbol{k}_p$; the subscript "H" refers to the horizontal component of a vector). On the contrary $(\boldsymbol{k}_c + \boldsymbol{k}_p)_H$ is always large.

A consequence of this is that a static model ($\boldsymbol{u} = 0$) exists only if these second order terms have a projection over the spherical harmonic of degree zero which is time independent. However, this is not the case. To nevertheless define a static model, we have to keep only their time independent part in the equations which describe the unperturbed model and to move the time fluctuating parts into the pulsation equations where they are source terms for non-linear excitations.

In this section, we will give the equations of the problem in which we immediately neglect the second order terms in $\boldsymbol{u}$ since we will consider linear perturbations only. We will look for their projection over low and very high degree spherical harmonics. We will also linearize the equations for convection (i.e. the projection over very high order spherical harmonics) as that approximation is required in order to recover the mixing length theory for the unperturbed convection.

2.1. *The equation of continuity*

Introducing Eq. (1) in the continuity equation, we obtain:

$$\frac{\partial(\rho_0+\Delta\rho)}{\partial t} + \boldsymbol{\nabla}.[(\rho_0+\Delta\rho)\boldsymbol{u}+\boldsymbol{q}] = 0 \tag{3}$$

If we project this equation over a low spherical order, we get the usual equation:

$$\frac{\partial\rho_0}{\partial t} + \boldsymbol{\nabla}.(\rho_0\boldsymbol{u}) = 0 \tag{4}$$

When there is no pulsation, Eq. (3) gives:

$$\frac{\partial\Delta\rho}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{q} = 0 \tag{5}$$

This equation has a projection on high degree spherical harmonics only and must be fulfilled by the unperturbed convection.

If we project Eq. (3) over a high degree spherical harmonic, we get:

$$\frac{\partial\Delta\rho}{\partial t} + \boldsymbol{\nabla}.[\Delta\rho\boldsymbol{u}+\Delta\boldsymbol{q}] = 0 \tag{6}$$

This equation describes the interaction between convection and pulsation. We define the time derivative following the mean motion as:

$$\frac{dy}{dt} = \frac{\partial y}{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla} y$$

Then, it is easily verified that Eq. (6) gives

$$\frac{d}{dt}\left(\frac{\Delta\rho}{\rho_0}\right) + \frac{1}{\rho_0}\boldsymbol{\nabla}.\Delta\boldsymbol{q} = 0 \tag{7}$$

2.2. *The equation of motion*

We start from:

$$\frac{\partial\rho\boldsymbol{v}}{\partial t} + \boldsymbol{\nabla}.\left[\rho\boldsymbol{v}\boldsymbol{v}\right] = -\rho\boldsymbol{\nabla}\phi - \boldsymbol{\nabla}p + \boldsymbol{\mathcal{D}_1} \tag{8}$$

in which we have to replace each variable taking Eq. (1) into account. $\boldsymbol{\mathcal{D}_1} = \boldsymbol{\nabla}.\mathcal{B}$ ($\mathcal{B}$ is the gas plus radiation viscous tensor) is the viscous dissipation term; its projection on low degree spherical harmonics is small and can be neglected while its projection over high degrees must be taken into account as it leads to dissipation of kinetic energy of turbulence into heat.

The projection over a low degree spherical harmonic is

$$\frac{\partial\rho_0\boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}.\left[\boldsymbol{R}\right]_p = -\rho_0\boldsymbol{\nabla}\phi - \boldsymbol{\nabla}p_0 \tag{9}$$

where $\boldsymbol{R}$ is the Reynolds tensor and the subscript "p" means that the projection over a low degree is to be taken. To illustrate the remark made above, we consider the projection over the spherical harmonic order of degree zero. It shows that in order to have a static model ($\boldsymbol{u} = 0$) the Reynolds tensor $\boldsymbol{R}_p$ must be time independent. This is not the case indeed and to define the static model we have to keep only its time independent part (which leads to the gradient of the turbulent pressure). The time fluctuating part will appear in the pulsation equation for radial modes and will be one of the source terms for non-linear excitations. (The time fluctuating part of the Reynolds tensor has a projection over all degrees)

When the model is perturbed the projection of the equation for low l gives:

$$\frac{\partial\rho\boldsymbol{u}}{\partial t} + \rho'\boldsymbol{\nabla}\phi_0 + \rho_0\boldsymbol{\nabla}\phi' + \boldsymbol{\nabla}p' + \boldsymbol{\nabla}.\left[\boldsymbol{R}'+\boldsymbol{R}_{fl}\right]_p = 0 \tag{10}$$

The term with the subscript "fl" is the source term which has been discussed in detail by Goldreich & Keeley (1977) and Goldreich & Kumar (1988, 1990). The other terms give the usual equation.

The projection of Eq. (8) over a high spherical order gives after eliminating $\frac{\partial\boldsymbol{u}}{\partial t}$:

$$\frac{\partial\Delta\boldsymbol{q}}{\partial t} + \boldsymbol{\nabla}.\left\{\boldsymbol{u}\Delta\boldsymbol{q}+\boldsymbol{R}_c\right\} + \Delta\boldsymbol{q}.\boldsymbol{\nabla}\boldsymbol{u} = \frac{\Delta\rho}{\rho}\left[\boldsymbol{\nabla}p_0+\boldsymbol{\nabla}\boldsymbol{R}_p\right] - \boldsymbol{\nabla}\Delta p + \boldsymbol{\mathcal{D}_1}(\boldsymbol{\Delta q}) \tag{11}$$

The subscript "c" means that the projection over a high degree is to be taken.

This equation can also be written as:

$$\rho\frac{d}{dt}\left(\frac{\Delta\boldsymbol{q}}{\rho}\right) + \Delta\boldsymbol{q}.\boldsymbol{\nabla}\frac{\Delta\boldsymbol{q}}{\rho} + \Delta\boldsymbol{q}.\boldsymbol{\nabla}\boldsymbol{u} = \frac{\Delta\rho}{\rho}\left[\boldsymbol{\nabla}p_0+\boldsymbol{\nabla}\boldsymbol{R}_p\right] - \boldsymbol{\nabla}\Delta p + \boldsymbol{\mathcal{D}_1}(\boldsymbol{\Delta q}) \tag{12}$$

To linearize this equation we replace $\boldsymbol{\mathcal{D}_1}(\boldsymbol{\Delta q}) - \Delta\boldsymbol{q}\boldsymbol{\nabla}\frac{\Delta\boldsymbol{q}}{\rho}$ by $-\alpha\frac{\Delta\boldsymbol{q}}{\tau}$. This term is supposed to take into account the energy losses through viscous dissipations and through the energy cascade. We obtain:

$$\rho\frac{d}{dt}\left(\frac{\Delta\boldsymbol{q}}{\rho}\right) + \Delta\boldsymbol{q}.\boldsymbol{\nabla}\boldsymbol{u} = \frac{\Delta\rho}{\rho}\left[\boldsymbol{\nabla}p_0+\boldsymbol{\nabla}\boldsymbol{R}_p\right] - \boldsymbol{\nabla}\Delta p - \alpha\frac{\Delta\boldsymbol{q}}{\tau} \tag{13}$$

2.3. The equation of thermal energy conservation

We start with

$$\frac{\partial \rho U}{\partial t} + \boldsymbol{\nabla}.(\rho H \boldsymbol{v}) - \boldsymbol{v}.\boldsymbol{\nabla} p = \rho\epsilon - \boldsymbol{\nabla}.\boldsymbol{F}_R + \mathcal{D}_2 \quad (14)$$

where U and H are respectively the energy and the enthalpy per unit mass while $\mathcal{D}_2 = \sum_{i,j} \mathcal{B}^i_j \boldsymbol{\nabla}_i \boldsymbol{v}^j$ takes the viscous dissipations into account.

Taking Eq. (1) into account and introducing the convective flux $\boldsymbol{F}_C = H\Delta\boldsymbol{q}$ and the dissipation rate of gravitational energy into convective energy $\rho\epsilon_3 = \frac{\Delta \boldsymbol{q}.\boldsymbol{\nabla} p}{\rho}$, we get for the projection over a low l:

$$\begin{aligned}\frac{\partial \rho_0 U_0}{\partial t} &+ \boldsymbol{\nabla}.(\rho_0 U_0 \boldsymbol{u}) - p_0 \boldsymbol{\nabla}.\boldsymbol{u} = \\ &(\rho\epsilon + \rho\epsilon_2 + \rho\epsilon_3 - \boldsymbol{\nabla}.[\boldsymbol{F}_R + \boldsymbol{F}_C])_p \quad (15)\end{aligned}$$

ϵ_2 is the rate of dissipation of kinetic energy of turbulence into heat (it comes from $\mathcal{D}_2$, see Ledoux & Walraven (1958)). To be consistent with the approximation made in Eq. (13), we must write:

$$\rho\epsilon_2 = \sum_{i,j} \mathcal{B}^i_j \boldsymbol{\nabla}_i \boldsymbol{v}^j = \alpha \frac{(\Delta \boldsymbol{q})^2}{\rho\tau}$$

Again to have a static model each of the terms in the right side member of this equation must be time independent. This is not the case and to define a static model, only their time independent part must be taken into account. Their fluctuating parts act as source terms for the oscillations.

For oscillations this equation leads to:

$$\begin{aligned}&\frac{\partial p'}{\partial t} - c_0^2 \frac{\partial \rho'}{\partial t} + \boldsymbol{u}.[\boldsymbol{\nabla} p_0 - c_0^2 \boldsymbol{\nabla} \rho_0] = \\ &(\Gamma_3 - 1)\{\rho\epsilon + \rho\epsilon_2 + \rho\epsilon_3 - \boldsymbol{\nabla}.[\boldsymbol{F}_R + \boldsymbol{F}_C]\}'_p + \\ &(\Gamma_3 - 1)\{\rho\epsilon + \rho\epsilon_2 + \rho\epsilon_3 - \boldsymbol{\nabla}.[\boldsymbol{F}_R + \boldsymbol{F}_C]\}_{p,fl} \quad (16)\end{aligned}$$

Again, the last term with the subscript "fl" is a source term for the oscillations. The contribution of ϵ_2 and ϵ_3 to the stability are often neglected without justification and should not be (Gabriel 1998b).

The projection over high degrees gives:

$$\begin{aligned}&\frac{\partial \Delta\rho U_0 + \rho_0 \Delta U}{\partial t} + \boldsymbol{\nabla}.[(H\Delta\boldsymbol{q})_c + (\Delta\rho H_0 + \rho_0 \Delta H)\boldsymbol{u}] \\ &- \left(\frac{\Delta \boldsymbol{q}}{\rho}.\boldsymbol{\nabla} p\right)_c - \boldsymbol{u}.\boldsymbol{\nabla}\Delta p = (\rho\epsilon - \boldsymbol{\nabla}.\boldsymbol{F}_R + \mathcal{D}_2)_c \quad (17)\end{aligned}$$

Assuming that $\boldsymbol{\nabla}.\Delta\boldsymbol{q} = 0$, that $\Delta p = 0$ except when its gradient is encountered and also taking a simple expression for the right side member (with $\epsilon = 0$), this equation gives:

$$\frac{d\Delta S}{dt} + \frac{\Delta(\rho T)}{\rho_0 T_0}\frac{dS_0}{dt} + \frac{T\Delta\boldsymbol{q}}{\rho_0 T_0}.\boldsymbol{\nabla} S = -\omega_R \Delta S + \frac{\mathcal{D}_2}{\rho_0 T_0} \quad (18)$$

Where S is the entropy, $\rho_0 T_0 \omega_R \Delta S$ is the usual approximation for the radiative dissipations $(\boldsymbol{\nabla}.\boldsymbol{F}_R)_c$. Introducing, as previously, the non-linear terms coming from the last term of the left side member and the term in $\mathcal{D}_{2,c}$ in $\Delta S/\tau$ to linearize the equation, we get:

$$\frac{d\Delta S}{dt} + \frac{\Delta(\rho T)}{\rho_0 T_0}\frac{dS_0}{dt} + \frac{\Delta\boldsymbol{q}}{\rho_0}.\boldsymbol{\nabla} S_0 = -\left(\frac{\omega_R \tau + 1}{\tau}\right)\Delta S \quad (19)$$

For ω_R, we use an expression a little more general than has been used previously. Now we take into account, in the same approximation as Henyey et al. (1965), that the convective globules are not necessarily optically thick. This leads to (l is the mixing length):

$$\omega_R = \frac{8\pi^2 ac}{3}\frac{T^3}{C_p \kappa \rho^2 l^2}\theta_1$$

with

$$\theta_1 = \frac{(\kappa\rho l)^2}{4\pi^2/3 + (\kappa\rho l)^2}$$

This section shows that, given the way we have chosen to split the variables (see Eq. (2)), the terms responsible for scattering are transfered in the equation for "convection" (i.e. in the projection over high degrees) because they have projections on these high degrees only. The terms mainly responsible for scattering are $\Delta\rho\boldsymbol{u}$ in Eq. (6) and $\Delta\boldsymbol{q}.\boldsymbol{\nabla}\boldsymbol{u} + \boldsymbol{\nabla}.\boldsymbol{u}\Delta\boldsymbol{q}$ in Eq. (11). None of them has been neglected and scattering is included in the equations which describe the linear interactions between convection and pulsation. Moreover scattering and the linear interactions between convection and pulsation cannot be separated, they both come both from the solution of the system obtained after taking the perturbation of Eqs. (6), (11) and (17) (or of Eqs. (7), (13) and (19)). The difficulty is to solve that problem with enough accuracy and we cannot claim that this has been done. Even the solution for unperturbed convection cannot in practice be obtained analytically and is known only through numerical simulations. As a result, stellar evolution codes use the mixing length theory which is the local, time independent solution of Eqs. (7), (13) and (19) (see appendix A). Coherence of the stability analysis requires that we start from the same equations to find both the static and the perturbed solutions for convection. Therefore, we have to use the same set of linearized equations obtained above for convection in stability analysis of models computed with the mixing length theory. This is what has always been done (see Gabriel et al. (1974, 1975), Gough (1977), Gabriel (1996)).

We will now discuss the consequences of this approximation. For this we will use a slightly modified form of Eqs. (6), (11) and (17). They are these for the usual Boussinesq's approximation and a perfect gas. We have:

$$\begin{aligned}\boldsymbol{\nabla}.\boldsymbol{V} &= 0 \\ \frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{V}.\boldsymbol{\nabla}\boldsymbol{V} + \boldsymbol{u}.\boldsymbol{\nabla}\boldsymbol{V} + \boldsymbol{V}.\boldsymbol{\nabla}\boldsymbol{u} &= \frac{\Delta\rho}{\rho}\frac{\boldsymbol{\nabla} p}{\rho} - \frac{\boldsymbol{\nabla}\Delta p}{\rho} \\ &+ \nu\nabla^2\boldsymbol{V} \\ \frac{\partial\theta}{\partial t} + (\boldsymbol{V} + \boldsymbol{u}).\boldsymbol{\nabla}\theta &= \chi\nabla^2\theta - \boldsymbol{\beta}.\boldsymbol{V} \quad (20) \\ \frac{\Delta\rho}{\rho} &= Q\frac{\Delta T}{T}\end{aligned}$$

where $\theta = \Delta T/T$, ν and χ are respectively the kinematic viscosity and the thermal diffusivity and βT is the super-adiabatic gradient.

3. The solution for unperturbed convection

If $\boldsymbol{y} = (\boldsymbol{V}, \theta)$ and $\boldsymbol{Y}$ is its Fourier transform and assuming that the coefficients of these equations are constant, after going to the Fourier space and eliminating Δp (and also keeping the same notation for the Fourier transform of $\boldsymbol{V}$ and θ) Eq. (20) become:

$$M.\boldsymbol{Y}(\omega, \boldsymbol{k}) + \imath \boldsymbol{I}_0(\omega, \boldsymbol{k}) = 0 \qquad (21)$$

with:

$$M = \begin{pmatrix} \imath\omega + \nu k^2 & 0 & 0 & -\left[\frac{Q\boldsymbol{\nabla}p}{\rho}.K\right]_r \\ 0 & \imath\omega + \nu k^2 & 0 & -\left[\frac{Q\boldsymbol{\nabla}p}{\rho}.K\right]_x \\ 0 & 0 & \imath\omega + \nu k^2 & -\left[\frac{Q\boldsymbol{\nabla}p}{\rho}.K\right]_y \\ \beta_r & 0 & 0 & \imath\omega + \chi k^2 \end{pmatrix}$$

$$\boldsymbol{I}_0 = \begin{pmatrix} \int \boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).\boldsymbol{k}\boldsymbol{V}(\boldsymbol{k}', \omega').K d\nu' d\underline{\boldsymbol{k}}' \\ \int \boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).\boldsymbol{k}\theta(\boldsymbol{k}', \omega') d\nu' d\underline{\boldsymbol{k}}' \end{pmatrix}$$

where $\underline{k}_i = k_i/(2\pi)$ and if E is the unit matrix:

$$K = E - \frac{\boldsymbol{k}\boldsymbol{k}}{k^2}$$

Since in M the coefficients have been assumed independent of the position in the star, the solution of Eq. (21) will be a local one. Therefore $Y_i(\boldsymbol{k}, \omega)$ is also a function of r.

The explicit solution of this integral system is not required for our work. Let us just note that it has the following properties. Since convection is a stationary process

$$E[Y_i^*(\boldsymbol{k}, \omega) Y_j(\boldsymbol{k}', \omega')] = \delta(\nu - \nu')\delta(\underline{\boldsymbol{k}} - \underline{\boldsymbol{k}}') \int f_{i,j}(\Delta\boldsymbol{r}\Delta t) \exp[\imath(\boldsymbol{k}.\Delta\boldsymbol{r} + \omega\Delta t)] d\Delta\boldsymbol{r} d\Delta t \qquad (22)$$

with $f_{i,j}(\Delta\boldsymbol{r}\Delta t) = E[y_i(\boldsymbol{r}, t) y_j(\boldsymbol{r}', t')]$, $\Delta\boldsymbol{r} = \boldsymbol{r} - \boldsymbol{r}'$ and $\Delta t = t - t'$. In relation to the local approximation, we must point out that Y_i and Y_j must be computed for the same position in the model. If it was not so, $f_{i,j}$ would be zero because we assume that correlations between convective variables are different from zero only over distances that are small compared to the model scale heights. It follows from this relation that $E[\mathcal{F}\{y_i(\boldsymbol{r}, t) y_j(\boldsymbol{r}, t)\}(\omega, \boldsymbol{k})]$ is equal to zero except when $\omega = 0$ and $\boldsymbol{k} = 0$.

4. The solution for the perturbed convection

If we introduce $\boldsymbol{Y}' = (\boldsymbol{V}', \theta')$, the perturbation of Eq. (20) gives in the Fourier space and after eliminating $(\Delta p)'$

$$M.\boldsymbol{Y}'(\omega, \boldsymbol{k}) + \imath \boldsymbol{I}(\omega, \boldsymbol{k}) = \boldsymbol{P}(\omega, \boldsymbol{k}) \qquad (23)$$

with:

$$\boldsymbol{I} = \begin{pmatrix} \int \boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).\mathcal{K}.\boldsymbol{V}'(\boldsymbol{k}', \omega') d\nu' d\underline{\boldsymbol{k}}' \\ \int [\boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).\boldsymbol{k}\theta'(\boldsymbol{k}', \omega') + \\ \theta^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)\boldsymbol{k}.\boldsymbol{V}'(\boldsymbol{k}', \omega')] d\nu' d\underline{\boldsymbol{k}}' \end{pmatrix}$$

$$\boldsymbol{P} = \begin{pmatrix} K.\int [\theta^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)\left(\frac{Q\boldsymbol{\nabla}p}{\rho}\right)'(\boldsymbol{k}', \omega') \\ +\imath(\boldsymbol{k} - \boldsymbol{k}')\frac{\Delta p^*}{\rho}(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)\frac{\rho'}{\rho}(\boldsymbol{k}', \omega') \\ -(\boldsymbol{k} - \boldsymbol{k}')^2 \boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)\nu'(\boldsymbol{k}', \omega')] \\ -\imath \boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)(\boldsymbol{k} - \boldsymbol{k}').\boldsymbol{u}(\boldsymbol{k}', \omega') \\ -\boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).(\boldsymbol{\nabla}\boldsymbol{u})(\boldsymbol{k}', \omega')] d\nu' d\underline{\boldsymbol{k}}' \\ - \int [(\boldsymbol{k} - \boldsymbol{k}')^2 \theta^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)\chi'(\boldsymbol{k}', \omega') \\ +\boldsymbol{V}^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega).\boldsymbol{\beta}'(\boldsymbol{k}', \omega') \\ +\imath\theta^*(\boldsymbol{k}' - \boldsymbol{k}, \omega' - \omega)(\boldsymbol{k} - \boldsymbol{k}').\boldsymbol{u}(\boldsymbol{k}', \omega')] d\nu' d\underline{\boldsymbol{k}}' \end{pmatrix}$$

with $\mathcal{K} = K\boldsymbol{k} + \boldsymbol{k}K$

Again, Eq. (23) is written in the local approximation which means that the coefficients of M and their perturbation (which appear in $\boldsymbol{P}$) are assumed independent of r. For the perturbations the local approximation can be expressed in two ways; it may be assumed that:

1. the perturbations of the model are independent of the angular variables (fully local approximation)
2. they are functions of the angular variables (r-local approximation)

This gives two slightly different set of equations. Also the solution for convection is as usual given in terms of trigonometric functions ($\exp(\imath\boldsymbol{k}.\boldsymbol{r})$) while the angular dependence of the model perturbations are given by spherical harmonics. Here we will use the r-local approximation and to avoid the technical difficulty of expressing one kind of function in terms of the other one, we will work in plane geometry. Therefore $\boldsymbol{k}'$ in the equations above is a horizontal wave number to be linked to the two parameters l and m of the spherical harmonics and it takes only discrete values.

If we are interested by the terms with a non-zero statistical mean (we will show that the other ones are small), the fully local approximation, which is good for low degree modes, allows us to avoid the difficulty linked to the spherical geometry and therefore will be very useful in practice.

To discuss the solution of Eq. (23), we split $\boldsymbol{Y}'$ in two parts $\boldsymbol{Y}' = \boldsymbol{Y}'_1 + \boldsymbol{Y}'_2$ where $\boldsymbol{Y}'_1$ is the solution of

$$M.\boldsymbol{Y}' = \boldsymbol{P}$$

and $\boldsymbol{Y}'_2$ is the solution of

$$M.\boldsymbol{Y}'_2 + \imath\boldsymbol{I}(\boldsymbol{Y}'_1 + \boldsymbol{Y}'_2) = 0$$

The homogeneous system $M.\boldsymbol{Y}'_2 + \imath\boldsymbol{I}(\boldsymbol{Y}'_2) = 0$ has no solution different from zero because $\boldsymbol{I} \neq \boldsymbol{I}_0$. Therefore, since we do not need the explicit form of the solution (which would be very difficult to obtain), we can write for that solution

$$\boldsymbol{Y}'_2(\omega, \boldsymbol{k}) = \int N(\omega, \boldsymbol{k}, \omega', \boldsymbol{k}').\boldsymbol{P}(\omega', \boldsymbol{k}') d\nu' d\underline{\boldsymbol{k}}'$$

where the matrix N is a complex function of the unperturbed solution for convection $\boldsymbol{Y}$.

The solution obtained using the linearized equations for convection corresponds essentially to $\boldsymbol{Y}'_1$ with the following modifications:

1. The term in ν is neglected (or included in the term obtained through the linearization, see point 3) while that in $\chi\nabla\theta$ is replaced by that in $\omega_R\theta$.
2. The convection spectrum is ignored.
3. To linearize the equations, the two nonlinear terms have been replaced by these in τ^{-1}. For this linearized system $\boldsymbol{I} \equiv 0$ and $\boldsymbol{Y}_2 \equiv 0$. It is hoped that this allows us to take the solution $\boldsymbol{Y}'_2$ into account through the modification of $\boldsymbol{Y}'_1$. This seems very unlikely but the constraint is somewhat relaxed as it is sufficient that the perturbation of the terms $y_\alpha y_\beta$ discussed just below is good.

The equations for global oscillations contain convective terms of the form $y_\alpha y_\beta$ (for instance $\boldsymbol{R}_p$ and $\boldsymbol{F}_C$) and their perturbations, terms of the form $y_\alpha y'_\beta + y'_\alpha y_\beta$. Their Fourier transform is given by:

$$\mathcal{F}(y_\alpha y'_\beta)(\omega, \boldsymbol{k}_P) = \int Y^*_\alpha(\boldsymbol{k}' - \boldsymbol{k}_P, \omega' - \omega) Y'_\beta(\boldsymbol{k}', \omega') d\nu' d\underline{\boldsymbol{k}}' \tag{24}$$

If σ_j and $\boldsymbol{u}_{a,j}$ are respectively one eigenvalue and one eigenfunction of the adiabatic problem, the solution can be written in the Fourier space:

$$\boldsymbol{u}(\omega) = \sum_i Q_i(\omega) \boldsymbol{u}_{a,i}$$

where the subscript "i" stands for the three numbers (n,l,m) or for (n, $\boldsymbol{k}_P$).

We are interested by the statistical average of Eq. (24). To obtain it, we replace Y'_β by its expression given in Eq. (23). Since $\boldsymbol{I}$ is a function of $\boldsymbol{Y}'$, we enter into an infinite iteration process which introduces terms with the following dependence in the Y_i and Q_i

$$\prod_{j=1}^{J} Y^*_{\alpha_j}(\omega^{(j)} - \omega^{(j-1)}, \boldsymbol{k}^{(j)} - \boldsymbol{k}^{(j-1)}) Q(\omega^{(J)}, \boldsymbol{k}^{(J)})$$

where J varies from 1 to ∞ and $\omega^0 = \omega$, $\boldsymbol{k}^0 = \boldsymbol{k}_P$. We have to compute the statistical average of the coefficient of Q. Going back to the ordinary $(\boldsymbol{r}, t)$ space, we make the natural hypothesis that $E[\prod_{j=1}^{J} y_{\alpha_j}(\boldsymbol{r}^{(j-1)}, t^{(j-1)})]$ is independent of the choice of the origin in space and time. It is then easily verified that $E[\prod_{j=1}^{J} Y^*_{\alpha_j}(\omega^{(j)} - \omega^{(j-1)}, \boldsymbol{k}^{(j)} - \boldsymbol{k}^{(j-1)})]$ is different from zero only when $\omega^{(J)} = \omega$ and $\boldsymbol{k}^{(J)} = \boldsymbol{k}_P$. Therefore, we have

$$\begin{aligned} \mathcal{F}(y_\alpha y_\beta)'(\omega, \boldsymbol{k}_P) \;&=\; \sum_j f_1(\omega, \boldsymbol{k}_P, \boldsymbol{u}_{a,j}) Q_j(\omega) \qquad (25) \\ &+\; \sum_i \int f_2(\omega', \omega, \boldsymbol{k}_P, \boldsymbol{u}_{a,i}) Q_i(\omega') d\nu' \end{aligned}$$

where f_1 is a non-fluctuating function (i.e. it is a statistical average) while f_2 is a zero mean fluctuating function. It is useful to note that f_2 is different from zero even if $\boldsymbol{Y}_2$ is assumed to be equal zero (this is done when the linearized equations for convection are used). This term is directly related to the turbulent spectrum. The subscript "j" means that the corresponding modes have $\boldsymbol{k}_j = \boldsymbol{k}_P$ while modes with the subscript "i" have any value of $\boldsymbol{k}$. We must stress that since we use the r-local approximation, only the radial dependence of the $\boldsymbol{u}$ appears in this equation.

From Eq. (23), we see that $\boldsymbol{Y}'$ is proportional to M^{-1}. Therefore $\mathcal{F}(y_\alpha y_\beta)'(\omega, \boldsymbol{k}_P)$ is very sensitive to the properties of convection at the resonant frequency of M^{-1} that is to say at the frequency for which the real part of its determinant cancels. It is given by $\omega^2 = \alpha\tau^{-2} + \nu\chi k^2$ (Eq. (A1) has been used) and the width of the resonance is $(\nu + \chi)k^4$. This resonance is also found in the simplified theory but its width is much broader as νk^2 and χk^2 are then replaced respectively by $\alpha\tau^{-1}$ and $(\tau^{-1} + \omega_R)$ which are much larger for the wave numbers containing most of the convection energy.

For readers who are not familiar with the subject, we discuss in appendix B the simple example of a harmonic oscillator with a damping coefficient which is a stochastic function of time. This kind of damping coefficient is to be related to the perturbation of the terms in $y_\alpha y_\beta$ just considered above. It may serve as an introduction to the following sections.

5. The perturbation for global oscillations

Taking the perturbation of Eq. (4), Eq. (10) and (16), we can obtain the wave equation:

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \boldsymbol{\mathcal{L}}_0(\boldsymbol{u}, \boldsymbol{r}) + \boldsymbol{\mathcal{L}}_1(\boldsymbol{u}, \boldsymbol{r}) + \boldsymbol{\mathcal{F}}(\boldsymbol{r}, t) \tag{26}$$

where $\boldsymbol{\mathcal{L}}_0$ is the usual adiabatic operator, $\boldsymbol{\mathcal{L}}_1$ takes the non-adiabatic effects into account and $\boldsymbol{\mathcal{F}}$ is the non-linear driving force. They are given by:

$$\boldsymbol{\mathcal{L}}_1 = -\frac{1}{\rho}\left[\frac{\partial \boldsymbol{M}_1}{\partial t} + \boldsymbol{\nabla} E_1\right] \tag{27}$$

with:

$$\boldsymbol{M}_1 \;=\; \boldsymbol{\nabla}.\boldsymbol{R}'_p \tag{28}$$

$$E_1 \;=\; (\Gamma_3 - 1)(\rho\epsilon + \rho\epsilon_2 + \rho\epsilon_3 - \boldsymbol{\nabla}.[\boldsymbol{F}_R + \boldsymbol{F}_C])'_p \tag{29}$$

$$\boldsymbol{\mathcal{F}} = -\frac{1}{\rho}\left[\frac{\partial \boldsymbol{M}_2}{\partial t} + \boldsymbol{\nabla} E_2\right] \tag{30}$$

with:

$$\boldsymbol{M}_2 \;=\; \boldsymbol{\nabla}.\boldsymbol{R}_{p,fl} \tag{31}$$

$$E_2 \;=\; (\Gamma_3 - 1)(\rho\epsilon + \rho\epsilon_2 + \rho\epsilon_3 - \boldsymbol{\nabla}.[\boldsymbol{F}_R + \boldsymbol{F}_C])_{p,fl} \tag{32}$$

We note that Eq. (26) is the projection over low spherical harmonics.

Making use of the decomposition made at the end of the previous section (see Eq. (25)), we split $\boldsymbol{\mathcal{L}}_1$ in two parts. $\boldsymbol{\mathcal{L}}_1$ is now an operator in which the time dependence comes from the global pulsation only (for convective terms it is associated to f_1) while the coefficients of $\boldsymbol{u}$ in $\boldsymbol{\mathcal{L}}_2$ are time dependent. $\boldsymbol{\mathcal{L}}_2$ includes convective terms only; these are associated with f_2. Eq. (26) is now written:

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \boldsymbol{\mathcal{L}}_0(\boldsymbol{u},\boldsymbol{r}) + \boldsymbol{\mathcal{L}}_1(\boldsymbol{u},\boldsymbol{r}) + \boldsymbol{\mathcal{L}}_2(\boldsymbol{u},\boldsymbol{r},t) + \boldsymbol{\mathcal{F}}(\boldsymbol{r},t) \quad (33)$$

We need to know the solution of this equation in lines only, that is to say for frequencies close to eigenvalues. We get that solution by the perturbation method to the first order, since $\boldsymbol{\mathcal{L}}_1$ and $\boldsymbol{\mathcal{L}}_2$ are small. We do it in the Fourier space and $\boldsymbol{L}_i$ and $\boldsymbol{F}$ are respectively the *time* Fourier transform of $\boldsymbol{\mathcal{L}}_i$ and of $\boldsymbol{\mathcal{F}}$.

From Eq. (25), we see that $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ can be written as:

$$\boldsymbol{L}_1 = \sum_j \boldsymbol{L}_1(\omega, \boldsymbol{k}_j, \boldsymbol{u}_j) Q_j(\omega') e^{\imath \boldsymbol{k}_j . \boldsymbol{r}}$$

$$\boldsymbol{L}_2 = \sum_{i,\boldsymbol{k}_P} \int \boldsymbol{L}_2(\omega, \omega', \boldsymbol{k}_P, \boldsymbol{u}_i) Q_i(\omega') d\nu' e^{\imath \boldsymbol{k}_P . \boldsymbol{r}}$$

5.1. Solution for the homogeneous problem

First, we compute the solution for the homogeneous problem assuming that the oscillation starts at $t = 0$ and is initiated by an impulse force $\boldsymbol{\mathcal{F}}(\boldsymbol{r},t) = \sum_i C_i \boldsymbol{u}_i \delta(t)$. It is given by:

$$Q_j(\omega) \;=\; \frac{C_j + \sum_{i \neq j} T_{1,i} + \sum_i T_{2,i}}{\sigma_j^2 - \left\langle \boldsymbol{L}_1(\boldsymbol{u}_{a,j}), \boldsymbol{u}_{a,j}^* \right\rangle - \omega^2} \quad (34)$$

with

$$T_{1,i} = \left\langle \boldsymbol{L}_1(\boldsymbol{u}_{a,i}), \boldsymbol{u}_{a,j}^* \right\rangle Q_i(\omega)$$

$$T_{2,i} = \int \left\langle \boldsymbol{L}_2(\omega, \omega', \boldsymbol{u}_{a,i}), \boldsymbol{u}_{a,j}^* \right\rangle Q_i(\omega') d\nu'$$

$$\left\langle \boldsymbol{L}_i(\boldsymbol{u}_{a,j}), \boldsymbol{u}_{a,k}^* \right\rangle = \frac{\int \boldsymbol{L}_i(\boldsymbol{u}_{a,j}).\boldsymbol{u}_{a,k}^* dm}{\int |\boldsymbol{u}_{a,k}|^2 dm}$$

It may be useful to recall that $\left\langle \boldsymbol{L}_i(\boldsymbol{u}_{a,j}), \boldsymbol{u}_{a,k}^* \right\rangle$ is independent of the amplitude of the perturbations and is only a function of the relative amplitude of $\boldsymbol{u}_{a,j}$ and $\boldsymbol{u}_{a,k}$.

The solution of this system must be found by iteration but, since $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ are small, just one iteration done with the Q_i in the right side member given by a Lorentzian profile will give a good approximation.

The eigenvalue of mode "j' is now $\sigma_j^2 - \left\langle \boldsymbol{L}_1(\boldsymbol{u}_{a,j}), \boldsymbol{u}_{a,j}^* \right\rangle$, it is complex indeed but its value is modified only by the dissipative effects of that mode. The presence of other excited modes does not modify the eigenvalue in opposition to the prediction of Goldreich & Murray (1994). Indeed scattering contributes to the perturbation of the eigenvalue as do other terms as their influence is not different from these of the other terms entering in $\boldsymbol{P}$. The only difference is that the frequency shift and the line width are now slowly varying functions of frequency and indeed their numerical value will differ from these found when neglecting the convection spectrum.

The terms in $\boldsymbol{L}_1$ are not fundamentally different from these found when the convection spectrum is ignored though they are more complex. The main difference is that now they are slowly varying functions of ω. The order of magnitude of each of them in the right member is smaller than about $|C_i 2\sigma_i \sigma_i' / (\sigma_j^2 - \sigma_i^2)|$. They will modify the line profile.

The terms in $\boldsymbol{L_2}$ appear because if we consider the stability of one mode of degree l_j and frequency ω_j and that many modes are excited it is always possible to find for each l_i and ω_i components of the convective spectrum which couple with that (l_i, ω_i) mode to give an oscillation of degree l_j and frequency ω_j. However, $\boldsymbol{L}_2$ has no influence on the eigenvalues it can only have an influence on the line profile.

5.2. Solution for the non-homogeneous problem

To get the solution for the non-homogeneous problem for $\omega \simeq \sigma_j$, we just have to replace C_j in Eq. (34) by $\langle \boldsymbol{F}.\boldsymbol{u}_{a,j} \rangle$.

As for the homogeneous system, a good approximation is obtained if we replace the Q_i in the right side member of Eq. (34) by:

$$Q_i(\omega) \;=\; \frac{\left\langle \boldsymbol{F}, \boldsymbol{u}_{a,i}^* \right\rangle}{\sigma_i^2 - \left\langle \boldsymbol{L}_1(\boldsymbol{u}_{a,i}).\boldsymbol{u}_{a,i}^* \right\rangle - \omega^2} \quad (35)$$

We are mainly interested by the power spectrum which is proportional to $|Q_i|^2$. First, it is easily verified that the statistical average of $\sum_i T_{2,i}$ is equal to zero when Q_i is given by Eq. (35). Therefore the p.d.f. of each point of the Fourier spectrum remains a χ_2^2 in agreement with the studies by Chaplin et al. (1997) and Chang & Gough (1998).

Let us now consider the statistical average $E[|Q_i|^2]$. The first term $E[\langle \boldsymbol{F}, \boldsymbol{u}_{a,j} \rangle]^2$ is the one discussed by Goldreich & Keeley (1977) (see also Kumar (1994)). It gives the largest contribution to Q_j

The terms in $E[T_{1,i} \langle \boldsymbol{F}, \boldsymbol{u}_{a,j} \rangle]$ with Q_i given by Eq. (35) contribute to the departures to Lorentz profiles and to line asymmetry. They have been discussed in Gabriel (1992, 1993, 1995). Since we have used the perturbation method, we can tell nothing concerning the comparison of asymmetries in velocity and intensity lines (see Gabriel (1998a). However there is little doubt that when the full non-adiabatic problem is solved including the influence of these new terms, velocity and intensity lines will have different profiles (though not necessarily with opposite asymmetry).

The main contribution of the new terms is given by $T = \sum_i T_i = E[\langle \boldsymbol{F}, \boldsymbol{u}_{a,j} \rangle \sum_i T_{2,i}]$ it introduces a new potential source of asymmetries. Unfortunately the T_i are very difficult to evaluate because this requires the computation of high order correlations between different convective variables and also a good solution for the interactions between convection and pulsations. Only one general remark can be made. It is required that the two eigenfunctions which appear in $\left\langle \boldsymbol{L}_1(\boldsymbol{u}_{a,j}).\boldsymbol{u}_{a,k}^* \right\rangle$ are such that those terms are not too small. This requires that the two eigenfunctions have not too different degrees and not too different radial variations in the outer convective envelope.

However a crude estimate of the ratio of T_i to the main term $E[\langle \boldsymbol{F}, \boldsymbol{u}_{a,j}\rangle]^2$ is given in appendix C. We find that its order of magnitude is given by the square of the ratio of the oscillation velocity amplitude to the convective velocity in the source region. It is therefore very small and these new terms have a very small contribution to the line profile.

6. Conclusions

We have shown that the equations which describe the linear interactions between convection and pulsations include all the terms responsible for scattering. However, these equations are usually solved using an oversimplified time independent picture for convection. When the time dependence and a spectrum are introduced in the solution for convection, we find that the perturbation of the Reynolds tensor and of the convective flux (and others often neglected) gives a contribution with a non zero statistical mean and one with a zero statistical mean. The first one influences the stability and the line profile in a way very similar to that found with the usual simplified theory though the numerical values will be different indeed. In opposition to Goldreich & Murray (1994), we find that the line width is not influenced by mode coupling. The second one has none analogous in the simplified theory and it couples all the global modes. These new terms modify neither the peak frequency nor the line width but influence the line asymmetry. They are very difficult to compute but a rough estimate suggests that they are very small. Nevertheless, it will be a difficult task to obtain a solution which takes the convection spectrum into account and allows a reliable evaluation of statistical average of the perturbation of the Reynolds tensor and of the convective flux.

Appendix A: equivalence of the linearized equations of convection with the mixing length theory

The time independent solution of Eqs. (7), (13) and (19) can be easily obtained in the local approximation. It is of the form:

$$y = y_a \exp(\imath \boldsymbol{k}.\boldsymbol{r})$$

where the wave vector $\boldsymbol{k}$ is arbitrary.

Introducing this expression in the equations, we get for the amplitude y_a (we drop the subscript "a")

$$\Delta q_r = \rho_0 V_r = \frac{1}{\alpha}\frac{k_H^2}{k^2}\nabla_r p_0 \frac{\Delta\rho}{\rho}\tau \tag{A.1}$$

$$\Delta q_i = \rho_0 V_i = -\frac{1}{\alpha}\frac{k_i k_r}{k^2}\nabla_r p_0 \frac{\Delta\rho}{\rho}\tau \qquad (i \neq r) \tag{A.2}$$

$$\Delta S = -\frac{\tau}{\omega_R \tau + 1}\nabla_r S_0 \frac{\Delta q_r}{\rho_0} \tag{A.3}$$

The value for ΔS is obtained from the condition that the total flux is equal to the sum of the radiative and convective fluxes.

If we eliminate Δq_r between (A.1) and (A.3), taking into account that:

$$\Delta S = C_p \frac{\Delta T}{T} = C_p Q^{-1}\frac{\Delta\rho}{\rho}$$

$$\nabla S_0 = C_p(\overline{\nabla} - \nabla_a)\nabla_r \ln p_0$$

we get with $\gamma = (\omega_R \tau)^{-1}$:

$$\begin{aligned}\gamma(\gamma+1) &= A(\overline{\nabla} - \nabla_a)\\ A &= -\frac{1}{\alpha}\frac{k_H^2}{k^2}\frac{Q}{\omega_r^2}\frac{p_0}{\rho_0}H_p^{-2}\end{aligned} \tag{A.4}$$

On the other hand, we have:

$$\begin{aligned}F &= -\frac{4ac}{3}\frac{T^4}{\kappa\rho}\nabla_R \nabla_r ln p_0\\ F_R &= -\frac{4ac}{3}\frac{T^4}{\kappa\rho f}\nabla\nabla_r ln p_0\\ F_C &= \frac{1}{2}T\Delta S \Delta q_r\end{aligned}$$

with (τ_{opt} is the optical depth)

$$f = \frac{3}{4T_{eff}^4}\frac{dT^4}{d\tau_{opt}}$$

Taking Eqs. (A.1), (A.4) and the definition of ω_R into account, $F = F_R + F_C$ gives:

$$\pi^2\theta_1 f\gamma^3 + \gamma(\gamma+1) - A(\nabla_R - \nabla_a) = 0 \tag{A.5}$$

This equation is identical to the one obtained by Henyey et al. (1965) if we take $\alpha^{-1}k_H^2/k^2 = 3/16$.

Appendix B: a simple example

To illustrate the problem discussed in the paper, we consider here the much simpler problem of the harmonic oscillator with one stochastic coefficient. We have to solve the equation

$$\ddot{q} + 2\gamma(t)\dot{q} + \omega_0^2 q = f(t) \tag{B.1}$$

$where\gamma(t)$ and $f(t)$ are stochastic functions. The average of $\gamma(t)$ is $\bar{\gamma}$ while that of $f(t)$ is zero. Also γ is assumed small compared to ω_0. Let $Q(\omega)$, $\Gamma(\omega)$ and $F(\omega)$ be the Fourier transforms of $q(t)$, $\gamma(t) - \bar{\gamma}$ and $f(t)$; in the Fourier space Eq. (B.1) gives:

$$Q(\omega) = \frac{F(\omega) - 2\imath\int \omega'\Gamma^*(\omega'-\omega)Q(\omega')d\nu'}{\omega_0^2 - \omega^2 + 2\imath\bar{\gamma}\omega}$$

Since γ is small, we have to first order:

$$Q(\omega) = \frac{F(\omega) - 2\imath\int \frac{\omega'\Gamma^*(\omega'-\omega)F(\omega')}{\omega_0^2 - \omega'^2 + 2\imath\bar{\gamma}\omega'}d\nu'}{\omega_0^2 - \omega^2 + 2\imath\bar{\gamma}\omega}$$

To compute the statistical average of the second term on the numerator, we go back to the time space and we assume indeed that $E[(\gamma(t) - \bar{\gamma})f(t')] = g(t-t')$. This gives if $h(\omega,\omega') = \omega'/(\omega_0^2 - \omega'^2 + 2\imath\bar{\gamma}\omega')$

$$\delta(\omega)\int h(\omega,\omega')g(t-t')e^{\imath\omega'(t-t')}d(t-t')d\nu'$$

which is equal to zero.

Following the same method, we can compute the average of the product of the two terms on the numerator and we get:

$$\int h(\omega,\omega')g_1 e^{\imath\omega(t-t')}e^{\imath\omega'(t'-t'')}d(t-t')d(t'-t'')$$

again we have assumed that $E[f(t)(\gamma(t')-\bar{\gamma})f(t'')] = g_1$ is independent of the choice of the time origin; g_1 may be different from zero.

Finally the average of the square of the second term leads to a very similar formula (though with more integrals) containing the statistical average $E[(\gamma(t)-\bar{\gamma})(\gamma(t')-\bar{\gamma})f(t'')f(t''')]$ which is again assumed independent of the choice of the time origin. This average is different from zero.

Appendix C: estimate of the contribution of the $\mathcal{L}_2$ terms to the line profile

We assume that $\mathcal{L}_1$ reduces to the (r,r) component of the Reynolds tensor and that

$$\mathcal{L}_{1,r} \simeq -\frac{2}{\rho}\frac{\partial^2}{\partial t\partial r}(\rho V_r V_r')$$

Then

$$L_{1,r}(\omega) = -\imath\frac{2\omega}{\rho} \sum_{\boldsymbol{k}_P}\frac{\partial}{\partial r}\int \rho V_r^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k}_P)V_r'(\omega',\boldsymbol{k}')d\nu' d\underline{\boldsymbol{k}}' e^{\imath \boldsymbol{k}_P.\boldsymbol{r}} \quad \text{(C.1)}$$

For V_r' we take $V_r' = (M^{-1}.\boldsymbol{P})_r$ and for $\boldsymbol{P}$ we take only one of the scattering terms

$$\boldsymbol{P}(\omega,\boldsymbol{k}) = -\imath\sum_{\boldsymbol{k}'}\int \boldsymbol{V}^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k}).K(\boldsymbol{k}-\boldsymbol{k}').\boldsymbol{u}(\omega',\boldsymbol{k}')d\nu' \quad \text{(C.2)}$$

which gives if we assume that the radial component of $\boldsymbol{u}$ is by far the largest

$$P_j(\omega,\boldsymbol{k}) = -\imath\sum_{\boldsymbol{k}'}\int V_j^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k})(k_r-k_r')u_r(\omega',\boldsymbol{k}')d\nu' \quad \text{(C.3)}$$

and

$$V_r'(\omega,\boldsymbol{k}) = g(\omega,\boldsymbol{k})P_r(\omega,\boldsymbol{k})$$

with

$$g(\omega,\boldsymbol{k}) = \frac{\imath\omega+\chi k^2}{\alpha\tau^{-2}+\nu\chi k^4-\omega^2+\imath\omega(\nu+\chi)k^2}$$

This gives

$$L_{1,r}(\omega) = \sum_{\boldsymbol{k}_P,i}\frac{2\omega}{\rho}\frac{\partial}{\partial r}\int\int g(\omega',\boldsymbol{k}')\frac{(k_r''-k_r')}{\sigma_{c,i}^2-\omega''^2}\rho\, V_r^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k}_P)V_r^*(\omega''-\omega',\boldsymbol{k}''-\boldsymbol{k}')u_{r,i}(r) \left\langle \boldsymbol{F}.\boldsymbol{u}_i^*(\boldsymbol{k}'')\right\rangle(\omega'')d\nu' d\nu'' d\underline{\boldsymbol{k}}' e^{\imath\boldsymbol{k}_P.\boldsymbol{r}} \quad \text{(C.4)}$$

The summation index "i' refers to modes with the horizontal wave number $\boldsymbol{k}''$, $\sigma_{c,i}$ is the eigenvalue corrected for non-adiabatic dissipations and

$$\left\langle \boldsymbol{F}.\boldsymbol{u}_i^*(\boldsymbol{k}_i)\right\rangle(\omega) \simeq \frac{\imath\omega}{N_i}\int\frac{du_{r,i}^*(r,\boldsymbol{k}_i)}{dr}V_r^2(\omega,\boldsymbol{k}_i)dm$$

$$N_i = \int |\boldsymbol{u}_i|^2 dm$$

$E[V_r^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k})V_r^*(\omega''-\omega',\boldsymbol{k}''-\boldsymbol{k}')] \neq 0$ only if $\omega''=\omega$ and $\boldsymbol{k}''=\boldsymbol{k}_P$. Therefore $L_{2,r}$ is obtained from the same formula with $\omega''\neq\omega$ *or* $\boldsymbol{k}''\neq\boldsymbol{k}_P$.

Then

$$\left\langle \boldsymbol{L}_2,\boldsymbol{u}_j^*\right\rangle(\omega) = \sum_i -\frac{2\omega}{N_j}\int\frac{du_{r,j}^*}{dr}u_{r,i}\rho(\omega'',\boldsymbol{k}'')g(\omega',\boldsymbol{k}') \frac{(k_r''-k_r')}{\sigma_{c,i}^2-\omega''^2}V_r^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k}_j)V_r^*(\omega''-\omega',\boldsymbol{k}''-\boldsymbol{k}') \left\langle \boldsymbol{F}.\boldsymbol{u}_i^*(\boldsymbol{k}")\right\rangle(\omega")d\nu' d\nu'' d\underline{\boldsymbol{k}}' d\mathcal{V} \quad \text{(C.5)}$$

The statistical average T of $[\left\langle \boldsymbol{F},\boldsymbol{u}_j^*\right\rangle^* \left\langle \boldsymbol{L}_2,\boldsymbol{u}_j^*\right\rangle]$ can now be computed.

$$T = \sum_i \frac{\omega^3}{N_j^2 N_i}\int\left(\frac{du_{r,j}}{dr}\right)^2\frac{du_{r,i}^2(r)}{dr}\rho^3 g(\omega',\boldsymbol{k}') \frac{(k_r''-k_r')}{\sigma_{c,i}^2-\omega''^2}Z d\nu' d\nu'' d\underline{\boldsymbol{k}}' d\mathcal{V} \quad \text{(C.6)}$$

with

$$Z = E[\int V_r^*(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k}_j)V_r^*(\omega''-\omega',\boldsymbol{k}''-\boldsymbol{k}') V_r^*(\omega^{(3)}-\omega'',\boldsymbol{k}^{(3)}-\boldsymbol{k}'')V_r(\omega^{(3)},\boldsymbol{k}^{(3)}) V_r(\omega^{(4)}-\omega^{(3)},\boldsymbol{k}^{(4)}-\boldsymbol{k}^{(3)})V_r^*(\omega^{(4)},\boldsymbol{k}^{(4)})d\nu^{(3)}d\nu^{(4)}] \quad \text{(C.7)}$$

As explained in Sect. 3, all the V_r in the above equation must be computed at the same location in the star in order that the statistical average be different from zero. This is why two integration over the volume are dropped.

On the other hand

$$Z \simeq E[|V_r(\omega'-\omega,\boldsymbol{k}'-\boldsymbol{k_j})|^2]E[|V_r(\omega',\boldsymbol{k}'|^2] E[|V_r(\omega''-\omega',\boldsymbol{k}''-\boldsymbol{k}')|^2] \quad \text{(C.8)}$$

Following Goldreich & Murray (1994), we take

$$E[|V(\omega,\boldsymbol{k})|^2] = \pi\overline{v^2}\tau\prod_l \Lambda_l e^{-\left(\frac{\omega\tau}{2}\right)^2}e^{-\sum_l\left(\frac{k_l\Lambda_l}{2}\right)^2}$$

with

$$\overline{v^2} = \lim_{T,L\to\infty}\int_0^T\int_o^L v^2(r,t)dtd\boldsymbol{r}$$

We can now integrate over ν', ν'' and $\boldsymbol{k}'$. The integration over ν' (ν'') is performed taking into account that g ($\sigma_{c,i}^2-\omega''^2$) has a resonance for $\omega' = \omega_1(k) = \left[\alpha\tau^{-2}+\nu\chi k^4\right]^{1/2}$ ($\omega" =$

$\mathcal{R}(\sigma_{c,i}) = \omega_i$). The integration over $\boldsymbol{k}'$ takes into account that $|\boldsymbol{k}_j| \simeq |\boldsymbol{k}"| \ll |\boldsymbol{k}'|$. We get

$$T \simeq \sum_i \frac{\pi^2\omega^2}{N_j^2} \int \left(\frac{du_{r,j}}{dr}\right)^2 \rho^2(\overline{v^2})^{3/2} H^4$$
$$e^{-(\frac{\omega\tau}{2})^2} \left\{ \frac{2}{3\pi} \frac{\omega}{\omega_i} \frac{H}{H_{u^2_{r,i}}} \frac{\rho H^3 u^2_{r,i}}{N_i} Z_1 Z_2 \right\} d\mathcal{V} \qquad \text{(C.9)}$$

where $H_{u^2_{r,i}}$ is the scale height of $u^2_{r,i}$ and Λ_l is taken equal to the pressure scale height H; also

$$Z_1 = 1 + \imath \frac{\Delta\omega_2(\Delta\omega_1 - \Delta\omega_2)}{\sqrt{2}\omega_1(\Delta\omega_1 + \Delta\omega_2)}$$

$$Z_2 = \exp\left\{ -\frac{\tau^2}{4} [3\omega_1^2(H^{-1}) + \omega_i^2 - 2\omega_1(H^{-1})(\omega + \omega_i)] \right\}$$

$$\Delta\omega_1 = \nu/H^2 \qquad \Delta\omega_2 = \chi/H^2$$

If the term in the curled brackets of Eq. (C9) was equal to one, we would have $T = |\left\langle \boldsymbol{F}, \boldsymbol{u}_j^* \right\rangle|^2$. However $\frac{\rho H^3 u^2_{r,i}}{N_i} \simeq \frac{\rho H^3}{M_i} \ll 1$ (M_i is the mass of the mode "i"). Also according to Goldreich & Keeley (1977) $\left(\frac{\rho H^3}{M_i}\right)^{1/2}$ is of the order of the velocity amplitude of the mode "i" divided by the convective velocity in the source region. Therefore T is much smaller than main term $|\left\langle \boldsymbol{F}, \boldsymbol{u}_j^* \right\rangle|^2$ and can be neglected. The statistical average of $|\langle \boldsymbol{L}_2, \boldsymbol{u}_j \rangle|^2$ is still much smaller.

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