

Density profiles of dark matter halos in an improved secondary infall model

A. Del Popolo^{1,2}, M. Gambera^{1,3}, E. Recami^{4,5,6}, and E. Spedicato²

¹ Istituto di Astronomia dell'Università di Catania, Viale A. Doria 6, I-95125 Catania, Italy

² Dipartimento di Matematica, Università Statale di Bergamo, Piazza Rosate 2, I-24129 Bergamo, Italy

³ Osservatorio Astrofisico di Catania and CNR-GNA, Viale A. Doria 6, I-95125 Catania, Italy

⁴ Facoltà di Ingegneria, Università Statale di Bergamo, Viale G. Marconi 5, I-24044 Dalmine, Bergamo, Italy

⁵ I.N.F.N., Sezione di Milano, Milano, Italy

⁶ C.C.S. and D.M.O./FEEC, State University at Campinas, S. P., Brazil

Received 1 December 1998 / Accepted 11 May 1999

Abstract. In this paper we calculate the density profiles of virialized halos both in the case of structure evolving hierarchically from a scale-free Gaussian δ -field having a power spectrum $P(k) \propto k^n$ in a $\Omega = 1$ Universe and in the case of the CDM model, by using a modified version of Hoffman & Shaham's (1985) (hereafter HS) and Hoffman's (1988) model. We suppose that the initial density contrast profile around local maxima is given by the mean peak profile introduced by Bardeen et al. (1986) (hereafter BBKS), and is not just proportional to the two-point correlation function, as assumed by HS. We show that the density profiles, both for scale-free Universes and the CDM model, are not power-laws but have a logarithmic slope that increases from the inner halo to its outer parts. Both scale-free, for $n \geq -1$, and CDM density profiles are well approximated by Navarro et al. (1995, 1996, 1997) profile. The radius a , at which the slope $\alpha = -2$, is a function of the mass of the halo and in the scale-free models also of the spectral index n .

Key words: cosmology: theory – cosmology: large-scale structure of Universe – galaxies: formation

1. Introduction

The collapse of perturbations onto local density maxima of the primordial density field is likely to have played a key role in the formation of galaxies and clusters of galaxies. The problem of the collapse has been investigated from two points of view, namely that of the statistical distribution of the formed objects (a question related to the biasing problem) (Kaiser 1984; Davis et al. 1985; BBKS) and that of the structure of these objects and its dependence on the statistical properties of the primordial density field (Gunn & Gott 1972; Gunn 1977; Fillmore & Goldreich 1984; Bertschinger 1985; West et al. 1987; HS; Hoffman 1988; Efstathiou et al. 1988; Quinn et al. 1986; Warren et al. 1991;

White & Zaritsky 1992; Evrard et al. 1993; Crone et al. 1994; Navarro et al. 1995, 1996, 1997; Avila-Reese et al. 1998).

To overcome the problem of the excessively steep density profiles, $\rho \propto r^{-4}$, obtained in numerical experiments of simple gravitational collapse Gunn & Gott (1972), Gott (1975) and Gunn (1977) were able to produce shallower profiles, $\rho \propto r^{-2}$ through the *secondary infall* process. Self-similar solutions were found by Fillmore & Goldreich (1984) and Bertschinger (1985), who found a profile of $\rho \propto r^{-2.25}$. HS considered a scale-free initial perturbation spectra, $P(k) \propto k^n$ and assumed that local density extrema are the progenitors of cosmic structures and that the density contrast profile around maxima is proportional to the two-point correlation function. They thus showed that $\rho \propto r^{-\alpha}$ with $\alpha = \frac{3(3+n)}{(4+n)}$, thus recovering Bertschinger's (1985) profile for $n = 0$ and $\Omega = 1$. They also showed that, in an open Universe, the slopes of the density profiles steepen with increasing values of n and with decreasing Ω , reaching a profile $\rho \propto r^{-4}$ for $\Omega \rightarrow 0$. Hoffman (1988) refined the calculations of HS and made a detailed comparison of the analytical predictions of the secondary infall model (hereafter SIM) with the simulations by Quinn et al. (1986) and Quinn & Zurek (1988). In spite of the high level of simplification of the SIM, and although the formation of dark matter halos in the numerical simulations seems to grow in mass by mergers with typically less massive halos, while the collapse in the SIM is spherical symmetric and gentle, those numerical simulations (Quinn et al. 1986; Frenk et al. 1988) were in agreement with the predictions of the SIM. The good results given by the SIM in describing the formation of dark matter halos seem to be due to the fact that in energy space the collapse is ordered and gentle, differently from what seen in N-body simulations (Zaroubi et al. 1996). If things go really in this way, it is possible that galaxies and clusters of galaxies retain memory of their initial conditions.

A great effort has been dedicated to study the role of initial conditions in shaping the final structure of the dark matter halos; but, if on large scales (evolution in the weakly non-linear regime) the growing mode of the initial density fluctuations can

be recovered if the present velocity or density field is given (Peebles 1989; Nusser & Dekel 1992), on small scales shell crossing and virialization contribute to make the situation less clear. To study the problem, three-dimensional large-scale structure simulations were run with often conflicting results. While Quinn et al. (1986) and Efstathiou et al. (1988) found a connection between the density profiles of collapsed objects and the initial fluctuation spectrum for Einstein-de Sitter universes [in particular Efstathiou et al. (1988) found density profiles steepening with increasing spectral index n], West et al. (1987) arrived at the opposite conclusion. In any case, the previous studies showed that the mass density profiles steepened with decreasing Ω . This result is in agreement with that of HS. More recent studies (Voglis et al. 1995; Zaroubi et al. 1996) showed a correlation between the profiles and the final structures. Finally Dubinski & Carlberg (1991), Lemson (1995), Cole & Lacey (1996), Navarro et al. (1996, 1997) and Moore et al. (1997) found that dark matter halos do not follow a power law but develop universal profile, a quite general profile for any scenario in which structures form due to hierarchical clustering, characterized by a slope $\beta = \frac{d \ln \rho}{d \ln r} = -1$ near the halo center and $\beta = -3$ at large radii. In that approach, density profiles can be fitted with a one parameter functional form:

$$\frac{\rho(r)}{\rho_b} = \frac{\delta_n}{\frac{r}{a} \left(1 + \frac{r}{a}\right)^2} \quad (1)$$

where ρ_b is the background density and δ_n is the overdensity [below we shall refer to Eq. (1) (Navarro et al. 1997) as the NFW profile]. The scale radius a , which defines the scale where the profile shape changes from slope $\beta < -2$ to $\beta > -2$, and the characteristic overdensity, δ_n , are related because the mean overdensity enclosed within the virial radius r_v is $\simeq 180$. The scale radius and the central overdensity are directly related to the formation time of a given halo (Navarro et al. 1997). The power spectrum and the cosmological parameters only enter to determine the typical formation epoch of a halo of a given mass, and thereby the dependence of the characteristic radius on the total mass of the halo. Also these last results are not universally accepted. Recently, Klypin et al. (1997) and Nusser & Sheth (1998) challenged the claim that a one parameter functional form could fit the density profiles using N-body simulations. Klypin et al. (1997), by using N-body simulations of CDM-like models, showed that the scatter about a one parameter fit is substantial, and that more than just one physical parameter is needed to describe the structure of halo density profiles in agreement with Nusser & Sheth's (1998) conclusions. In short, the question of whether galaxies and clusters mass density profiles retain information on the initial conditions and the evolutionary history that led to their formation remains an open question.

In this paper, we introduce a modified version of HS and Hoffman's (1988) models to study the shapes of the density profiles that result from the gravitational collapse. In particular, we relax the hypothesis that the initial density profile is proportional to the two-point correlation function, and use the density profiles given by BBKS.

The plan of the paper is the following: in Sect. 2 we show the reasons why the HS (1985) model must be improved and how it can be improved. In Sect. 3 we introduce our model and in Sect. 4 we show our results and finally in Sect. 5 we draw our conclusions.

2. Limits of the SIM and Hoffman & Shaham's approaches

Most analytic work has focussed on studying the evolution of isolated spherical systems (Gunn & Gott 1972, Gott 1975; Gunn 1977; Fillmore & Goldreich 1984; Bertschinger 1985; HS; Hoffman 1988; White & Zaritsky 1992) because there is no analytical technique to study the evolution of a system starting from general initial conditions. It is known that, if the initial density profile of a halo is a power law in radius, $\delta(r) \propto r^{-m}$, ($\delta(r)$ is the mean density at a distance r from a peak) then the density profile of the collapsed halo is also a power law:

$$\rho \propto r^{-\alpha} \quad (2)$$

where $\alpha = \frac{3m}{1+m}$, if $m \geq 2$, and $\alpha = -2$ if $m < 2$ (Fillmore & Goldreich 1984; Bertschinger 1985). The restriction on m can be relaxed if non-radial orbits are permitted (White & Zaritsky 1992). HS and Hoffman (1988) related this spherical solution to dark matter halos, that form from initial gaussian density fields. In their paper they noted that the density profiles of very high peaks for $r > r_v$ have the same radial dependence as the correlation function of the initial field, $\delta \propto \xi \propto r^{-(3+n)}$. Then HS set $m = 3 + n$, obtaining:

$$\alpha = \frac{3(3+n)}{(4+n)} \quad (3)$$

for $n \geq -1$, and $\alpha = -2$ for $n < -1$. This last conclusion ($\alpha = -2$ for $n < -1$) is not a direct consequence of the HS model, but it was an assumption made by the quoted authors following the study of self-similar gravitational collapse by Fillmore & Goldreich (1984). In fact, as reported by the same authors, in deriving the relation between the density at maximum expansion and the final one (see next section) HS assumed that each new shell that collapses can be considered as a small perturbation to the gravitational field of the collapsed halo. This assumption breaks down for $n < -1$. In reality, all the slopes of the power spectrum satisfying the condition $n < -1$ constitute a big problem for SIM (Zaroubi et al. 1996). In fact, as shown in Zaroubi's et al. (1996) simulations and Lokas' et al. (1996) paper, the slope $n = -1$ marks the transition between two different dynamical regimes. In the cosmological context, such regimes correspond to a primordial perturbation field whose power spectrum is dominated by the high wavenumber modes, $n > -1$, or by the low wavenumber modes, $n < -1$, respectively.

In the regime $n < -1$, the energy of the particles changes violently in time, and one expects at the end a state of statistical equilibrium, similar to the violent relaxation described by Lynden-Bell (1967). In this regime there is a strong dependence on the boundary conditions and the dynamics strongly depends on the last collapsing shell (Zaroubi et al. 1996). According to

Lokas et al. (1996) for $n < -1$ the fluctuations grow faster than in the linear theory.

In the $n > -1$ regime, the dynamics of the collapsed shells is hardly affected by the ongoing collapse of more distant shells. In this case, order is preserved in energy space and according to Lokas et al. (1996) a slowdown in the growth rate of perturbations is expected. In such a case, the SIM is expected to be a useful tool for calculating the final virialized structure of collapsed halos. For these reasons in this paper we suppose that $-1 \leq n \leq 0$.

Let us also add that Eq. (3) is true only for regions outside the virial radius of a dark matter halo (see Peebles 1974; Peebles & Groth 1976; Davis & Peebles 1977; Bonometto & Lucchin 1978; Peebles 1980; Fry 1984). In the inner regions of the halo, scaling arguments plus the stability assumption tell us that $\xi(r) \propto r^{-\frac{3(3+n)}{5+n}}$ and we expect a slope different from Eq. (3). Syer & White (1996) found for the inner regions of the halo a profile $\rho(r) \propto r^{-\frac{3(3+n)}{5+n}}$, coincident with the slope of the correlation function. Nusser & Sheth (1998) found $\frac{3(3+n)}{5+n} \leq \alpha \leq \frac{3(3+n)}{4+n}$, while Sheth & Jain (1996) found $\alpha = \frac{3(4+n)}{5+n}$. In other words, HS's (1985) solution applies only to the outer regions of collapsed halos and consequently the conclusion, obtained from that model, that dark matter halos density profiles can be approximated by power-laws on their overall radius range is not correct. It is then necessary to introduce a model that can make predictions also on the inner parts of halos.

Another problem of HS's work is the assumption that $\delta(r) \propto \xi(r) \propto r^{-(3+n)}$. This is true only for very high peaks (see Ryden & Gunn 1987). As the peak height decreases, the peak profile becomes steeper than the correlation function (BBKS) and consequently the final density profile becomes steeper. Moreover, according to BBKS, the mean peak profile depends on a sum involving the initial correlation function, $\xi(r) \propto r^{-(5+n)}$, and its Laplacian, $\nabla^2 \xi(r) \propto r^{-(3+n)}$ (BBKS; Ryden & Gunn 1987). This means that there are at least two reasons why the density profile outside the virial radius must be steeper than in HS's model:

- peaks that give origin to structures have height $\nu = 2, 3$, and not $\nu \rightarrow \infty$;
- $\delta(r)$ depends on a sum of $\xi(r)$ and its Laplacian, and for peaks $\nu = 2, 3$ is steeper than the correlation function.

Inside the virial radius HS's model cannot be used to predict the virialized density profile because it was a priori constructed to estimate density profiles at $r > r_v$.

That model suffers also from another drawback: namely, the assumption that the accreting matter is not clumpy, while in hierarchical scenarios for structure formation halo grows in mass by merger with less massive halos. As previously told according to Zaroubi et al. (1996) probably this last problem is not so difficult it, compared with the other two.

In the following, we want to show that the predictive power of the SIM is greatly improved when the problems reported at point a) and b) are removed. These two problems are not

problems of the SIM but only problems introduced by the HS's implementation of it.

3. The model

In the most promising cosmological scenarios, structure formation in the universe is generated through the growth and collapse of primeval density perturbations originated from quantum fluctuations (Guth & Pi 1982; Hawking 1982; Starobinsky 1982; BBKS) in an inflationary phase of early Universe. The growth in time of small perturbations is due to gravitational instability. The statistics of density fluctuations originated in the inflationary era are Gaussian, and can be expressed entirely in terms of the power spectrum of the density fluctuations:

$$P(k) = \langle |\delta_{\mathbf{k}}|^2 \rangle \quad (4)$$

where

$$\delta_{\mathbf{k}} = \int d^3x \exp(-i\mathbf{k}\mathbf{x}) \delta(\mathbf{x}) \quad (5)$$

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_b}{\rho_b} \quad (6)$$

and ρ_b is the mean background density. In biased structure formation theory it is assumed that cosmic structures of linear scale R_f form around the peaks of the density field, $\delta(\mathbf{x})$, smoothed on the same scale.

If we suppose we are sitting on a $\nu\sigma$ extremum in the smoothed density field, we have that:

$$\delta(0) = \nu\xi(0)^{1/2} = \nu\sigma \quad (7)$$

together with:

$$\nabla\delta(r) \big|_{r=0} = 0 \quad (8)$$

If the Laplacian of $\delta(r)$ is unspecified, that means that the extremum may be a maximum or a minimum, the mean density at a distance r from the peak is then:

$$\delta(r) = \nu\xi(r)/\xi(0)^{1/2} \quad (9)$$

(Peebles 1984; HS). If we calculate the mean density around maxima, as done by BBKS, by adding the constraint:

$$\nabla^2\delta(r) \big|_{r=0} < 0 \quad (10)$$

we find that the mean density around a peak is given by:

$$\langle \delta(r) \rangle = \frac{\nu\xi(r)}{\xi(0)^{1/2}} - \frac{\vartheta(\nu\gamma, \gamma)}{\gamma(1-\gamma^2)} \left[\gamma^2\xi(r) + \frac{R_*^2}{3} \nabla^2\xi(r) \right] \cdot \xi(0)^{-1/2} \quad (11)$$

(BBKS; Ryden & Gunn 1987), where ν is the height of a density peak, $\xi(r)$ is the two-point correlation function:

$$\xi(r) = \frac{1}{2\pi^2 r} \int_0^\infty P(k) k \sin(kr) dk \quad (12)$$

γ and R_* are two spectral parameters given respectively by:

$$\gamma = \frac{\int k^4 P(k) dk}{\left[\int k^2 P(k) dk \int k^6 P(k) dk \right]^{1/2}} \quad (13)$$

$$R_* = \left[\frac{3 \int k^4 P(k) dk}{\int k^6 P(k) dk} \right]^{1/2} \quad (14)$$

while $\vartheta(\gamma\nu, \gamma)$ is:

$$\theta(\nu\gamma, \gamma) = \frac{3(1 - \gamma^2) + (1.216 - 0.9\gamma^4) \exp\left[-\left(\frac{\gamma}{2}\right)\left(\frac{\nu\gamma}{2}\right)^2\right]}{\left[3(1 - \gamma^2) + 0.45 + \left(\frac{\nu\gamma}{2}\right)^2\right]^{1/2} + \frac{\nu\gamma}{2}} \quad (15)$$

In order to calculate $\delta(r)$ we need a power spectrum, $P(k)$. In the following, we restrict our study to an Einstein-De Sitter ($\Omega = 1$) Universe with zero cosmological constant and scale-free density perturbation spectrum $P(k)$

$$P(k) = Ak^n \quad (16)$$

with a spectral index in the range $-1 \leq n \leq 0$, and also to a CDM Universe with spectrum given by BBKS:

$$P(k) = Ak^{-1} [\ln(1 + 4.164k)]^2 (192.9 + 1340k + 1.599 \times 10^5 k^2 + 1.78 \times 10^5 k^3 + 3.995 \times 10^6 k^4)^{-1/2} \quad (17)$$

We normalized the spectrum by imposing that the mass variance at $8h^{-1}Mpc$ is $\sigma_8 = 0.63$. In the case of a scale-free power spectrum, it is easy to show that the two-point correlation function can be expressed in terms of the confluent hypergeometric function, ${}_1F_1$, and of the Γ function as:

$$\xi(r) = \frac{1}{2\pi^2} \exp\left(-\frac{r^2}{4\beta}\right) \frac{1}{2\beta^{(n+3)/2}} \times \Gamma\left(\frac{n+3}{2}\right) {}_1F_1\left(\frac{-n}{2}; \frac{3}{2}; \frac{r^2}{4\beta}\right) \quad (18)$$

where $\beta = R_f^2/2$, being R_f the filtering radius, and the Laplacian of $\xi(r)$ as:

$$\nabla^2 \xi(r) = -\frac{1}{2\pi^2} \exp\left(-\frac{r^2}{4\beta}\right) \frac{1}{2\beta^{(n+5)/2}} \Gamma\left(\frac{n+5}{2}\right) {}_1F_1\left(\frac{-n-2}{2}; \frac{3}{2}; \frac{r^2}{4\beta}\right) \quad (19)$$

and finally $\delta(r)$ is:

$$\delta(r) = \left(\frac{\nu}{\xi(0)^{1/2}} - \frac{\theta(\nu\gamma, \gamma)\gamma}{(1 - \gamma^2)\xi(0)^{1/2}}\right) \frac{1}{2\pi^2} \exp\left(-\frac{r^2}{4\beta}\right) \times \frac{1}{2\beta^{(n+3)/2}} \cdot \Gamma\left(\frac{n+3}{2}\right) \cdot {}_1F_1\left(\frac{-n}{2}; \frac{3}{2}; \frac{r^2}{4\beta}\right) + \frac{\theta(\nu\gamma, \gamma)R_*^2}{3\gamma(1 - \gamma^2)\xi(0)^{1/2}} \frac{1}{2\pi^2} \exp\left(-\frac{r^2}{4\beta}\right) \frac{1}{2\beta^{(n+5)/2}} \times \Gamma\left(\frac{n+5}{2}\right) {}_1F_1\left(\frac{-n-2}{2}; \frac{3}{2}; \frac{r^2}{4\beta}\right) \quad (20)$$

In the case that ν is very large Eq. (15) reduces to

$$\theta \rightarrow 3(1 - \gamma^2)/(\nu\gamma) \quad (21)$$

and the mean density is well approximated by Eq. (9), which is the approximation used by HS to calculate $\delta(r)$. In reality, for peaks having $\nu = 2, 3, 4$, the mean expected density profile is different from a profile proportional to the correlation function both for galaxies and clusters of galaxies (see BBKS). For example for galaxies the CDM profile is steeper than that proportional to $\xi(r)$ as shown by Ryden & Gunn (1987) with a discrepancy increasing with decreasing ν . As shown by Gunn & Gott (1972), a bound mass shell having initial comoving radius x will expand to a maximum radius:

$$r_m = x/\bar{\delta}(r) \quad (22)$$

where the mean fractional density excess inside the shell, as measured at current epoch t_0 , assuming linear growth, can be calculated as:

$$\bar{\delta} = \frac{3}{r^3} \int_0^r \delta(y)y^2 dy \quad (23)$$

At initial time t_i and for a Universe with density parameter Ω_i , a more general form of Eq. (22) (Peebles 1980) is:

$$r_m = r_i \frac{1 + \bar{\delta}_i}{\bar{\delta}_i - (\Omega_i^{-1} - 1)} \quad (24)$$

The last equation must be regarded as the main essence of the SIM. It tells us that the final time averaged radius of a given Lagrangian shell does scale with its initial radius. Expressing the scaling of the final radius, r , with the initial one by relating r to the turn around radius, r_m , it is possible to write:

$$r = Fr_m \quad (25)$$

where F is a constant that depends on α :

$$F = F(\alpha) = 0.186 + 0.156\alpha + 0.013\alpha^2 + 0.017\alpha^3 - 0.0045\alpha^4 + 0.0032\alpha^5 \quad (26)$$

(Zaroubi et al. 1996). If energy is conserved, then the shape of the density profile at maximum of expansion is conserved after the virialization, and is given by (Peebles 1980; HS; White & Zaritsky 1992):

$$\rho(r) = \rho_i \left(\frac{r_i}{r}\right)^2 \frac{dr_i}{dr} \quad (27)$$

The density profile is a function of three parameters: the spectral index n , the density parameter Ω , and the height of the density peak, ν . In the limit $\nu \gg 1$, the overdensity $\delta(r)$ is proportional to the two-point correlation function and the density profile is a function of n and Ω only, and then the expected profile is that by HS.

4. Results and discussion

By using the model introduced in the previous section we have studied the density profiles of halos in scale-free universes with

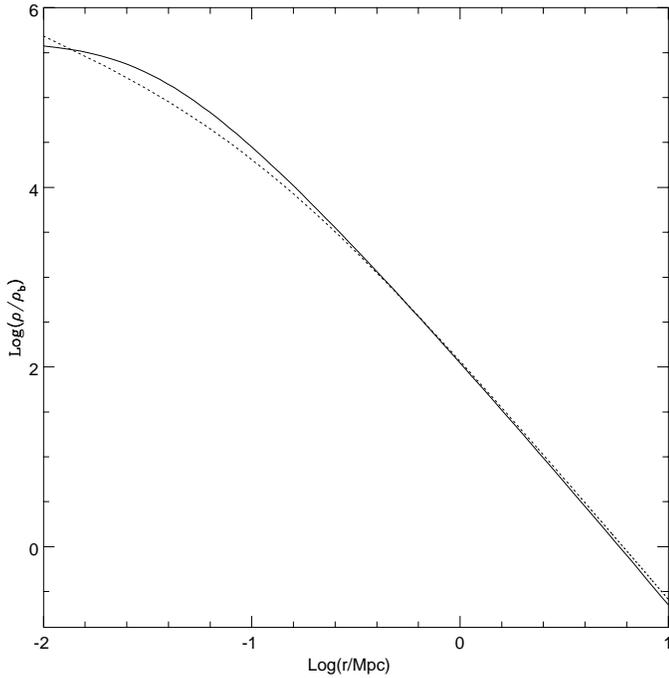


Fig. 1. Density profile for a scale-free spectrum with $n = -1$ (solid line) and the NFW fit (dashed line). The mass of the halo is $\sim 2 \times 10^{15} M_{\odot}$.

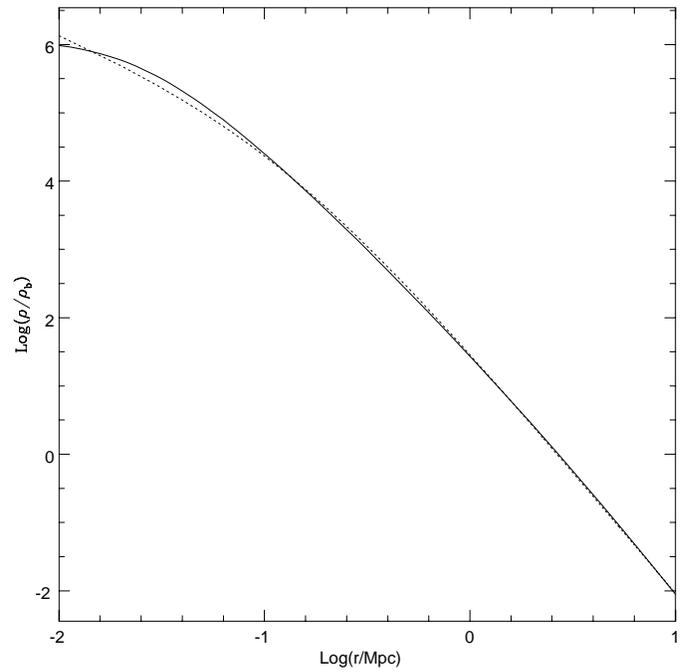


Fig. 2. As in Fig. 1, but now $n = 0$. The mass of the halo is $\sim 0.5 \times 10^{15} M_{\odot}$.

$-1 \leq n \leq 0$, and for a CDM model characterized by a BBKS spectrum. As previously quoted, the chosen range of n is dictated by the limits of the SIM and by the values of n interesting in the cosmological context. The results of our calculations are shown in Figs. 1–6.

In Figs. 1–3 we have calculated the density profiles of halos in a scale-free universe with $n = -1$, $n = 0$, and in a CDM model.

In Fig. 1 we show the density profile for $n = -1$. The dashed line is the NFW fit, while the solid line represents the profile obtained from our model. The NFW profile fits well the density profile, except in the inner part. In the case $n = 0$, see Fig. 2, the NFW (dashed line) gives a good fit to the the density profile (solid line) also in the inner part of the density profile. The situation for the halo obtained from the CDM spectrum, smoothed on clusters scales $R_f = 5h^{-1} \text{Mpc}$, (see Fig. 3) is similar to that of the case $n = -1$. This slope is in fact similar to that of the standard CDM power spectrum on cluster scales.

In Fig. 4 we plot the slope of the density profile for several values of n and ν . The fundamental aim of the picture is to show how for small values of ν ($\nu = 2, 3$), out from the inner region, the slope is larger than HF result. We began by finding the HS solution, that was recovered as expected in the limit $\nu \gg 1$ or equivalently $\delta(r) \simeq \nu \xi(r) / \xi(0)^{1/2}$. This solution is represented by the short-dashed line which coincides with the result by HS, namely $\alpha = \frac{3(3+n)}{(4+n)}$, indicating an increase in the slope α with increasing n . Moreover the value of α is independent on the radius chosen to compute the slope which means that halos are described by pure power-laws, as described in the HF. Because of the rarity of extremely high peaks, most galaxies and clusters

will form from peaks of height 2 or 3 σ (BBKS; Ryden & Gunn 1987): so we repeated the calculation of α for these values. If we choose a value of $\nu = 3$, the logarithmic slope of the density profile, calculated at $1h^{-1} \text{Mpc}$ (we calculated the slope at a fixed distance because the density profiles are not power-laws) is steeper for all values of n (solid line) than that obtained by HS, and it is well approximated by Shet & Jain (1996) (dotted line), $\alpha = \frac{3(4+n)}{(5+n)}$, obtained using stable clustering and neglecting halo-halo correlations. At the same time the dependence of α on n is weaker than that shown by HS. We compared our result to that of Shet & Jain (1996) because their simple analytical formalism describes reasonably well NFW profiles on scales $r \geq 0.1r_v$ (at least till to $1h^{-1} \text{Mpc}$). The choice of calculating the slope, α , at $1h^{-1} \text{Mpc}$ is suggested from the consideration that at that radius NFW profiles are well fit by Shet & Jain (1996) model.

Obviously changing the radius at which the slope is calculated this reflects on the relation α - n , because the density profiles described by our model (excluding the case $\nu \gg 1$) are not power laws. For values larger than $1h^{-1} \text{Mpc}$, the value of the slope is obviously larger. In any case, we have to remember that the comparison with Shet & Jain (1996) model is displayed only to show that density profiles have larger slopes than those found by HF.

For $\nu = 2$ the slope is even steeper than the previous case (long-dashed line) and it is well approximated by Crone's et al. (1994) result, which is also consistent with the results by Navarro et al. (1997) (see their Fig. 13). As in the previous case the dependence of α on n is weaker with respect to that shown in HS. More massive halos $\nu = 3$ have flatter density profiles than less massive ones in agreement with Tormen et al. (1997).

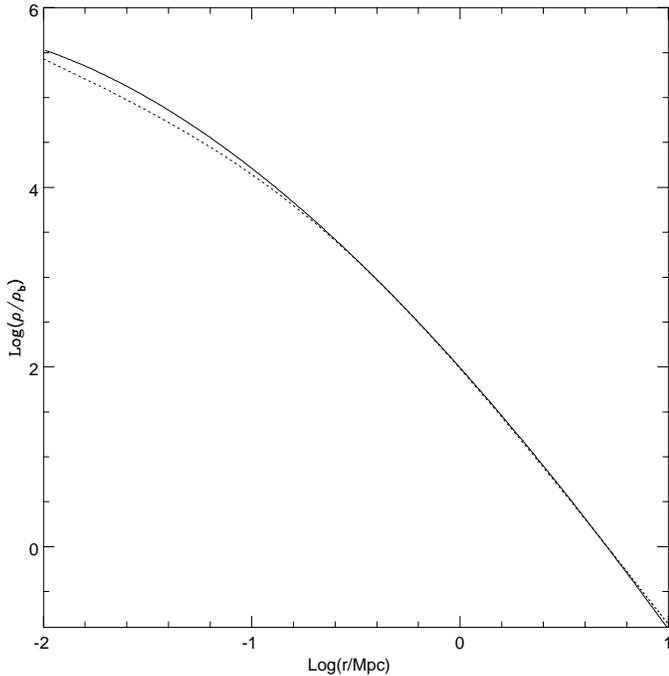


Fig. 3. Density profiles for a CDM spectrum smoothed on a scale $R_f = 5h^{-1} Mpc$ (solid line) and the NFW fit (dashed line).

In Fig. 5 and Fig. 6 we plot a/r_v , the variation of the ratio of the scale parameter, a and the virial radius, r_v , versus M/M_* for a scale-free spectrum and a CDM spectrum, respectively. We remember that according to Navarro et al. (1996, 1997), a is linked to a dimensionless “concentration” parameter, c , by the relation $a = \frac{r_v}{c}$ and the parameter c is linked to the characteristic density, δ_n , by the relation:

$$\delta_n = \frac{200}{3} \frac{c^3}{\ln(1+c) - c/(1+c)} \quad (28)$$

The behaviour of the characteristic density of a halo, increasing towards lower masses in all the cosmological models, supports the idea that the M_v - δ_n relation is a direct result of the higher redshift of collapse of less massive systems. For scale-free models this implies $\delta_n \propto M^{-(n+3)/2}$ (same scaling relating M_* and the mean cosmic density at a fixed redshift z).

In the scale-free case, masses are normalized by the characteristic mass M_* , which is defined at a time t as the linear mass on the scale currently reaching the non-linear regime:

$$M_*(t) = \frac{4\pi}{3} R_*^3 \rho_b(t) \quad (29)$$

where the scale R_* is such that the linear density contrast on this scale is $\delta(R_*) = 1.69$. Once known that the mass variance, σ_M , for a power spectrum $P(k) \propto k^n$ is given by $\sigma_M \propto R^{-(3+n)}$ and remembering our normalization $\sigma_M(8h^{-1} Mpc) = 0.63$, the value of M_* for $n = -1$ results to be $M_* = 6 \times 10^{13} M_\odot$. In the CDM case, the normalization mass, M_* , is obtained imposing the condition $\sigma_0[M_*(z)] = 1.69(1+z)$. For our adopted normalization of the CDM power spectrum, $M_* = 3 \times 10^{13} M_\odot$, at $z = 0$.

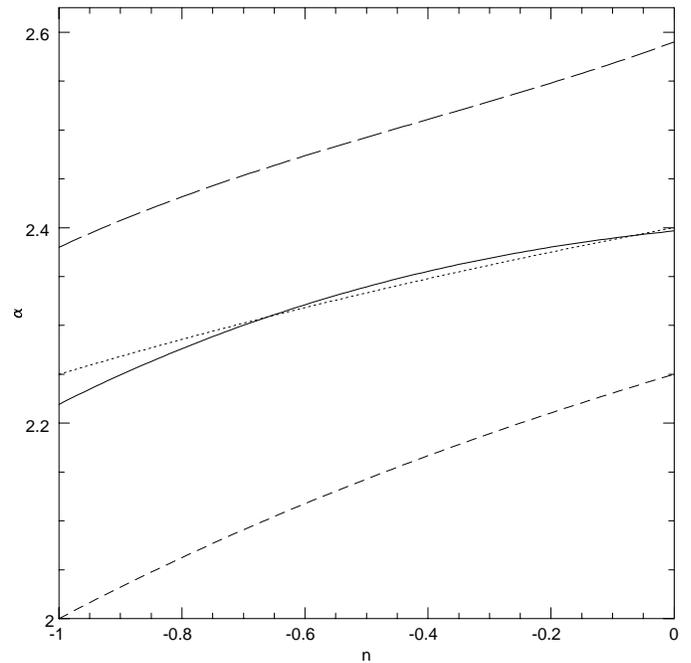


Fig. 4. The slope α of density profiles as a function of the spectral index n and ν . The short-dashed line represents α in the limit $\nu \gg 1$. It coincides with the HS result. The solid line represents the logarithmic slope for $\nu = 3$, while the dotted line is Shet & Jain’s (1996) result. The long-dashed line represents α for $\nu = 2$.

In the scale-free case, Fig. 5, the value of the scale radius a correlates strongly with halo mass and with spectral index n . The solid line represents a for $n = -1$. As shown in the figure, more massive halos have a larger scale radius a , or equivalently less massive halos are more concentrated. The dotted line shows a for $n = 0$. Also for this value of n more massive halos are less centrally concentrated. Finally from Fig. 5 we also see that in models with more small-scale power (or equivalently larger values of n) the haloes tend to have denser cores. These results were expected because halos with mass $M \ll M_*$ form much earlier than haloes with $M \gg M_*$ and then are more centrally concentrated. Moreover, for a fixed value of M/M_* , haloes form earlier in models with larger values of n and then have denser cores. This result is in qualitative agreement with those by Navarro et al. (1997), Cole & Lacey (1996), Tormen et al. (1997). The filled squares and the filled exagons, in Fig. 5, represents a/r_v for $n = -1$ and $n = 0$ respectively obtained by Navarro et al. (1997) in the case $f = 0.01$ (see their paper for a definition of this parameter) which give the best fit to the results of their simulations.

The virial radius r_v is obtained by using Navarro et al. (1997) equation:

$$r_v = 1.63 \times 10^{-2} \left(\frac{M}{h^{-1} M_\odot} \right)^{1/3} \left(\frac{\Omega_0}{\Omega(z)} \right)^{-1/3} \times (1+z)^{-1} h^{-1} kpc \quad (30)$$

where Ω_0 is the actual value of the density parameter and $h = 0.5$. The virial radius determines the mass of the halo through:

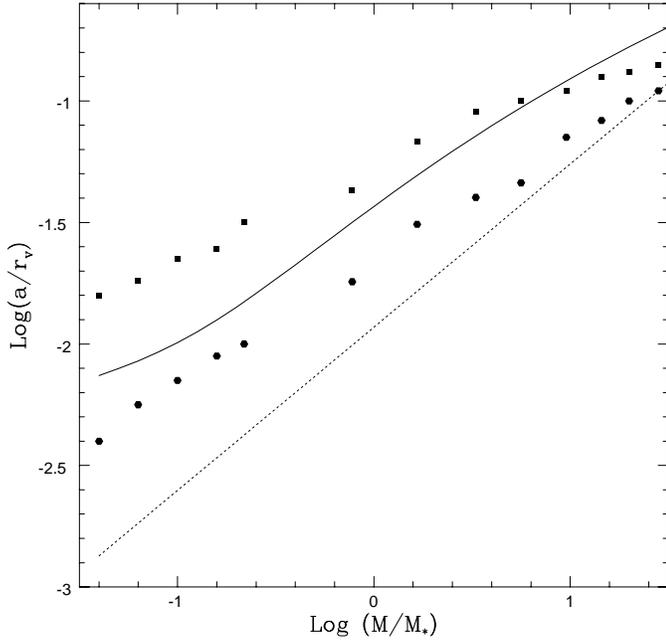


Fig. 5. Trend of the scale radius a versus the mass of the halos in the case $n = -1$ (solid line) and $n = 0$ (dotted line). The filled squares and the filled exagons represents a/r_v , for $n = -1$ and $n = 0$ respectively, obtained by Navarro et al. (1997) in the case $f = 0.01$.

$$M_v = 200\rho_b \frac{4\pi}{3} r_v^3 \quad (31)$$

In the case $n = -1$, our model gives less concentrated halos till $M \simeq 10M_*$ and after this value the tendency is reversed. Maximum deviations of ~ 2.5 , between Navarro et al. (1997) data and our model, are found in the low mass domain, $< 0.25M_*$. The situation is similar to that described by Tormen et al. (1997) in his *most relaxed* dynamical configuration (Fig. 16 second row in the left panel). In the case $n = 0$, our model gives halos slightly more concentrated in the overall studied mass range. In this case the discrepancies reach values of ~ 3 .

Also in the CDM case, Fig. 6, the value of the scale radius a correlates strongly with halo mass. The solid line represents a for our model and the filled exagons the N-body results of Navarro et al. (1996,1997). As for scale-free spectrum, large halos are significantly less concentrated than small ones. This trend, a decreasing with decreasing M , can be explained in terms of the formation times of halos (Navarro et al. (1996,1997)). The dependence of concentration on mass is weak: there is only a change of a factor of ~ 4 while M varies by 4 orders of magnitude. In this case the discrepancies between data and model is ~ 2 .

The result obtained is a remarkable improvement of the SIM being it able to reproduce almost all the prediction of N-body simulations with discrepancies of the same magnitude of those shown in Tormen et al. (1997) (this for the case $n = -1$) and surely much smaller than those found in Cole & Lacey (1996). Our model is based on spherical symmetry, and as we previously stressed, halos accretion does not happen in spherical shells but by aggregation of subclumps of matter which have already

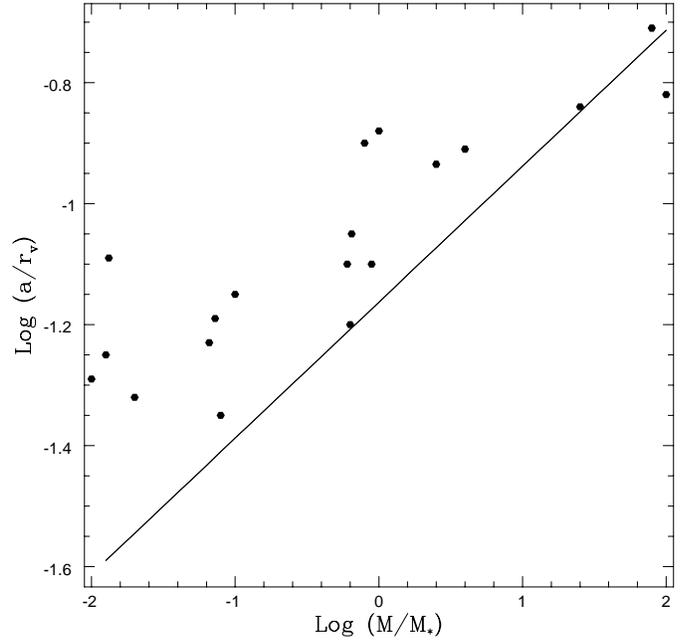


Fig. 6. Trend of the scale radius a (solid line) versus the mass of the halos in the case of a CDM smoothed on a scale $R_f = 5h^{-1} Mpc$ and with normalization $\sigma_8 = 0.63$. The filled exagons represents a/r_v , obtained by Navarro et al. (1996,1997).

collapsed. In other words it seems that the halos structures does not depend crucially on hierarchical merging, in agreement with Huss et al. (1998). The SIM seems to have more predictive power than that till now conferred to it.

5. Conclusions

In this paper we have developed an improved version of the HS model to study the structure of the dark matter halos. We assumed that the initial density profile is given by the average profile given by BBKS and, solving the spherical collapse model, we obtained the virialized density profile. Our results can be summarized as follows:

- a) Differently from HS's (1985) model the density profiles are not power-laws but have a logarithmic slope that increase from the inner halo to its outer parts. In the outer parts of the halo, the density profiles are steeper than that found by HS and are consistent with $\rho \propto r^{-3}$, while in the inner part of the halo we find $\rho \propto r^{-1}$. The analytic model proposed by Navarro et al. (1995) is a good fit to the halo profiles.
- b) The radius, a , at which the slope equals -2 is a function of the mass of the halo and of the spectral index n . Lower mass halos are more centrally concentrated than the higher ones. For a given mass M , halos having larger values of n have denser cores.
- c) The good agreement of our (spherical symmetric) model with several N-body simulations lead us to think, in agreement with Huss' et al. (1998) paper, that the role of merging in the formation of halos is not as crucial as generally believed.

References

- Avila-Reese V., Firmani C., Hernandez X., 1998, preprint SISSA astro-ph/9710201
- Bardeen J.M., Bond J.R., Kaiser N., Szalay A.S., 1986, ApJ 304, 15 (BBKS)
- Bertschinger E., 1985, ApJS 58, 39
- Bonometto S.A., Lucchin F., 1978, A&A 67, L7
- Cole S., Lacey C., 1996, MNRAS 281, 716
- Crone M.M., Evrard A.E., Richstone D.O., 1994, ApJ 434, 402
- Davis M., Peebles P.J.E., 1977, ApJS 34, 425
- Davis M., Efstathiou G., Frenk C.S., White S.D.M., 1985, ApJ 292, 371
- Dubinski J., Carlberg R., 1991, ApJ 378, 496
- Efstathiou G., Frenk C.S., White S.D.M., Davis M., 1988, MNRAS 235, 715
- Evrard A.E., Mohr J.J., Fabricant D.G., Geller M.J., 1993, ApJ 419, L9
- Filmore J.A., Goldreich P., 1984, ApJ 281, 1
- Frenk C.S., White S.D.M., Davis M., Efstathiou G., 1988, ApJ 351, 10
- Fry J.N., 1984, ApJ 279, 499
- Gott J.R., 1975, ApJ 201, 296
- Gunn J.E., 1977, ApJ 218, 592
- Gunn J.E., Gott J.R., 1972, ApJ 176, 1
- Guth A.H., Pi S.Y., 1982, Phys. Letters 49, 1110
- Hawking S.W., 1982, Phys. Letters 115B, 295
- Hoffman Y., 1988, ApJ 328, 489
- Hoffman Y., Shaham J., 1985, ApJ 297, 16 (HS)
- Huss A., Jain B., Steinmez M., 1998, preprint SISSA astro-ph/9803117
- Kaiser N., 1984, ApJ 284, L9
- Klypin A.A., Holtzman J., 1997, preprint SISSA astro-ph/9712217
- Lemson, G., 1995, Ph.D. Thesis, Riksuniversiteit Groningen
- Lokas E.L., Juszkiewicz R., Bouchet F.R., Hivon E., 1996, ApJ 467, 1
- Lynden-Bell, 1967, In: Ehlers J., (ed.) Relativity Theory and Astrophysics II, Galactic Structure. American Mathematical Society
- Moore B., Governato F., Quinn T., Stadel J., Lake G., 1997, preprint SISSA astro-ph/9709051
- Navarro J.F., Frenk C.S., White S.D.M., 1995, MNRAS 275, 720
- Navarro J.F., Frenk C.S., White S.D.M., 1996, ApJ 462, 563
- Navarro J.F., Frenk C.S., White S.D.M., 1997, ApJ 490, 493 (NFW)
- Nusser A., Dekel A., 1992, ApJ 362, 14
- Nusser A., Sheth R.K., 1998, preprint SISSA astro-ph/9803281
- Peebles P.J.E., 1974, ApJ 189, L51
- Peebles P.J.E., 1980, The large scale structure of the Universe. Princeton University Press
- Peebles P.J.E., 1984, ApJ 284, 439
- Peebles P.J.E., 1989, ApJ 344, L53
- Peebles P.J.E., Groth E.J., 1976, A&A 53, 131
- Quinn P.J., Zurek W.H., 1988, ApJ 331, 1
- Quinn P.J., Salmon J.K., Zurek W.H., 1986, Nat 322, 329
- Ryden B.S., Gunn J.E., 1987, ApJ 318, 15
- Sheth R.K., Jain, B., 1996, preprint SISSA astro-ph/9602103
- Starobinsky A.A., 1982, Phys. Letters 117B, 175
- Syer D., White S.D.M., 1996, preprint SISSA astro-ph/9611065
- Tormen G., Diaferio A., Syer D., 1997, preprint SISSA astro-ph/9712222
- Voglis N., Hiodelis N., Harsoula M., 1995, Ap&SS 226, 213
- Warren M.S., Zurek W.H., Quinn P.J., Salmon J.K., 1991, In: Holt S., Trimble V., Bennett C. (eds.) After the First Three minutes. AIP, New York, 210
- West M.J., Dekel A., Oemler A., 1987, ApJ 316, 1
- White S.D.M., Zaritsky D., 1992, ApJ 394, 1
- Zaroubi S., Naim A., Hoffman Y., 1996, ApJ 457, 50