

Gravitational waves from galaxy cluster distributions

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Abstract. Galaxy clusters are sources of gravitational radiation. The main aim of this paper is to give numerical estimates and theoretical description of the relevant features of the gravitational radiation coming from an appropriate spatial distributions of galaxy clusters. Since no analytical approaches are currently available to describe the strongly nonlinear regime, our numerical approach – combining numerical simulations with statistical arguments – seems to be an useful way of studying the main features of that radiation. Although far to be detectable with present technology, we advance some ideas about future observational strategies and its cosmological implications.

Key words: gravitational waves – methods: numerical – cosmology: theory – cosmology: large-scale structure of Universe

1. Introduction

The cosmological stochastic gravitational-wave background produced by mildly nonlinear evolution of density fluctuations can be estimated using analytical perturbative methods (see, e.g., Matarrese & Mollerach 1997 and references cited therein). However, the estimation of that emission, in the strongly nonlinear regime, would require non-perturbative approaches which are not currently available. Galaxy clusters are evolving beyond the mildly nonlinear regime and they undergo the so-called virialization process (violent relaxation). Then, the following question arises (Matarrese & Mollerach 1997): have the clusters produced a significant amount of gravitational radiation? Since the problems with the *analytical* calculation of the gravitational radiation coming from strongly nonlinear galaxy clusters has not been still solved, we have estimated that radiation numerically.

In a previous paper (Quilis et al. 1998b), the gravitational radiation released during the formation of an isolated galaxy cluster was analyzed. In this paper, we have extended that calculation to the case of the emission coming from spatial distributions of galaxy clusters, thus we have performed, for the first time, a quantitative nonlinear calculation of this emission. Hence, we have obtained a numerical estimate of the contribution to the stochastic gravitational-wave background due to a

given spatial distribution of galaxy clusters. Although the resulting emission has appeared to be weak in our calculations, we want to stress that, at least from the theoretical point of view, it is worthy to know the main features of that radiation.

Cluster simulations including both baryonic and dark matter have undergone a high level of development (e.g. Kang et al. 1994, Metzler & Evrard 1994, Navarro et al. 1995, Anninos & Norman 1996, Gheller et al. 1998). The baryonic component is evolved either with an Eulerian or with a Lagrangian code, the treatment of the dark matter component is based on appropriate N-body techniques, and both components are gravitationally coupled through Poisson's equation, which is usually solved using the Fast Fourier Transform. Simulations based on a 3D Eulerian code (Quilis et al. 1998a) were used to estimate the gravitational radiation generated by individual galaxy clusters (Quilis et al. 1998b). Computations were performed in the framework of the standard Cold Dark Matter (CDM) scenario. The Hubble constant and the density parameter were assumed to be $H_0 = 50 \text{ Kms}^{-1}\text{Mpc}^{-1}$ and $\Omega_0 = 1$, respectively. The same assumptions and method have been used in the cluster simulations used in this paper (this procedure is justified below). Appropriate initial conditions have been chosen to get both rich and standard galaxy clusters. Our rich clusters have a X-ray luminosity (L_x) of the order of $\sim 10^{44} \text{ erg/s}$, a temperature $T \geq 3 \times 10^7 \text{ K}$ and, a total mass inside the Abell radius (3 Mpc) $M \geq 4 \times 10^{14} M_\odot$. Our standard cluster has $L_x \sim 10^{43} \text{ erg/s}$, $T \leq 6 \times 10^6$ and, $M \leq 10^{14} M_\odot$.

As it was discussed in Quilis et al. (1998b), the gravitational radiation from a galaxy cluster produces progressive deformations on some material systems, this is what was referred to as *the secular effect*. In the particular case of a system formed by two test particles, the deformation reduces to a relative variation of their separation distance. Since this variation is proportional to time, then, leaving aside the question on the characteristic frequency (crucial for detectability), it reaches values which are in the range of space-based laser interferometric observatories of gravitational waves.

There are various conclusions of Quilis et al. (1998b) to be taking into account here: (i) The secular effect is approximately proportional to D^{-1} , where D is the distance from the cluster to the observer, (ii) clusters located at a distance $D > 600 \text{ Mpc}$ only produce a small effect which is neglected in the simulations

of this paper, (iii) for distances $D < 600$ Mpc, gravitational waves can be considered as propagating in the Minkowskian space tangent to the Friedman-Robertson-Walker spacetime at the emission point. Although the secular effect is only significant in the near zone ($D < 600$ Mpc), no differentiation is made – along the paper – between this effect and gravitational waves (in the radiation zone), this is because, in both cases, the deformations produced on detectors have the same sources (galaxy clusters) and the same transverse signature and, also, because these deformations are estimated with the same formulae holding in both the near and the radiation zones, (iv) the secular effect produced by each cluster does not appear isolated, but superimposed to the effect due to other clusters, and (v) the internal dynamics of clusters having similar features (L_x , T , and M) is expected to depend on the cosmological parameters (Ω_0 , H_0, \dots) weakly, while the spatial distributions of clusters is sensitive to these parameters. Consequently with this last point, we have fixed Ω_0 and H_0 for simulating individual clusters, while the chosen spatial distributions mimic some features of cluster catalogues.

2. Simulation strategy and results

Only very crude simulations of the total secular effect produced by cluster distributions were given in Quilis et al. (1998b), where some numbers were found with the essential aim of proving that the effects of many clusters do not cancel among them. The main goal of this paper is, exactly, this one: to present improved calculations showing the features of the signal coming from an appropriate spatial distribution of galaxy clusters. This spatial distributions of clusters could be chosen either mimicking the observed one or simulating them in a certain scenario of structure formation in the Universe. In any case, the distribution can be restricted inside a sphere of 600 Mpc radius. According to Dalton et al. (1994), the analysis of the APM galaxy survey leads to a mean density of rich clusters of $4.25 \times 10^{-6} \text{ Mpc}^{-3}$ and a two-point correlation function of the form $\xi_{cc} = (r_0/r)^2$, with $r_0 = 28.6$ Mpc. All the distributions of rich galaxy clusters used in this paper – to estimate the total secular effect – have been constrained (Pons-Borderia et al. 1999) to have these features.

In order to give a complete and clear physical description of the secular effect, a simple detector formed by two test particles, A and B, is appropriate. Let us analyze the response of this detector to the gravitational waves from a cluster (secular effect). In the Transverse Traceless (TT) gauge, the relative motion of these particles is fully given (Misner et al. 1973) by the quantities h_{ij}^{TT} , which describe a propagating small perturbations of the space-time structure (gravitational wave). Direction $x^3 \equiv z$ coincides with the line of sight of the cluster producing the gravitational radiation, while directions $x^1 \equiv x$ and $x^2 \equiv y$ are in a plane orthogonal to this line. There are four nonvanishing components of h_{ij}^{TT} satisfying the relations $h_{xy}^{TT} = h_{yx}^{TT}$ and $h_{yy}^{TT} = -h_{xx}^{TT}$; hence, only $h_{xx}^{TT} \equiv h_+$ and $h_{xy}^{TT} \equiv h_\times$ are independent quantities defining two polarization states. In TT gauge, there is a system of coordinates attached to A, in

which the coordinate of the particle B undergoes the following variations from present time t_0 to time $t_0 + \Delta t$:

$$\Delta x = \frac{1}{2} \{x_0 \dot{h}_+(t_0) + y_0 \dot{h}_\times(t_0)\} \Delta t \quad (1)$$

$$\Delta y = \frac{1}{2} \{x_0 \dot{h}_\times(t_0) - y_0 \dot{h}_+(t_0)\} \Delta t \quad (2)$$

$$\Delta z = 0 \quad (3)$$

where Δt is much smaller than the period of the gravitational waves emitted by clusters ($\Delta t \ll 10^9$ yr), the subscript “0” stands for the initial coordinates of the particle B at time t_0 , the overdot stands for a time derivative and, the quantities \dot{h}_+ and \dot{h}_\times are computed at point A and at present time.

Each cluster produces a secular effect described by Eqs. (1) – (3). These equations show that the effect is proportional to Δt . The estimate of Δx and Δy requires the knowledge of \dot{h}_+ and \dot{h}_\times for the chosen cluster. An explicit computation of these quantities is only possible if cluster evolution is known, as it occurs in the case of numerically simulated clusters. The total secular effect produced by a distribution of clusters can be obtained from the \dot{h}_+ and \dot{h}_\times quantities corresponding to each cluster. Obviously, the simulation of each one of the ~ 4000 rich clusters located inside a sphere of 600 Mpc radius is out of current computational capabilities; therefore, some type of statistical treatment of the problem is necessary. Quantities \dot{h}_+ and \dot{h}_\times must be assigned to each one of the clusters belonging to some spatial distribution using adequate criteria. Let us motivate these criteria listing various considerations:

(a) The gravitational radiation from a cluster is the superposition of the radiation produced by the motion of many particles of dark and baryonic matter inside the cluster; hence, low levels of polarization are expected and the assumption that quantities \dot{h}_+ and \dot{h}_\times are independent is good enough. Furthermore, clusters radiate incoherently and, consequently, the waves from different clusters have distinct uncorrelated phases. At emission time, each cluster should be in an evolution state different from and independent on the state of any other cluster.

(b) Several simulations of rich clusters have been analyzed in order to define an interval where the values of \dot{h}_+ and \dot{h}_\times are distributed. Let us assume a sphere of 100 Mpc radius centered in one of our simulated clusters. We could observe the central cluster from any point on the sphere. The radius passing through the observation point would be our z axis and, taking into account the evolution law of the cluster – which has been numerically simulated –, quantities \dot{h}_+ and \dot{h}_\times can be computed for each observation point. Then, a great number of these points can be considered and the maximum and minimum of these quantities can be easily estimated. The mean of the maxima obtained from all the simulated rich clusters, $\bar{\dot{h}} \sim \langle \dot{h}_+(max) \rangle \sim \langle \dot{h}_\times(max) \rangle$, has appeared to be $\sim 9. \times 10^{-21} \text{ yr}^{-1}$; hence, we assume that quantities \dot{h}_+ and \dot{h}_\times are distributed in the interval $(-\bar{\dot{h}}, \bar{\dot{h}})$.

(c) The same study has been done for standard clusters. In this case, the resulting $\bar{\dot{h}}$ value is $\sim 1. \times 10^{-21} \text{ yr}^{-1}$; hence, rich clusters produces an effect which is one order of magnitude

greater than that of the standard clusters. On account of this fact, only the distributions of rich clusters are considered in this paper; nevertheless, more work should be done to estimate the contribution of standard and small clusters, which are abundant structures producing weak secular effects.

After these comments, the following method seems to be appropriate to give values to \dot{h}_+ and \dot{h}_\times : According to (a), these quantities are generated as statistically independent numbers and, on account of (a) and (b), each of these quantities is assumed to be a random number (η for h_+ and ξ for h_\times) uniformly distributed in the interval $(-1,1)$ multiplied by the mean value \bar{h} . Finally, if a given cluster is not located at 100 Mpc from the observer, but at a distance D , number \bar{h} must be multiplied by the factor $100/D$. These criteria plus Eqs. (1) – (3) allow us to compute the relative variation of the distance AB produced by an arbitrary cluster located at distance D (in Mpc) from the detector. After trivial algebra, the following key equation is easily found:

$$\frac{\Delta l}{l} = \pm \frac{50}{D} \dot{h} (\eta^2 + \xi^2)^{1/2} (\sin^2 \theta) \Delta t \quad (4)$$

where θ is the angle formed by the segment AB and the cluster line of sight. For $\theta = 0$, the segment AB is aligned with the line of sight and no deformation is produced at all. In the case $\theta = \pi/2$, the segment AB is orthogonal to the line of sight and the cluster produces a maximum $\frac{\Delta l}{l}$ independent on the orientation of the AB segment in the plane orthogonal to the line of sight.

Given a spatial distribution of clusters and an orientation of the segment AB, Eq. (4) allows us to assign a small $\frac{\Delta l}{l}$ to each cluster and, then, all these values must be added to find the total secular effect produced by the cluster distribution. If this distribution is not altered, but the orientation of the segment AB is changed, the total secular effect measured by the detector changes (*anisotropy*). According to Eq. (4), this change occurs because numbers η , ξ , and \bar{h} keep unaltered, but the angle θ corresponding to each cluster varies. Given a spatial distribution of rich clusters, we have all the ingredients necessary to generate full maps of the sky; namely, maps where all the AB orientations are considered. These maps completely describe the anisotropy of the total secular effect.

One map of the full sky is displayed in Fig. 1, where quantity $\mu = l^{-1} \frac{\Delta l}{\Delta t}$ is given for all the directions joining the center of a sphere (A) with the points (B) located on a hemisphere. There are anisotropies in the sense that, for the chosen cluster distribution, the μ value –displayed by the grey scale– depends on the orientation of the segment AB in the space. The map of this Figure has been expanded in spherical harmonics turning out in the superposition of a monopole $a_{0,0} = -8.17 \times 10^{-20}$ and the following independent quadrupole components: the real component $a_{2,0} = -1.38 \times 10^{-20}$, and the complex ones $a_{2,2} = a_{2,-2}^* = (1.44 \times 10^{-20}, -1.78 \times 10^{-20})$ and $a_{2,1} = -a_{2,-1}^* = (-4.25 \times 10^{-20}, 1.81 \times 10^{-20})$. Any other multipole appears to be negligible. These numerical results are easily understood taking into account Eq. (4), where the depen-

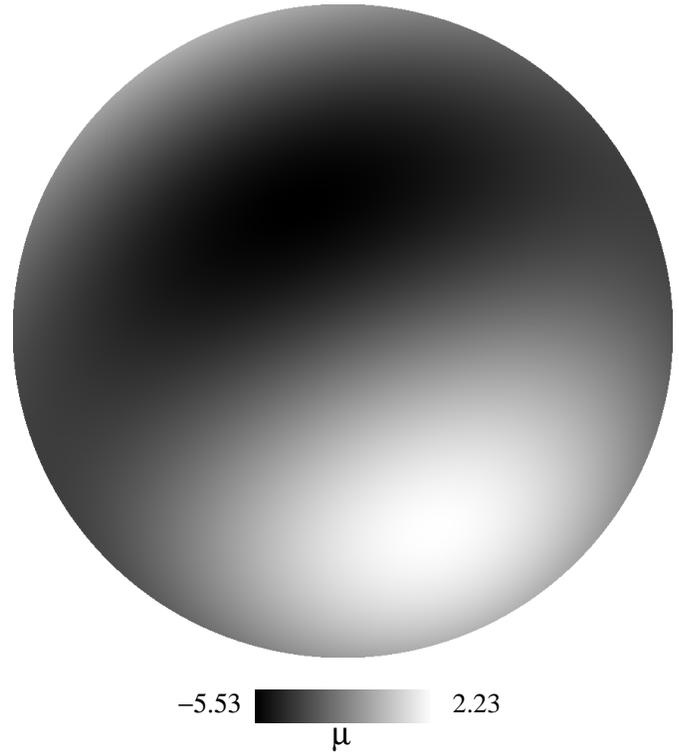


Fig. 1. Map of the secular effect produced by a distribution of rich clusters. The quantity $\mu \times 10^{20}$, with $\mu = l^{-1} \frac{\Delta l}{\Delta t}$ in units of yr^{-1} , is plotted for a semisphere. Lambert's projection is used.

dence on $\sin^2 \theta$ shows that the effect of each cluster is the superposition of a monopole and a quadrupole. We can conclude that the total secular effect can be completely described by the six components of the quantities $(a_{0,0}, a_{2,2}, a_{2,1}, a_{2,0})$, which depend on the chosen cluster distribution.

Various simulations have been done using different distributions of clusters. Many orientations of the segment AB have been considered, and the mean and standard deviation σ of the predicted μ values have been calculated. The mean value of μ changes from simulation to simulation and the standard deviation is a rather stable quantity. This quantity, which measures typical deviations with respect to the mean and, consequently, anisotropy, ranges in the interval $(1. \times 10^{-20} \text{ yr}^{-1}, 2. \times 10^{-20} \text{ yr}^{-1})$. In the worst case, $\frac{\Delta l}{l} = \mu \Delta t$ takes on the values $\sim 10^{-26}$, $\sim 10^{-22}$ and $\sim 10^{-19}$ in time intervals of 30 s, 3.6 days and a decade, respectively.

3. Discussion and speculations on observational strategies

The efficiency of the gravitational wave emission from a rich cluster is very low, $\approx 10^{-16}$ (see Quilis et al. 1998b). This means that during all the age of the Universe a single cluster would radiate a gravitational energy of $\approx 10^{-16}$ times the cluster mass. The contribution of that energy to the present density parameter, Ω_{gw} , would be of the order of $\approx 10^{-16} \Omega_c$, being Ω_c the contribution of all the clusters to the density parameter. That rough argument suggests that Ω_{gw} would range between $10^{-17} - 10^{-18}$. A very low value.

The gravitational waves from galaxy clusters produce anisotropies on the Cosmic Microwave Background (CMB). Other gravitational waves generated during inflation or in another early process would lead to “primary” CMB anisotropies because they were present at recombination time; however, the gravitational waves coming from galaxy clusters were emitted after recombination and they can only produce “secondary” anisotropies, which are due to the motion of the CMB photons in the time varying gravitational field associated to these waves. Although these anisotropies exist (detailed estimates are in progress), they are expected to be too small –for detection– due to the low values of Ω_{gw} given above. This means that observations of the CMB seem not to be appropriate for detecting the gravitational background from galaxy clusters. Could we use interferometry, as in standard detectors of gravitational waves, to detect this background?

3.1. Some clues for future observational strategies

Measurements of the total secular effect do not require a fixed orientation of the AB segment; suppose, for instance, that particles A and B move with the terrestrial equator; then, the AB direction depends on time in a well known way and it covers all the directions of the equatorial plane during a day. Every day, the total relative variation $\Delta l/l$ is given by the integral

$$(\Delta l/l)_{\text{day}} = \frac{1}{W} \int_0^{2\pi} \mu(\beta) d\beta \quad (5)$$

where angle β defines the AB direction inside the equatorial plane and W is Earth’s angular velocity. We have fixed a cluster distribution and, then, various planes playing the role of the equatorial one have been considered. See, for instance, Fig. 2, where functions $\mu(\beta)$ are displayed for three of these planes. Using a large number of planes and Eq. (5), many possible values of $(\Delta l/l)_{\text{day}}$ have been calculated. The mean of these values and the standard deviation with respect to it are 4.93×10^{-23} and 1.8×10^{-23} . Other cluster distributions have been considered and the means and standard deviations appear to be different; nevertheless, the orders of magnitude keep unaltered. We can conclude that the detector would undergo successive pulses with amplitudes of the order of 10^{-23} and periods of 6 hours (the period is $T = \pi/2W$, see Fig. 2). The pulse appears as a result of the detector rotation in the anisotropic gravitational field created by the clusters.

Since the sensitivity of laser interferometry-based detectors decreases with the signal frequency ν , they cannot observe –directly– secular effects with almost negligible ν ; nevertheless, as discussed above, *rotating* detectors would receive effective pulses – with frequency $\nu = 2W/\pi$ – whose possible detectability would require further analysis. The angular velocity of the detector fixes the pulse frequency $2W/\pi$ and its amplitude (which is proportional to W^{-1} according to Eq. (5)). According to our estimates, the amplitude reaches the order 10^{-24} for pulse frequencies of the order of 10^{-3} . As the angular velocity increases, the pulse frequency increases – gaining in sensitivity – but the pulse amplitude decreases. Taking into

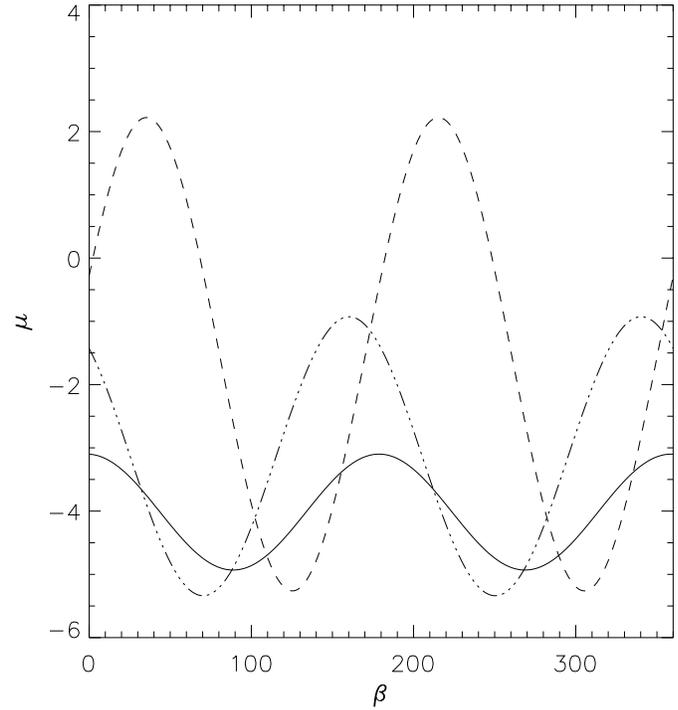


Fig. 2. Plot of the quantity $\mu \times 10^{20}$ (in yr^{-1}) as a function of the angle β (in degrees) defining the orientation of the AB segment inside a fixed plane. Three different planes have been chosen at random.

account these considerations and the fact that interferometry is a very accurate technique, the design of specific experiments for detecting the above systematic and repetitive pulses –produced by the clusters on rotating detectors– deserves attention.

3.2. Cosmological consequences

The secular effect and its anisotropy are expected to be dependent on the value of some cosmological parameters (see above). Distinct values of these parameters would lead to different spatial distributions of clusters and, then, to distinct secular signals. For instance, the dependence of the secular effect on the Hubble constant can be easily analyzed. As this constant changes, all the distances are multiplied by the factor H_0^{-1} , and the angles (θ) between the AB segment and the cluster lines of sight do not change; hence, the secular effect, which is roughly proportional to D^{-1} , is approximately proportional to H_0 . The secular signals could be predicted for a wide range of values of the involved parameters and, then, comparisons of the resulting predictions and future observations could give new bounds on some of these parameters. These bounds would be very interesting from the cosmological point of view; however, taking into account that: (i) the secular effect is small and, (ii) its detection is not expected to be easy, our considerations about the cosmological consequences of detecting the gravitational radiation from clusters have to be read wisely. Other observations could be much more effective to constraint the cosmological parameters; for instance, CMB observations from future spatial missions as MAP and PLANCK.

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