Astron. Astrophys. 353, 473–478 (2000)

ASTRONOMY AND ASTROPHYSICS

Particle acceleration by rotating magnetospheres in active galactic nuclei

F.M. Rieger and K. Mannheim

Universitäts-Sternwarte, Geismarlandstrasse 11, 37083 Göttingen, Germany

Received 15 July 1999 / Accepted 20 October 1999

Abstract. We consider the centrifugal acceleration of charged test particles by rotating magnetospheres which are widely believed to be responsible for the relativistic jet phenomenon in active galactic nuclei (AGN). Based on an analysis of forces the equation for the radial accelerated motion is derived and an analytical solution presented under the assumption of an idealized spherical magnetosphere. We show that the rotational energy gain of charged particles moving outwards along rotating magnetic field lines is limited in general by (i) inverse-Compton losses in the radiation field of the disk in which the magnetosphere is anchored and (ii) the breakdown of the bead-on-the-wire approximation which occurs in the vicinity of the light cylinder. The corresponding maximum Lorentz factor for electrons is of the order of a few hundred for the sub-Eddington conditions regarded to be typical for BL Lacs. In AGN with enhanced accretion rate the acceleration mechanism seems to be almost inefficient due to increasing inverse-Compton losses.

Key words: acceleration of particles – radiation mechanisms: non-thermal – gamma rays: theory – galaxies: active

1. Introduction

The origin of the nonthermal, highly variable emission in active galactic nuclei (AGN) has been widely discussed. Several acceleration mechanisms have been proposed which may explain the observed high energy emission extending up to TeV energies at least in three blazars (Mkn 421, Mkn 501, 1ES 2344+514: e.g. Catanese 1999). Fermi-type particle acceleration mechanisms in relativistic jets seem to be very effective but require a seed population of electrons with Lorentz factors of at least 100. Up to now, it remains a problem to be solved, how this pre-acceleration is achieved (e.g. Kirk et al. 1994).

Since the pioneering work of Gold in the late 1960s (Gold 1968, 1969), centrifugal driven outflow of matter has often been discussed in the context of pulsar emission theory (for recent contribution see e.g. Machabeli & Rogava 1994; Chedia et al. 1996; Gangadhara 1996; Contopoulos et al. 1999). In the case of accreting black hole systems (e.g. AGN) Blandford and Payne (1982) first pointed out that centrifugal driven outflows (jets) from accretion disks are possible, if the poloidal field direction is inclined at an angle less than $60°$ to the radial direction. In such models, a rotating magnetosphere could emerge from an accretion disk or the rotating black hole itself (Blandford & Znajek 1977) initiating a plasma outflow with initially spherical shape until the flow is collimated on a scale of less than a few hundred Schwarzschild radii (e.g. Camenzind 1995, 1996; Fendt 1997). For a rapidly rotating black hole system, the critical angle mentioned above could be as large as $90°$ (Cao 1997).

In magnetohydrodynamical scenarios for the origin of relativistic jets, centrifugal acceleration is rather limited, leading to maximum bulk Lorentz factors of the order of 10 (Camenzind 1989). Nevertheless, it seems quite interesting whether supra-thermal test particles (e.g. from magnetic flares on the accretion disk) could be accelerated to even higher energies by such rotating magnetospheres. Recently, Gangadhara & Lesch (1997) proposed a model for spinning active galactic nuclei in which charged test particles are accelerated to very high energies by the centrifugal force while moving along rotating magnetic field lines. According to their calculations, the nonthermal X-ray and γ-ray emission in AGN could arise via the inverse-Compton scattering of UV-photons by centrifugal accelerated electrons.

In this paper, we reinvestigate the acceleration of charged test particles in an idealized two-dimensional model where the magnetic field rotates rigidly with a fraction of the rotational velocity of the black hole (cf. Fendt 1997). Centrifugal acceleration occurs as a consequence of the bead-on-the-wire motion. A charged particle gains rotational energy as long as it is directed outwards but we show that its energy gain is substantially limited not only by inverse-Compton losses but also by the effects of the relativistic Coriolis force.

Based on an analysis of forces, the special relativistic equation of motion is derived in Sect. 2. This equation is solved in closed form in Sect. 3. Sect. 4 gives an estimate for the maximum Lorentz factor attainable in the case of AGN. The results are discussed in the context of the particle acceleration problem for rotating AGN jets in Sect. 5.

2. Analysis of forces in a rotating reference frame

Usually the motion of a particle along rotating magnetic field lines is treated in the bead-on-the-wire approximation where a bead is assumed to follow the rotating field line and experiences centrifugal acceleration (or deceleration) while moving in the

Send offprint requests to: F.M. Rieger (frieger@uni-sw.gwdg.de)

outward direction (e.g. Machabeli & Rogava 1994; Chedia et al. 1996; Cao 1997). This simple approach yields interesting results, though, as we will show further below, such an approximation breaks down in the region near the light cylinder.

Let us consider the forces acting on a particle in a rotating frame of reference (Gangadhara 1996; Gangadhara & Lesch 1997). A particle with rest mass m and charge q, which is injected at time t_0 and position r_0 with initial velocity v_0 parallel to the magnetic field line $B_{\rm r}(t_0)$ experiences a centrifugal force in the radial direction given by

$$\boldsymbol{F}_{\rm cf} = m\,\gamma\,(\boldsymbol{\Omega}\times\boldsymbol{r})\times\boldsymbol{\Omega}\,, \tag{1}$$

where γ is the Lorentz factor of the particle and $\boldsymbol{\Omega} = \Omega\,\boldsymbol{e}_z$ is the angular velocity of the field. Additionally, there is also a relativistic Coriolis force in the noninertial frame governed by the equation

$$\boldsymbol{F}_{\rm cor} = m\,\left(2\,\gamma\frac{dr}{dt} + r\,\frac{d\gamma}{dt}\right)\,(\boldsymbol{e}_{\rm r}\times\boldsymbol{\Omega})\,, \tag{2}$$

which acts as a deviation-force in the azimuthal direction. In the inertial rest frame the particle sees the field line bending off from its initial injection position. Hence, it experiences a Lorentz force, which may be written as

$$\boldsymbol{F}_{\rm L} = q\,(\boldsymbol{v}_{\rm rel}\times\boldsymbol{B})\,, \tag{3}$$

where $v_{\rm rel}$ is the relative velocity between the particle and the magnetic field line and where the convention $c = 1$ is used. Due to the Lorentz force a charged particle tries to gyrate around the magnetic field line. Initially, the direction of the Lorentz force is perpendicular to the direction of the Coriolis force, but as a particle gyrates, it changes direction and eventually becomes antiparallel to the Coriolis force. Hence one expects that the bead-on-the-wire approximation holds, if the Lorentz force is not balanced by the Coriolis force. In this case, the accelerated motion of the particle's guiding center due to the centrifugal force is given by

$$\gamma\frac{{\rm d}^2r}{{\rm d}t^2} + \frac{{\rm d}r}{{\rm d}t}\,\frac{{\rm d}\gamma}{{\rm d}t} = \gamma\,\Omega^2\,r\,, \tag{4}$$

where r denotes the radial coordinate and $\gamma = 1/\sqrt{1-\Omega^2r^2-\dot{r}^2}$. The bead-on-the-wire motion for the guiding center breaks down, if the Coriolis force exceeds the Lorentz force, i.e. if the following inequality, given by the azimuthal components of the forces, holds:

$$\frac{{\rm d}\gamma}{{\rm d}t} > \frac{1}{r}\left(\frac{B\,q\,v_{\rm rel}}{m\,\Omega} - 2\,\gamma\frac{{\rm d}r}{{\rm d}t}\right)\,. \tag{5}$$

3. Analytic solution for the radial acceleration

The general solution of Eq. (4) can be found using the simple argument that the energy E of the particle in the rotating reference frame is constant. If E_0 denotes the energy of the particle in the inertial rest frame, then the energy E in the uniformly rotating frame (angular velocity Ω) is given in the non-relativistic case by: $E = E_0 - m\,\Omega^2\,r^2$ (e.g. Landau & Lifshitz 1960). The generalisation of this equation to the relativistic case is straightforward and leads to the transformation

$$E = \gamma\,m\,(1-\Omega^2\,r^2)\,, \tag{6}$$

where γ is the Lorentz factor defined above.

Assume now, that in the general case a particle is injected at time $t = t_0$ and position $r = r_0$ with initial velocity $v = v_0$. Then, from Eq. (6) it follows that the time-derivative of the radial coordinate r may be written as

$$\frac{{\rm d}r(t)}{{\rm d}t} = \sqrt{(1-\Omega^2r^2)[1-\tilde{m}\,(1-\Omega^2r^2)]}\,, \tag{7}$$

where $\tilde{m} = (1-\Omega^2r_0^2-v_0^2)/(1-\Omega^2r_0^2)^2$. In the case $r_0 = 0$, this expression reduces to the equation given in Henriksen & Rayburn (1971).

The equation for the radial velocity $v_{\rm r} = {\rm d}r/{\rm d}t$, Eq. (7), can be solved analytically (Machabeli & Rogava 1994), yielding

$$r(t) = \frac{1}{\Omega}\,{\rm cn}(\lambda_0 - \Omega\,{\rm t})\,, \tag{8}$$

where ${\rm cn}$ is the Jacobian elliptic cosine (Abramowitz & Stegun 1965, p.569ff), λ_0 a Legendre elliptic integral of the first kind:

$$\lambda_0 = \int_0^{\phi_0}\frac{{\rm d}\theta}{(1-\tilde{m}\,\sin\theta)^{1/2}}\,, \tag{9}$$

and where ϕ_0 is defined by $\phi_0 = \arccos(\Omega\,r_0)$. By using Eq. (8), the time-derivative of r can be expressed as $\dot{r} = {\rm dn}(\lambda_0-\Omega\,{\rm t})\,{\rm sn}(\lambda_0-\Omega\,{\rm t})$. Note that the Jacobian elliptic functions ${\rm sn}$ and ${\rm dn}$ are usually defined by the relations ${\rm sn}^2+{\rm cn}^2 = 1$ and $\tilde{m}\,{\rm sn}^2 + {\rm dn}^2 = 1$.

Using Eq. (7), the Lorentz factor may be written as a function of the radial coordinate r:

$$\gamma = \frac{1}{\sqrt{\tilde{m}}\,(1-\Omega^2r^2)}\,. \tag{10}$$

Thus, in terms of Jacobian elliptic functions one gets

$$\gamma = \frac{1}{\sqrt{\tilde{m}}\,[{\rm sn}(\lambda_0-\Omega\,{\rm t})]^2}\,. \tag{11}$$

For the particular conditions where the injection of a test particle is described by $r(t_0 = 0) = 0$ and $v(t_0 = 0) = v_0$, the time-dependence of the radial coordinate is given by a much simpler expression: In this situation, λ_0 reduces to a complete elliptic integral of the first kind and therefore, after a change of the arguments, the time-dependence of the radial coordinate becomes

$$r(t) = \frac{v_0\,{\rm sn}(\Omega\,{\rm t})}{\Omega\,{\rm dn}(\Omega\,{\rm t})}\,. \tag{12}$$

If one considers non-relativistic motions, where $\tilde{m} \simeq 1$, and the special case $r_0 = 0$, Eq. (8) reduces to (cf. Abramowitz & Stegun 1965)

$$r(t) = v_0\tanh(\Omega\,{\rm t})/\Omega\,{\rm sech}(\Omega\,{\rm t}) = v_0\sinh(\Omega\,{\rm t})/\Omega\,. \tag{13}$$

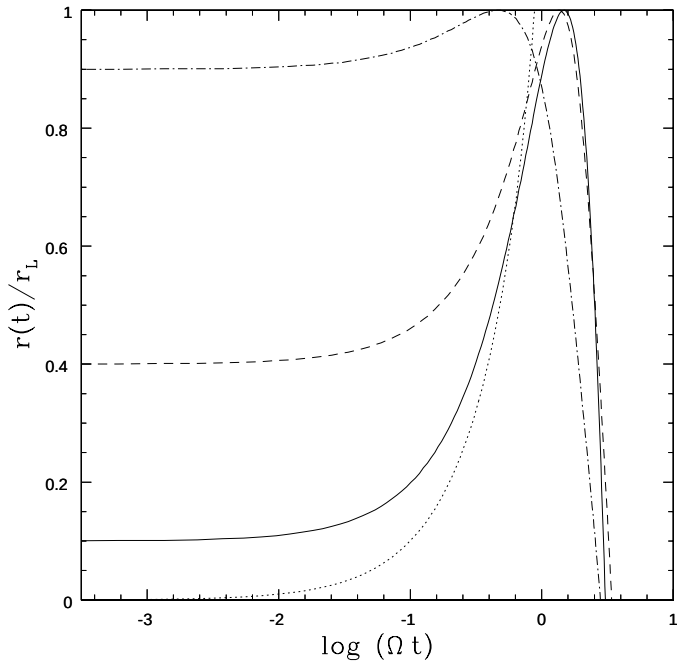

Fig. 1. The time-dependence of the radial coordinate r, plotted for the initial conditions $v_0 = 0.99\,c$ and $r_0 = 0.1\,r_{\rm L}$ (solid line), $v_0 = 0.6\,c$ and $r_0 = 0.4\,r_L$ (short dashed), $v_0 = 0.4\,c$ and $r_0 = 0.9\,r_L$ (dotted-short dashed); also indicated is the non-relativistic limit: $r(t)\,\Omega/v_0 = \sinh(\Omega\,{\rm t})$ (dotted).

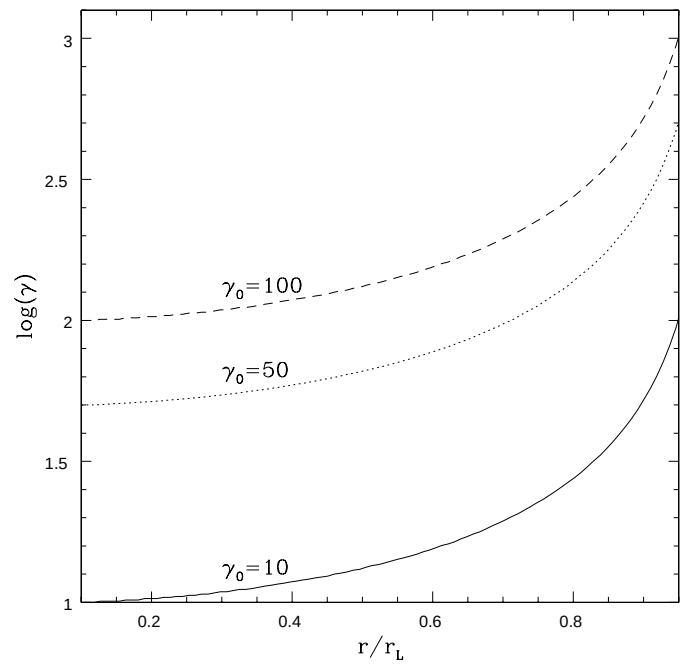

Fig. 2. The relativistic Lorentz factor γ for a particle approaching the light cylinder $r_{\rm L}$ using $r_0 = r_{\rm L}/10$ and injection Lorentz factors $\gamma_0 = 10$ (solid line), $\gamma_0 = 50$ (dotted) and $\gamma_0 = 100$ (dashed).

This expression is known to be the general solution of the equation

$$\ddot{r} - \Omega^2\, r = 0\,, \tag{14}$$

which describes the motion of a particle due to the centrifugal force in the non-relativistic limit. In Fig. 1, we compute the time-dependence of the radial coordinate r for different initial conditions under the (unphysical) assumption that the bead-on-the-wire motion continues until the light cylinder (with radius $r_{\rm L}$) is reached. Note that in the relativistic case all particles would turn back at the light cylinder due to the reversal of the centrifugal acceleration (e.g. Machabeli & Rogava 1994).

Using the definition of the Lorentz factor, the equation for the accelerated motion, Eq. (4), may also be written as

$$\frac{{\rm d}^2 r}{{\rm d}t^2} = \frac{\Omega^2 r}{1-\Omega^2\, r^2}\left[1 - \Omega^2\, r^2 - 2\left(\frac{{\rm d}r}{{\rm d}t}\right)^2\right]\,. \tag{15}$$

(cf. Chedia et al. 1996; Kahniashvili et al. 1997). By inserting the above relations, the solution for the radial acceleration could be expressed in terms of the Jacobian elliptic functions:

$$\ddot{r} = \Omega\,\cdot\,{\rm cn}(\lambda_0 - \Omega\,{\rm t})\,[1 - 2\,{\rm dn}^2(\lambda_0 - \Omega\,{\rm t})]. \tag{16}$$

According to our simple model, one expects that a charged test particle gains energy due to rotational motion as long as it is directed outwards. Therefore the relativistic Lorentz factor increases with distance r as the particle approaches the light cylinder. This is illustrated in Fig. 2 where we plot the evolution of the relativistic Lorentz factor γ as a function of r for different initial velocities v_0 and fixed $r_0 = r_{\rm L}/10$ (using a typical light cylinder radius of $r_{\rm L} \simeq 10^{15}$cm). Note that $\gamma(r/r_{\rm L})$ is not scale-invariant with respect to the injection velocity v_0 (i.e. the injection energy). If one identifies Eq. (15) with the general expression for the centrifugal force, which reduces in the non-relativistic limit to the well-known classical expression, the centrifugal force changes its signs and becomes negative for $r^2/r_{\rm L}^2 > 1 - (2\,\tilde{m})^{-1}$ (see Fig. 3; cf. also Machabeli & Rogava 1994). Hence, if one assumes that the bead-on-the-wire approximation holds in the vicinity of the light cylinder, the radial velocity becomes zero at the light cylinder and changes direction in any case. Accordingly, a crossing of the light cylinder within the bead-on-the-wire approximation, as mentioned for example in Gangadhara & Lesch 1997, is not physical (cf. Fig. 1). The reversal of the direction of centrifugal force according to which the centrifugal force may attract rotating matter towards the centre is well-known in strong gravitational fields (for Schwarzschild geometry: Abramowicz 1990; Abramowicz & Prasanna 1990; for Kerr geometry: Iyer & Prasanna 1993; Sonego & Massar 1996). For illustration, we compute in Fig. 3 the evolution of the effective radial acceleration $a_{\rm r} = {\rm d}^2 r/{\rm d}t^2$ as a function of the radial coordinate r for different initial velocities. Obviously, there exists a point where the effective acceleration, i.e. the centrifugal force, becomes negative.

4. Estimate of the maximum Lorentz factor

Consider now the acceleration of electrons via rotating magnetospheres in AGN. Imagine an electron which moves along a rotating magnetic field line towards the light cylinder. Gener-

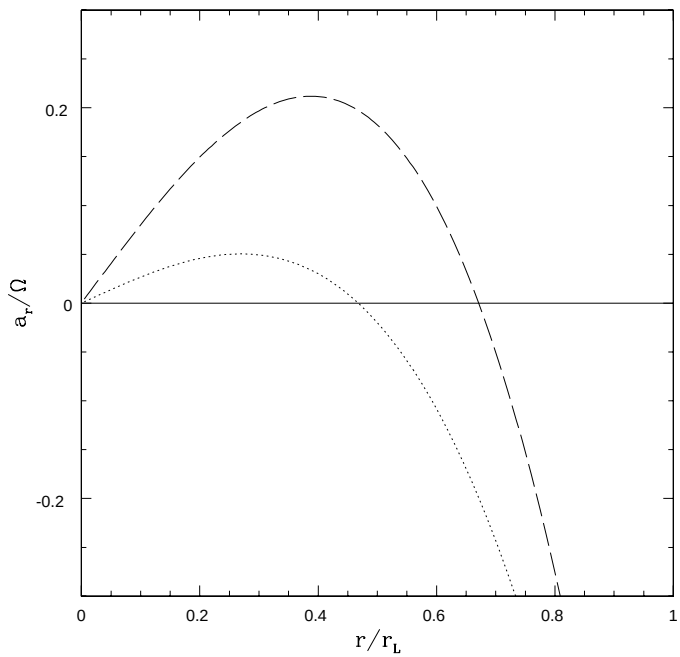

Fig. 3. The radial acceleration $a_{\rm r}$ as a function of $r/r_{\rm L}$ for the initial conditions $r_0 = 0$ and $v_0 = 0.3\,c$ (dashed), $v_0 = 0.6$ c (dotted).

ally, one expects that there are two processes which could limit the energy gain of a particle:

First, there are inverse-Compton energy losses due to interaction with accretion disk photons: low energy accretion disk photons are scattered to higher energies by the accelerated electrons so that the photons gain energy while the electrons lose energy. Near the disk the electrons might encounter a very strong disk radiation field, which substantially limits the maximum attainable energy (this need not be the case if electrons are accelerated far away from the disk, e.g. Bednarek et al. 1996). The maximum energy, which an electron is able to reach under the influence of inverse-Compton scattering is given at the point where the acceleration time scale equals the cooling time scale. In the case, where the energy of the photon in the electron rest frame is small compared to the energy of the electron (Thomson scattering), the cooling time scale for inverse-Compton losses can be approximated by (e.g. Rybicki & Lightman 1979)

$$t_{\rm cool}^{\rm IC} = 3 \times 10^7 \, \frac{\gamma}{(\gamma^2 - 1)\, U_{\rm rad}} \, [{\rm s}] \,, \tag{17}$$

where $U_{\rm rad} = \tau\, L_{\rm disk}/4\,\pi\, r_{\rm L}^2$ is the energy density of the disk radiation field and $\tau \leq 1$.

If one uses Eq. (10), the acceleration time scale $t_{\rm acc}$ may be written as:

$$t_{\rm acc} = \gamma/\dot{\gamma} = \frac{\sqrt{1 - \Omega^2\, r^2}}{2\,\Omega^2\, r\, \sqrt{1 - \tilde{m}\,(1 - \Omega^2\, r^2)}} \,. \tag{18}$$

By equating this two time scales we obtain an estimate for the maximum electron Lorentz factor $\gamma_{\rm max}$.

A second, general constraint, which was neither considered by Machabeli & Rogava (1997) nor used in the calculation by Gangadhara & Lesch (1997), is given by the breakdown of the bead-on-the-wire approximation which occurs in the vicinity of the light cylinder. Beyond this point, where the Coriolis force exceeds the Lorentz force [see condition Eq. (5)], the particle leaves the magnetic field line so that the rotational energy gain ceases. Hence the acceleration mechanism becomes ineffective. In the case of AGN, where the magnetic field strength is much smaller than in pulsars, this constraint may be quite important.

For illustration, we apply our calculations in the following to a typical AGN using a central black hole mass $M_{\rm BH} = m_8\, 10^8\, M_\odot$ and a light cylinder radius $r_{\rm L} \simeq 10^{15} m_8$ cm, where $M_\odot$ denotes the solar mass. The Eddington luminosity, i.e. the maximum luminosity of a source of mass $M_{\rm BH}$ which is powered by spherical accretion, is given by $L_{\rm Edd} \simeq 10^{46}\,{\rm ergs\ s^{-1}}$. Typically, we may express the disk luminosity as $L_{\rm disk} = l_{\rm e} \times L_{\rm Edd}$, with $10^{-4} < l_{\rm e} \leq 1$. Thus, the equipartition magnetic field strength at the radius r is given by $B(r)^2 = 2\, L_{\rm disk}/r^2$. Electrons are assumed to be injected at an initial position $r_0 \simeq 0.4\, r_{\rm L}$ with a characteristic escape velocity from the last marginally stable orbit around a black hole of $v_0 \simeq 0.6$ c. By applying the two constraints above, we get three generic regimes for the acceleration of electrons by rotating magnetospheres:

1. the region, in which inverse-Compton losses dominate entirely over the energy gains, leading to an inefficient acceleration (generally in the case of Eddington accretion, i.e. $l_{\rm e} \sim 1$).
2. the region, in which inverse-Compton losses are important but not dominant (generally the sub-Eddington range: $l_{\rm e} \leq 2 \times 10^{-2}$). In this case the acceleration mechanism works, but there exists a maximum Lorentz factor given at the position where the energy gain is exactly balanced by losses. This is illustrated in Fig. 4, where we calculate the cooling and the acceleration time scale as a function of the Lorentz factor γ for $l_{\rm e} = 5 \times 10^{-3}$. For this value, the maximum Lorentz factor is roughly $\gamma \simeq 150$. Typically, the maximum Lorentz factors in this range are of the order of 100 to 1000 (see Fig. 5).
3. the region, in which the inverse-Compton losses are rather unimportant (generally $l_{\rm e} < 10^{-3}$). In this case, the maximum Lorentz factor is determined by the breakdown of the bead-on-the-wire approximation [see Eq. (5)], which yields a general upper limit for the Lorentz factor of the order of 1000. This limit is found if one approximates $v_{\rm rel}$ by the light velocity which amounts to the highest value for the Lorentz forces. The results are shown in Fig. 6, where we also allow the injection position to vary. We wish to note, that the results, presented in Fig. 6, depend essentially on the assumed intrinsic magnetic field strength and the size of the light cylinder radius (i.e. the angular velocity). Generally, for a sufficient approximation, the maximum Lorentz factor is given by:

$$\gamma_{\rm max} \simeq \frac{1}{\tilde{m}^{1/6}} \left(\frac{B(r_{\rm L})\, q}{2\, m\, \Omega\, c} \right)^{2/3} \,. \tag{19}$$

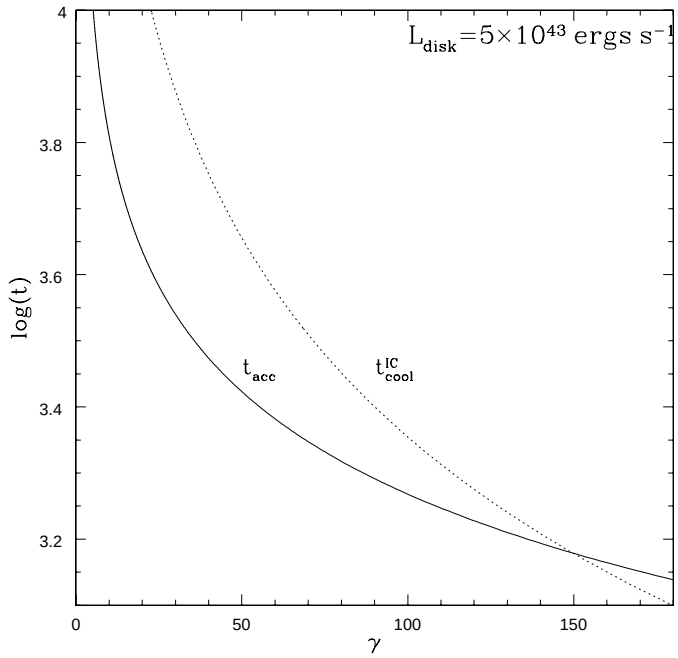

Fig. 4. Cooling times scale $t_{\rm cool}^{\rm IC}$ for inverse-Compton scattering, Eq. (17), and acceleration time scale $t_{\rm acc}$, Eq. (18), as a function of the Lorentz factor γ using $l_{\rm e} = 5 \times 10^{-3}$ and $\tau = 1$. The maximum electron Lorentz factor, given at the position where the cooling time scale equals the accelerations time scale, is approximately 150.

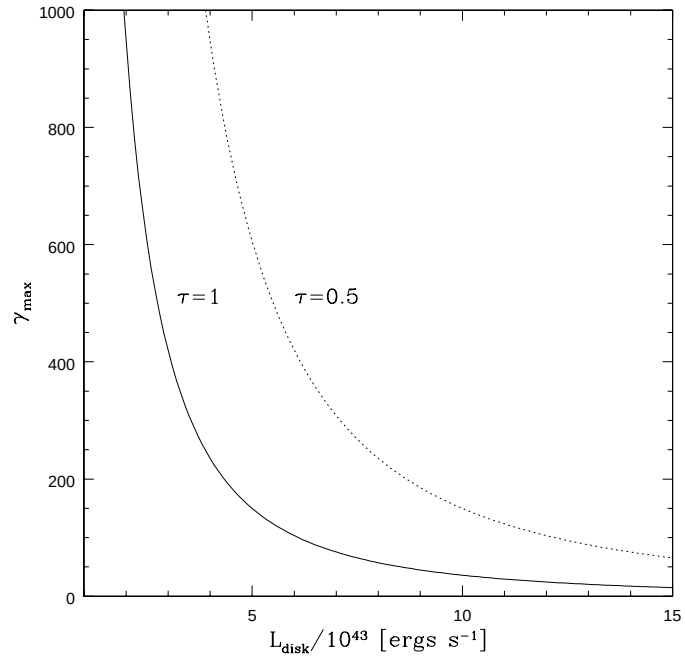

Fig. 5. Maximum electron Lorentz factor $\gamma_{\rm max}$ attainable under the influence of inverse-Compton losses as a function of the disk luminosity $L_{\rm disk}$ for $\tau = 0.5$ (dotted) and $\tau = 1$ (solid), where $\tau = 4\,\pi\, r_{\rm L}^2 U_{\rm rad}/L_{\rm disk}$ and $U_{\rm rad}$ being the energy density of the disk radiation field.

Thus, even if one uses a magnetic field strength of $B(r_{\rm L}) = 100\,{\rm G}$, which is roughly three times the corresponding equipartition field, the maximum Lorentz factor does not exceed 2.5×10^3.

5. Discussion

We have considered the acceleration of charged test particles via rotating magnetospheres based on a model topology which is motivated by the standard model for AGN (cf. Begelman 1994; Camenzind 1995; Fendt 1997). Accordingly, the jet magnetosphere originates very close to the central black hole from an accretion disk, with initially spherical profile until the relativistic jet is collimated to a cylindrical shape outside the light cylinder.

The centrifugal particle acceleration model described in this paper extends the calculations by Machabeli & Rogava (1994) and Gangadhara & Lesch (1997). We find that the maximum Lorentz factor attainable for an electron moving along a rotating magnetic field line is substantially limited not only by radiation losses (e.g. inverse-Compton) but also by the breakdown of the bead-on-the-wire approximation which occurs in the vicinity of the light cylinder. Due to these limiting effects, the acceleration of particles by rotating magnetospheres seems to be rather less important in the case of AGN. Our current calculations indicate, that for sub-Eddington accreting black holes, such as black holes with advection-dominated accretion flows (e.g. Narayan & Yi 1994; Narayan 1997), efficient pre-acceleration of electrons to Lorentz factors of the order of a few hundred might be possible, at least under the highly idealized conditions of our analytical toy model. It seems interesting that the highest energy gamma rays have been discovered from AGN of the BL Lac type which very likely accrete in a sub-Eddington mode (e.g. Celotti et al. 1998). Under such conditions, inverse-Compton scattering of accretions disk photons with energy ~ 0.1 keV produces gamma rays with a maximum energy of $\sim 100\,(\gamma_{\rm max}/10^3)^2$ MeV, which is in general too low to explain the observed high-energy gamma rays in blazars (e.g. Kanbach 1996; Catanese 1999).

We wish to mention that the results in this paper essentially depend on the assumed intrinsic magnetic field and the angular frequency Ω, i.e. on the size of the light cylinder radius ($\gamma_{\rm max} \propto (B/\Omega)^{2/3}$). Therefore, one could find a way out of the problem above, for example, by assuming a light cylinder radius in BL Lac type objects which is much greater than 10^{15} cm for a black hole mass of $M_{\rm BH} = 10^8\,M_\odot$. However, in view of magnetohydrodynamic models already existing, this seems to be rather improbable. In any case, the acceleration of suprathermal test particles by rotating magnetospheres might possibly provide an interesting explanation for the pre-acceleration which is required for efficient Fermi-type particle acceleration at larger scales in radio jets.

There are several restrictions on our approach, e.g. we have assumed a projected, two-dimensional geometry and rigid rotation of magnetic field lines almost up to the light cylinder, hence, concerning the last point, neglected a kind of toroidal twist (Begelman 1994), when the inertial forces overcome the tension in the field line so that the field line is swept back opposite to the sense of rotation. However, one would not expect

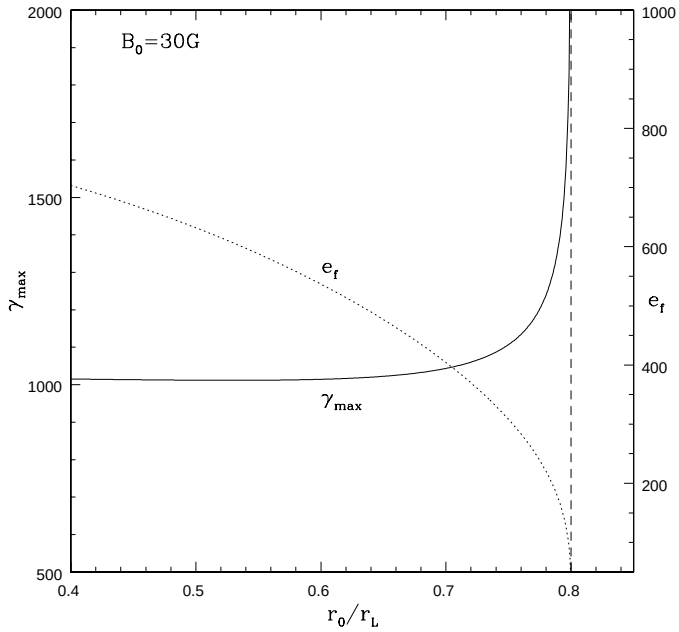

Fig. 6. Maximum electron Lorentz factor $\gamma_{\max}$ as a function of the initial injection position r_0 for $v_0 = 0.6$ c and $B(r_{\rm L}) = 30\,$G (i.e. a disk luminosity $L_{\rm disk} \simeq 1.35 \times 10^{43}$ergs s^{-1}). The dotted line shows the decrease in efficiency of energy gain $e_f = \gamma_{\max}/\gamma_0$, while the dashed line indicate the relativistic limit for injection given by the condition $1 - v_0^2 - \Omega^2\, r_0^2 > 0$.

that these restrictions alter our conclusions essentially since they should lower the upper limit for the maximum Lorentz factor by making the acceleration mechanism ineffective somewhat earlier. Another restriction is the use of special relativity in our analysis, which is only justified far away from the black hole. A detailed general relativistic model is needed to assess whether this might affect the results very strongly or not.

Acknowledgements. We would like to thank Harald Lesch for comments. F.M.R. acknowledge support under DFG Ma 1545/2-1.

References

Abramowicz M.A., 1990, MNRAS 245, 733
Abramowicz M.A., Prasanna A.R., 1990, MNRAS 245, 720
Abramowitz M., Stegun I.A. (eds.), Handbook of Mathematical Functions. Dover Publications, New York
Bednarek W., Kirk J.G., Mastichiadis A., 1996, A&A 307, L17
Begelman M.C., 1994, In: Robinson A., Terlevich R. (eds.), Compact Objects in Active Galactic Nuclei. Cambridge University Press, Cambridge, p. 361
Blandford R.D., Payne D.G., 1982, MNRAS 199, 883
Blandford R.D., Znajek R.L., 1977, MNRAS 179, 433
Camenzind M., 1989, In: Belvedere G. (ed.) Accretion Disks and Magnetic Fields in Astrophysics. Kluwer, Dordrecht, p. 129
Camenzind M., 1995, In: Klare, G. (ed.) Reviews in Modern Astronomy 8, Hamburg, p. 201
Camenzind M., 1996, In: Tsinganos K.C. (ed.) Solar and Astrophysical Magnetohydrodynamic Flows. Kluwer, Dordrecht, p. 699
Cao X., 1997, MNRAS 291, 145
Catanese M., 1999, In: Takalo L.O., Sillanpää A. Dultzin-Hacyan D. (eds.), PASP Conf. Ser. Nr. 159
Celotti A., Fabian A.C., Rees M.J., 1998, MNRAS 293, 239
Chedia O.V., Kahniashvili T.A., Machabeli G.Z., Nanobashvili I.S., 1996, Ap&SS 239, 57
Contopoulos I, Kazanas D., Fendt C., 1999, ApJ 511, 351
Fendt C., 1997, A&A 319, 1025
Gangadhara R.T., 1996, A&A 314, 853
Gangadhara R.T., Lesch H., 1997, A&A 323, L45
Gold T., 1968, Nat 218, 731
Gold T., 1969, Nat 221, 25
Henriksen R.N., Rayburn D.R., 1971, MNRAS 152, 323
Iyer S., Prasanna A.R., 1993, Class. Q. Grav. 10, L13
Kahniashvili T.A., Machabeli G.Z., Nanobashvili I.S., 1997, Physics of Plasmas 4, 1132
Kanbach G., 1996, In: Kirk J.G., Camenzind M., v. Montigny C., Wagner S. (eds.) Proc. of the Heidelberg Workshop on Gamma-Ray Emitting AGN, Heidelberg, October 1996 (MPI H-V37-1996), p. 1
Kirk J.G., Melrose D.B., Priest E.R., 1994, In: Benz A.O., Couvoisier T.J.-L. (eds.) Plasma Astrophysics. Springer, Berlin, p. 166
Landau L.D., Lifshitz E.M., 1960, Mechanics. Pergamon Press, Oxford, p. 128
Machabeli G.Z., Rogava A.D., 1994, Phys. Rev. A 50, 98
Narayan R., 1997, In: Proc. IAU Symp. 188, The Hot Universe, p. 41
Narayan R., Yi I., 1994, ApJ 428, L13
Rybicki G.B., Lightman A.P., 1979, Radiative Processes in Astrophysics. Wiley, New York
Sonego S., Massar M, 1996, MNRAS 281, 659