

*Letter to the Editor***Differential piston angular anisoplanatism for astronomical optical interferometers**

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Abstract. This letter considers the problem of measuring the differential piston of a two-pupil astronomical optical interferometer which uses a reference source located at a different position with respect to the observed scientific target. The differential piston error as a function of the angular distance between the reference and astronomical target can be calculated using existing formulae for Zernike polynomial correlation coefficients. This allows the definition of an isopiston angle that resembles the isoplanatic angle for Adaptive Optics System wavefront correction. Values obtained for the isopiston angle are comparable to the isoplanatic angle showing that sky-coverage could be a potential problem for an astronomical interferometer aiming to correct for atmospheric fringe jitter.

Key words: instrumentation: interferometers – atmospheric effects

1. Introduction

One of the most challenging and promising classes of astronomical instruments is that of large ground based optical interferometers made up of two or more very large telescopes as in the case of VLTI (Mariotti et al. 1998), Keck (Colavita et al. 1998) and LBT (Angel et al. 1998). Each of these instruments aims to correct for the atmospheric effects that reduces fringe visibility. This is done by first phasing the wavefront on each of the various interferometer pupils using Adaptive Optics Systems (AOSs) (Bonaccini et al. 1998; Thatte et al. 1998; Salinari & Sandler 1998; Brusa et al. 1998) and then co-phasing the various wavefronts, actively controlling for differential phase delay. In order to accomplish this second task and thereby allow for proper fringe stabilization, the atmospheric differential piston on the interferometric combined wavefronts has to be sensed and corrected. Usually, in order to increase the number of astronomical targets observed using AOSs, a bright nearby star is used as a reference for wavefront sensing (Beckers 1993; Sandler et al. 1994;

Parenti & Sasiela 1994). The same has to be done to measure the differential piston. However, whereas angular anisoplanatism effects on AOS performance has been extensively investigated by several authors (Fried 1982; Chassat et al. 1989; Rigaut & Gendron 1992), the piston term of the wavefront perturbation has not been taken into account since it is irrelevant in the image formation process of single aperture telescopes. This is no longer true when considering arrays of astronomical telescopes used as interferometers, as pointed out by Mariotti (1994).

In Sect. 2, we describe the concept of differential piston angular anisoplanatism (hereafter angular anisoplanatism), and we present the formulae to evaluate it. Moreover, we obtain the angular anisoplanatism error in the case of a single turbulent layer. In Sect. 3, we apply these results to some astronomical interferometers such as VLTI, Keck and LBT. The isopiston angle (θ_p) is defined and calculated as a function of the turbulence outer scale and wavelength. Finally, Sect. 4 contains the conclusion of the present work.

2. Differential piston error: concept and numerical evaluation

Consider the case of a single turbulent layer located at an altitude h above an astronomical interferometer pointing to a scientific target on axis. We assume the instrument to be made up of two pupils, each of diameter D and having a center to center distance (baseline) equal to Δ . An off-axis reference source, at an angular distance θ from the astronomical target, is used to sense the differential piston in order to remove the fringe jitter of the astronomical target. In Fig. 1, the two couples of pupils P_1, P_2 and P'_1, P'_2 are shown projected onto the considered turbulent layer along the astronomical target and reference star directions, together with all the relevant geometrical elements.

The quantity δp which we want to correct for is the differential piston between the two apertures P_1 and P_2 given by

$$\delta p = p_1 - p_2, \quad (1)$$

where p_1 and p_2 represent the piston terms of the optical path wavefront error on the pupils P_1 and P_2 respectively. The signal

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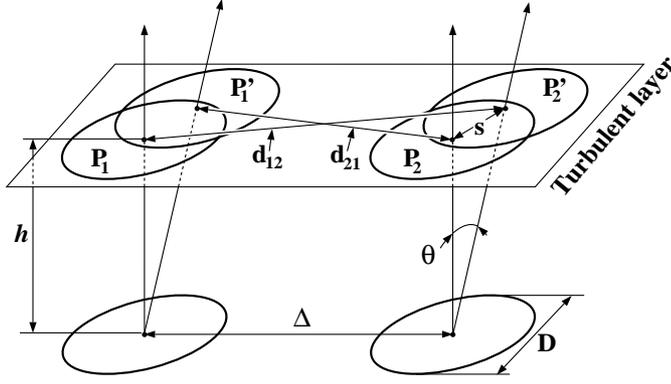


Fig. 1. Geometrical elements considered for the calculations.

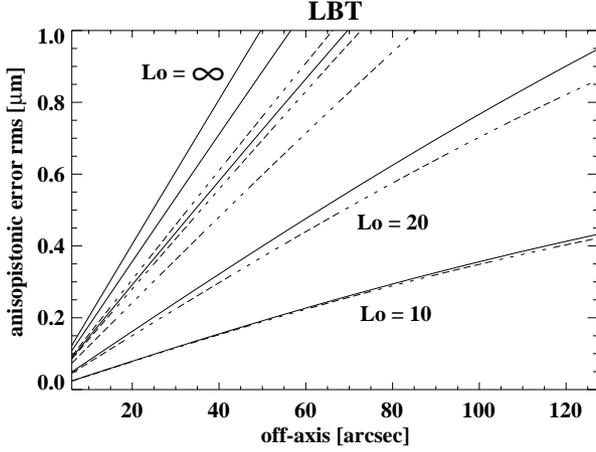


Fig. 2. Anisopistic error rms σ_p as a function of the off-axis angle of the reference source for a single-layer turbulent atmosphere (see text for the details on the rest of parameters). Plotted curves refer to $L_0 = 10, 20, 50, 100$ m and ∞ from the least to the most steep curve. Solid and dashed lines represent the cases where the reference source is displaced along and orthogonally to the baseline, respectively.

$\delta p'$ we use for the correction is obtained observing the off-axis reference star so that

$$\delta p' = p'_1 - p'_2, \quad (2)$$

where, as above, p'_1 and p'_2 represent the piston terms on the pupils P'_1 and P'_2 respectively.

The variance σ_p^2 of the differential piston error (anisopistic error variance in the following) has the form

$$\begin{aligned} \sigma_p^2 &= \langle (\delta p - \delta p')^2 \rangle \\ &= 2 [2 \langle p_1^2 \rangle - 2 (\langle p_1 p_2 \rangle + \langle p_1 p'_1 \rangle) \\ &\quad + \langle p_2 p'_1 \rangle + \langle p_1 p'_2 \rangle], \end{aligned} \quad (3)$$

where we used the following relationships $\langle p_1^2 \rangle = \langle p_2^2 \rangle$, $\langle p_1 p_2 \rangle = \langle p'_1 p'_2 \rangle$ and $\langle p_1 p'_1 \rangle = \langle p_2 p'_2 \rangle$ by assuming homogeneity and isotropy of atmospheric turbulence. Spatial correlation of the coefficients of the Zernike modes representing the wavefront distortion were first calculated by Valley & Wandzura (1979) where the outer scale of turbulence was considered infinite. A more general expression with a finite outer scale is given

by Takato & Yamaguchi (1995). Considering Eq. 12 of this last paper, in the special case of $j=j'=1$, the spatial covariance of the piston mode in terms of optical path difference is:

$$\begin{aligned} \langle p p(s) \rangle &= 0.0229 \, 4\pi^{2/3} \lambda^2 \left(\frac{D}{r_0} \right)^{5/3} \\ &\times \int_0^\infty \frac{J_0(2x \frac{s}{D}) J_1^2(x)}{x \left[x^2 + \left(\pi \frac{D}{L_0} \right)^2 \right]^{11/6}} dx, \end{aligned} \quad (4)$$

where J_n is the Bessel function of n -th order; s , D , λ , r_0 , L_0 , are the linear pupil separation, the telescope diameter, the wavelength, the Fried parameter and the outer scale of turbulence for the considered single layer respectively. Substituting this formula into Eq. (3) we can evaluate σ_p^2 for a single layer at altitude h . Referring to the symbols introduced in Fig. 1, we obtain

$$\begin{aligned} \sigma_p^2 &= 0.0229 \, 4\pi^{2/3} \lambda^2 \left(\frac{D}{r_0} \right)^{5/3} \int_0^\infty \frac{J_1^2(x)}{x \left[x^2 + \left(\pi \frac{D}{L_0} \right)^2 \right]^{11/6}} \\ &\times \left\{ 2 \left[1 - J_0 \left(2x \frac{s}{D} \right) - J_0 \left(2x \frac{\Delta}{D} \right) \right] \right. \\ &\quad \left. + J_0 \left(2x \frac{d_{12}}{D} \right) + J_0 \left(2x \frac{d_{21}}{D} \right) \right\} dx. \end{aligned} \quad (5)$$

Fig. 2 shows the behavior of σ_p for the LBT interferometer ($D = 8.4$ m, $\Delta = 14.4$ m) in the case of a single turbulent layer at altitude $h = 10$ km above the telescope for $r_0(0.55 \mu\text{m}) = 0.2$ m and various L_0 values. The dependance of σ_p on $s = h\theta$ is quasi-linear so that, especially for large outer scale values, we have

$$\sigma_p \propto \lambda \left(\frac{D}{r_0} \right)^{5/6} \frac{h\theta}{D}. \quad (6)$$

The behavior of σ_p when the reference source is displaced along the interferometer baseline (dashed line) or orthogonal to it (solid line) is shown in the same figure. The difference is due to the cross-correlation terms $\langle p_1 p'_2 \rangle$ and $\langle p_2 p'_1 \rangle$ represented by the last two terms in Eq. (5).

3. Application to some astronomical interferometers

The formulae obtained in the previous section quantify the anisopistic error variance (σ_p^2) due to a single layer. They can easily be used to evaluate σ_p^2 for a multi-layer atmosphere by assuming that the phase perturbations at the various layers are uncorrelated. With this assumption the overall error variance will simply be the sum of the single layer error variances. We apply this procedure to evaluate the anisopistic error rms for the Keck, VLTI (UT1–UT4) and LBT interferometers. For Keck and VLTI we use available SCIDAR measurements for the atmospheric turbulence profiles above Mauna Kea (Racine & Ellerbroek 1995) and Cerro Paranal (Sarazin 1995). In absence of atmospheric turbulence profiles

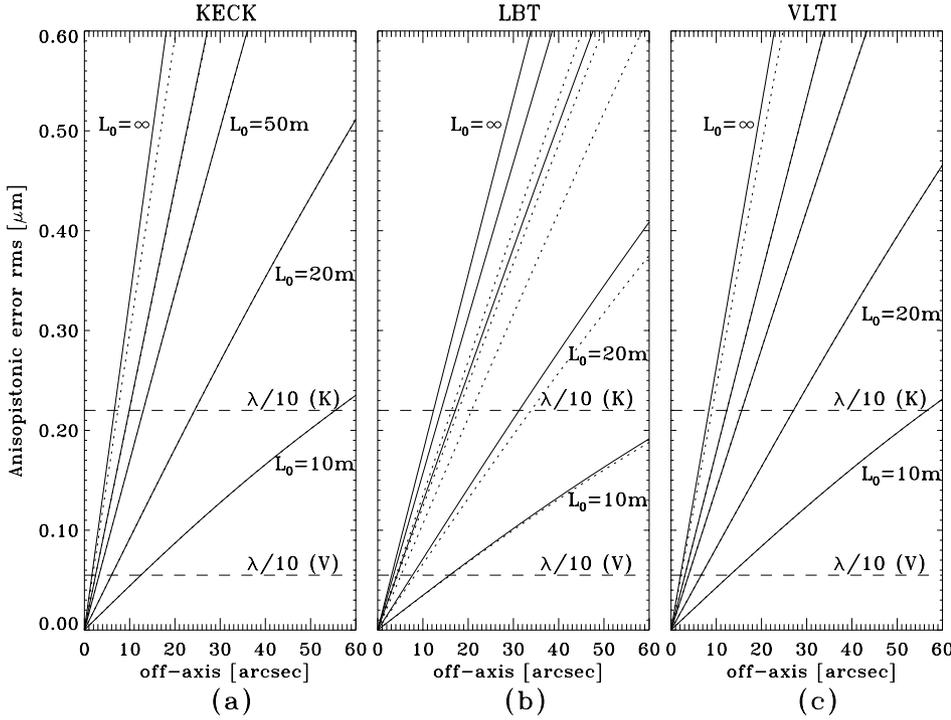


Fig. 3a – c. Anisopistic error rms σ_p as a function of the off-axis angle of the reference source. The solid and dashed curves have the same meaning as in Fig. 2 and, for small values of the outer scale are overposed. The $\lambda/10$ error level is shown as a dashed horizontal line in the case of the K and V bands.

Table 1. Geometrical elements used in the calculations. VLTI interferometer refer to UT1 and UT2 telescopes.

	Single tel. Diam. [m]	Telescope altitude [m]	Baseline [m]
VLTI	8.2	2635	130
Keck	10	4200	80
LBT	8.4	3200	14.4

for LBT we have considered a modified Hufnagel-Valley model (Beckers 1993). These atmospheric turbulence profiles have been normalized to the median $r_0(0.5\mu\text{m})$ values above each site: 20.4, 13.3 and 14.1 cm for the Keck, LBT and VLTI, respectively (Racine & Ellerbroek 1995, Ulich & Davison 1985, Le Louarn et al. 1998). Moreover we consider five outer scale values of 10, 20, 50, 100 m and ∞ . Geometrical parameters of these interferometers are reported in Table 1. Panels (a), (b) and (c) in Fig. 3 show the behavior of σ_p for Keck, LBT and VLTI, respectively. Two curves are plotted for each L_0 value. These curves represent σ_p values when the reference source is located along the interferometer baseline or orthogonal to it. Intermediate position angles of the reference source give values within these two curves. For the considered cases only the LBT is sensitive to the off-axis direction of the reference source. This is because, only in the case of the LBT, the normalized distances d_{12}/D , d_{21}/D and Δ/D contained in Eq. (5) are significantly different in the two considered cases of reference source displaced along and orthogonally to the baseline. Data shown in Fig. 3 allow to calculate the isopistic angle θ_p defined as the angular radius of a circular region where the anisopistic error reduces the fringe visibility to no more than 80% of the unper-

turbed value. The 20% visibility reduction happens when the residual piston error σ_p is about $\lambda/10$.

Table 2 summarizes the θ_p values obtained in the K and V bands. As an example, assuming $L_0 = 50$ m, we obtain, in the K band, a relatively small azimuthally averaged isopistic angle of about 14, 20 and 16 arcsec for Keck, LBT and VLTI respectively. In the V band, the isopistic angles reduce to 3.4, 4.9 and 4.0 arcsec. Keck exhibits the smallest values for θ_p irrespective of L_0 , because of the larger amount of turbulence located at high altitude in Mauna Kea (Racine & Ellerbroek 1995). The isopistic angles can be compared with the isoplanatic angles (θ_0) listed in the last column of Table 2 for both K and V bands. We note that the isopistic angle, which is about twice the isoplanatic angle, is substantially smaller than the size of the currently planned fields of view, of about an arcminute radius (Mariotti et al. 1998, Colavita et al. 1998, Angel et al. 1998), for fringe tracking reference star acquisition. These values for the isopistic angle suggest that sky-coverage could be a potential problem for atmospheric fringe tracking purposes.

4. Conclusion

Calculations of the present letter allow the quantification of the error in the fringe tracking process due to differential piston angular anisoplanatism. This error leads to isopistic patches as small as about 13.5 arcsec (K-Band) and 3.4 arcsec (V-Band) for $L_0 = 50$ m. These values show that sky-coverage could be a matter of concern for astronomical interferometers which aim to correct atmospheric differential piston. Finally, we note that the anisopistic error could probably be reduced by using tomographic wavefront reconstruction as proposed by Tallon &

Table 2. Isopiston angle values in arcsec for different values of outer scale in K($2.2\mu\text{m}$) and V($0.55\mu\text{m}$) bands. The two values quoted for each L_0 value refer to reference star position angle of 0 deg (right) and 90 deg (left) with respect to the interferometer baseline. The last column gives the isoplanatic angle at 2200 and 550 μm .

L_0		10 m		20 m		50 m		100 m		∞	θ_0	
K Band	Keck	56.1	56.1	25.8	25.8	13.5	13.5	10.2	10.2	6.7	7.5	7.0
	LBT	68.9	70.2	32.3	35.2	17.6	21.4	14.3	18.3	12.6	16.8	12.8
	VLTI	57.1	57.1	28.4	28.4	16.1	16.1	12.6	12.6	8.5	9.2	9.9
V Band	Keck	14.0	14.0	6.5	6.5	3.4	3.4	2.6	2.6	1.7	1.9	1.3
	LBT	17.2	17.6	8.1	8.8	4.4	5.4	3.6	4.6	3.2	4.2	2.4
	VLTI	14.3	14.2	7.1	7.1	4.0	4.0	3.2	3.2	2.1	2.3	1.9

Foy (1990) considering an approach similar to the one reported by Ragazzoni et al. (1998).

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