Astron. Astrophys. 354, 1–6 (2000)

ASTRONOMY
AND
ASTROPHYSICS

On the existence of the intrinsic anisotropies in the angular distributions of gamma-ray bursts

A. Mészáros[1], **Z. Bagoly**[2], **and R. Vavrek**[3,4]

[1] Department of Astronomy, Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic (meszaros@mbox.cesnet.cz)
[2] Laboratory for Information Technology, Eötvös University, Pázmány Péter sétány 1/A, 1117 Budapest, Hungary (bagoly@ludens.elte.hu)
[3] Konkoly Observatory, Box 67, 1505 Budapest, Hungary (vavrek@ogyalla.konkoly.hu)
[4] Observatoire de Paris, 92195 Meudon Cedex, France

Received 5 July, 1999 / Accepted 21 September 1999

Abstract. This article is concerned primarily with the intrinsic anisotropy in the angular distribution of 2281 gamma-ray bursts (GRBs) collected in Current BATSE Gamma-Ray Burst Catalog until the end of year 1998, and, second, with intrinsic anisotropies of three subclasses ("short", "intermediate", "long") of GRBs. Testing based on spherical harmonics of each class, in equatorial coordinates, is presented. Because the sky exposure function of BATSE instrument is not dependent on the right ascension α, any non-zero spherical harmonic proportional either to $P_n^m(\sin\delta)\sin m\alpha$ or to $P_n^m(\sin\delta)\cos m\alpha$ with $m \neq 0$ (δ is the declination), immediately indicates an intrinsic non-zero term. It is a somewhat surprising result that the "intermediate" subclass shows an intrinsic anisotropy at the 97% significance level caused by the high non-zero $P_3^1(\sin\delta)\sin\alpha$ harmonic. The remaining two subclasses, and the full sample of GRBs, remain isotropic.

Key words: cosmology: large-scale structure of Universe – gamma rays: bursts

1. Introduction

The discovery of an anisotropy in the angular distribution of 2025 gamma-ray bursts (GRBs) collected in Current BATSE Gamma-Ray Burst Catalog (Meegan et al. 1998) up to the end of year 1997, has recently been announced (Balázs et al. 1998). Of course, the existence of an observed anisotropy is expected from the non-uniform sky exposure function of BATSE instrument (Fishman et al. 1994). Nevertheless, Balázs et al. (1998) argue that the detected anisotropy should *not* be a pure instrumental effect; i.e. there also exists an *intrinsic* anisotropy (see also Balázs et al. 1999). The exact details of the nature of the intrinsic anisotropy remains open to debate.

Recently, Horváth (1998) and Mukherjee et al. (1998) have proposed that the long GRBs should be further separated into two classes; the limit separating the "long" subclass should be at $T_{90} = 10$ s, where T_{90} is the "90%" duration (Fishman et al. 1994). Hence, in the studies of subclasses it is required to consider *three* subclasses separately ("short" subclass with $T_{90} < 2$ s; "intermediate" subclass with 2 s $< T_{90} < 10$ s; "long" subclass with $T_{90} > 10$ s).

In principle, for GRBs, there are 16 possible types which can occur, because for "all" GRBs, and for the three subclasses as well, there are two eventualities: either they are distributed isotropically or they are not.

The purpose of this paper is to clarify the situation concerning these possibilities. We will again use the analysis based on the spherical harmonics (Balázs et al. 1998).

In the study by Balázs et al. (1998), an analysis of GRBs having a peak flux greater than 0.65 photons/(cm^2 s) was performed. Hence, the GRBs having a peak flux smaller than 0.65 photons/(cm^2 s) were omitted from the sample. This truncation did not give any new result; except for the fact that the number of GRBs was lowered, and hence the corresponding significance levels were also lowered. The High-Energy versus Non-High-Energy separation (Pendleton et al. 1997) did not give any new result either; hence, we will not use this separation here.

It is well-known that there is no general agreement concerning the significance level required to reject a null hypothesis. In an ad hoc manner, we will take, in accordance with the propositions of standard statistical textbooks (Trumpler & Weaver 1953, Kendall & Stuart 1969), for the "doubtless" or "definite" rejection, the $\geq 99.9\%$ significance level. (This level of significance is obtained for all GRBs in the sample examined by Balázs et al. 1998; but together with the instrumental effects.) We will treat a significance level of $\geq 99\%$ as a *practically* sure result, with the $\geq 95\%$ significance level result being considered as *noteworthy*.

2. Mathematical considerations

The key ideas of the analysis by Balázs et al. (1998) were based on spherical harmonics up to the quadrupole terms. In this section, we generalize these considerations to include higher order harmonics. In addition, we introduce new statistical tests.

As we note in the introduction, it is necessary to eliminate the BATSE instrumental effects coming from the non-uniform

Send offprint requests to: A. Mészáros sky exposure. Several methods exist. For example, in Briggs et al. (1996), Monte Carlo simulations are used: N points are randomly scattered on the sky many times, and the scattering is "deformed" in accordance with the sky exposure function; N is the number of GRBs. Then the spherical harmonics calculated from these simulations are compared with the spherical harmonics of the observed BATSE data. Moreover, in Briggs et al. (1996), a further quite simple method is proposed: to use the equatorial coordinates α and δ ($0 \leq \alpha \leq 2\pi; -\pi/2 \leq \delta \leq \pi/2$) instead of the Galactic coordinates, because the sky exposure function is not dependent on α. In this paper we will use these coordinates and the proposed "equatorial" method.

The key ideas of this method may be seen as follows. Let $\omega(\alpha, \delta)\ \cos\delta\ d\delta\ d\alpha$ be the probability of finding a GRB in the infinitesimal solid angle $\cos\delta\ d\delta\ d\alpha$. If we assume that there is an isotropy in the sky distribution of GRBs, then ω should reflect the sky exposure function (Balázs et al. 1998). The function ω can be decomposed into its spherical harmonics. One then has

$$\omega(\alpha, \delta) = \sum_{n=0}^{\infty} \omega_{n,0} P_n(\sin\delta) +$$

$$\sum_{n=1}^{\infty} \sum_{m=1}^{n} P_n^m(\sin\delta)(\omega_{n,-m} \sin m\alpha + \omega_{n,m} \cos m\alpha), \qquad (1)$$

where $P_n^m(\sin\delta)$ are the Legendre polynomials.

If there is an intrinsic isotropy, then $\omega(\alpha, \delta) = \omega(\delta)$, and one must have $\omega_{n,-m} = \omega_{n,m} = 0$ for any $n \geq m \geq 1$. Conversely, if even one single $\omega_{n,\pm m}$ for $m \geq 1$ were non-zero, then there would be an intrinsic anisotropy. *This is the key idea of this method.* Simply, if ω is not dependent on α, then all $m \neq 0$ harmonics must be identically zero independent of the concrete form of function $\omega(\delta)$. Then, of course, this theoretical prediction should be compared to the observational data.

Concerning the $m = 0$ terms one may use either the Monte Carlo method (Briggs et al. 1996), or - if possible - to calculate from the known $\omega(\delta)$, numerically or even analytically, the expected theoretical values of $\omega_{n,0}$. They will not be zeros in the general case. (They would be zeros, if ω were constant; i.e. if the sky exposure function were constant.)

In this paper we will not study these terms in detail, and an eventual observed non-zero $\omega_{n,0}$ will simply be assumed to be that caused exclusively by the instrumental effects.

In our case the null hypothesis of intrinsic isotropy will be identical to the assumption $\omega(\alpha, \delta) = \omega(\delta)$; i.e. to the assumption that for all $\omega_{n,\pm m} = 0$ for $m \neq 0$. Hence, to test the null hypothesis of intrinsic isotropy one has to test these theoretically expected zeros for the given spherical harmonics with $m \neq 0$.

This may be done similarly to Balázs et al. (1998). This means that, if one has N GRBs with measured $[\alpha_j, \delta_j]$ ($j = 1, 2, \dots, N$) coordinates, one has to calculate for the given n, m the values (Peebles 1980, Eq. (46.1); Balázs et al. 1998)

$$\sum_{j=1}^{N} P_n^m(\sin\delta_j) \cos m\alpha_j \qquad (2)$$

and

$$\sqrt{\sum_{j=1}^{N} (P_n^m(\sin\delta_j) \cos m\alpha_j)^2}. \qquad (3)$$

(For $n, -m$ one simply has to substitute cosine by sine.) Then one can apply Student's t test, where the expected mean is zero. Here t is simply the ratio of quantities from Eqs. (2-3), because for $N \gg 1$ (this is always the case here) one may use $N - 1 \simeq N$. Note also that there are no problems which will arise from the normalization constant of a spherical harmonic. (If, instead of P_n^m, one uses $const.P_n^m$, where $const.$ is an arbitrary positive number, then t will not change, if $N - 1 \simeq N$, and if the expected mean is zero.) Technically, there can also occur several computational numerical problems with the definition of P_n^m itself for large n on the computer (see Press et al. 1992, Chapt. 6.8). In our calculations, up to the used n, we encountered no such problems. We used the algorithm described in Chapt. 6.8 of Press et al. (1992).

We will also use a modification of this test. From the symmetry properties of $P_n^m(\sin\delta) \cos m\alpha$ and $P_n^m(\sin\delta) \sin m\alpha$ it follows that for $m \neq 0$ in exactly one half of the sky the sign of the harmonic is positive, and in the other half it is negative. For $m = 0$ this is true for odd n only, but not for even n. Hence, except for even n with $m = 0$, one may proceed as follows. One simply calculates the number $k_{\rm obs}$ of GRBs having - say - negative values. Then one may apply the Bernoulli test described in Mészáros (1997) and Balázs et al. (1998). (It is essential to note that the half of the sky, where the given harmonic has the given sign, need not be a "connected" compact region (Balázs et al. 1998).) Then for the given $k_{\rm obs}$ the calculation of the significance level is described in Balázs et al. (1998). In essence, one can take - as the theoretical expectation - a normal distribution with mean $N/2$ and dispersion $\sqrt{N}/2$. Hence, if $|k_{\rm obs} - N/2| > \sqrt{N}$, then the significance level is larger than 2σ, i.e. 95.4% (Balázs et al. 1998). Note that - trivially - in this "sign test" the choice sign does not matter. Obviously, the "sign-test" and Student's t test need not give the same result for a given harmonic.

This method based on spherical harmonics seems to be extremely powerful because, at least in principle, one single non-zero harmonic for $m \neq 0$ should be enough to reject the null hypothesis of intrinsic isotropy (Peebles 1980, Chapt. 46; Tegmark et al. 1996). This follows from the orthogonality of spherical functions (Peebles 1980, Chapt. 46). On the other hand, it is not excluded that - testing several harmonics simultaneously with two different tests -, some of these tests can give a higher than $\alpha\%$ significance level "by chance". For example, if $\alpha = 95$, then one may expect that 5% of tests can give a greater than 95% significance level "by chance". Therefore, to be maximally careful, we will reject the null hypothesis only in the case, when the number of tests giving $\geq \alpha\%$ significance level is far above $(100 - \alpha)\%$ fraction. Concretely, this means the following. We take the absolute value of the difference between the number of tests giving $\geq \alpha\%$ significance level and the theoretically expected number of tests giving $\geq \alpha\%$ significance level. This value must be so large that the probability of a chance of this difference is smaller than $(100 - \alpha)\%$. This probability is simply calculated from the binomial (Bernoulli) distribution.

In our case we have infinite harmonics and hence in principle we may apply an infinite number of tests. Nevertheless, due to the discrete character of measured $\omega(\alpha, \delta)$, there is a limitation to the harmonics, which can be tested. Having the angular coordinates of N GRBs, we have $2N$ independent measured quantities. Hence, clearly, we cannot obtain more than $2N$ *independent* harmonics. Which harmonics are these? In addition, even for these $2N$ independent harmonics there may occur a further problem. Of course, the mean, variation and the t for a given harmonic can be calculated from the positions using Eqs. (2-3). But what is the accuracy of this harmonic calculated for finite N compared with the ideal case, when one would have $N \to \infty$? In other words, is the calculated t a "true" estimator of the Student t? Obviously, we would need to compare the measured t obtained for the "infinite" case with the theoretically predicted zero mean.

Several authors have investigated the answers to these and similar questions (Trumpler & Weaver 1953, Kendall & Stuart 1969, Bracewell 1978, Peebles 1980, Press et al. 1992, Tegmark et al. 1996). Note that some conclusions can be deduced for this case immediately from the well-discussed case of discrete Fourier transform (cf. Bracewell 1978, Chapt. 14; Press et al. 1992, Chapts. 12-13); in α we have in fact a Fourier decomposition. The key result of all these studies is the statement that for the lowest-order harmonics (dipole, $n = 1$; quadrupole, $n = 2$; etc. ...) the Student t calculated from the observational data can be considered as if it were $N \to \infty$ (Horack et al. 1993, Tegmark et al. 1996). Hence, for these low-order harmonics Student's t test gives the correct conclusion. This result also suggests that the $2N$ independent harmonics, which should be tested, should be the lowest possible ones with $n \leq (\sqrt{2N} - 1)$. (Clearly, up to n, we have $1 + 3 + \ldots + (2n + 1) = (n + 1)^2$ harmonics.) In addition, one should note that, even for $n > \sqrt{2N} - 1$ some harmonics, being expressed by the lower ones, can also be used with care. For example, in Tegmark et al. (1996), N was $= 1122$ and even $n = 65$ was discussed. Nevertheless, in this paper, in order to avoid any complications with higher n, we restrict ourselves to $n \leq (\sqrt{2N} - 1)$.

A rough estimate of the inaccuracy of quantities in Eqs. (2-3) for the given n, m, N may be quite simply performed. The typical angular scale belonging to n is $\simeq \pi/n$ (Peebles 1980, Chapt. 46), and hence the corresponding solid angle is $\simeq (\pi/n)^2$ steradians. Clearly, on a solid angle $\simeq 4\pi/N$ steradian the discreteness is crucial, and hence for $(\pi/n)^2 \simeq (4\pi/N)$ the inaccuracies can be even of order unity (they need not be, but they can be for the worst case). All this suggests that for n the inaccuracy coming from discreteness should be maximally $\simeq (4\pi/N)/(\pi/n)^2 \simeq n^2/N$. Of course, in the inaccuracy formula $\simeq n^2/N$ there is an uncertainty concerning the numerical factor itself. (We have roughly taken $4 \simeq \pi$; in Eq. (3) $\cos^2 n\alpha$ may be expressed by $\cos 2n\alpha$, and hence here one will have generally a four-times larger uncertainty; the typical π/n angle belonging to n is a rough estimate; etc.) Recently, this question has usually been studied by numerical simulations (cf. Tegmark et al. 1996, and references therein). But, classical mathematical treatments are also known (cf. Boas 1983, Chapt. 16.6; Press et al. 1992, Chapt. 13.4; Bracewell 1978, Chapt. 14). All these studies suggest that for $n \ll \sqrt{2N}$ the discreteness leads to negligible inaccuracies, which is in accordance with our estimate $\simeq n^2/N$. On the other hand, Tegmark et al. (1996) imply that even for $n^2 \simeq N$ the analysis based on spherical harmonics can be good. This suggests that the inaccuracy should be much smaller that n^2/N. (Note that, trivially, the dependence on m need not be mentioned especially. Clearly $m \leq n$, and there are $2n+1$ harmonics for a given n. Then the inaccuracy for a given n will simply be the largest inaccuracy among these $2n + 1$ harmonics.) We will not go into the details of this discussion, and in this paper we will simply assume that the inaccuracy is maximally $\simeq n^2/N$. We will also keep in mind the fact that the estimate of this inaccuracy is never complete. Therefore, one should be careful with any conclusions derived, for instance, when $n^2 \ll N$ does not apply. To avoid problems with higher harmonics, we will carry out a further drastic truncation. We will, ad hoc, consider only $n \leq \sqrt{N}/3$. This means that, instead of the possible $2N$ harmonics, we will study only $\simeq N/9$. This (together with the avoiding of $m = 0$ terms) means that we will test only $\simeq (5-6)\%$ of all allowed independent harmonics.

Concerning the "sign-test" the accuracy seems to be much better than for Student's t test. By this we mean that there is no problem with the correctness of a conclusion coming from the Bernoulli test. Therefore, the "sign-test" may be used for the same harmonics for which Student's t test was used.

For the used values of n the positional errors (of sizes $\simeq 1-4$ degrees, Fishman et al. 1994) should give no further complications. We will consider only such values of n, when the typical angular size belonging to an n ($\simeq \pi/n$; Peebles 1980, Chapt. 46) will be much larger than the size of the positional error.

The detailed discussion presented by Tegmark et al. (1996) shows that for such values of n no complications should arise from the positional errors.

3. Results

In order to test the isotropy of 2281 GRBs collected in Current BATSE Gamma-Ray Burst Catalog (Meegan et al. 1998) until the end of 1998, we calculated their spherical harmonics up to $n = 15$. This maximum n value was chosen in accordance with the discussion in Sect. 2, where we restricted ourselves to $n \leq \sqrt{N}/3$. Here $N = 2281$, and $\sqrt{N}/3 = 15.9$. In other words, instead of $2 \times 2281 = 4562$ possible independent spherical harmonics we tested only $(16^2 - 16) = 240$ both with Student's t test and sign tests. The results are collected in Table 1, in which we present all dipole and quadrupole terms (in order to compare them with Balázs et al. 1998), but for larger harmonics only the terms for which either $|t| > 1.96$ or $k_{\rm obs} < 1093$ or $k_{\rm obs} > 1188$. For $|t| > 1.96$ there is a smaller than 5% probability that there is still an isotropy; for $|t| > 2.58$ this probability is smaller than 1% (Trumpler & Weaver 1953). For

Table 1. Results of Student's t and the sign tests, respectively, of 2281 GRBs from spherical harmonics up to $n = 15$. All dipole + quadrupole terms are presented, but for higher order harmonics only the components with either $|t| > 1.96$ or $k_{\rm obs} < 1093$ or $k_{\rm obs} > 1188$. * (**) means that the test gives a $> 95\%$ ($> 99\%$) probability of anisotropy. For $m > 0$ ($m < 0$) one has the cosine (sine) term in Eq. (1). For n even with $m = 0$ the sign test is not applied.

n	m	t	$k_{\rm obs}$
1	0	$*2.29$	1098
1	1	-0.44	1155
1	-1	0.12	1148
2	0	$**4.26$	. . .
2	1	-0.88	1149
2	-1	-0.30	1149
2	2	-0.29	1137
2	-2	0.85	1137
4	4	0.89	$*1092$
5	4	$*2.20$	1094
9	-6	0.53	$*1092$
9	8	$**2.67$	$**1072$
9	-8	$*2.44$	1106
10	-1	1.42	$*1089$
10	-10	$*-2.14$	1184
11	8	$**2.88$	1136
11	-8	$*2.03$	1095
12	0	$*2.31$	. . .
12	-12	$**2.69$	1094
13	8	$**2.81$	$**1072$
13	9	$*2.18$	1147
14	-9	0.09	$*1092$
14	-12	$**3.36$	1103
15	8	$*2.39$	1115
15	9	$*2.43$	1110
15	11	0.28	$*1091$

the "sign test" one needs $|k_{\rm obs} - N/2| > \sqrt{N}$ to have a $> 95.4\%$ significance level for anisotropy, where $k_{\rm obs}$ is the observed number of GRBs having the given harmonic negative value. We also present the $m = 0$ cases for comparison as well, but these values will not be taken as indications of intrinsic anisotropy. For $m = 0$ the "sign test" is applied only for odd n, of course.

Table 1 shows that there are three $m = 0$ components with $|t| > 1.96$ ($n = 1, 2, 12$). The biggest value is the $n = 2, m = 0$ quadrupole term with $t = 4.26$. This means that there exists an anisotropy with a certainty; the probability that this quadrupole term is non-zero due to chance is much smaller than 0.1% probability ($t = 3.29$ corresponds to a 0.1% probability). In fact, this is nothing new (Balázs et al. 1998); it again confirms that the observed distribution of all GRBs is without doubt anisotropic.

We will interpret these three terms as the anisotropy that follows *exclusively* from the non-uniform sky exposure function of BATSE instrument.

There are no further dipole and quadrupole anisotropies above $|t| > 1.96$; the sign test also gives no such anisotropies. On the other hand, there are 19 $m \neq 0$ harmonics above the 95% significance level. (In 12 cases $|t| > 1.96$; in 7 cases the sign test gives anisotropy above the 95% significance level; for $n = 9, m = 8$ and $n = 13, m = 8$, respectively, both tests give anisotropy.) In addition, for five harmonics one has $|t| > 2.58$. For two of them one has $k_{\rm obs} < 1093$ as well. Concerning the 95% significance level, one expects that in $2 \times 240/20 = 24$ cases, one can obtain this "by chance", too. This is more than the 19 cases obtained, and the difference could well be due to chance. Concerning the 99% significance level, one expects 4.8 cases theoretically, and we obtain 7 ones. This can also be due to chance. The dispersion arising from the binomial distribution is $\sqrt{4.8 \times 0.99} = 2.18$; this is practically identical to $(7 - 4.8) = 2.2$. There is also a $t = 3.36$ value, which should give a higher than 99.9% significance level. This single value is hardly enough to reject the null hypothesis. One expects 0.48 such cases. The dispersion is 0.69; hence one single test may well occur "by chance" as well. Hence, we conclude that the null-hypothesis of intrinsic isotropy for all GRBs should *not* be rejected.

Table 2. Results of the Student t + sign tests, respectively, for 419 short GRBs from spherical harmonics up to $n = 6$. The notation is the same as in Table 1; for $k_{\rm obs}$ here we needed either $k_{\rm obs} < 190$ or $k_{\rm obs} > 231$.

n	m	t	$k_{\rm obs}$
1	0	0.38	207
1	1	-0.61	211
1	-1	1.52	200
2	0	$**2.69$	. . .
2	1	-1.07	218
2	-1	-0.05	207
2	2	-0.08	201
2	-2	1.74	199
3	1	0.98	$*186$
5	-1	1.60	$*188$
5	2	0.78	$*189$
5	3	$*-2.19$	229

In Current BATSE Catalog there are 1670 GRBs having measurements both for the positions and T_{90}. (The durations are catalogued for GRBs detected before August 1996.) Among these, 419 objects have $T_{90} < 2$ s ("short" subclass), 253 objects have 2 s $< T_{90} <$ 10 s ("intermediate" subclass), and 998 objects $T_{90} > 10$ s ("long" subclass). For these three subclasses we repeated the above procedure.

The results for the 419 short GRBs having $T_{90} < 2$ s are collected in Table 2. (Here $N = 419$, and therefore we consider only $n \leq 6$.) Again there is a clear non-zero term for $n = 2$, $m = 0$ expected from the BATSE's sky exposure function. There are no dipole and quadrupole terms above $|t| > 1.96$. On the other hand, there are 4 further tests determining non-zero harmonics above the $> 95\%$ significance level. Nevertheless, from the 84 tests performed here one expects theoretically that 4.2 tests will be above the 95% significance level "by chance".

Table 3. Results of the Student t + sign tests, respectively, for 253 intermediate GRBs from spherical harmonics up to $n = 5$. The notation is the same as in Table 1; for $k_{\rm obs}$ here we needed either $k_{\rm obs} < 111$ or $k_{\rm obs} > 142$.

n	m	t	$k_{\rm obs}$
1	0	1.33	116
1	1	−0.83	136
1	−1	∗ − 2.24	142
2	0	1.74	...
2	1	1.17	120
2	−1	−0.68	132
2	2	−0.49	129
2	−2	−0.43	132
3	−1	∗∗ −3.37	∗∗ 154
5	−1	∗ − 2.25	130
5	1	0.20	∗108
5	4	∗2.25	112

Again, similarly to all GRBs, the null-hypothesis of intrinsic isotropy of short GRBs should *not* be rejected.

The results for the 253 intermediate GRBs having 2 s $< T_{90} < 10$ s are collected in Table 3. (Here $N = 253$, and therefore we consider only $n \leq 5$.) Interestingly, there are no non-zero $m = 0$ terms expected from the BATSE sky exposure function. There are 6 tests determining non-zero harmonics above the $> 95\%$ significance level. From the 60 tests done here one expects theoretically that 3 tests will be above the 95% significance level "by chance". The obtained 6 cases - instead of the expected 3 ones - can just still be by chance. The dispersion is $\sqrt{3 \times 0.95} = 1.69$. $6 - 3 = 3$ is still smaller than $2 \times 1.69 = 3.38$. Hence the obtained 6 tests instead of the expected 3 can still be by chance with a probability larger than 5%. Nevertheless, we find that for the intermediate subclass the null hypothesis of isotropy can be rejected at the $> 95\%$ significance level. This follows from the large value of t and $k_{\rm obs}$ for $n = 3$, $m = -1$. Student's t test yields a probability of zero value for this harmonic of 0.08%. From the "sign-test" this value is even smaller, because - from the Bernoulli distribution - the zero value is rejected at the $(154 - 126.5)/7.95 = 3.46\sigma$ significance level; the probability of a chance is only 0.06%. From the considered 60 tests, we obtain 2 cases giving a $\geq 99.92\%$ significance level; the expected value is maximally 0.48. Then the dispersion is 0.7, and $(1 - 0.48) = 1.52 > 2 \times 0.7$. Hence, there is a $1.52/0.7 = 2.17\sigma$ - i.e. 97% - probability that this is not by chance. Taking into account this estimate we conclude that this single spherical harmonic alone is enough to reject the assumption of isotropy at a higher than 95% significance level. This significance level is minimally 97%.

The results for the 998 GRBs having $T_{90} > 10$ s are collected in Table 4. (Here $N = 998$, and therefore here we consider only $n \leq 10$.) First, from these results it follows that there are the $n = 4, 7$, $m = 0$ anisotropy terms expected from the BATSE sky exposure function. There are no further dipole and quadrupole terms suggesting anisotropy on these scales.

Table 4. Results of the Student t and sign tests, respectively, for 998 long GRBs from spherical harmonics up to $n = 10$. The notation is the same as in Table 1; for $k_{\rm obs}$ here we needed either $k_{\rm obs} < 468$ or $k_{\rm obs} > 530$.

n	m	t	$k_{\rm obs}$
1	0	0.73	487
1	1	0.57	494
1	−1	0.83	486
2	0	1.88	...
2	1	−0.51	499
2	−1	0.81	481
2	2	0.20	488
2	−2	0.38	498
4	0	∗ − 2.19	...
4	1	−1.82	∗534
4	−1	∗2.18	474
4	−4	1.36	∗466
5	4	∗2.44	468
6	−1	∗2.06	498
6	2	1.33	∗460
6	−4	0.61	∗466
6	−5	∗2.60	∗463
6	6	∗ − 2.04	523
7	0	∗2.20	469
7	−2	−0.91	∗536
8	−1	∗2.31	492
8	−5	∗2.04	496
8	−7	∗1.96	528
10	−1	∗2.47	∗467

Second, there are 16 tests giving anisotropies between 95% and 99% significance levels. Third, there are no tests giving anisotropy above the 99% probability level. Here, except for the $m = 0$ terms, 220 test were done, and from them theoretically 220/20 = 11 tests should give $> 95\%$ probabilities by "chance". Instead of this, there are 16 such cases, which can still be by chance. The dispersion is $\sqrt{0.95 \times 11} = 3.23$, and $2 \times 3.23 = 6.46 > (16 - 11) = 5$. The probability to obtain 16 cases instead of 11 by chance is bigger than 5%. Again, similarly to all GRBs and to the short subclass, the null-hypothesis of intrinsic isotropy of long GRBs should *not* be rejected.

4. Conclusions

The results of paper may be summarized as follows.

First, the $n = 2, m = 0$ spherical harmonic shows that there is a clear anisotropy on the significance level $> 99.9\%$ in the distribution of all 2281 GRBs. This fact is expected from the BATSE sky exposure function, and is interpreted as an artificial "instrumental" anisotropy.

Second, there is a clear intrinsic anisotropy of 253 "intermediate" GRBs at the $\geq 97\%$ significance level due to the $n = 3$ $m = -1$ term.

Third, both the 419 short GRBs and the 998 long GRBs, respectively, and also all 2281 GRBs do *not* exhibit anisotropies on statistically high enough significance levels.

All these results are interesting, because the departure from intrinsic isotropy just for the new "intermediate" subclass having the smallest number of GRBs is surprising. Of course, the significance level should still be increased, because the 99% level (or even the 99.9%) is desirable. However, the 97% significance level is already remarkable, and hence surely should be announced. The refinement of significance level may follow either from the methods used in this paper and in Balázs et al. (1998) (for example, a better estimation of inaccuracy will not need the drastic truncation $n < \sqrt{N}/3$), or from wholly different statistical methods (Kendall & Stuart 1969, Bagoly et al. 1998).

Acknowledgements. We thank the valuable discussions with L.G. Balázs, I. Horváth, D. James, J. Katgert, P. Mészáros, D. Vokrouhlický and with the referee, S. Sigurdsson. This article was partly supported by GAUK grant 36/97, by GAČR grant 202/98/0522, and by Domus Hungarica Scientiarium et Artium grant (A.M.).

References

Bagoly Z., Mészáros A., Horváth I., Balázs L.G., Mészáros P., 1998, ApJ 498, 342
Balázs L.G., Mészáros A., Horváth I., 1998, A&A 339, 1
Balázs L.G., Mészáros A., Horváth I., Vavrek R., 1999, A&AS 138, 417
Boas M.L., 1983, Mathematical Methods in the Physical Sciences, Second Edition, Wiley
Bracewell R.N., 1978, The Fourier Transform and its Applications, Second Edition, McGraw-Hill, Auckland
Briggs M.S., Paciesas W.S., Pendleton G.F., et al., 1996, ApJ 459, 40
Fishman G.J., Meegan C.A., Wilson R.B., et al., 1994, ApJS 92, 229
Horack J.M., Meegan C.J., Fishman G.J., et al., 1993, ApJ 413, 293
Horváth I., 1998, ApJ 508, 757
Kendall M.G., Stuart A., 1969, Advanced Theory of Statistics, Vol. 1., Griffin, London
Meegan C.A., Pendleton G.N., Briggs M.S., et al., 1998, Current BATSE Gamma-Ray Burst Catalog, http://www.batse.msfc.nasa.gov/data
Mészáros A., 1997, A&A 328, 1
Mukherjee S., Feigelson E.D., Babu G.J., et al., 1998, ApJ 508, 314
Peebles P.J.E., 1980, The large-scale structure of the Universe, Princeton University Press, Princeton
Pendleton C.N., Paciesas W.S., Briggs M.S., et al., 1997, ApJ 489, 175
Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1992, Numerical Recipes in Fortran, Second Edition, Cambridge University Press, Cambridge
Tegmark M., Hartmann D.H., Briggs M.S., Hakkila J., Meegan C.A., 1996, ApJ 468, 214
Trumpler R.J., Weaver H.F., 1953, Statistical Astronomy, University of California Press, Berkeley