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# An investigation of the revised CORS method based on theoretical models

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**Abstract.** Direct measurements of Cepheid radii are a key for understanding the physical structure of these variables, and in turn for constraining their pulsation properties. In this paper we discuss the numerical experiments we performed for testing the accuracy of Cepheid radii obtained by adopting both the pure Baade-Wesselink (BW) method and the revised CORS method, as well as the consistency of the physical assumptions on which these methods are based. We applied both the BW and the revised CORS methods to the synthetic light, color and radial velocity curves predicted by Cepheid full amplitude, nonlinear, convective models at solar chemical composition.

We found that these methods systematically either underestimate or overestimate "theoretical" radii if radius determinations are based on optical $(BVR)$ bands or on $(VJK)$ bands, respectively. At the same time, current simulations suggest that CORS radii are in very good agreement with "theoretical" radii if the surface brightness is calibrated by adopting a bidimensional fit of atmosphere models which accounts for temperature, gravity, and bolometric correction variations along the pulsation cycle.

Finally, a slight discrepancy between "computed" and "theoretical" radii of a Bump Cepheid supports the exclusion of these pulsation phases in both BW and CORS analyses. In fact, we found that the assumption of quasi-static approximation is no longer valid during the pulsation phases in which appears the bump.

**Key words:** stars: distances – stars: fundamental parameters – stars: oscillations – stars: variables: Cepheids

## 1. Introduction

The Baade-Wesselink (BW, Baade 1926; Wesselink 1946) method on the basis of luminosity, color, and radial velocity variations along the pulsation cycle provides the key opportunity to estimate both radii and distances of variable stars. Both the physical assumptions on which this method relies and the intrinsic drawbacks have been thoroughly discussed in the literature (Oke et al. 1962; Gautschy 1987; Bono et al. 1994; Butler et al. 1996).

Different approaches have been suggested for improving both accuracy and consistency of the BW method:

a) the radius variations are estimated by adopting a maximum likelihood method (Balona 1977; Laney & Stobie 1995) which accounts for observational errors.

b) The use of light and velocity variations together with two color indices to account for both temperature and gravity changes along the pulsation cycle (Caccin et al. 1981; Sollazzo et al. 1981; Onnenbo et al. 1985, the CORS group). However, the CORS method needs photometric calibration and a good sampling of both light and velocity curves for evaluating temperatures and gravities.

c) The use of a surface brightness relation (Barnes & Evans 1976; Gieren et al. 1989; Fouqué & Gieren 1997; Gieren et al. 1997, hereinafter GFG). However, this method needs accurate calibration of the surface brightness parameter in order to provide simultaneous estimates of radii and distances.

d) A detailed comparison between theory and observations brought out that Cepheid radii and distance determinations should be based on atmosphere models constructed by adopting a microturbulence velocity of the order of 4 km $s^{-1}$ (Bersier et al. 1997). Thus confirming the result originally pointed out by Lub & Pel (1977) and Pel (1978).

e) In a recent paper Ripepi et al. (1997, hereinafter RBMR) revised the CORS method by including the surface brightness calibration suggested by Barnes & Evans (1976). This method has been applied to a large sample of Galactic Cepheids and the radius estimates they obtained are in fair agreement with previous evaluations.

f) Krockenberger et al. (1997) adopted a Fourier analysis of both light and velocity curves to account for individual measurement errors.

On the basis of these developments and of accurate photometric and spectroscopic data it has been suggested that the most recent estimates of Cepheid radii are affected by very small internal errors (Di Benedetto 1997; GFG). Moreover, in

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a recent investigation based on new Cepheid models Bono et al. (1998, hereinafter BCM) settled a long-standing discrepancy between theoretical and empirical Period-Radius (PR) relations (Laney & Stobie 1995)[1]. In fact, they found very good agreement between theory and observations in the period range $0.9 \leq \log P \leq 1.8$. However, they also found that outside this range, at both shorter and longer periods theoretical predictions attain intermediate values between empirical radii estimated by adopting different BW methods and/or photometric bandpasses.

Even though, it has been recently suggested that period and radii of Cepheids obey to a *universal* PR relation, theoretical predictions support the evidence that both the slope and the zero point of this relation depend on metallicity (BCM). Moreover, it has also been estimated that the accuracy of Cepheid radii based on infrared colors is of the order of 3% (GFG) and therefore the metallicity dependence, if any, should have already been detected. However, preliminary results (Laney 1999, 2000) based on a large sample of Galactic and MC Cepheids for which multiband photometric data are available seem to support theoretical predictions, and indeed he found that the radii of MC Cepheids are, at one $\sigma$ level, systematically larger than the radii of the Galactic ones.

A similar discrepancy has been found between theoretical and empirical estimates of the Cepheid intrinsic luminosity. In fact, recent theoretical investigations support the evidence that the Cepheid PL relation depends on the metallicity, since at fixed period metal-rich Cepheids are fainter than metal-poor ones (Bono et al. 1999a, hereinafter BMS; Bono et al. 1999b, hereinafter BCCM). However, these predictions are at odds with current empirical estimates based on the BW method or on other approaches, since the latter show that the PL relation is either unaffected by the metallicity (GFG), or it presents a mild dependence but with an opposite sign, i.e. metal-rich Cepheids seem to be brighter than metal-poor ones (Sasselov et al. 1997; Kennicutt et al. 1998).

The main aim of this investigation is to test both physical and numerical assumptions adopted for developing the revised CORS method by performing a set of numerical experiments based on theoretical light, color and radial velocity curves. The pulsation models and the static atmosphere models adopted for transforming theoretical observables into the observative plane are discussed in Sect. 2. In Sect. 3 we briefly summarize the leading equations on which the revised CORS method is based and then we describe the approach adopted for testing the method. The results of the numerical experiments we performed are presented in Sects. 4.1 and 4.2, together with a detailed analysis of the dependence of radius estimates on the photometric bands currently adopted.

A new calibration of the surface brightness based on atmosphere models, which accounts for both temperature and gravity changes of classical Cepheids, is discussed in Sect. 4.3. In this section the improvements in CORS radii obtained by adopting the theoretical instead of the empirical calibration are also presented together with the limits of the quasi-static approximation close to the bump phases. A brief discussion on future developments closes the paper.

[1] The theoretical PR relations provided by BCM were estimated by adopting a large set of full amplitude, nonlinear, convective models which cover a wide range of stellar masses ($5 \leq$ M/$M_\odot \leq 11$), and effective temperatures ($4000 \leq T_{\rm e} \leq 7000$ K). The models were constructed by adopting three chemical compositions which are representative of Galactic ($Y = 0.28$, $Z = 0.02$), Large Magellanic Cloud (LMC, $Y = 0.25$, $Z = 0.008$), and Small Magellanic Cloud (SMC, $Y = 0.25$, $Z = 0.004$) Cepheids. On the basis of predicted periods and radii, BCM derived analytical PR relations at fixed chemical composition of the type $\log P = \alpha + \beta \log R$.

**Table 1.** Physical properties of the selected Cepheid models

| model | Mass $\mathrm{M/M_\odot}$ | Luminosity $\log \mathrm{L/L_\odot}$ | $T_{\rm e}$ K | Radius $\mathrm{R_\odot}$ | Period Days |
|---|---|---|---|---|---|
| 5m1 | 5 | 3.07 | 5800 | 34.3 | 3.5231 |
| 5m2 | 5 | 3.07 | 5600 | 36.7 | 3.9569 |
| 7m1 | 7 | 3.65 | 5300 | 81.0 | 12.1307 |
| 7m2 | 7 | 3.65 | 5000 | 91.3 | 14.7877 |
| 7m3 | 7 | 3.65 | 4800 | 97.3 | 16.8658 |
| 9m1 | 9 | 4.00 | 4900 | 141.1 | 27.2763 |
| 9m2 | 9 | 4.00 | 4700 | 152.8 | 31.3729 |
| 9m3 | 9 | 4.00 | 4500 | 164.5 | 36.0966 |
| 11m1 | 11 | 4.40 | 4800 | 230.2 | 59.7157 |
| 11m2 | 11 | 4.40 | 4300 | 282.7 | 86.3676 |
| 11m3 | 11 | 4.40 | 4000 | 310.6 | 106.5060 |

## 2. The synthetic curves

In order to test both the accuracy and the consistency of the revised CORS method we adopted the observables predicted by hydrodynamical models of variable stars. The reader interested in a detailed discussion on the physical assumptions adopted to construct these models and on the comparison between theory and observations is referred to BCM, BMS, and BCCM. Among the different sequences of nonlinear models we selected canonical models[2] at solar chemical composition (Y=0.28, Z=0.02) and stellar masses ranging from 5 to 11 $M_\odot$. At fixed stellar mass we generally selected three models which are located in the middle of the instability strip as well as close to the blue and the red edge. The period of the selected models roughly ranges from 3.5 to 106 days. The input parameters and the pulsation periods are summarized in Table 1 which gives, from left to right, (1) the model identification, (2) the stellar mass, (3) the luminosity, (4) the effective temperature, (5) the nonlinear time average radius along a full pulsation cycle, (6) the nonlinear pulsation period.

Theoretical observables have been transformed into the observational plane by adopting the bolometric corrections (BC) and the color-temperature relations by Castelli et al. (1997a,b). We assumed $M_{\rm Bol}(\odot)$=4.62 mag. The main difference between the static atmosphere models constructed by the quoted authors

[2] The canonical models were constructed by adopting a mass-luminosity relation based on evolutionary tracks which neglect the convective core overshooting during hydrogen burning phases (Castellani et al. 1992).

and the grid of models computed by Kurucz (1992) is that overshooting was neglected. In fact, they found that for temperatures and gravities typical of the Cepheid instability strip both the color indices and the Balmer profiles of the models constructed by neglecting overshooting are in better agreement with observational data. Unfortunately the set of atmosphere models provided by Castelli et al. (1997a,b) was constructed by adopting a fixed value of microturbulence velocity $\xi = 2$ km s$^{-1}$. Even though it has been recently suggested by Bersier et al. (1997) that theoretical colors based on atmosphere models which adopt higher microturbulent velocities are in better agreement with observational data, we plan to investigate the dependence on this parameter as soon as homogeneous sets of atmosphere models constructed by adopting different $\xi$ values become available.

To account for the effect of the gravity on both magnitudes and colors, the luminosity and temperature variations along the pulsation cycle have been transformed by adopting static and effective gravities[3]. In the following the models transformed by adopting $g_{\rm stat}$ and $g_{\rm eff}$ will be referred to as "static" and "effective" models, respectively. For each model we have taken into account two magnitudes -$V$, $K$- and four colors, namely $(B-V)$, $(V-R)$, $(V-K)$, and $(J-K)$. Fig. 1 shows these curves together with the variations of radius, effective temperature, and gravity for the model at 7 $M_\odot$ and $T_{\rm e} = 5300$ K. The curves plotted in this figure show quite clearly that both magnitude and colors present a negligible dependence on gravity. In fact, even though static and effective gravities attain different values along the cycle and present a difference of the order of 0.05 dex close to the bump phases, the two synthetic curves are almost identical (for a detailed analysis of the dependence of bump Cepheids on static and effective gravities see Sect. 4).

## 3. Test of the revised CORS method

In the following we briefly summarize the main features of the CORS method. The reader interested in a comprehensive discussion on the adopted physical and numerical assumptions is referred to RBMR and references therein. The CORS method relies on the definition of surface brightness $S_V$:

$$S_V = V + 5 \cdot \log \alpha \tag{1}$$

$$\Updownarrow$$

$$M_V - S_V + 5 \cdot \log(R/R_\odot) = cost. \tag{2}$$

The solution is found by differentiating Eq. (2) with respect to the phase, then by multiplying the result for a color index, e.g. $(B-V)$, and eventually by integrating along the full cycle.

Since the radial velocity is tightly connected with the pulsation velocity according to:

$$\dot{R}(\phi) = -p \cdot P \cdot u(\phi) \tag{3}$$

[3] The static gravity is defined as $g_{\rm stat} = GM/R^2$, while the effective gravity as $g_{\rm eff} = g_{\rm stat} + du/dt$, where $u$ is the radial velocity. For a detailed discussion concerning the dependence on the effective gravity the interested reader is referred to Lub & Pel (1977) and to Bersier et al. (1997).

we obtain the following equation:

$$a \int_0^1 \log\{R_0(\phi) - p \cdot P \int_{\phi_0}^{\phi} u(\phi') \cdot d\phi'\} (B \dot{-} V)(\phi) \cdot d\phi + -B + \Delta B = 0 \tag{4}$$

$$B = \int_0^1 (B-V)(\phi) \cdot \dot{V}(\phi) \cdot d\phi \tag{5}$$

$$\Delta B = \int_0^1 (B-V)(\phi) \cdot \dot{S_V}(\phi) \cdot d\phi \tag{6}$$

where $\phi$ is the phase, $P$ is the pulsation period, $R_0$ is the radius (in solar units) at a given phase $\phi_0$, $u$ is the radial velocity, $p$ is the radial velocity projection factor (Parsons 1972; Gieren et al. 1989; Sabbey et al. 1995) and $a$ is a constant equal to $5 \cdot \log_e 10$.

The numerical solution of Eq. (4) supplies the unknown quantity $R_0$. In order to evaluate the radius as a function of the phase we adopt Eq. (3) and finally the mean radius is estimated by averaging along the radius curve. By neglecting the $\Delta B$ term in Eq. (4), we obtain the pure Baade-Wesselink method which requires a radial velocity, a magnitude and a color curve for each individual variable. A more precise radius determination can be obtained by including the $\Delta B$ term. In fact, Sollazzo et al. (1981) and RBMR demonstrated that the inclusion of this term improves the accuracy of radius estimates, provided that $S_V$ is evaluated at each pulsation phase. The $\Delta B$ term was included in the original CORS method (Sollazzo et al. 1981) by adopting the empirical photometric calibration of the Walraven system provided by Pel (1978), and in the revised CORS method (see Sect. 2.3 in RBMR) by adopting the empirical calibration of the *reduced* surface brightness $F_V$ ($S_V = const - 10 \cdot F_V$) as a function of $(V-R)$, provided by Barnes & Evans (1976). This change allowed RBMR to apply the CORS method to a large sample of Cepheids for which photometric data in the conventional $BVRI$ bands were available. It is worth underlining that both the original and the revised CORS method do require two color curves but the latter method, thanks to the new calibration, can supply radius estimates of Cepheids for which are available two different pairs of Johnson/Cousins color indices.

In order to test the accuracy of the $\Delta B$ term evaluation we apply the two previous approaches to synthetic light, color, and radial velocity curves. In particular, we adopted theoretical periods and the synthetic curves, covered with 125 points, were fitted with Fourier series which include up to 31 terms (15 sine, 15 cosine plus a constant term) and eventually the quantities B and $\Delta B$ were evaluated as well. We adopted a large number of both points and Fourier terms, since we are interested in testing the accuracy of the revised CORS method by adopting theoretical templates which should not be affected, within the intrinsic uncertainties, by systematic and/or deceptive errors.

Since the modified CORS method requires an empirical estimation of the surface brightness as a function of a color, in this investigation we applied the calibrations provided by Fouqué & Gieren (1997) on the basis of stellar angular diameter measurements collected by Di Benedetto (1993) and Dyck et al. (1996),

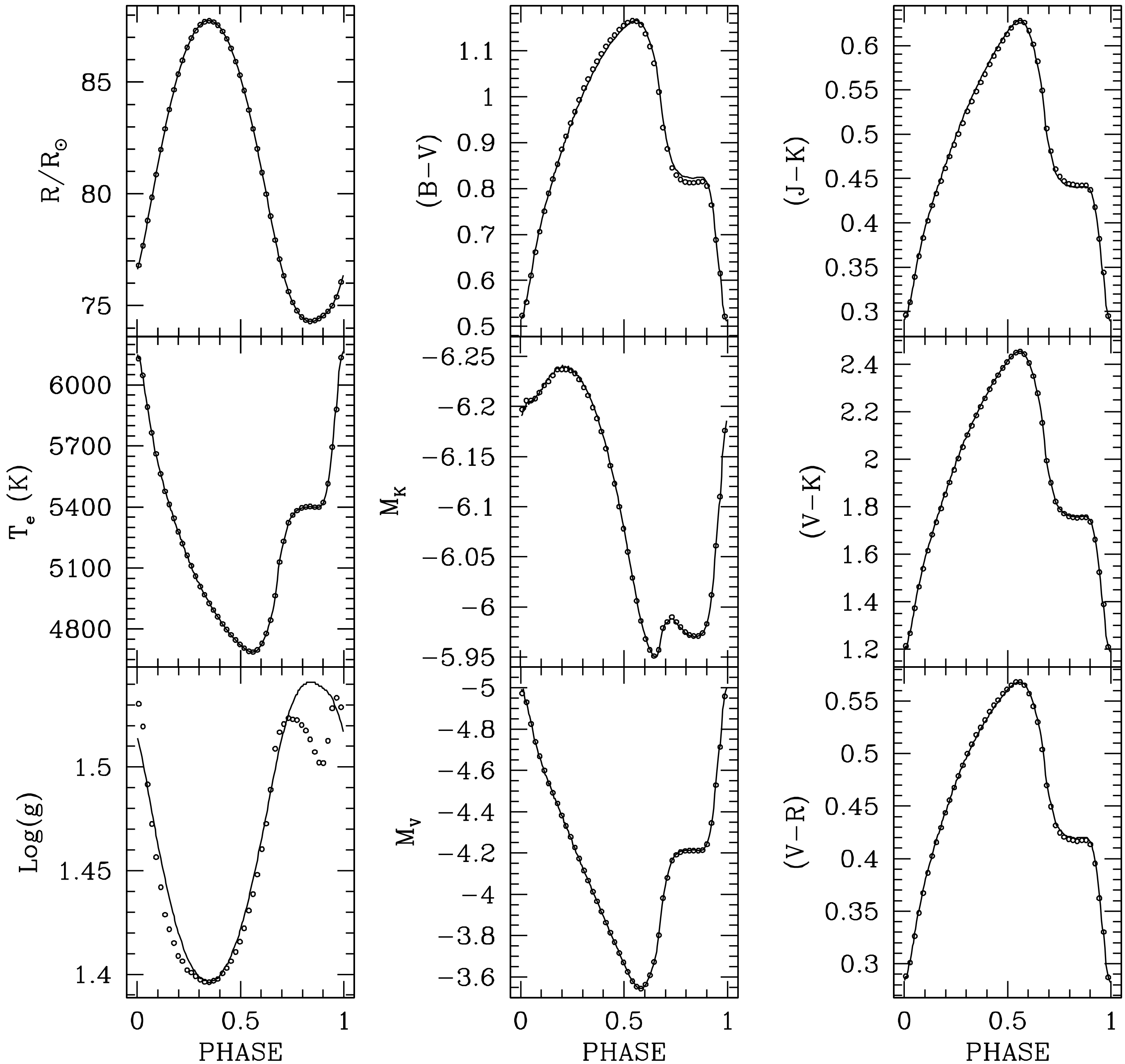


**Fig. 1.** Variations along a full pulsation cycle of several theoretical observables for the model at 7 $M_{\odot}$ and $T_{\mathrm{e}} = 5300$ K. This model presents a well-defined bump soon after the phase of luminosity maximum. Solid lines and dots display magnitudes and colors transformed by adopting static and effective gravities respectively.

i.e.:

$$F_{V_0} = 3.947 - 0.380(V - R_J)_0 \quad (7)$$

$$F_{V_0} = 3.947 - 0.131(V - K)_0 \quad (8)$$

$$F_{K_0} = 3.947 - 0.110(J - K)_0 \quad (9)$$

where $F$ is the *reduced* surface brightness and $R_J$ is the Johnson $R$ band. However, it is worth noting that the $R$ photometric bandpass adopted by Castelli et al. (1997a,b) is the Cousins band. Therefore to account for the color difference between $(V - R_J)$ and $(V - R_C)$ the slope in Eq. (7) has to be replaced with 0.521 according to the transformation provided by Bessel (1979). The uncertainty on this color transformation, due to a twofold fortunate circumstance, has negligible effects on radius estimates. In fact, as discussed by RBMR, only the slope of the $F_V$ versus color calibration is taken into account in the revised CORS method, since the zero point does not affect the derivatives. On the other hand, a change of the order of 30% in the slope of Eq. (7), due to the additive nature of the $\Delta B$ term, causes a change of only 4% in the radius estimates.

A similar calibration -$F_V$ versus $(V-K)$- was originally suggested by Di Benedetto (1995). However, for applying the revised CORS method to different colors, we adopted the multiband calibrations provided by Fouqué & Gieren (1997). We emphasize once again that the modified CORS method adopts one magnitude and two color curves (cases 2, 3, 5 below), whereas the pure BW method adopts one magnitude and one color curve (cases 1, 4, 6 below). On the basis of the selected bands we investigated the following combinations of magnitudes and colors:

1. $V$, $(B-V)$
2. $V$, $(B-V)$, $(V-R)$
3. $V$, $(B-V)$, $(V-K)$
4. $V$, $(V-K)$
5. $K$, $(V-K)$, $(J-K)$
6. $K$, $(J-K)$

These bands were selected because they are quite common in the current literature, and also because they give a proper coverage of both optical and NIR wavelenghts.

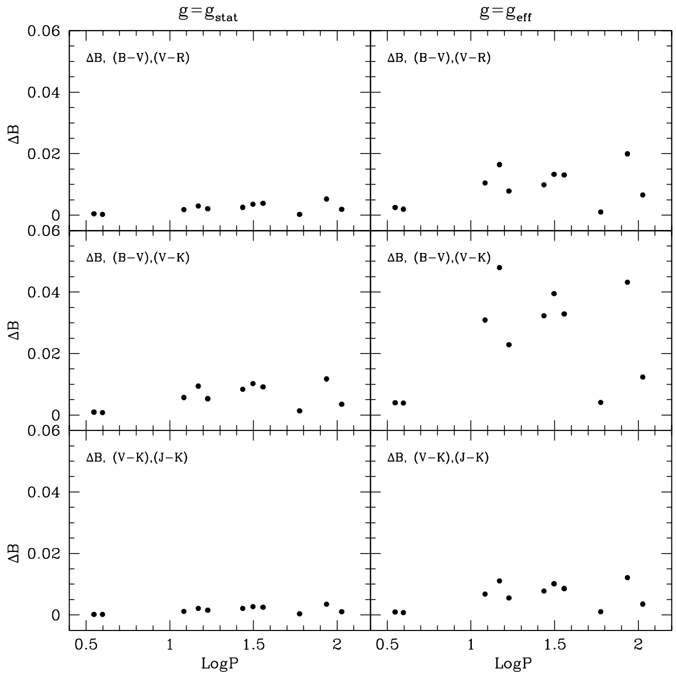


**Fig. 2.** $\Delta B$ values as a function of the logarithmic period for the whole sample of Cepheid models. The left panels show the models transformed into the observational plane by adopting the static gravity, while the right ones the models transformed by adopting the effective gravity.

## 4. Results

### *4.1. Dependence of the $\Delta B$ term on photometric bands*

The main aim of the present analysis is to provide tight constraints on the $\Delta B$ term adopted in the CORS method. This term quantifies the area of the loop performed by the variable in the $S_V$-color plane and therefore provides an estimate of the failure of the BW assumption that phases of equal color are also phases of equal temperature. In fact, the area of the loop described by the Cepheid in this plane supplies fundamental information on the variation of both effective gravity and effective temperature values along the pulsation cycle (Caccin et al. 1981).

Fig. 2 shows the $\Delta B$ values we obtained by adopting the selected magnitude and color combinations (see labels) for both static (left panels) and effective (right panels) models. The first interesting outcome is that $\Delta B$ values attained by static models are systematically smaller than the values of the effective models. This can be easily explained by the fact that the area of the color-color loops performed by effective models is larger than that of the static ones. This difference is caused by the sudden changes in the acceleration term (see Sect. 2) during the phases of rapid expansion and/or contraction. This larger excursion implies not only a difference in the area of the loop but also a change of its shape.

The $\Delta B$ values of effective models present substantial differences between different color pairs, and indeed the values attained for $[(V-K),(J-K)]$ colors are at least a factor of three smaller than the values for $[(B-V),(V-K)]$ colors. However, we note that radius evaluations based on colors which present large $\Delta B$ values are not *a priori* more reliable than the evaluations based on colors with small $\Delta B$ values. In fact, in the following we show that radii based on $[(V-K),(J-K)]$ colors are more in agreement with theoretical radii than the radii based on $[(B-V),(V-K)]$ colors. At the same time, large $\Delta B$ values do not *a priori* imply that the CORS method is a major breakthrough in radius evaluations when compared to the BW method. In fact, radius estimates based on the BW method in $[V,(V-K)]$ and on the CORS method in $[V,(B-V),(V-K)]$ are in very good agreement with theoretical radii, and the discrepancy for both of them is smaller than 10%.

### *4.2. Dependence of radius estimates on photometric bands*

The radius estimates of effective models in different photometric bands are plotted in Fig. 3. Left and right panels show the radii evaluated by neglecting (pure BW method) and by including (revised CORS method) the $\Delta B$ term respectively. To evaluate the accuracy of radius estimates based on different methods, in this figure we plotted the ratio between "computed" and "theoretical" radii. Data plotted in the left panels show quite clearly that BW estimates based on $[V,(V-K)]$ and $[K,(J-K)]$ bands are in very good agreement with theoretical radii, and indeed the discrepancy is systematically smaller than 10%. On the other hand, the radius evaluations in $[V,(B-V)]$ are systematically smaller than the predicted ones and the discrepancy is of the order of 30% close to $\log P \approx 1.6$. Thus confirming the empirical evidence originally pointed out by Welch (1994) and by Laney & Stobie (1995) that the use of $(V-K)$ colors or infrared bands ensures more accurate measurements of Cepheid radii.

This result is further strengthened by radius estimates obtained by means of the revised CORS method (left panels).

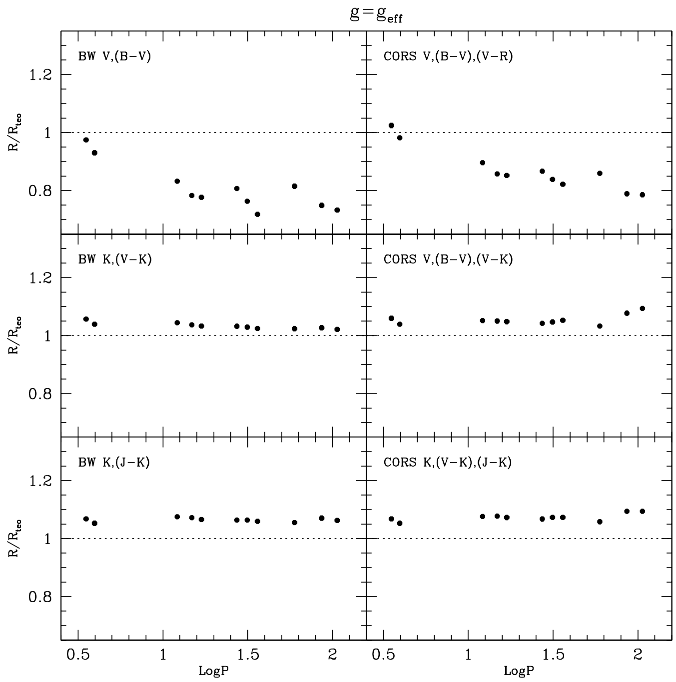


**Fig. 3.** Ratio between "computed" and "theoretical" radii as a function of the logarithmic period. The left panels display the radius evaluations based on a pure BW method, while the right ones the radius evaluations based on the revised CORS method. The radius estimates plotted in this figure refer to models transformed by adopting $g = g_{\rm eff}$.

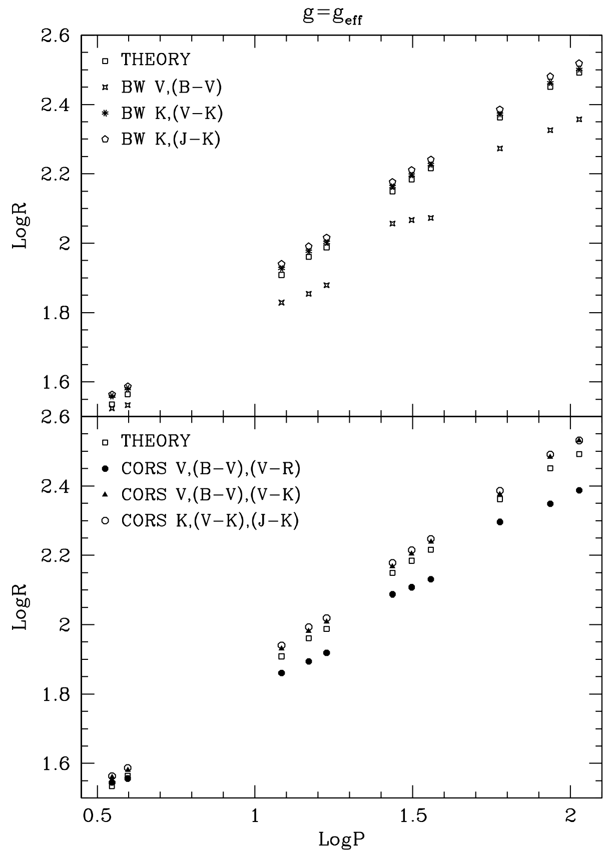


**Fig. 4.** "Computed" and "theoretical" Cepheid radii as a function of the pulsation period in a $\log - \log$ plane. The top panel shows the radius estimates obtained by adopting a pure BW method, while the radii plotted in the bottom panel by adopting the revised CORS method. Radius estimates based on different magnitudes and/or colors are displayed with different symbols.

In fact, the discrepancy between "computed" and "theoretical" radii is generally smaller than 10% when both $[V, (B-V), (V-K)]$ and $[K, (V-K), (J-K)]$ bands are adopted. At the same time, it is worth noting that radius determinations based on optical bands -$[V, (B-V), (V-R)]$- present a discrepancy smaller or equal to 20% over the entire period range. The results of our numerical experiments suggest that by adopting NIR bands the radii evaluated through the revised CORS method present on average the same accuracy of the radii based on the pure BW method. However, the former method supplies more accurate radius determinations than the latter one when optical bands are adopted. Thus supporting the plausibility of physical and numerical assumptions adopted in the revised CORS method.

We will now focus our attention on the choice of the photometric bands which should be adopted for providing accurate radius determinations. Fig. 4 shows the comparison between "computed" and "theoretical" radii in the $\log P - \log R$ plane. Data plotted in the top and in the bottom panels display radius estimates based on the pure BW and on the revised CORS method respectively. The main outcomes of this comparison are the following: 1) the slope of the PR relation, as already noted by Laney & Stobie (1995), becomes steeper when moving from optical to NIR bands. 2) Radius estimates based on NIR/optical bands overestimate/underestimate theoretical radii. These results apply to radius evaluations based on the pure BW method and on the revised CORS method, thus supporting the evidence that this "photometric drift" is not an artifact of the method adopted for estimating the radius. At the same time, data in the bottom panel of Fig. 4 suggest that radii obtained by averaging the estimates in the $[V, (B-V), (V-R)]$ and in the $[K, (V-K), (J-K)]$ bands are much less affected by systematic errors than the radius evaluations only based on NIR bands or on optical bands.

### *4.3. A theoretical estimate of the $\Delta B$ term*

Even though previous results supply useful suggestions for improving the accuracy of radius measurements, the collection of both NIR and optical data for a large Cepheid sample is not a trivial observational effort. As a consequence, we decided to improve the approach suggested by RBMR for evaluating the $\Delta B$ term. Since $\Delta B$ is the area of the loop performed by each variable in the $S_V$-color plane, the idea is to compute $S_V$ along the pulsation cycle directly from observations. However, the surface brightness depends on both $T_{\rm e}$ and $g_{\rm eff}$, and therefore two relations should be inverted for deriving $S_V$:

$$C_1 = f\,(T_{\rm e}, g_{\rm eff})$$
$$C_2 = g\,(T_{\rm e}, g_{\rm eff})$$

where $C_1$ and $C_2$ are two arbitrary colors. Unfortunately this problem does not admit a general solution over the whole parameter space, since the same color can be obtained for different pairs of $T_{\rm e}$ and $g_{\rm eff}$ values. This notwithstanding, it is still possible to find a local solution. Fig. 5 shows the surface covered by synthetic models in the 3D spaces $[(B-V), \log T_{\rm e}, \log g_{\rm eff}]$ and $[(V-R), \log T_{\rm e}, \log g_{\rm eff}]$ respectively. Data plotted in these figures show quite clearly that theoretical models populate a well-defined region of the quoted space. Therefore by performing a $4^{th}$ degree polynomial fit to the data it is possible to invert the two relations governing the $(B-V)$ and the $(V-R)$ colors as a function of temperature and gravity. The results of the polynomial approximations are presented in the appendix.

On the basis of these relations we can estimate the surface brightness $S_V$ directly from the following equation:

$$S_V = const. - 10 \log T_{\rm e} - BC \tag{10}$$

where the symbols have their usual meaning. The constant term depends on the photometric system used, but in our application it is not relevant, since the surface brightness in Eq. (6) appears as a derivative.

By taking into account this new theoretical calibration we applied once again the CORS method to the sample of synthetic models for estimating the Cepheid radii. Fig. 6 and 7 show the results of these calculations. Data plotted in these figures support the evidence that:

1) the theoretical calibration of the surface brightness we developed is intrinsically correct. In fact, the discrepancy between "computed" and "theoretical" Cepheid radii is systematically smaller than 7%. The only exception to this behavior is the radius of the model at $7M_\odot$ and $T_{\rm e} = 5300$ K which shows a well-defined bump along the rising branch. This evidence suggests that CORS estimates of Bump Cepheid radii could be affected by systematic errors. However, data plotted in Fig. 1 show that the outermost layers of this model undergo sudden gravity changes close to the bump phases. During these pulsation phases the assumption of hydrostatic equilibrium is no longer valid and therefore both the bolometric corrections and the colors obtained by adopting static atmosphere models should be regarded as suitable average estimates of the actual properties. It can be easily shown (Bono 1994) that this limit is mainly due to the effective gravity, since this quantity is estimated by assuming both radiative and hydrostatic equilibrium. The effective temperature only depends on the assumption of radiative equilibrium but the departures from the radiative equilibrium are, under the typical conditions of a pulsation cycle, quite small. These leading physical arguments suggest that the bump phases should be neglected in the CORS analysis.

2) In comparison with "theoretical" radii the "computed" radii do not show any systematic shift. This result suggests that the $\Delta B$ terms evaluated by adopting the theoretical calibration - based on the polynomial approximations of $T_{\rm e}$ and $g_{\rm eff}$ in the color-color plane $[(B-V)-(V-R)]$ and of the BC in the $[\log T_{\rm e} - \log g_{\rm eff}]$ plane- are more accurate than the $\Delta B$ terms obtained by means of the empirical calibration.

3) Accurate radius determinations can be obtained by adopting photometric data in three optical bands.

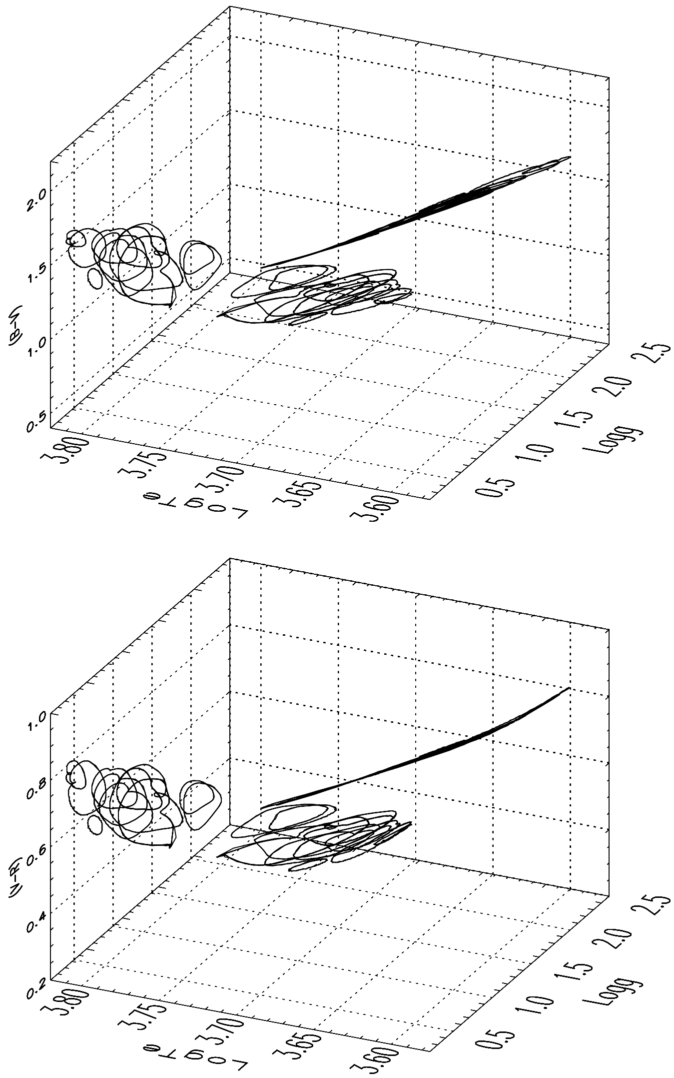


**Fig. 5.** Top plot: surface covered by the sample of theoretical models adopted in this investigation in the 3D space $[(B-V), \log T_{\rm e}, \log g_{\rm eff}]$. In order to make clear the dependence of $(B-V)$ colors on both temperature and gravity the loop performed by each variable in the $(B-V) - \log g_{\rm eff}$ plane (left panel) and in the $(B-V) - \log T_{\rm e}$ plane (right panel) are also plotted. Bottom plot: same as above, but in the 3D space $[(V-R), \log T_{\rm e}, \log g_{\rm eff}]$.

Finally, we mention that following a referee's suggestion we applied the revised CORS method to theoretical curves which mimic real observations. In particular, we performed several numerical experiments by sampling the theoretical $M_V$, $(B-V)$, $(V-K)$, and radial velocity curves with 20-30 phase points randomly distributed along the pulsation cycle. To account for observational uncertainties the points were spread out by assuming typical photometric and spectroscopic errors and then fitted with up to 7 Fourier terms. Interestingly enough, we find that radius estimates based on these curves still present a discrepancy $\leq$ 7% when compared with theoretical radii. Therefore this uncertainty can be assumed as a plausible upper limit to the intrinsic accuracy of radius determinations based on the revised CORS method.

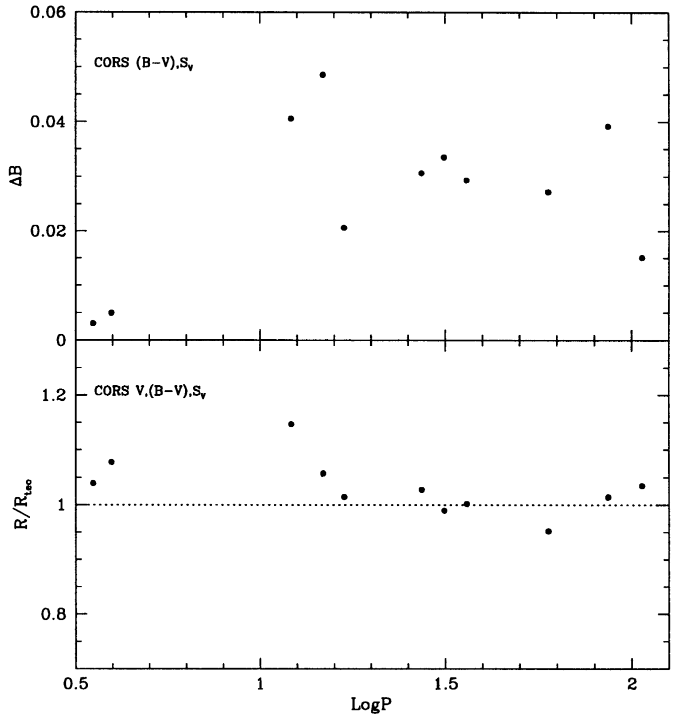


**Fig. 6.** Top panel: $\Delta B$ terms as a function of the logarithmic period obtained by adopting the revised CORS method and the theoretical calibration of the surface brightness. Bottom panel: similar to top panel but refers to the ratio between "computed" and "theoretical" radii.

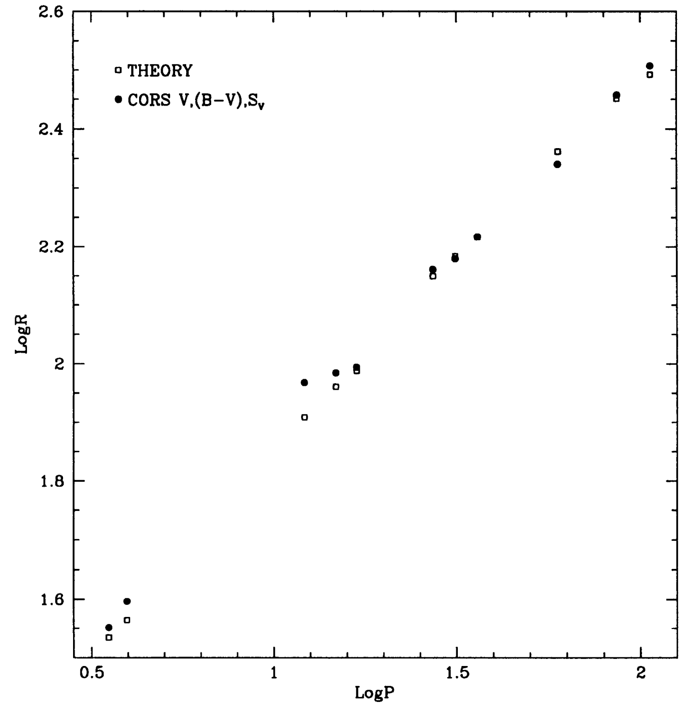


**Fig. 7.** Period-Radius relation in a log-log plane. The radius estimates have been obtained by adopting the revised CORS method and the theoretical calibration of the surface brightness. See text for further details.

## 5. Conclusions

In this paper we present the results of a detailed investigation on the intrinsic accuracy of radius estimates obtained by adopting both the revised CORS method and the pure BW method. In order to avoid systematic errors the numerical experiments were performed by adopting theoretical observables -light, color, and radial velocity variations- predicted by nonlinear, convective models of classical Cepheids at solar chemical composition. The main findings of this analysis are the following:

1) the revised CORS method and the pure BW method applied to NIR data provide radius estimates characterized on average by the same accuracy. However, the former method supplies more accurate radius determinations than the latter one when applied to optical bands.

2) In agreement with current empirical evidence (Laney & Stobie 1995) the PR relations obtained by adopting theoretical predictions are affected by the "photometric drift", i.e. the slope becomes steeper when moving from optical to NIR bands. Thus suggesting that at fixed period radius determinations based on NIR/optical bands overestimate/underestimate "true" radii.

At the same time, in order to develop a method which can be applied to a large sample of Cepheids, we provided a new theoretical calibration of the $\Delta B$ term included in the revised CORS method. On the basis of this new calibration we find that the computed radii are affected by a discrepancy when compared with theoretical radii that is $\leq$ 7%. Moreover and even more importantly, we also find that computed radii based on optical bands do not show any systematic difference with theoretical radii.

Obviously before any firm conclusion on the accuracy of the current Cepheid PR relations can be reached, this method should be applied directly to empirical data. However, the main interesting feature of the current calibration is that it only relies on theoretical models. Therefore the comparison between theory and observations can allow us to supply tight constraints on the systematic uncertainties which affect radius estimates such as metallicity, reddening, and microturbulence velocity.

Finally, we mention that direct measurements of Cepheid angular diameters through optical interferometry are becoming available (Nordgren et al. 2000, and references therein). In the near future, new and more accurate interferometric data can allow us to assess on a firm physical basis the calibration of the CORS method. At the same time, it is worth emphasizing that the development of a homogeneous theoretical framework to be compared with new empirical data can also supply sound suggestions on the plausibility and the accuracy of the physical assumptions adopted for constructing both pulsation and atmosphere models.

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**Table A.1.** Coefficients for the polynomial fits described in the Appendix.

| $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
|---|---|---|---|---|---|---|---|---|
| 3.896 | -0.012 | 0.034 | -0.199 | 0.270 | -0.319 | -1.249 | 0.831 | 0.032 |
| $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ |
| 0.963 | -168.126 | 124.432 | 251.995 | -5.932 | -147.224 | -80.090 | -14.874 | 75.675 |
| $c_0$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ |
| -81.064 | 35.986 | -3.827 | -286.189 | 157.642 | -21.706 | 104.568 | -57.152 | 7.809 |

## Appendix A: polynomial approximations

The $4^{th}$ degree polynomial fit to effective temperature, effective gravity, and bolometric corrections mentioned in Sect. 4.3 are the following:

$$\begin{aligned}\log T_{\rm e} &= a_0 + a_1(B-V) + a_2(B-V)^2 \\ &+ a_3(V-R) + a_4(B-V)(V-R) \\ &+ a_5(B-V)^2(V-R) + a_6(V-R)^2 \\ &+ a_7(B-V)(V-R)^2 + a_8(B-V)^2(V-R)^2\end{aligned} \quad (A.1)$$

$$\begin{aligned}\log g_{\rm eff} &= b_0 + a_1(B-V) + b_2(B-V)^2 \\ &+ b_3(V-R) + b_4(B-V)(V-R) \\ &+ b_5(B-V)^2(V-R) + b_6(V-R)^2 \\ &+ b_7(B-V)(V-R)^2 + b_8(B-V)^2(V-R)^2\end{aligned} \quad (A.2)$$

$$\begin{aligned}BC &= c_0 + c_1\log T_{\rm e} + c_2\log T_{\rm e}^2 \\ &+ c_3\log g_{\rm eff} + c_4\log T_{\rm e}\log g_{\rm eff} \\ &+ c_5\log T_{\rm e}^2\log g_{\rm eff} + c_6\log g_{\rm eff}^2 \\ &+ c_7\log T_{\rm e}\log g_{\rm eff}^2 + c_8\log {T_{\rm e}}^2\log g_{\rm eff}^2\end{aligned} \quad (A.3)$$

the coefficients $a_i, b_i, c_i$ of the previous relations are listed in Table A.1 and the other symbols have their usual meaning. Note that the r.m.s. of the previous relations are 0.0008, 0.04 and 0.003 respectively.

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