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On the radio spectral index of galaxies

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Abstract. The radio emission of a galaxy consists of thermal bremsstrahlung, synchrotron emission from discrete supernova remnants, and diffuse synchrotron emission from cosmic ray electrons spread over the galactic disk and halo. Each of these components has a different spectral index so that the total radio spectral index of a galaxy depends sensitively on their relative contribution and on the processes shaping the diffuse synchrotron emission. In the present paper we calculate the contribution of supernova remnants to the total synchrotron emission and conclude that it is about 10%. This moderate contribution has a noticeable effect on the nonthermal spectral index, lowering it by $\simeq 0.1$ for steep spectra. We then calculate the diffuse synchrotron emission in two simple models, a diffusion and a convection model. We find that the spatially integrated nonthermal spectral index is in general a poor diagnostic for the type of propagation or the importance of energy losses. Spatially resolved radio data for the halo of galaxies are necessary in order to draw firm conclusions. The steepening of the spectrum away from the disk is a clear indication that synchrotron and inverse Compton losses are taking place during the propagation of cosmic ray electrons in the halo. In those galaxies for which spatially resolved data for the halos exist such a steepening has been found. We conclude therefore that energy losses are generally important and that cosmic ray electrons cannot freely escape from galaxies.

Key words: ISM: cosmic rays – ISM: supernova remnants – radio continuum: galaxies – radio continuum: ISM

1. Introduction

The total radio emission of a galaxy consists of different components: The thermal radio emission, $P_{\rm therm}$, representing bremsstrahlung from H II regions and the nonthermal radio emission which is synchrotron emission from cosmic ray electrons (CREs). As far as the nonthermal radio emission, $P_{\rm nth}$, is concerned, a further division can be made by distinguishing between the diffuse radio emission, $P_{\rm diff}$, from CREs that are spread over the galactic disks and halos and the emission from discrete supernova remnants (SNRs), $P_{\rm SNR}$. The three components possess very different spectral indices: Whereas the thermal radio emission has a flat spectral index ($\propto \nu^{-0.1}$), the nonthermal radio spectrum is steeper. The spectral index of SNRs is on average $\approx \nu^{-0.5}$ (Green 1998) and the diffuse radio emission can have, depending on the dominant process for the CR propagation, spectral indices[1] , α, between about 0.5 and 1.1 which can, additionally, vary over the frequency range. Therefore, the total radio spectral index depends very sensitively on the relative contribution of the different components.

The spectrum of the diffuse synchrotron emission is shaped by the physical processes that are characterizing the CR propagation, in particular the type of propagation (diffusion or convection), of energy losses (synchrotron, inverse Compton, adiabatic losses or bremsstrahlung), and the confinement of CREs. Here, we consider CREs confined to a galaxy if they loose their energy through combined synchrotron and inverse Compton losses, down below the energy level corresponding to the observing frequency, before they are able to leave the galactic halo. The question whether CREs are confined to galaxies or can escape from the halos without suffering considerable energy losses is important for the interpretation of the far-infrared (FIR)/radio correlation. Different models have been proposed, explaining this correlation by advocating either the situation that CREs are confined (Völk 1989; Lisenfeld et al. 1996a) or that they can escape more or less freely (Chi & Wolfendale 1990; Helou & Bicay 1993; Niklas & Beck 1997).

The interpretation of the radio spectrum in terms of the above mentioned processes is difficult because they are hard to disentangle without spatially resolved observations. Furthermore, it is necessary to separate the diffuse radio emission from the thermal radio emission and the radio emission from SNRs. The observational separation of the three contributions based on their different spectral indices is difficult, especially for the radio emission of SNRs which has a similar spectral index as the diffuse synchrotron emission. The separation of the thermal radio emission is possible, if high-frequency radio data ($\nu \approx 10-20$ GHz) are available, because its spectral index is very different.

In this paper we are going to examine the influence of these various processes on the radio spectral index. For this, we will start by estimating the contribution of SNRs to the radio emission and to calculate its effect on the spectral index. Then, we discuss within a simple model the influence of energy losses, the type of propagation and escape of CREs on the spectral index.

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[1] Here and in the following we define the spectral index α as $S(\nu) \propto \nu^{-\alpha}$

2. Radio emission of SNRs

2.1. Theoretically expected value

We estimate the fraction of the nonthermal radio emission of a galaxy that is expected to come from SNRs in the framework of a simple leaky-box model. The number of CREs in a certain energy range $\mathrm{d}E$, $N(E)$, can then be calculated as:

$$\frac{\partial N(E)}{\partial t} + \frac{N(E)}{\tau} = Q(E). \qquad (1)$$

Q(E) is the production rate of CREs and τ the life-time of CREs. The main sources of CREs are gravitational supernovae (SN) (SN II and Ib/c) (e.g. Xu et al. 1994a and discussion in Xu et al 1994b). The energy spectrum of the CREs produced can be described by a power law:

$$Q(E) = \nu_{\rm SN}\, q_{\rm SN} \left(\frac{E}{E_0}\right)^{-\gamma}, \qquad (2)$$

where $\nu_{\rm SN}$ is the SN-rate, $q_{\rm SN}$ the number of CREs produced per energy interval per SN and γ the energy injection spectral index. For CREs in SNRs τ is given by the life-time of the SNR, $\tau_{\rm SNR}$, and for the diffuse CREs $\tau = \tau_{\rm diff}$ can be expressed as:

$$\frac{1}{\tau_{\rm diff}} = \frac{1}{\tau_{\rm esc}} + \frac{1}{\tau_{\rm loss}}, \qquad (3)$$

where $\tau_{\rm esc}$ is the escape time-scale from the galaxy and $\tau_{\rm loss}$ is the energy loss time-scale. We take into account a possible energy dependence of $\tau_{\rm diff}$ according to:

$$\tau_{\rm diff}(E) = \tau_{\rm diff}(E_0) \left(\frac{E}{E_0}\right)^{-y}. \qquad (4)$$

The exponent y can have different values depending on the characteristics of the propagation of CREs. It is $y = 1$ if $\tau_{\rm diff} = \tau_{\rm loss}$ and synchrotron and inverse Compton losses are the dominant energy loss processes. If $\tau_{\rm diff} = \tau_{\rm esc}$ and CREs propagate mainly by diffusion, the energy dependence of the diffusion coefficient, $D(E) \propto D_0 E^{-\mu}$, gets reflected in $y = \mu \approx 0.5$. If, on the other hand, CREs can escape freely ($\tau_{\rm diff} = \tau_{\rm esc}$) by convection or if adiabatic and bremsstrahlung losses are dominant, we expect $y = 0$.

In the case of a steady state ($\partial N/\partial t = 0$) the solution of Eq. (1) is simply:

$$N(E) = Q(E)\, \tau. \qquad (5)$$

We make the simplification that an electron with energy E emits all the synchrotron radiation at the frequency

$$\nu = 0.435 \left(\frac{E}{m_{\rm e}c^2}\right) \nu_{\rm G} = 4.65 \left(\frac{E}{1\rm GeV}\right)^2 \left(\frac{B}{\mu\rm G}\right) \rm MHz \qquad (6)$$

(Longair 1992, Vol. 1 p. 332) where $\nu_{\rm G}$ is the gyrofrequency. Then the total synchrotron emission is given by:

$$P(\nu)\mathrm{d}\nu = N(E) \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\rm syn} \left(\frac{\mathrm{d}E}{\mathrm{d}\nu}\right) \mathrm{d}\nu, \qquad (7)$$

with the energy losses by synchrotron radiation

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\rm syn} = \frac{4}{3}\sigma_{\rm T} c \left(\frac{E}{m_{\rm e}c^2}\right)^2 U_{\rm B} = 2.5\,10^{-18} \left(\frac{B}{\mu\rm G}\right)^2 \left(\frac{E}{\rm GeV}\right)^2 \rm GeV s^{-1}, \qquad (8)$$

where $\sigma_{\rm T}$ is the Thompson scattering cross section, B the magnetic field strength and $U_{\rm B}$ its energy density.

With the above equations, we can express the ratio of the diffuse synchrotron emission to the emission from SNRs as:

$$\frac{P_{\rm diff}(\nu)}{P_{\rm SNR}(\nu)} = \frac{N(E)_{\rm diff}\,(\mathrm{d}E/\mathrm{d}t)_{\rm syn,diff}\,(\mathrm{d}E/\mathrm{d}\nu)_{\rm diff}}{N(E)_{\rm SNR}\,(\mathrm{d}E/\mathrm{d}t)_{\rm syn,SNR}\,(\mathrm{d}E/\mathrm{d}\nu)_{\rm SNR}} = \frac{E^{-\gamma}\tau_{\rm diff} E^2 B^2_{\rm diff}\,(\mathrm{d}E/\mathrm{d}\nu)_{\rm diff}}{E^{-\gamma}\tau_{\rm SNR} E^2 B^2_{\rm SNR}\,(\mathrm{d}E/\mathrm{d}\nu)_{\rm SNR}} = \left(\frac{\nu}{\nu_0}\right)^{-\frac{y}{2}} \left(\frac{B_{\rm diff}}{B_{\rm SNR}}\right)^{\frac{\gamma+1}{2}} \frac{\tau_{\rm diff}(\nu_0)}{\tau_{\rm SNR}}. \qquad (9)$$

Thus, the ratio of the diffuse synchrotron emission to the emission from SNRs is mainly determined by the ratio of the life-times of CREs in the two zones and by the ratio of the magnetic field strengths. The magnetic field strength in SNRs is expected to be higher than in the diffuse ISM because the gas in which the magnetic field is frozen-in gets compressed by the shock. In general, the magnetic field strength found in SNRs exceeds the value expected by this compression which would be a factor of 4 in a strong shock, indicating that the magnetic field is not only compressed, but enhanced by the shock. Especially in young SNRs, the inferred values span a wide range from a few μG (SN 1006; Tanimori et al. 1998) to much higher values of 70 μG (Kepler's SNR; Matsui et al. 1984), > 80 μG (Cas A; Cowsik & Sarkar 1980) and even 300 μG (Crab Nebula; Kennel & Coroniti 1984). Taking these values as a guide, we assume for our present estimate $B_{\rm SNR} = 75\mu$G and $B_{\rm diff} = 5\mu$G. Furthermore, we adopt for the injection spectral index $\gamma = 2.2$ (Völk et al. 1988), for the life-time of a SNR its adiabatic phase $\tau_{\rm SNR} \approx 3\,10^4$ yr and for τ the energy loss time scale due to synchrotron and inverse Compton losses. The latter yields for a magnetic field of 5 μG and a radiation field of energy density 1 eV cm^{-3} $\tau_{\rm diff} = 2.5\,10^7$ yr at $\nu_0 = 1.5$ GHz. With these values we get $P_{\rm SNR}/P_{\rm diff} = 0.09$ at a frequency 1.5 GHz.

A further contribution to the radio emission of a galaxy are radio supernovae (RSNe, e.g. Weiler et al. 1986). Their contribution is difficult to estimate due to uncertainties in their luminosity and life-time. Pérez-Olea & Colina (1995) have included RSNe in a model for the radio emission of starburst galaxies by generalizing and extrapolating the luminosities of 9 observed RSNe. Adopting their numbers, we calculate that for a galaxy in a steady state the total radio emission of RSNe is, for a life-time of a RSNe of 100 yr, $1.5\times$ the total radio emission of SNRs. For a shorter life-time of 10 yr, their total radio emission would be a factor of 2 lower. Thus, in spite of the uncertainties, it seems plausible that RSNe contribute at a level of at least a few percent to the total radio emission of a galaxy.

2.2. Comparison to observations

In order to compare this theoretically derived ratio to observations we calculate the expected average radio emission of a SNR:

$$P_{\rm SNR}(\nu){\rm d}\nu = q_{\rm SN}\left(\frac{E}{E_0}\right)^{-\gamma}\left(\frac{{\rm d}E}{{\rm d}t}\right)_{\rm syn}\left(\frac{{\rm d}E}{{\rm d}\nu}\right){\rm d}\nu$$

$$= 1.07\,10^{-25}\left(\frac{q_{\rm SN}}{{\rm eV}^{-1}}\right)\left(\frac{\nu}{\rm MHz}\right)^{\frac{1-\gamma}{2}}\left(\frac{B}{\mu{\rm G}}\right)^{\frac{1+\gamma}{2}}{\rm WHz}^{-1} \quad (10)$$

where we have inserted the relations from Eqs. (6) and (8). For $\gamma = 2.2$ this predicts a radio spectral index of SNRs of 0.6, in reasonable agreement with the observations. The total number of CREs per energy interval produced by a SN, $q_{\rm SN}$, can be estimated from the total energy released by a SN which we take as 10^{51} erg. Assuming that 10% of this energy goes into CR acceleration and of this 1% into the electron component, a total energy of $E^{\rm e}_{\rm SN} = 10^{48}$erg is available per SN to produce the relativistic electrons following a power-law spectrum:

$$E^{\rm e}_{\rm SN} = q_{\rm SN}\int_{\rm E_{min}}^{\infty} E\left(\frac{E}{E_0}\right)^{-\gamma}{\rm d}E. \quad (11)$$

With $\gamma = 2.2$, assuming for the lower energy limit of the produced spectrum $E_{\rm min} = 1$ GeV and adopting the scaling parameter $E_0 = 1$ GeV this yields

$$q_{\rm SN} = \frac{E^{\rm e}_{\rm SN}}{E_0^2}\left(\frac{E_{\rm min}}{E_0}\right)^{\gamma-2}(\gamma-2) = 1.25\times10^{41}{\rm eV}^{-1}. \quad (12)$$

In total, we obtain for the radio emission of a SNR, taking $B_{\rm SNR} = 75\mu$G,

$$P_{\rm SNR} = 2.1\,10^{17}\left(\frac{\nu}{\rm GHz}\right)^{-0.6}{\rm WHz}^{-1} \quad (13)$$

This can be compared to the average observed emission of a SNR which can be derived from the relation between the surface brightness Σ and the diameter D of SNRs. We adopt the relation found by Huang et al. (1994) who fitted SNRs from the Galaxy, M 82 and the Magellanic Clouds:

$$\Sigma_{\rm 8.4GHz} = 6.6\,10^{-16}\left(\frac{D}{\rm pc}\right)^{-3.6}{\rm Wm^{-2}Hz^{-1}sr^{-1}}. \quad (14)$$

Following Condon & Yin (1990) one can connect the diameter and the age of the radio emitting SNR by:

$$\left(\frac{D}{\rm pc}\right) = 0.43\left(\frac{E_{50}}{n}\right)\left(\frac{t}{\rm yr}\right)^{\frac{2}{5}}, \quad (15)$$

where E_{50} is the energy of a SN going into particle acceleration in units of 10^{50} erg and n is the gas density in particles per cm^3. In the following we assume $E_{50} = 1$ and $n = 1$. Using $L = 4\pi^2(D/2)^2\Sigma$ we then calculate the average luminosity of a SNR by integrating the luminosity evolution from $t = 0$ to $t = \tau_{\rm SNR} = 3\,10^4$ yr and adopting the frequency dependence of Eq. (13), $P_{\rm SNR} \propto \nu^{-0.6}$:

$$\langle P_{\rm SNR}\rangle = 4\pi^2\left(\frac{D}{2}\right)^2\left(\frac{1}{\tau_{\rm SNR}}\right)$$

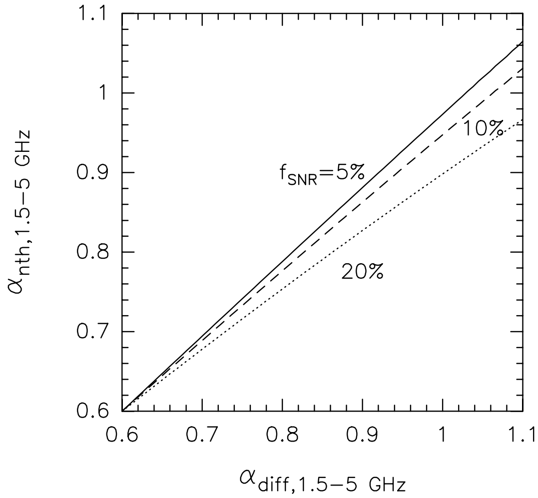

Fig. 1. The nonthermal spectral index, $\alpha_{\rm nth}$, as a function of the diffuse radio spectral index, $\alpha_{\rm diff}$, between 1.5 and 5 GHz, for different contributions of the radio emission from SNRs to the radio emission: $f_{\rm SNR}$ =5% (full line), 10% (dashed line), 20% (dotted line).

$$\int_0^{3\,10^4}\Sigma(D(t)) = 3.3\,10^{17}\left(\frac{\nu}{\rm GHz}\right)^{-0.6}{\rm WHz}^{-1}, \quad (16)$$

which is very close to the above theoretical value (Eq. 13).

We can now estimate the total emission of SNRs expected for our Galaxy: Adopting a SN rate of 1 SN per 30 yr (Berkhuijsen 1984), the total number of the SNRs present in our Galaxy is, with $\tau_{\rm SNR} = 3\,10^4$ yr, $N_{\rm SNR} = \nu_{\rm SN}\tau_{\rm SN} = 1000$. This yields, together with Eq. (13), a total radio emission from SNRs of $P_{\rm SNR,tot}(1.5\,{\rm GHz}) = P_{\rm SNR}(1.5\,{\rm GHz})N_{\rm SNR} = 2.6\,10^{20}{\rm WHz}^{-1}$. This can be compared to the total radio emission of our Galaxy which is at 408 MHz $8.7\,10^{21}{\rm WHz}^{-1}$ (Beuermann et al. 1985). Extrapolating this value, with a radio spectral index of 0.8, to 1.5 GHz we get $3\,10^{21}$ W Hz^{-1}. From this calculation we derive a contribution of about 9% from SNRs to the total radio emission, in satisfactory agreement with the value derived in the previous section in spite of the crudeness of both estimates. Therefore, we conclude that $\simeq$ 10% is a reasonable estimate for the contribution of SNRs to the total radio emission. This value is also in agreement with previous estimates by other authors, who compared the expected total synchrotron emission from SNRs with the observed radio emission of galaxies, and have found that SNRs alone can only explain about 10% of the total radio emission (Biermann 1976; Ulvestad 1982).

2.3. Influence on the spatially integrated radio spectral index

In Fig. 1 we show the total (i.e. spatially integrated) nonthermal spectral index, $\alpha_{\rm nth}$, as a function of the spectral index of the diffuse radio emission, $\alpha_{\rm diff}$, for different contributions of SNRs to the radio emission, expressed by $f_{\rm SNR} = P_{\rm SNR}/P_{\rm nth}$. We assume, as above, an (energy) injection spectral index for CREs of $\gamma = 2.2$, yielding a (frequency) spectral index of SNRs of $\alpha_{\rm SNR} = (\gamma-1)/2 = 0.6$ (Eq. (10)). A lower source spectral index of $\gamma = 2.0$ to 2.1 is indicated by recent calculations by Berezhko and Völk (1997). This would give $\alpha_{\rm SNR} = 0.5$ to 0.55 and would somewhat increase the difference between $\alpha_{\rm nth}$ and $\alpha_{\rm diff}$ for a given $f_{\rm SNR}$.

The radio emission of SNRs has a noticeable effect on the nonthermal radio spectral index. Due to the contribution of SNRs it gets flatter, especially for large values of $\alpha_{\rm diff}$ where the spectral index is lowered by ~ 0.1 for $f_{\rm SNR} = 0.1$.

3. CR propagation and energy losses

The types of propagation of CREs in a galaxy are diffusion, i.e. the CREs are scattered randomly by magnetic field irregularities, and convection, a systematic movement of the scattering centres outwardly. While propagating through a galaxy, CREs suffer energy losses. Eventually, CREs might escape from the galaxy.

In order to discuss the radio spectral index in the various cases, we calculate two simple models, one that considers diffusion and one in which we study convection. In both models we take into account energy losses by synchrotron and inverse Compton losses. Here, we ignore adiabatic and bremsstrahlung losses which might also be important, especially in starburst galaxies, but since they do not change the spectral index, they are in the framework of the present study equivalent to no energy losses at all.

3.1. Diffusion model

The diffusion model can be described by the following equation:

$$D(E)\frac{\partial^2 N(E,z)}{\partial z^2} + \frac{\partial}{\partial E}\left(b(E)N(E,z)\right) = -Q(E,z), \quad (17)$$

where z is the height above the galactic midplane. We assume that the sources of CRs are situated in the midplane at $z = 0$ and that their energy dependence is given by a power-law according to Eq. (2). The synchrotron and inverse Compton losses, $b(E)$, are assumed to be spatially homogeneous and are given by:

$$b(E) = -\left.\frac{{\rm d}E}{{\rm d}t}\right|_{\rm syn+iC} = \frac{4}{3}\sigma_{\rm T} c \left(\frac{E}{m_{\rm e}c^2}\right)^2 (U_{\rm rad} + U_B). \quad (18)$$

$U_{\rm rad}$ is the energy density of the radiation field below the Klein-Nishina limit (e.g. Longair 1992). We adopt a free escape boundary at $|z| = z_{\rm h}$, the outer boundary of the halo, so that $N(E, |z_{\rm h}|) = 0$. An approximate solution is given by (for further details see Lisenfeld et al. 1996a):

$$\begin{aligned} N(E,z) &= \frac{q_{\rm SN}\cdot\nu_{\rm SN}}{2D_0}\left(\frac{E}{E_0}\right)^{-\gamma}\left(\frac{E}{E_0}\right)^{-\mu} \\ &\left(\frac{{}_1F_1(p+\frac{1}{2},\frac{3}{2},s_{\rm h})}{{}_1F_1(p,\frac{1}{2},s_{\rm h})}\,{}_1F_1(p,\frac{1}{2},s)\cdot|z_{\rm h}| \right. \\ &\left. -\,{}_1F_1(p+\frac{1}{2},\frac{3}{2},s)\cdot|z|\right) \end{aligned} \quad (19)$$

with

$$p = -\left(\frac{2\gamma+\mu-3}{2(1-\mu)}\right), \quad (20)$$

${}_1F_1(a,b,x)$ denoting the confluent hypergeometric function (or Kummer function), and

$$s = -\frac{c_1(U_{\rm rad}+U_{\rm B})(1-\mu)}{4D_0} z^2 \left(\frac{E}{m_{\rm e}c^2}\right)\left(\frac{E}{1{\rm GeV}}\right)^{-\mu} \quad (21)$$

and

$$s_{\rm h} = -\frac{c_1(U_{\rm rad}+U_{\rm B})(1-\mu)}{4D_0} z_{\rm h}^2 \left(\frac{E}{m_{\rm e}c^2}\right)\left(\frac{E}{1{\rm GeV}}\right)^{-\mu} \quad (22)$$

where

$$c_1 = \frac{4}{3}\frac{\sigma_{\rm T} c}{m_{\rm e}c^2}. \quad (23)$$

3.2. Convection model

The convection model is described by the following equation:

$$\begin{aligned} Q(E,z) \;=\; & \frac{\partial}{\partial z}\left(VN(E,z)\right) \\ & -\;\frac{\partial}{\partial E}\left\{\left(\frac{1}{3}\frac{\partial V}{\partial z}E + b(E)\right)N(E,z)\right\} \end{aligned} \quad (24)$$

where V is the convection speed, assumed here to be constant. The solution of this equation can be obtained with the method of characteristics:

$$N(E,z) = \frac{\nu_{\rm SN}\cdot q_{\rm SN}}{2V}\left(\frac{E}{E_0}\right)^{-\gamma}\left(1-\frac{b\cdot z}{E\cdot V}\right)^{\gamma-1}. \quad (25)$$

3.3. Asymptotic spectral indices

The diffuse synchrotron emission is calculated from $N(E,z)$ in the same way as for SNRs (Eqs. (7) and (8)). The total diffuse synchrotron emission is obtained by integrating $P_{\rm diff}$ over the extent of the halo, $z_{\rm halo}$, which in the case of the diffusion model coincides with the free-escape boundary, $z_{\rm h}$.

We define the escape probability, $P_{\rm esc}(\nu)$, as the ratio of the measured total nonthermal radio emission (including the diffuse synchrotron emission and the radio emission from SNRs), $P_{\rm nth}(\nu)$, to the maximum possible nonthermal radio emission, $P_{\rm nth,max}(\nu)$, that would be measured if the halo were infinitely large.

The expected diffuse and nonthermal spectral indices for different asymptotic cases are:

Case (1): Synchrotron and inverse Compton losses are very strong so that no escape is possible. Then, independent of whether diffusion or convection is the principle mode of propagation, we have $\alpha_{\rm diff} = \gamma/2 = 1.1$. Including the contribution of SNRs, the spectral index becomes (see Fig. 1) $\alpha_{\rm nth} \simeq 1.0$. For a lower injection spectral index, $\gamma = 2.0$, we would get $\alpha_{\rm nth} \simeq 0.9$.

Case (2): CREs can escape freely from the galaxy by diffusion without suffering synchrotron and inverse Compton losses. The diffusion coefficient can be energy dependent, $D(E) \propto E^{\mu}$. For our Galaxy, μ has been determined from the energy-dependence of the fraction of secondary to primary

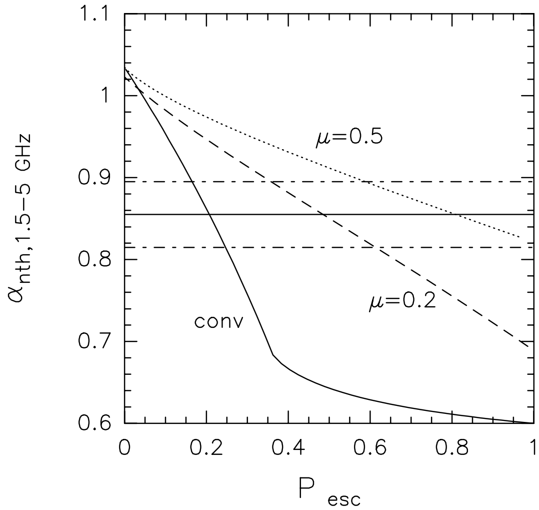

Fig. 2. The nonthermal spectral index between 1.5 and 5 GHz, assuming $\gamma = 2.2$ and $f_{\rm SNR} = 0.1$, is shown as a function of the escape probability $P_{\rm esc}$. The full line is calculated for a convection model, the dashed line for a diffusion model with $\mu = 0.2$ and the dotted line for a diffusion model with $\mu = 0.5$. The full and dashed-dotted horizontal lines give the observational value for $\alpha_{\rm nth}$ and its dispersion, calculated as the average of the results by Niklas et al. (1997) and Klein (1988). For $\gamma = 2.0$ all model curves would be lower by about 0.1.

CR nuclei. For CRs with energies above a few GeV, the energy range relevant for radio observations in the GHz range, $\mu = 0.4 - 0.7$ has been found (García-Muñoz et al. 1987). The expected diffuse spectral index is then, adopting $\mu = 0.5$, $\alpha_{\rm diff} = (\gamma + \mu - 1)/2 = 0.85$, and including the contribution of SNRs it becomes $\alpha_{\rm nth} \simeq 0.82$, with corresponding reductions for $\gamma = 2.0$.

Case (3): CREs propagate by convection and do not suffer considerable synchrotron or inverse Compton losses. In this case we expect $\alpha_{\rm diff} = \alpha_{\rm nth} = (\gamma - 1)/2 = 0.6$ for $\gamma = 2.2$; the spectral index remains unchanged when including the emission from SNRs.

3.4. *The spatially integrated spectral index*

The results of the models are shown in Fig. 2 where the nonthermal spectral index (assuming a contribution of SNRs of 10% at 1.5 GHz) is plotted as a function of the escape probability for the case of diffusion and of convection. The values for $\alpha_{\rm nth}$ predicted by the two models are compared to the observed mean nonthermal spectral index which is about 0.85 ± 0.04. This observational value is obtained as the average of the results from results by Klein (1988) (0.88 ± 0.06) and Niklas et al. (1997) (0.83 ± 0.02) who determined the average $\alpha_{\rm nth}$ of samples of galaxies by subtracting the thermal radio emission from the observed total radio emission.

Within the pure diffusion model with $\mu = 0.5$ the observed $\alpha_{\rm nth}$ is explained by a high escape probability ($P_{\rm esc} \gtrsim 50\%$). For a weaker energy dependence of the diffusion coefficient, $\mu = 0.2$, however, the predicted escape probability is lower, $P_{\rm esc} = 30 - 60\%$. The conclusion from the convection model is very different: Here the observed $\alpha_{\rm nth}$ is explained by $P_{\rm esc} \simeq 25\%$. The different conclusions with respect to the escape probability given by the diffusion and the convection model are due to the fact that (i) convection is an energy independent process and (ii) in the case of convection the relation between the spectral index and the escape probability is not linear, in the sense that low escape probabilities are predicted for almost the whole range of spectral indices.

We conclude that from the observed average radio spectral index of galaxies it can neither be decided to which degree CREs can escape from galaxies nor whether diffusion or convection is the dominant mode of propagation. Only for asymptotic cases partial conclusions can be drawn: If the spectral index of a galaxy is very steep ($\gtrsim 0.9$) energy losses are important and the escape rate is low. Yet, it cannot be determined whether diffusion or convection is the dominant type of propagation. A very flat overall spectral index ($\lesssim 0.7$), on the other hand, indicates strong convection or almost energy independent diffusion. Since measurements in our Galaxy indicate that diffusion does depend significantly on energy ($\mu = 0.4 - 0.7$, García-Muñoz et al. 1987), convection is the more likely explanation. In this case, however, we cannot draw any conclusions with respect to the confinement of CREs.

3.5. *Spatially resolved spectral index*

The question of how important energy losses are for the propagation of CREs in a galaxy can be decided if the spectral index distribution in the halo is known. If the spectrum steepens considerably away from the disk, it is a clear sign that CREs suffer considerable synchrotron and inverse Compton losses as they propagate outwardly. This is illustrated in Figs. 3 and 4, where we show the diffuse radio spectral index, $\alpha_{\rm diff,z}$, calculated from the convection and diffusion models of Sect. 3.1 and 3.2, as a function of the distance from the galactic plane, z. In Fig. 3 we show the results for a large halo ($z_h = 15$ kpc) in which the energy losses affect the CREs considerably before they manage to escape. The escape probabilities are here 9% for convection and 6% ($\mu = 0.2$), respectively 10% ($\mu = 0.5$) for diffusion. In Fig. 4 the opposite case is shown: A small halo ($z_h = 3$ kpc) where the CREs can escape without suffering significant energy losses, so that the escape probabilities lie between 78% for convection and 85% ($\mu = 0.2$), respectively 87% ($\mu = 0.5$) for diffusion. We find that important energy losses are reflected clearly in a steepening of the spectral index with increasing distance from the plane. On the contrary, if the CREs can escape almost freely, or if only adiabatic or bremsstrahlung losses occur, no such steepening is seen.

3.6. *Comparison to observations: Are energy losses important?*

The distribution of the spectral index in the halo is a definite diagnostic of whether energy losses are important. The observational study of radio halos in external galaxies is difficult, because the halos are extended and intrinsically faint (see Dahlem

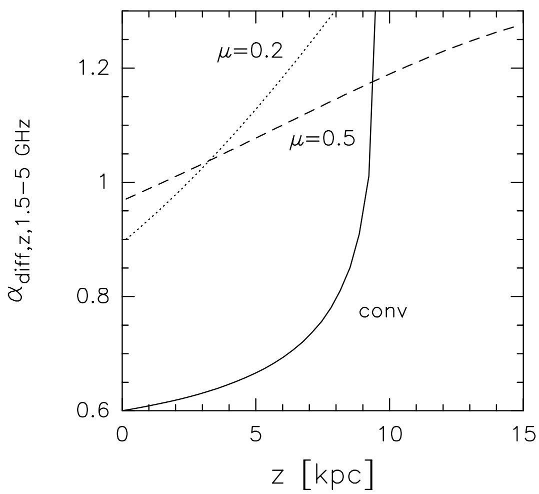

Fig. 3. The diffuse radio spectral index as a function of the height over the galactic plane, z, in the case of important energy losses. The full line is calculated for a convection model, the dashed line for a diffusion model with $\mu = 0.5$ and the dotted line for a diffusion model with $\mu = 0.2$. We have assumed a magnetic field $B = 5\,\mu$G, a radiation field of energy density 1 eV cm^{-3}, convection velocity $V = 200$ km s^{-1} and diffusion coefficient $D_0 = 10^{29}$ cm^2 s^{-1}.

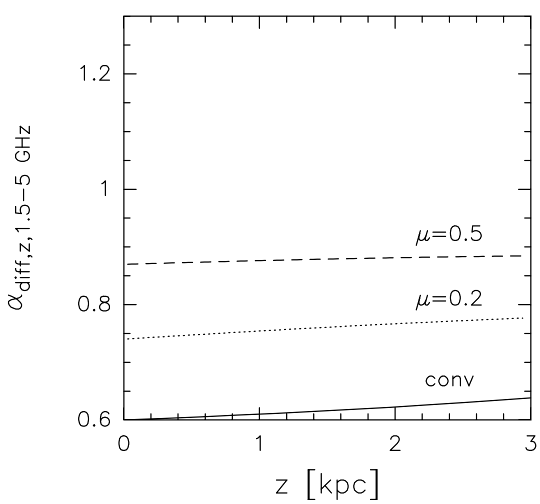

Fig. 4. The diffuse radio spectral index as a function of the height over the galactic plane, z, in the case of almost free escape. The line coding and parameters are as in Fig. 3.

1997). Therefore, not many galaxies with multi-frequency radio data for the halo are known.

One of the best studied radio halos is that of the spiral galaxy NGC 891 (Hummel et al. 1991). Its spatially integrated non-thermal radio spectral index is 0.78 (Niklas et al. 1997), only slightly below the average spectral index of galaxies (see Fig 2). In Fig. 5 we show its radio spectral index distribution in the halo. The data are at a resolution of 40", corresponding to 1.8 kpc at the distance of 9.5 kpc of NGC 891. The spectrum steepens very quickly within the first 2-3 kpc. It continues to steepen more slowly outside this range, especially in the eastern side of the halo. Here, the steepening becomes more pronounced at $z \gtrsim 6$ kpc.

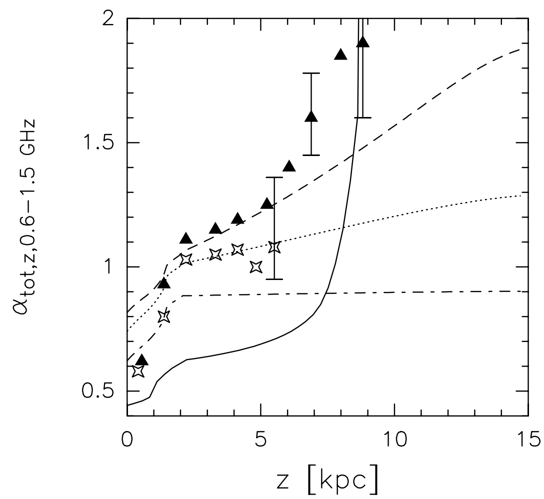

Fig. 5. A comparison of our model results to the data of NGC 891 (Hummel et al. 1991). The triangle refer to the spectral index in the western part of the halo, and the stars to the eastern side of the halo. The full line corresponds to the convection model and the other lines to the diffusion model with $\mu = 0.5$ and different values for the diffusion coefficient and thus escape probability (see text).

In Fig. 5 we also show the results of our model in which we have included, in order to correspond to the data, apart from the diffuse radio emission, the contribution from SNRs and the thermal radio emission. For the latter two components we have assumed, $f_{\rm SNR} = 0.1$, as before, and a contribution of the thermal radio emission of 10% at 1.5 GHz, a typical value for normal galaxies (Condon 1992). We furthermore assume that the thermal radio emission and the radio emission from SNRs is restricted within 100 pc above the disk (the exact value of this spatial extent is not important because of the low resolution of the data). Finally, we convolve the model results with a Gaussian beam of HPBW 1.8 kpc, corresponding to the resolution of the data.

We have adjusted the diffusion coefficient and convection velocity such that we achieved the best possible fit to the data. Because of the simplicity of the models described here, we cannot expect to describe the data in all detail, we are aiming rather at a qualitative comparison with the main features. For a quantitative comparison a more complex model would be necessary, combining diffusion and convection, taking into account spatial variations of the parameters (B, D_0, etc.) and describing more realistically the distribution of SNRs and the contribution of the thermal radio emission. Nevertheless, the diffusion model is able to describe a large range of the data reasonably well. The fast steepening of the spectral index within the first 2-3 kpc above the disk can to a large extent be attributed to the contribution of SNRs and the thermal radio emission close to the disk. The steepening outside this range requires the presence of dominant energy losses: The diffusion models that describe the data well have escape probabilities of 11% (dotted line) and 0% (dashed line). For comparison a model with a high escape rate is shown (78% – dashed-dotted line) which is not able to describe the data. In order to reproduce the fast steepening of the spectral index at $z \gtrsim 6$ kpc in the western part of the halo, we would possibly have combine the diffusion model with the convection model.

Radio halos have been observed in several other galaxies, although mostly not in such detail. In all of these galaxies a steepening of the spectral index at least up to $\alpha_{\rm nth} \gtrsim 1.0$ is found with increasing distance from the disk. This is the case for both starburst galaxies, as M 82 (Seaquist & Odegard 1991; Reuter et al. 1992); NGC 253 (Carilli et al. 1992); NGC 2146 (Lisenfeld et al. 1996b); NGC 4666 (Sukumar et al. 1988; Dahlem et al. 1997) and non-starburst spiral galaxies like NGC 4631 (Hummel & Dettmar 1990); NGC 5775 (Duric et al. 1998); NGC 5055 and NGC 7331 (Hummel & Bosma 1982). We conclude therefore that for all galaxies for which a multi-frequency radio halo has been observed, energy losses are important and it seems likely that this is generally the case in galactic halos.

Niklas & Beck (1997) have found a trend that actively star-forming galaxies tend to have lower spectral indices and more quiescent galaxies steeper ones. They interpret this as an indication that in galaxies with a high star-formation efficiency (SFE) CREs escape more easily due to a galactic wind. On the basis of the above consideration, however, it seems more likely that in these galaxies convection causes the flat spatially integrated spectrum. Energy losses play an important role also in galaxies with a high SFE which is shown by the steepening of the spectral index in the halo. A good example is the starburst galaxy M 82. For this galaxy the observed overall synchrotron spectral index $\alpha_{\rm nth} = 0.66$ (Niklas et al. 1997) indicates strong convection (see Fig. 2) whereas at the same time the steepening of the spectral index in the halo shows the presence of important energy losses (Seaquist & Odegard 1991; Reuter et al. 1992).

3.7. Consequences for the FIR/radio correlation

The question whether CREs can escape freely from galaxies or whether synchrotron and inverse Compton losses determine their spectra has been asked in the framework of the FIR/radio correlation. Völk (1989) has proposed a calorimeter model in which the tightness of the correlation can be explained by the fact that CREs loose their energy by synchrotron and inverse Compton losses below the energy level corresponding to the observing frequency before they are able to escape from a galaxy. Lisenfeld et al. (1996a) have generalized this model, allowing for moderate escape and a spatially inhomogeneous magnetic field. Other authors have claimed the opposite, i.e. that CREs can escape more or less freely from the galaxy (Chi & Wolfendale 1990; Helou & Bicay 1993; Niklas & Beck 1997).

It has been argued (Niklas & Beck 1997) that the observed spectral index of galaxies, $\alpha_{\rm nth} \simeq 0.85$, is in contradiction with the calorimeter model which predicts in its asymptotic version $\alpha_{\rm nth} \simeq 1.0 - 1.1$, corresponding to case (1) of Sect. 3.3. The difference between the prediction of the calorimeter model and observations of the radio spectral index can be decreased by allowing for moderate escape and a spatially inhomogeneous magnetic field (Lisenfeld et al. 1996a) and even more by including the contribution of SNRs (this work), so that such a modified calorimeter model would predict $\alpha_{\rm nth} \simeq 0.85 - 0.95$, corresponding to the observed average value. However, even in this case this model is not able to account for the flatter spectra that some of the galaxies (e.g. M 82) show because it is based on diffusion. In this paper, we have shown, that for galaxies with such a low spectral index, convection is likely to be important. With respect to the question of whether confinement or escape is taking place the overall spectral index is not a good diagnostic. Spatially resolved observations are necessary and these indicate that energy losses indeed do play a dominant role. Thus, even for galaxies with a low overall spectral index, like M 82, escape seems to be negligible as predicted by the calorimeter model.

4. Conclusions and summary

We have estimated and discussed different processes and components that shape the radio spectral index of a galaxy. In particular, we have considered the contribution of SNRs, the influence of different types of CR propagation (diffusion and convection) and of energy losses (inverse Compton and synchrotron). Our conclusions can be summarized as follows:

(i) We have estimated that the radio emission of SNRs represents about 10% of the nonthermal radio emission of a galaxy. This moderate contribution has a noticeable effect on the nonthermal radio spectral index, lowering it by 0.1 for steep spectra.

(ii) The spatially integrated radio spectral index of a galaxy is difficult to interpret, as it is influenced by various processes. Only for asymptotic cases conclusions can be drawn: In galaxies with very steep spectral indices ($\gtrsim 0.9$) energy losses are high and the escape rate of CRE is low. A very flat spectral index ($\lesssim 0.7$), on the other hand, is a sign for strong convection.

(iii) In order to draw more general conclusions about the CR propagation and the importance of energy losses, spatially resolved radio observations of galactic halos are necessary. A steepening of the spectral index away from the galactic disk is a clear indication that inverse Compton and synchrotron losses are important.

(iv) In all galaxies with multi-frequency radio data for the halo such a steepening has been found and therefore in these galaxies synchrotron and inverse Compton losses take place. Thus, it seems likely that these energy losses are generally important in galactic halos and that escape rates are low. The low overall spectral index that is frequently found in starburst galaxies, at the same time as the steepening of the spectrum away from the disk, can be interpreted as convection.

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