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Calibration of ι Pegasi system

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Abstract. Recent observations provide determinations of individual masses, chemical composition and metallicity of the components of the spectroscopic and interferometric binary ι Peg (Boden et al. 1999). Using updated physics, to calibrate the system, we have computed using the stellar evolutionary code CESAM (Morel 1997), evolutionary sequences of stellar models with the masses of ι Peg A $1.326\,M_\odot$ and ι Peg B $0.819\,M_\odot$ (Boden et al. $loc.\,cit$) and with different values of the mixing-length parameter α, the helium Y and the heavy element Z initial mass fraction with the constraint of the observed metallicity. Adopting effective temperatures and luminosities, as derived from observations with the bolometric corrections, and the empirical scale of temperatures of Alonso et al. (1995, 1996), we find $\alpha_{\rm A} = 1.46$, $\alpha_{\rm B} = 1.36$, $Y = 0.278$, $Z = 0.017$. The evolution time, including pre-main sequence, is found within $\sim 40\,{\rm My} \lesssim t_{\rm ev} \lesssim 0.5\,{\rm Gy}$. The calibrated models of ι Peg. A and B are non homogeneous zero age main sequence stars with the evolutionary time $t_{\rm ev} = 56$ My. Due to the large uncertainties of their determinations, the values derived for the mixing-length parameters are smaller than the solar one but however marginally compatible with it. Our results ought to be improved as soon as a more accurate value of the magnitude difference in the V filter will be available. Detailed spectroscopic analysis for both components looks practicable, so it is urgently needed.

Key words: stars: binaries: spectroscopic – stars: binaries: visual – stars: fundamental parameters – stars: individual: ι Peg

1. Introduction

Along the last decade, owing to significant improvements of our knowledge of fundamental stellar data, trigonometric parallaxes with HIPPARCOS (ESA 1997), spectroscopic-speckle mass determination (e.g. Scarfe et al. 1994), stellar diameters (e.g. Koechlin & Rabbia 1985; Di Benetto & Bonneau 1991), photometric (Mermillod et al. 1997) and spectroscopic data (Cayrel de Strobel et al. 1997), the constraints on stellar modeling became more stringent leading to improvements of stellar evolution theory. Meanwhile, physics, e.g. opacities (Iglesias & Rogers 1996), equation of state (Rogers et al. 1996; Däppen 1996), thermonuclear reaction rates (Adelberger et al. 1998, Angulo et al. 1999), have been improved. Most of these new data are nowadays available in free access in data banks.

With the exception of the Sun, modeling a single star is not a closed problem because the number of indeterminate parameters is larger than the observed ones. For some well known binaries, e.g. α Cen (Demarque et al. 1986; Noels et al. 1991), with the reasonable hypothesis of a common origin for both components, i.e. same initial chemical composition and age, it is possible to confront observations and theoretical stellar evolution sequences. One can derive estimates for age, helium content and metallicity, fundamental quantities for our understanding of the galactic chemical evolution. One derives also the mixing-length parameter α – also named "convection parameter".

For a *fixed* physics, the value of the mixing-length parameter $1.0 \lessapprox \alpha_\odot \lessapprox 2.5$ derived from a calibrated solar model is currently used to model other stars. But, one can ask why the value adjusted for the present Sun should be applied to stars with different masses, ages and/or chemical composition. Fixing the physics, Fernandes et al. (1998) have shown that $\alpha_\odot$ is valuable for a sample of low mass binaries $0.6\,M_\odot \lesssim M_\star \lesssim 1.0\,M_\odot$ in the metallicity range $-0.65 \lesssim [{\rm Fe/H}] \lesssim +0.0$. Indeed, for α Cen Fernandes & Neuforge (1995) obtained a value close to the solar one. More recently, Pourbaix et al. (1999) using improved mass and gravity determinations for both components of α Cen were able to relax the assumption of a unique value for the convection parameter. They obtained $\alpha_{\rm A} = 1.86 \pm 0.80$ ($resp.$ $\alpha_{\rm B} = 2.1 \pm 0.80$) for the A ($resp.$ B) component. Due to the range of uncertainties, this result does not rule out the assumption of a constant value for the mixing-length parameter for masses and metallicities smaller or slightly larger to solar values.

Up to now, the calibrations of double stars have especially concerned main sequence stars with ages of few Gy. As soon as new data become available, it is important to extend the sample of calibrated binaries. In most cases the individual masses are inaccurately known and the isochrones are not really constrained. The spectroscopic and interferometric binary ι Peg. is a particularly interesting object since the two masses are accurately known (Boden et al. $loc.\,cit.$). Moreover, the mass and

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the metallicity of the primary are slightly larger than solar. It is a "solar like" star whose seismological properties have been studied (e.g., Houdek et al. 1995, Houdek 1996, Michel et al. 1999). Some stars of this group are COROT targets (Baglin 1998). The values of the mixing-length parameters, as derived from calibration for ι Peg. A and B, will provide a test for the 2-D numerical calculations of compressible convection (Ludwig et al. 1999).

Some authors have already derived estimates for the age of the ι Peg. binary system. A value of ~ 80 My was derived from the observed lithium depletion (Fekel & Tomkin 1983, Gray 1984). Lyubimkov et al. 1991 have obtained the slightly larger value of 170 ± 80 My, from comparison with evolutionary models. None of these estimates is based on evolutionary models taking explicitly into account, as we do, the microscopic diffusion and gravitational settling, basic phenomena in stellar modeling.

In this paper we report an attempt to calibrate ι Peg. using stellar evolution models including pre-main sequence and microscopic diffusion. It is organized as follows: In Sect. 2 we collect and discuss the observations. Sect. 3 is devoted to derivations of effective temperatures and luminosities. The method of calibration is described in Sect. 4. Sect. 5 presents the stellar modeling procedure. In Sect. 6 we give the results with emphasis on a seismological analysis. We discuss the results and conclude in Sect. 7.

2. Observations of ι Peg

ι Pegasi (HR 8430, HD 210027) is a nearby ($d \simeq 11.75$ pc), a wonderful double lined spectroscopic and interferometric double star. The binarity was discovered more than a century ago by Campbell (1899), who notices "The velocity of ι Pegasi in the line of sight is variable". Table 1 shows the astrometric, photometric and spectroscopic observational data retained for the present investigation.

2.1. Orbit, masses, parallax and diameters

Recently, Boden et al. ($loc.\ cit.$) using observations of the Palomar Testbed Interferometer computed the visual orbit of ι Peg. and derived the individual masses. The orbital period is accurately known $p = 10.213033 \pm 1.3\, 10^{-5}$ day (Fekel & Tomkin 1983). The spectroscopic and visual orbits for ι Peg. are each statically consistent with being circular. That corroborates the result of Zahn & Bouchet (1989) who predict, for close binary systems with masses ranging from 0.5 to $1.25\, M_\odot$, that the orbit is circularized during the Hayashi phase. Moreover, since the period of ι Peg. is larger than ≈ 8 days, it is expected that the main sequence is reached with components rotating faster than the orbital rate, Zahn & Bouchet ($loc.\ cit.$).

The orbital parallax derived by Boden et al. ($loc.\ cit.$), as seen in Table 1, is in close agreement with the trigonometric $\varpi_{\rm trig} = 85.06 \pm 0.71$ mas (ESA 1997) determination by HIPPARCOS. Though the accuracy of the HIPPARCOS parallax is slightly higher, we have used the Boden et al. value for global consistency. Boden et al. ($loc.\ cit.$), assuming two different limb darkening laws, have obtained consistent estimates of diameters for each component: $\oslash_{\rm A} = 0.99 \pm 0.05$ mas, $\oslash_{\rm B} = 0.70 \pm 0.10$ mas. The illuminating Fig. 1 of Boden et al. ($loc.\ cit.$) shows that the previous values of diameters are not large enough for eclipsing, despite the high inclination $i = 95.85° \pm 0.22°$ and the small semi-major axis $a = 10.33 \pm 0.10$ mas. From the previous interferometric data and orbital parallax one derives the "interferometric" radii of $R_{\rm A\,in} = 1.224 \pm 0.025\, R_\odot$ and $R_{\rm B\,in} = 0.872 \pm 0.050\, R_\odot$, respectively, for primary and secondary. As emphasized by Boden et al. ($loc.\ cit.$), the sum of the radii is consistent with the absence of evidence of photometric eclipse. For the gravities one obtains: $\log_{10}\, g_{\rm A\,in} = 4.39 \pm 0.23$ and $\log_{10}\, g_{\rm B\,in} = 4.47 \pm 0.65$.

Table 1. Astrometric, photometric and spectroscopic adopted data of ι Peg. $m_{\rm V}$, $\Delta m_{\rm V}$ ($resp.$ $m_{\rm K}$, $\Delta m_{\rm K}$) are the global apparent magnitudes and magnitude differences in the V ($resp.$ K) band. $\varpi_{\rm orb}$ is the orbital parallax. $\mathcal{M}_{\rm A}$, $\mathcal{M}_{\rm B}$ are the masses, $[{\rm Li}]_{\rm A}$, $[{\rm Li}]_{\rm B}$ the lithium abundances (H=12), of A and B components, respectively. $[{\rm Fe/H}]$ is the metallicity. The flag "‡" signals a value estimated by *ourselves*.

Spectra	F5V – G8V	Fekel & Tomkin (1983)
$m_{\rm V}$	$3.771 \pm (0.001)^\ddagger$	Mermillod et al. (1997)
$\Delta m_{\rm V}$	$2.68 \pm (0.05)^\ddagger$	Fekel & Tomkin (1983)
$m_{\rm K}$	2.656 ± 0.002	Bouchet et al. (1991)
$\Delta m_{\rm K}$	1.610 ± 0.021	Boden et al. (1999)
$\varpi_{\rm orb}$	86.91 ± 1.0 mas	Boden et al. (1999)
$\mathcal{M}_{\rm A}$	$1.326 \pm 0.016\, M_\odot$	Boden et al. (1999)
$\mathcal{M}_{\rm B}$	$0.819 \pm 0.009\, M_\odot$	Boden et al. (1999)
$[{\rm Li}]_{\rm A}$	$2.9 \pm (0.4)^\ddagger$ dex	Fekel & Tomkin (1983)
$[{\rm Li}]_{\rm B}$	2.6 ± 0.6 dex	Fekel & Tomkin (1983)
$[{\rm Fe/H}]$	$(+0.00 \pm 0.1)^\ddagger$ dex	See text

2.2. Spectroscopy

From analyses of 2.5 Å per mm CCD spectra, Lyubimkov et al. (1991) have derived effective temperatures $T_{\rm eff\,A\,sp} = 6750 \pm 150$ K, $T_{\rm eff\,B\,sp} = 5350 \pm 350$ K, and gravities $\log_{10}\, g_{\rm A\,sp} = 4.35 \pm 0.05$, $\log_{10}\, g_{\rm B\,sp} = 4.57 \pm 0.10$. With the values of the masses derived by Boden et al. ($loc.\ cit.$), that correspond to a "spectroscopic" luminosity of $L_{\rm A\,sp} = 3.024 \pm 0.62\, L_\odot$ and $L_{\rm B\,sp} = 0.444 \pm 0.219\, L_\odot$, respectively for the primary and secondary, and to radii $R_{\rm A\,sp} = 1.274 \pm 0.07\, R_\odot$ and $R_{\rm B\,sp} = 0.772 \pm 0.089\, R_\odot$. Another effective temperature value $T_{\rm eff} = 6488$ K, derived from spectroscopic analysis, was just recently listed by Boesgaard et al. (1999). Note that the constraint on the diameters, due to the absence of evidence of photometric eclipse (Boden et al. $loc\, cit.$), is also fulfilled with the "spectroscopic" values of the radii.

2.3. Chemical composition

Lyubimkov et al. (1991) have qualified the chemical composition as "close to normal". The Catalog of [Fe/H] (Cayrel de Strobel 1997) recommends $[\frac{\rm Fe}{\rm H}] = +0.10$. Another value of the metallicity $[\frac{\rm Fe}{\rm H}] = -0.13$ is due to Duncan (1981). According to Boesgaard et al. (1999) the metallicity is $[\frac{\rm Fe}{\rm H}] = -0.08$. Faced with these rather scattered data, we have adopted for our calculations $[\frac{\rm Fe}{\rm H}] = 0.00 \pm 0.10$ (Cayrel de Strobel, private communication). Except for lithium (see next), none of the authors has reported significant differences between the surface chemical composition of primary and secondary. It is an indication that both components of ι Peg. are newly formed stars, since the gravitational settling, which acts more efficiently in a $1.326\,M_\odot$ than in a $0.819\,M_\odot$, still has not have enough time to differentiate significantly the surface chemical composition of primary and secondary (e.g. Morel & Baglin 1999). Therefore, the initial metallicity of ι Peg. is certainly very close to the nowadays observed value.

Table 1 shows the adopted values of lithium abundances. They are in agreement with the measurements of Lyubimkov et al. (1991): $[{\rm Li}]_{\rm A} = 3.25$ for primary and $[{\rm Li}]_{\rm B} = 2.58$ for secondary. According to the standard evolution theory, for an isolated star of solar metallicity and less massive than $M_\star \lessapprox 0.85\,M_\odot$, all lithium is destroyed at the end of the fully convective phase of the pre-main sequence. That results from how fast the convective zone recedes from the core where the lithium is depleted – recall that lithium burns as soon as the temperature is greater than $T \gtrapprox 2.4\,$MK. In the core of a late type star, the increase of temperature is slow, so are the decreases of opacity and radiative gradient. Then, the fully convective state lasts longer than in a more massive star. With respect to an isolated star, ι Peg. B is lithium under-depleted. For not isolated stars under-depletion can occur on the main-sequence in binary systems which are tidally locked for orbital periods below ≈ 8 days (Zahn 1994). The period of ι Peg. (10 days) is slightly larger than this limit, as supported by the no-synchronous rotation found by Fekel & Tomkin (1983). But, Gray (1984) deduced, from the analysis of higher resolution spectra, that primary and secondary are slow rotators $v_{\rm A} \sin i = 6.5 \pm 0.3\,{\rm km\,s^{-1}}$, $v_{\rm B} \sin i = 5. \pm 1\,{\rm km\,s^{-1}}$, and argued that both components are in synchronous rotation. As previously conjectured, the components may have reached the main sequence with rotation velocities larger than the orbital one and the individual rotation velocities may have decreased, the system being again tidally locked. Fig. 1 shows that the locus of ι Peg. B in the HR diagram excludes the possibility that the secondary is nowadays still in the pre-main sequence, before the lithium burning phase which occurs soon after the end of Hayashi's track. Therefore, the under-depletion of lithium, observed in ι Peg. B, is unexplained. There is an alternative either, i) along the pre-main sequence, for some unknown reason, the mixing of the convection zone is slowed and lithium is not destroyed or, ii) at the beginning of the main sequence, coming from somewhere, lithium has been provided to the outer convection zone of the secondary. Due to the distance between the components $a \approx 0.12\,$AU there is no possibility for mass exchange between the components (F. van 't Veer, private communication). Then the first part (i) of the alternative appears to be the most reliable. That reinforces the conjecture that ι Peg. is certainly composed of two newly formed stars. In our calculations, to mimic the unknown process leading to the under-depletion of lithium in the secondary, the convective zones will be only mildly mixed in the course of the pre-main sequence (see Sect. 5.2).

Table 2. Physical parameters derived from data of Table 1 using $\varpi_{\rm orb}$. $Bc_{\rm K}$ is the bolometric correction relative to the K magnitude. $M_{\rm bol}$, $M_{\rm V}$ ($resp.$ $M_{\rm K}$) are respectively the bolometric and absolute magnitudes in the V ($resp.$ K) band.

Parameter	ι Peg. A	ι Peg. B
$Bc_{\rm K}$	$+0.97$	$+1.82$
$M_{\rm bol}$	3.54 ± 0.08	6.01 ± 0.09
$M_{\rm V}$	3.56 ± 0.03	6.24 ± 0.07
$M_{\rm K}$	2.57 ± 0.03	4.18 ± 0.03
$T_{\rm eff}$	$6642 \pm 52\,$K	$4992 \pm 115\,$K
$L/L_\odot$	3.041 ± 0.23	0.3145 ± 0.027

2.4. Photometry

ι Peg. A is a MK standard and its photometry is accurately known. Another value of the K apparent magnitude $m_{\rm K} = 2.623 \pm 0.016$ is due to Carrasco et al. (1991). For the magnitude difference $\Delta m_{\rm V}$ in the V band we have adopted the estimate of Fekel & Tomkin (1983). Their analysis is based on spectroscopic and photometric arguments. Comparing the strength of iron lines of ι Peg. B and ϵ Eri, a K2V star, they derive, for ι Peg. an estimate of the magnitude difference $\Delta m_{\rm R} = 2.5 \pm 0.3$ in the R band. Such a value of $\Delta m_{\rm R}$ characterizes stars between G5V and K0V. Then, they deduce $\Delta m_{\rm V} = 2.68$ from the sensitivity of the global K magnitude for binary systems composed of a F5V primary and secondaries with spectral types between G5V and K0V. Fekel & Tomkin ($loc.\ cit.$) do not give the uncertainty $\delta\Delta m_{\rm V}$ of their estimation of $\Delta m_{\rm V}$; it is difficult to derive such value from their subtle analysis. We have *estimated* $\delta\Delta m_{\rm V} = \pm 0.05$. This quantity is the most poorly known parameter of our calibration. Lyubimkov et al. (1991) have reported the estimate $\Delta m_{\rm V} = 2.13$ which significantly differs from the value derived by Fekel & Tomkin ($loc.\ cit.$).

3. Luminosities and effective temperatures

Faced with the scattered values of luminosities and effective temperatures, derived either from spectroscopic (Sect. 2.2) or photometric (Sect. 2.4) observations, we make the choice to use the photometric data alone. Table 2 gives the physical parameters of primary and secondary as derived from data of Table 1. The bolometric corrections for the K band are obtained using the bolometric corrections of Alonso et al. (1995) with a correction of the zero point giving a bolometric magnitude of 4.75 to the Sun (Cayrel, private communication). The uncertainty of the method is of the order of $\approx \pm 0.05$ magnitude. The effective temperatures are derived from the Alonso et al. (1996) empirical scale of temperature, valid for the low main sequence from F0V to K0V. The uncertainty of the fit is of the order of $\widetilde{\delta T_{\rm eff}} \approx \pm 30\,$K. We are aware that many tidally locked binary systems, having a lithium under-depleted

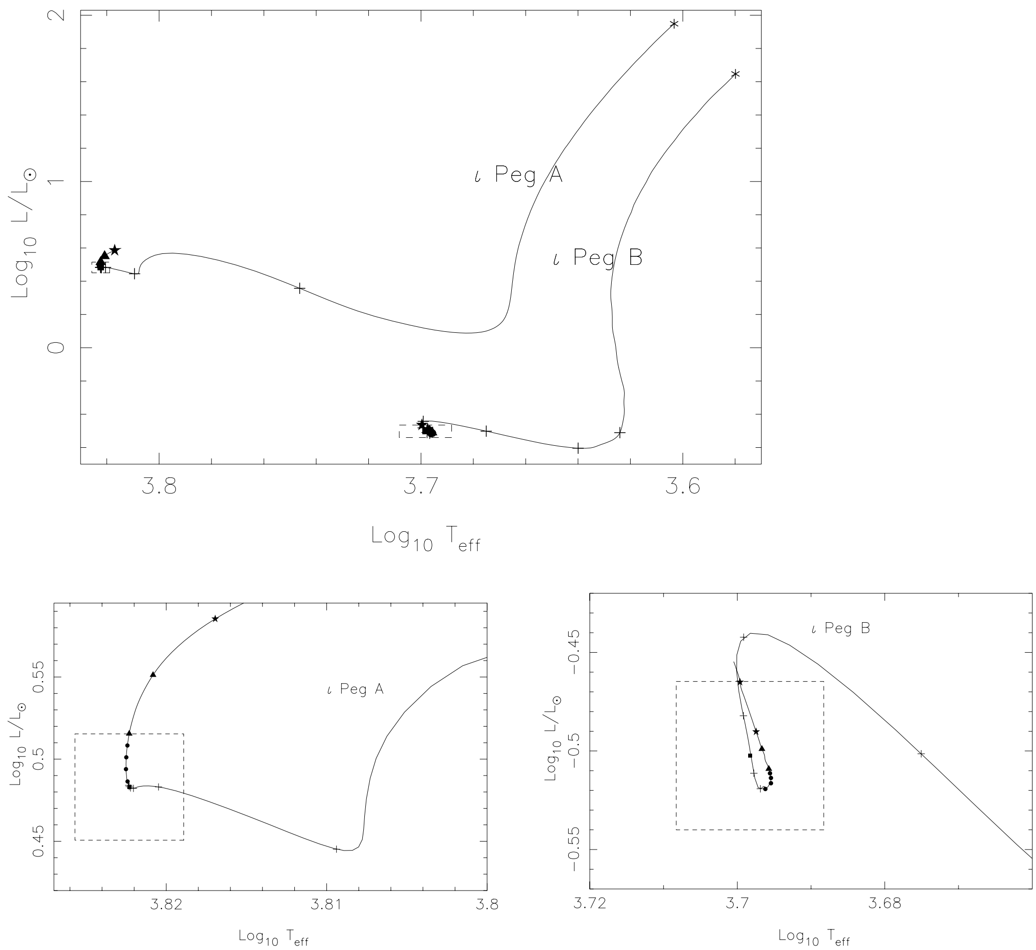

Fig. 1. Top: Evolutionary tracks in the HR diagram for ι Peg. A and B. Bottom: enlargements around the observational points (full square) with the uncertainty domains. The stellar evolution sequences are initialized (stick mark $*$) on the pre-main sequence soon after the deuteron ignition. Along the stellar tracks the time intervals between the marks are respectively 10 MY ($+$), 100 MY (●), 500 MY (full triangle), 1500 MY ($\star$).

late type component, are recognized as chromospherically active (Barrado y Navascués et al. 1997). As ι Peg. *is not* identified as chromospherically active (Barrado y Navascués et al. *loc. cit.*) the use of the Alonso et al. (1995, 1996) procedures is valid. From the previous photometric and orbital data one derives the "photometric" radii of $R_{\rm A\,ph} = 1.319 \pm 0.023 R_\odot$ for primary and $R_{\rm B\,ph} = 0.751 \pm 0.027 R_\odot$ for secondary. Indeed, for the gravities one obtains: $\log_{10}\, g_{\rm A\,ph} = 4.32 \pm 0.02$, $\log_{10}\, g_{\rm B\,ph} = 4.60 \pm 0.03$, values in agreement with their "spectroscopic" determination of Sect. 2.2. Note that the constraint on the diameters, due to the absence of evidence of photometric eclipse (Boden et al. *loc cit.*), is also fulfilled with the "photometric" values of radii.

The effective temperatures, luminosities and radii of F5V and G8V standard stars are respectively 6440 K, 3.20 $L_\odot$, 1.44 $R_\odot$ and 5570 K, 0.66 $L_\odot$, 0.874 $R_\odot$ (Schmidt-Kaler 1982). According to radii and luminosities, derived previously either from photometric data or from spectroscopic data (Sect. 2.2), both components of ι Peg. are under luminous and have smaller diameters than standards. That reinforces the conjecture of a small age. ι Peg. being certainly a young binary system, the models have to take properly into account the pre-main sequence evolution.

Uncertainty domains. For each component the luminosity and the effective temperature, as derived from Alonso et al. (*loc. cit.*) semi empirical adjustments, are functions of absolute magnitudes $M_{\rm V}$ and $M_{\rm K}$. Then, the luminosity and the effective temperature have the following *non-linear* functional dependency with respect to the observed parameters:

$$L = L(m_{\rm V}, \Delta m_{\rm V}, m_{\rm K}, \Delta m_{\rm K}, \varpi, [{\rm Fe/H}]),$$
$$T_{\rm eff} = T_{\rm eff}(m_{\rm V}, \Delta m_{\rm V}, m_{\rm K}, \Delta m_{\rm K}, \varpi, [{\rm Fe/H}]).$$

Only the uncertainties on the observed magnitudes, the distance and the color index can be assumed to be Gaussian. The statistical behavior of the metallicity determination, of the estimated V magnitude difference (see Sect. 2.4) and of the fits of Alonso et al. (*loc. cit.*) are unknown. Therefore, due to the non-linear dependence of the effective temperature with respect to variables some of which are correlated and not Gaussian, we have estimated the uncertainty on the effective temperature using the sum:

$$\begin{aligned}\Delta T_{\rm eff} \;\approx\; & \left|\frac{\partial T_{\rm eff}}{\partial m_{\rm V}}\right| \delta m_{\rm V} + \left|\frac{\partial T_{\rm eff}}{\partial \Delta m_{\rm V}}\right| \delta \Delta m_{\rm V} + \\ & + \left|\frac{\partial T_{\rm eff}}{\partial m_{\rm K}}\right| \delta m_{\rm K} + \left|\frac{\partial T_{\rm eff}}{\partial \Delta m_{\rm K}}\right| \delta \Delta m_{\rm K} + \\ & + \left|\frac{\partial T_{\rm eff}}{\partial \varpi}\right| \delta \varpi + \left|\frac{\partial T_{\rm eff}}{\partial [{\rm Fe/H}]}\right| \delta [{\rm Fe/H}] + \widetilde{\delta T_{\rm eff}},\end{aligned}$$

instead of the standard square root of the sum of the square of the standard deviations. A similar relation holds for the luminosity. Here, δx is the uncertainty on x. Table 2 shows the uncertainties derived in such a way for effective temperatures and luminosities. They are maybe slightly optimistic but give a reasonable order of magnitude.

4. The method of calibration

Fixing the initial mass and the physics, the calculation of a stellar model needs 4 unknowns namely, the mixing-length parameter α, the helium Y_0 and heavy elements Z_0 initial mass fractions and the age. To model a binary, assuming a common origin to each component, the age and the initial chemical composition are identical for primary and secondary. There are only $4 \times 2 - 3 = 5$ unknowns. There are 6 observables: luminosity, effective temperature and metallicity for each component. In case of ι Peg., many presumptions of a small age allow to assume that the observed surface metallicity $(\frac{Z}{X})_{\iota\,{\rm Peg.}}$ is the same for both components and then close to its initial value (see Sect. 2.3). Therefore, there remain 5 observables and 5 unknowns: the mixing-length parameters $\alpha_{\iota\,{\rm Peg.\,A}}$ and $\alpha_{\iota\,{\rm Peg.\,B}}$ of primary and secondary, the initial helium Y_0 and metal Z_0 contents and the age $t_{\iota\,{\rm Peg.}}$. As $1 \equiv X + Y + Z$, the initial values of helium and metals are related by:

$$Z_0 = \frac{(1 - Y_0)}{1 + (\frac{Z}{X})_{\iota\,{\rm Peg.}}} \times (\frac{Z}{X})_{\iota\,{\rm Peg.}}.$$

This relationship holds *only* because the initial and present day metallicities are alike for both components; otherwise Y_0 and Z_0 need to be derived independently. Therefore, 4 calibration parameters remain:

$$\alpha_{\iota\,{\rm Peg.A}},\; \alpha_{\iota\,{\rm Peg.B}},\; Y_0,\; t_{\iota\,{\rm Peg.}}.$$

Since we have adopted $[\frac{\rm Fe}{\rm H}]_{\iota\,{\rm Peg.}} = 0.0$, we used $(\frac{Z}{X})_{\iota\,{\rm Peg.}} = (\frac{Z}{X})_\odot = 0.0245$ (Grevesse & Noels. 1993).

Search of a solution. In a first time we have explored systematically the domain of the calibration parameters. In a second step, more or less empirically, we have refined the convection parameters and the initial helium mass fraction. Doing that we are guided by two facts, i) in the HR diagram an increase of the convection parameter moves the evolutionary track towards larger temperatures at about constant luminosity and, ii) an increase of the helium content shifts the evolutionary track towards larger effective temperatures *and* luminosities. For stars with convective envelope, the first point (i) results from the fact that an increase of the efficiency of the convection deepens the base of the convection zone and does not affect the core. The luminosity is then almost unchanged, while the adiabat of the external convection zone is adjusted on a larger temperature value at the base of the convection zone, with the result of an increase of the effective temperature. The second point (ii) can be understood in the following way: an increase of the helium mass fraction leads to an increase of the mean molecular weight which, by turn, diminishes the pressure gradient. The quasi-static equilibrium is conserved via a temperature increase. A concomitant luminosity increase results from the enlargement of the nuclear energy generation magnified by the large power law dependence of nuclear reaction rates, with respect to temperature. In summary, in the HR diagram a change of α translates the locus of a model in effective temperature at constant luminosity, while one can adjust changes of α *and* Y in order to translate the locus of a model in luminosity at constant effective temperature.

Finally, the steepest descent algorithm (e.g. Conte & de Boor 1987, Noels et al. 1991) is used to improve the calibration parameters. As this gradient method converges in the vicinity of *stable* solutions we have only consider as a "solution" any set of calibration parameters for which the gradient method converges. We have kept for the *calibrated models* the solution which best fits the central values of observational boxes in the HR diagram for each component. Then we have estimated the domains of variations allowed to the parameters in order that the locii of the models remain in these observational boxes.

5. The physics of the models

Basically the physics used in the models is the same as in Morel et al. (1997). The standard assumptions of stellar modeling are made, i.e. spherical symmetry, no rotation, no magnetic field, no mass loss. Each evolution is initialized with a homogeneous zero-age pre-main sequence model in quasi-static contraction with a temperature at center close to the onset of the deuteron burning, i.e. $T_{\rm c} \sim 0.5\,{\rm MK}$. We shall call the "time of evolution" of a model the time $t_{\rm ev}$ elapsed from initialization. In a sequence of models we shall designate by "model of zero age main-sequence" (ZAMS) the first model, if any, where the nuclear reactions dominate gravitation by more than 99% as the primary energy source. Typically ZAMS occurs after ~ 40 My of evolution for ι Peg. A and ~ 70 My for ι Peg. B.

5.1. Nuclear and diffusion network

The general nuclear network we used contains the following species: $^{1}{\rm H}$, $^{2}{\rm H}$, $^{3}{\rm He}$, $^{4}{\rm He}$, $^{7}{\rm Li}$, $^{7}{\rm Be}$, $^{12}{\rm C}$, $^{13}{\rm C}$, $^{14}{\rm N}$, $^{15}{\rm N}$, $^{16}{\rm O}$, $^{17}{\rm O}$ and Ex; Ex is an "Extra" fictitious mean non-CNO heavy element with atomic mass 28 and charge 13 (${\rm Ex} \sim {}^{28}{\rm Al}$) which complements the mixture i.e., $X_{\rm Ex} = 1 - \sum_{i={}^1{\rm H}}^{^{17}{\rm O}} X_i$ with X_i as the mass fraction of the species labeled with $i = {}^1{\rm H}, \ldots, {}^{17}{\rm O}$. With respect to time, due to microscopic diffusion processes, the abundances of heavy elements are enhanced towards the center; Ex mimics that enhancement for the non CNO metals which contribute to changes of Z, then to opacity variations but not to the nuclear burning and nuclear energy generation. We have taken into account the most important nuclear reactions of PP+CNO cycles (Clayton, 1968). The relevant nuclear reaction rates are taken from the NACRE compilation (Angulo et al. 1999) with the reaction $^7\mathrm{Be}(e^-,\nu\gamma)^7\mathrm{Li}$ taken from the compilation of Adelberger et al. (1998). Weak screening (Salpeter 1954) is assumed. Owing to the fact that the nowadays observed surface chemical composition is solar and close to the initial mixture (see Sect. 4), the initial fractions between the heavy elements within Z are set to their photospheric solar present day values. More details about this procedure can be found in Morel & Baglin (1999). We have used the meteoritic value (Grevesse & Sauval 1998) for the initial lithium abundance, $[\mathrm{Li}]_0 = 3.31 \pm 0.04$ (H $\equiv$ 12). For the calculations of the depletion, the lithium is assumed to be in its most abundant isotope ^{7}Li form. The initial abundance of each isotope is derived from isotopic fractions and initial values of $Y \equiv {}^3\mathrm{He} + {}^4\mathrm{He}$ and Z in order to fulfill the basic relationship $X + Y + Z \equiv 1$ with $X \equiv {}^1\mathrm{H} + {}^2\mathrm{H}$. Microscopic diffusion is described by the simplified formalism of Michaud & Proffitt (1993) with each heavy element as a trace element.

5.2. *Equation of state, opacities, convection and atmosphere.*

We have used the OPAL equation of state (Rogers et al. 1996) and the opacities of Iglesias & Rogers (1996) for the solar mixture of Grevesse & Noels (1993) complemented, at low temperatures by Alexander & Ferguson (1994) opacities.

In the convection zones the temperature gradient is computed according to the standard mixing-length theory (Böhm-Vitense 1958). The mixing-length is defined as $l \equiv \alpha H_\mathrm{p}$, where H_p is the pressure scale height. The convection zones are mixed via a strong full mixing turbulent diffusion coefficient $d_\mathrm{fm} = 10^{13}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$ which produces a homogeneous composition (Morel 1997). As quoted in Sect. 2, during the pre-main sequence, we used a soft mixing of the convection zones. We found empirically that the mixing generated by a turbulent diffusion coefficient $d_\mathrm{mm} = 10^6\,\mathrm{cm}^2\,\mathrm{s}^{-1}$ provides lithium depletions close to the observed values for both components. We are aware that this procedure is not fully satisfactory: first, as already quoted, we do not know why and how the lithium destruction is annihilated in ι Peg. B during the pre-main sequence, second, we suppose *a priori* that the observed depletion corresponds to the onset of the main sequence and, third, we do not know how the unknown physical process which acts on lithium performs on other chemicals.

Following the prescriptions of Schaller et al. (1992) the models take into account overshooting of mixed convective cores over the distance $O_\mathrm{v} = 0.2\min(H_\mathrm{p}, R_\mathrm{co})$; R_co is the radius of the convective core.

An atmosphere is restored using Hopf's $T(\tau)$ law (Mihalas 1978). The connection with the envelope is made at the Rosseland optical depth $\tau_\mathrm{b} = 10$, where the diffusion approximation for radiative transfer becomes valid (Morel et al. 1994). In the convective part of the atmosphere, a numerical trick (Henyey et al. 1965) is employed in connection with the purely radiative Hopf law in order to ensure the continuity of gradients at the limit between the atmosphere and the envelope. At each time step, the radius $R_\star$ of the model is taken at the optical depth $\tau_\star \simeq 0.6454$ where $T(\tau_\star) = T_\mathrm{eff}$; the mass of the star $M_\star$, is the mass inside the sphere of radius $R_\star$. The external boundary is located at the optical depth $\tau_\mathrm{ext} = 10^{-4}$, where the density is fixed to $\rho(\tau_\mathrm{ext}) = 3.55\,10^{-9}$ g cm^{-3}.

5.3. *Numerics*

The models have been computed using the CESAM code (Morel 1997). The numerical schemes are fully implicit and their accuracy is first order for the time and third order for the space. Typically, each model is described by about 600 mass shells, it increases up to 2100 for the models used in seismological analysis.

6. Results

6.1. *Age and chemical composition*

In the HR diagram Fig. 1 (bottom left) shows that the locus of ι Peg. A belongs to its uncertainty domain for evolutionary times in the interval $30\,\mathrm{My} \lessapprox t_\mathrm{ev\,A} \lessapprox 500\,\mathrm{My}$. Effective temperature and luminosity are very close to their observed values for $t_\mathrm{ev\,A} \simeq 45$ My. The locus of ι Peg. B (Fig. 1, bottom right) belongs to its uncertainty domain for times $45\,\mathrm{My} \lessapprox t_\mathrm{ev\,B} \lessapprox 3\,\mathrm{Gy}$. The distances between the loci of theoretical effective temperatures and luminosities and their observed values are smaller for evolutionary times close to $t_\mathrm{ev\,B} \sim 55$ My and $t_\mathrm{ev\,B} \sim 1$ Gy.

According to our definition in Sect. 5, the ZAMS occurs as soon as the nuclear dominates the gravitation as the primary source of energy. That occurs at evolutionary times $t_\mathrm{zams\,A} \simeq 36$ My and $t_\mathrm{zams\,B} \simeq 70$ My for ι Peg. A and B, respectively. Therefore, the age of the system ι Peg. is between $\sim$ ZAMS and ~ 0.5 Gy. For the calibrated models, within our definition of the theoretical zero age main sequence, the age of the binary system is about zero i.e. ι Peg. A and ι Peg. B are inhomogeneous ZAMS stars. The arguments in favor of this calibration are the following:

- the low amount of lithium depletion observed in the secondary is in agreement with the observations,
- in the HR diagram the loci of theoretical effective temperatures and luminosities are close to the observations at time $t_\mathrm{ev} \approx 56$ My for primary and secondary, while the differences are significantly larger as soon as $\sim 0.1\,\mathrm{Gy} \lesssim t_\mathrm{ev} \lesssim 0.5\,\mathrm{Gy}$,
- the fact that the method of steepest descent converges simultaneously for primary and secondary models reveals a stable solution.

Table 3 shows the characteristics of ι Peg. A and B calibrated models. The primary and the secondary have a convective core. The lithium depletion at the surface of both components is close to the observed values.

Table 4 shows the calibration parameters obtained for ι Peg. and solar models computed with the same physics. The un-

Table 3. Characteristics of a calibration of the ι Peg. system. $R_{\rm cz}$ and $R_{\rm co}$ are respectively the radius of the base of the external convection zone and of convective core (including overshoot). At the center, $T_{\rm c}$, $\rho_{\rm c}$, $X_{\rm c}$, $Y_{\rm c}$ are respectively the temperature (in M K), density (in g cm^{-3}), hydrogen and helium mass fractions.

	ι Peg. A	ι Peg. B
$T_{\rm eff}$	6642 K	4991 K
$L/L_\odot$	3.041	0.3148
$R/R_\odot$	1.320	0.7499
$R_{\rm cz}/R_\star$	0.929	0.696
$R_{\rm co}/R_\star$	0.100	0.166
$T_{\rm c}$	17.21	12.19
$\rho_{\rm c}$	86.83	79.22
$X_{\rm c}$	0.699	0.682
$Y_{\rm c}$	0.283	0.297
[Li]	+3.27 dex	+2.69 dex

certainties are derived empirically by changing the calibration parameters around their values derived of the calibrated model (the observed *solar* metallicity of ι Peg. is assumed to be error free). Considering first, the non linearity of the solution around ZAMS and second, the large uncertainty domains, chiefly due to the inaccurate value of $\Delta m_{\rm V}$, more elaborated estimations of calibration parameters uncertainties via, e.g. Monte Carlo procedure, will need heavy computations, which are not relevant of the scope of this paper.

6.2. Seismological analysis of ι Peg. A

The star ι Peg. A belongs to the solar-like stars region of the HR diagram. The oscillations of such stars may be stochastically excited by the convection, as it occurs in the Sun. The amplitudes of solar-like oscillations have been estimated by Houdek (1996) for stars of different masses and ages. Here, we consider the oscillation spectrum in the frequency range where the predicted amplitude is maximum, i.e. between 1 and 3 mHz.

The oscillations of a non rotating star are characterized by two numbers: the degree ℓ which is inversely proportional to the horizontal wavelength and the radial order n (Christensen-Dalsgaard & Berthomieu 1991). Here the rotation is small, hence neglected. The frequencies which may be observed, according to Houdek, are on the low-degree (ℓ=0 to 3) high-frequency p-modes (i.e. large radial order p-modes). The frequency spectrum is characterized by a few global quantities, $\Delta\nu_0$ which describes the quasi-equidistance of the frequencies in the spectrum and $\delta_{\ell,\ell+2}$ the small frequency separations between frequencies of modes with degrees of same parity and consecutive radial orders $\nu_{n,\ell}$ and $\nu_{n-1,\ell+2}$. They can be derived from the following simplified polynomial expression (Berthomieu et al. 1993):

$$\nu_{n,\ell} = \nu_{0\ell} + \Delta\nu_0(n + \frac{\ell}{2} - n_0).$$

The quantity $\nu_{0\ell}$ varies little with the degree ℓ. We have taken n_0= 21, approximately in the middle of the range of the expected excited frequencies (i.e. $1-3$ mHz) according to Houdek (*loc. cit.*). The large frequency separation $\Delta\nu_0$ is the distance between peaks of consecutive radial order for a given degree. It corresponds to the sound travel time across a stellar diameter and is mainly sensitive to the outer layers of the star. The small frequency separations around $n = n_0$, defined by:

$$\delta_{\ell,\ell+2} = \nu_{n_0,\ell} - \nu_{n_0-1,\ell+2} \sim \nu_{0\ell} - \nu_{0\ell+2},$$

are very sensitive to the core of the star.

Table 5 shows the quantities $\nu_{0,0}$, $\Delta\nu_0$ and $\delta_{0,2}$ $(n_0 = 21)$ which have been computed for three models A0, A1, A2 of ι Peg. A in the observational uncertainty HR box, at different evolutionary times $t_{\rm ev} = 50,\ 100,\ 500$ My. These characteristic quantities decrease with the age. ν_{00} and $\Delta\nu_0$ vary proportionally to $R^{-\frac{3}{2}}$, as expected, since the structure of the three models are very similar. The large separation $\Delta\nu_0$ which is the first quantity to be derived by observations varies significantly with the age of the star. The relative variation of $\delta_{0,2}$ is larger, due to the differences of structure in the core, but more difficult to be observed.

Table 4. Calibration parameters of a ι Peg. model lying within the uncertainty boxes and of a calibrated solar model using the same physics. The initial values are labeled by "$_0$". The uncertainties are empirically estimated (see text).

	ι Peg.	Sun
$\alpha_{\rm A}$	$1.46^{+0.20}_{-0.15}$	1.76
$\alpha_{\rm B}$	$1.36^{+0.30}_{-0.10}$	
Y_0	$0.278^{+0.007}_{-0.007}$	0.2722
age	0.00^{+500}_{-10} My	4.6 Gy
X_0	0.705	0.7081
Z_0	0.017	0.0197

Table 5. Theoretical global characteristics of the low degree p-mode spectrum of the star ι Peg. A. All the quantities are given in μHz (see text).

	age (My)	ν_{00}	$\Delta\nu_0$	$\delta_{0,2}$
A0	50	2234.91	102.0	11.76
A1	150	2222.57	101.50	11.47
A2	500	2106.61	96.30	10.61

7. Discussion and conclusions

With the individual masses derived from spectroscopic and visual orbits (Boden et al. 1999), we have attempted to calibrate the close binary system ι Peg. We have adopted effective temperatures and luminosities as derived from photometric data with the K bolometric correction and the empirical scale of temperature of Alonso et al. (1995, 1996). We have fixed the initial metallicity of primary and secondary to the nowadays observed (*solar*) value. The evolutionary sequences include the pre-main sequence. Microscopic diffusion and gravitational settling of chemical species are allowed for. To fit the observed lithium depletion in ι Peg. B we have used a mildly mixing of convection zones in the course of the pre-main sequence. The convective cores are extended by $0.2\,H_{\rm p}$, the standard amount of overshoot.

We have obtained adjustments within the uncertainty domains for effective temperatures and luminosities for both components with times of evolution within $\sim 40\,{\rm My} \lesssim t_{\rm ev} \lesssim 0.5\,{\rm Gy}$. Owing to the lack of lithium depletion observed in ι Peg. B, a solution is obtained for the evolutionary time $t_{\iota\,\rm Peg.} \sim 56\,$My. The age of the system is then close to zero and we argue that the components of the binary system ι Peg. are non-homogeneous ZAMS stars.

Separate values are found for the mixing-length parameter of each component. Even for ι Peg. B, of mass smaller than solar, Table 3 shows significant differences between the convection parameter derived for each component and those of the calibrated solar model computed with the same physics. This result appears in disagreement with the universality of the solar convection parameter for stellar masses between $0.6 M_\odot \lesssim M_\star \lesssim 1.0 M_\odot$ with metallicity close to solar (Fernandes et al. 1998). Nevertheless, owing to the large observational uncertainty domains, our result does not totally rule out the paradigm of the universality of the solar convection parameter.

We find a larger mixing-length parameter for ι Peg. A ($T_{\rm eff} = 6642\,$K) than for the cooler ι Peg. B ($T_{\rm eff} = 4991\,$K). The calculations of Ludwig et al. (1999) seem to predict exactly the opposite. We do not elucidate the reasons of this difference. We are aware that our models are not strongly constrained by the observations. The largest uncertainty comes from the difficulty to estimate the magnitude difference in the V band, a fundamental quantity for the determination of luminosities and effective temperatures based on photometric data. Improvements of the calibration of ι Peg. will necessitate a more accurate value for the V magnitude difference. Chemical composition determinations and estimates of effective temperatures and gravities by spectroscopic detailed analysis of each component are perhaps realistic with modern technology (C. van 't Veer, private communication) and are urgently needed.

We must also emphasize the fact that ι Peg. is probably tidally locked or, at least, recently unlocked and then it maybe a chromospherically active binary. Therefore to model such a pair as two standard isolated single stars is perhaps too simplistic. In some years from now, one can expect that asteroseismology from space e.g. COROT mission (Baglin 1998) will provide more constraints on stellar models as it is done nowadays with the Sun. This preliminary work also shows the powerful interest of interferometric and spectroscopic binaries such as ι Peg. as tests of transport process theories in stellar modeling.

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