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# Global and local bias in the FK5 from the Hipparcos data

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**Abstract.** Extensive analyses of the FK5 Catalogue with respect to the Hipparcos reference frame have been made in order to determine the overall and local corrections to be applied to any FK5-based secondary frame. The global rotation in position and proper motion is significant, and for the latter, interpreted in term of lack of inertiality of the FK5 frame and compared to a similar analysis carried out with the PPM proper motions. Zonal corrections are large compared to the random errors and difficult to represent with a small number of spherical functions. The differences (including the global rotation) are then given in the form of easy-to-use tables.

**Key words:** astrometry – reference systems

## 1. Introduction

The Hipparcos Catalogue is referred to the International Celestial Reference System (ICRS) and constitutes the optical counterpart of the inertial system materialized by a set of radio sources observed by the VLBI technique. Its internal precision is typically below 1 mas in right ascension and declination for all the bright stars and about 1 mas/year for the components of the proper motion (ESA, 1997). In addition it is expected to be free of regional errors at the level of 0.1–0.2 mas, a level much lower than any existing global catalogue. This provides the opportunity to use Hipparcos astrometry as a virtually error-free reference to determine the true errors of other catalogues at the Hipparcos epoch and to devise rules to correct for their systematic errors.

Up to now, the FK5 provided the basic stellar reference frame as adopted by the IAU in 1976. It was considered as the best realization of an inertial system through the accurate coordinates and proper motions of 1535 bright stars (Fricke et al., 1988). Although the ICRS has been built in order to be as much as possible consistent with the FK5 for the mean equinox and equator of the standard epoch J2000, there is still a global rotation between the two reference systems due to the uncertainty of the stellar positions in the FK5. Estimate of the misalignment was made possible only with the availability of the Hipparcos Catalogue.

Beyond this rotation, regional systematic differences between the FK5 stars (and hence PPM) and the Hipparcos positions do exist up to 100 mas. These systematic differences are evaluated and characterized as a function of the right ascension and declination. Due to the global nature of the construction of the Hipparcos Catalogue and its intrinsic accuracy (on a global scale better than 0.2 mas) there is no doubt that these regional differences must be attributed to the FK5 (and hence PPM) and not to Hipparcos. Eventually the knowledge of these differences might allow to reprocess old astrometric observations tied to the FK5 frame or at least to assess the magnitude of possible errors following from the defects of the reference catalogue.

This paper finalizes a study initiated for the Hipparcos documentation (Mignard et al., 1997) and fulfills the promise to provide easy-to-use corrections tables between the FK5 system and the ICRS (Mignard & Frœschlé, 1998).

## 2. The FK5 catalogue

The FK5 Catalogue has provided until the advent of the ICRF (Ma et al., 1998) a practical realization of a dynamical frame through the position and proper motion of carefully measured stars. It represents a revision of the FK4 and results from the determination of systematic and individual corrections to the mean positions and proper motions of the FK4, the elimination of the error in the position and motion of the FK4 equinox, and the introduction of the IAU (1976) system of astronomical constants (Fricke et al., 1988). In the course of his work, Fricke (1982) has derived the following correction in mas, $E(T) = (525 \pm 45) + (12.75 \pm 1.5)(t - 1950.0)$, to the zero point of the right ascension. One must bear in mind that any error in the time dependent term propagates directly into a systematic effect in the system of proper motion and into a defect of inertiality, impossible to detect before the availability of a more reliable system.

The expected positional accuracy of the 1535 bright stars is 0.03 arcsec at the mean epoch of the catalogue and 0.6 mas/yr for the proper motion of the northern hemisphere and 1.0 mas/yr for the southern. By propagating the FK5 positions to the Hipparcos epoch J1991.25, this leads to an expected error in right ascension and declination of 40 to 60 mas according to the hemisphere.

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The 1535 stars of the FK5 have been observed successfully by Hipparcos and their positions are known in 1991.25 with an accuracy typically $0.4\pm0.1$ mas in declination and $0.6\pm0.2$ mas in right ascension, with in the latter case a marked dependence with the declination. The corresponding figures for the proper motions are $0.7 \pm 0.2$ mas/yr and $0.55 \pm 0.15$ mas/yr.

The Hipparcos Catalogue has been constructed in such a way that the reference frame materialized by the positions coincides with the International Celestial Reference Frame, within the measurement errors. The Hipparcos positions and proper motions provide a realization of an optical non-rotating reference frame with axes pointing to fixed extragalactic directions to within 0.6 mas for the alignment and 0.25 mas $\mathrm{yr}^{-1}$ in the rate of rotation, resulting into a similar uncertainty in the inertiality of the system of proper motion (ESA 1997). In term of consistency, the optical reference frame defined by the Hipparcos positions and proper motions is believed to have a global accuracy of about 0.1 mas and 0.1 mas/yr with no regional distortion. The quality and stability of the Hipparcos link (rigid-body rotation only) has been recently confirmed by new observational material (Stone 1998) consisting of nearly 700 Hipparcos stars observed in optics with respect to visual counterparts of ICRF radiosources.

## 3. Comparison of the FK5 to Hipparcos

### *3.1. The settings*

All the multiple stars and objects with questionable solution in Hipparcos have been excluded from the analysis and the selection has ended up with 1233 stars with reliable solutions. The reason to exclude the double stars and the astrometric binaries, is not due to a mistrust of their astrometric solution, but the fact the Hipparcos and FK5 astrometric solutions may correspond to two different points on the sky and that the Hipparcos proper motion determined over a timespan shorter than the orbital period does not represent the systematic motion of the barycentre (Wielen et al., 1999). Thus these stars were considered not suitable for this analysis.

The stellar positions and proper motions in the FK5 are given for the epoch J2000 in the FK5 system, while the Hipparcos Catalogue being an observation catalogue is referred to an epoch close to the average observation time, namely $T_0 = \mathrm{J}\,1991.25$(TT). All the FK5 positions have been transported from J2000 to the epoch $T_0$ by using straightforwardly the FK5 proper motions. No attempt has been made to estimate the errors in the FK5 coordinates at the epoch $T_0$ for each star. Instead we are using an overall and global estimate of the errors in the following discussion. In the following these positions (transported to $T_0$) are denoted by $\alpha_{\mathrm{F}}$, $\delta_{\mathrm{F}}$ while the Hipparcos positions are labelled $\alpha_{\mathrm{H}}$, $\delta_{\mathrm{H}}$.

The comparison is performed at $T_0 = 1991.25$ and for each star one has computed the positional differences

$$(\alpha_{\mathrm{F}} - \alpha_{\mathrm{H}}) \cos\delta$$

and

$$\delta_{\mathrm{F}} - \delta_{\mathrm{H}}$$

**Table 1.** Global orientation and spin between the Hipparcos and FK5 Catalogues.

| | Orientation (mas) | | | | Spin(mas/yr) | | |
|---|---|---|---|---|---|---|---|
| | J1991.25 | J2000 | $\sigma$ | | | $\sigma_{\mathrm{HIP}}$ | $\sigma_{\mathrm{ICRS}}$ |
| $\epsilon_x$: | -17.3 | -19.9 | 2.3 | $\omega_x$: | -0.30 | 0.10 | 0.27 |
| $\epsilon_y$: | -14.3 | - 9.1 | 2.3 | $\omega_y$: | 0.60 | 0.10 | 0.27 |
| $\epsilon_z$: | 16.8 | 22.9 | 2.3 | $\omega_z$: | 0.70 | 0.10 | 0.27 |

and the proper motion differences,

$$(\mu_\alpha)_{\mathrm{F}} - (\mu_\alpha)_{\mathrm{H}},\ (\mu_\delta)_{\mathrm{F}} - (\mu_\delta)_{\mathrm{H}}.$$

### *3.2. The overall orientation and spin*

The Hipparcos Catalogue was referred to the ICRS after the final astrometric solution has been rotated as explained in the Hipparcos documentation (Vol III, chap. 18). Nominally the ICRS was to maintain the continuity with the previous dynamical reference system realized by the FK5 Catalogue. However due to its limited accuracy the alignment of the ICRS pole and origin of right ascension with the corresponding ones in the FK5 could not be achieved with consistency better than 20 mas for the pole and 80 mas for the origin of the right ascension. The final Hipparcos solution, ICRS(Hipparcos) and the optical reference frame defined by the FK5, J2000(FK5), differ by a pure rotation and numerous zonal differences at various wavelengths.

The positional and proper motion differences have been projected on a set of orthogonal vectorial harmonics. The first degree of these harmonics represents the pure rotation while the harmonics of higher degree account for the zonal differences at decreasing wavelengths with increasing degree. The value of the three angles of the global orientation and of the components of the spin vector are given in Table 1 with their respective uncertainties.

The orientation is given at the Hipparcos epoch J1991.25 and at J2000. The standard errors are similar at both epochs and there is no need to consider the possible systematic difference of the order of 0.6 mas between the Hipparcos induced frame and the ICRS.

The sign convention for this global rotation is such that the rotation represents the orientation of the Hipparcos triad with respect to the FK5 triad or equivalently the three consecutive rotations to be applied to align the FK5 triad to the Hipparcos axes. In term of stellar coordinates the equivalent transformation between the coordinates of a star in the FK5 and the same star in the Hipparcos frame is as follows,

$$\begin{aligned}(\alpha_F - \alpha_H)\cos\delta &= -\epsilon_x \cos\alpha\sin\delta - \epsilon_y \sin\alpha\sin\delta + \epsilon_z\cos\delta \\ (\delta_F - \delta_H) &= \epsilon_x \sin\alpha - \epsilon_y \cos\alpha\end{aligned}$$

The global rotation has no deep physical meaning. It gives only the relative orientation of the FK5 frame with respect to the ICRS at the reference epoch, insofar as the latter can be represented by the Hipparcos Catalogue. The three angles of rotation

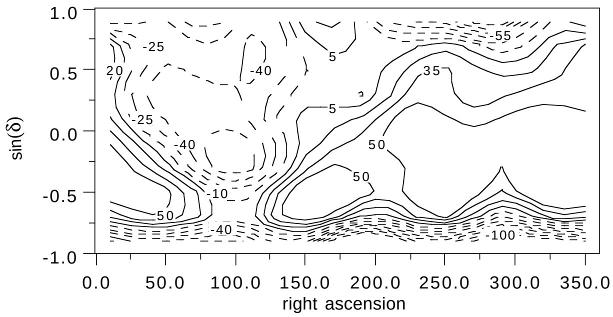


**Fig. 1.** Difference FK5-Hipparcos in right ascension. The curves are labelled in mas.

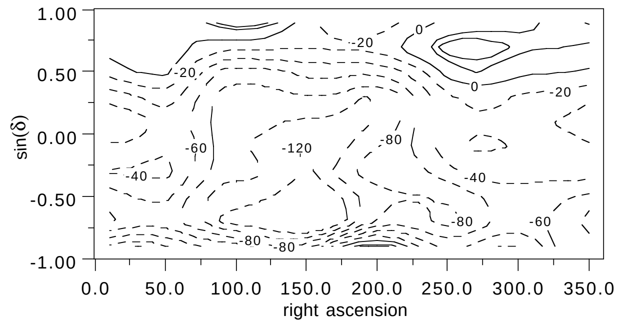


**Fig. 2.** Difference FK5-Hipparcos in declination. The curves are labelled in mas.

are consistent with the IAU recommendations stipulating that the direction of the Conventional Celestial Pole relative to the FK5's should be within 50 mas, (Arias et al. 1995). Due to the remaining uncertainty of the Hipparcos frame with respect to the ICRF ($\sim 0.6$ mas), the uncertainty of the alignment with respect to the Hipparcos frame or to the ICRF are similar.

It goes differently with the spin between the two systems of proper motion. A spin component between two frames tells that a time dependant rotation between the two triads does exist, that is to say that one frame is rotating at constant angular speed with respect to the other. Thus, here, the components of the spin are related to the lack of inertiality of the FK5 frame with respect to the non–rotating extragalactic frame. This translates into a well defined signature in the difference of the stellar proper motions projected in each frame, mathematically expressed by

$$
\begin{aligned}
(\mu_{\alpha_F} - \mu_{\alpha_H})\,\cos\delta \;&=\; -\omega_x \cos\alpha \sin\delta - \omega_y \sin\alpha \sin\delta + \\
&\quad\;\; \omega_z \cos\delta \\
(\mu_{\delta_F} - \mu_{\delta_H}) \;&=\; \omega_x \sin\alpha - \omega_y \cos\alpha
\end{aligned}
$$

The components of $\omega$ have been determined from a least-squares fit of the proper motion difference to the mathematical model.

The uncertainty with respect to Hipparcos ($\sigma_{\rm HIP}$ in the table) is the formal result of the fit, while we have taken into account the uncertainty of the link of 0.25 mas/yr to determine the true default of inertiality of the FK5 proper motion system ($\sigma_{\rm ICRS}$ in the table). It was combined quadratically, although the difference between the Hipparcos induced frame and the ICRS should be considered as a systematic bias. Comparisons of the Hipparcos stars with a selection of bright stars (comparable to the average magnitude of the FK5 stars) from the PPM Catalogue (see Sect. 6) has yielded comparable values for the spin, irrespective to whether the FK5 stars were removed or not from this selection. The rotation between the two Catalogues is then real and should be traced back to the construction of the FK5.

We suggest a tentative interpretation based on a correction to the precession constant, $\Delta p$ and a combination of a non precessional motion of the equinox of the FK5 $\Delta E$ and a correction $\Delta\lambda$ to the planetary precession, although the latter is likely to be negligible. With,

$$
\begin{aligned}
\omega_y \;&=\; -\Delta p \,\sin\epsilon \\
\omega_z \;&=\; \quad \Delta p \,\cos\epsilon - (\Delta\lambda + \Delta E)
\end{aligned}
$$

which gives,

$$
\begin{aligned}
\Delta p \;&=\; -1.5 \pm 0.7 \text{ mas/yr} \\
\Delta\lambda + \Delta E \;&=\; -2.1 \pm 0.7 \text{ mas/yr}
\end{aligned}
$$

The correction to the precession constant is not consistent with the recent determination based on the VLBI (Charlot et al. 1995) or LLR (Chapront et al. 1999) which are both close to -3 mas/yr with few tenth of mas in uncertainty. A similar conclusion was also reached by Zhu & Yang (1999) from their analysis of different catalogues (PPM, ACRS) with respect to Hipparcos. A possible clue lies in the variation of the spin with the magnitude of the stars. We have divided the FK5 Catalogue in two halves of comparable size, one with the brightest stars up to $V = 4.9$ and the second with the 'faint' end and determined the spin components as before. While there is no marked difference in $\omega_x$ and $\omega_z$ between the two sets, one gets $\omega_y = 0.33$ mas/yr for the bright end and $0.70$ mas/yr for the faint half. This is marginally conclusive, because the number of stars ($\approx$ 600 per group) is rather small to ensure a perfect orthogonality of the zonal effects over the sphere, but sufficient to indicate that a single global rotation is not the best model to characterize the defect of inertiality of the FK5 system. The magnitude dependence is investigated further in Sect. 6 with the proper motions of the PPM Catalogue, which has the advantage of having been constructed in the FK5 system and of including many more stars than the FK5. Finally one must also add that the correction to the non precessional motion of the equinox is statistically significant and is also compatible with the uncertainty of $\pm 1.5$ mas/yr obtained by Fricke.

### *3.3. The zonal differences*

Although the overall rotation is rather small, the local deviations of the Fk5 system with respect to the Hipparcos are much larger, reaching 100 mas in position and 2.5 mas/yr in proper motion. The differences are shown graphically in Figs. 1-2 for the positions and in Figs. 3-4 for the proper motion components of the FK5. There are major features in these plots, like the large

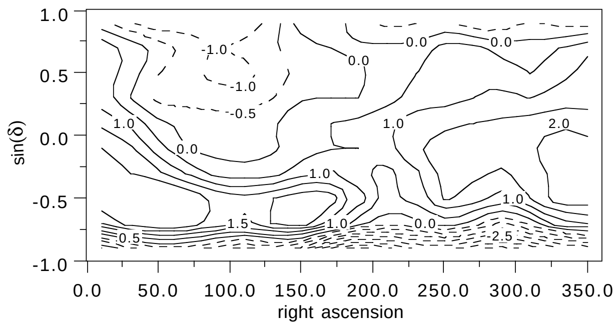


**Fig. 3.** Difference FK5-Hipparcos in proper motion in right ascension. The curves are labelled in mas/yr.

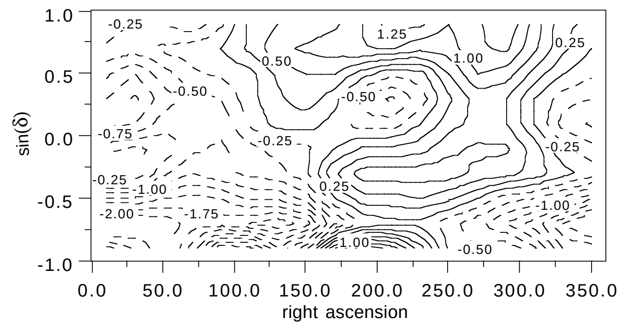


**Fig. 4.** Difference FK5-Hipparcos in proper motion in declination. The curves are labelled in mas/yr.

deviations in the southern polar region. The plots of differences in right ascension (resp: declination) and proper motion in right ascension (resp: declination) resemble each other, indicating that part of the zonal differences in positions is in fact due to the positions being updated with the proper motion.

No convenient and accurate analytical representation of these differences was found which could be simpler to use than tables and linear interpolation. The actual numbers are given in tabular form for the FK5 in Tables 3. Both the plots and the tables include the overall rotation, that is to say they give the observed differences in position in 1991.25 and the differences in proper motion, so that the tables can be used to carry out simultaneously a system transformation and correct for the largest systematic zonal differences.

## 4. Comparison of Hipparcos to the PPM

The PPM Star Catalogue (Röser & Bastian 1991; Bastian et al. 1993) was designed to provide a more accurate net of reference stars on the J2000(FK5) system based on multiple positional epochs rather than the usual two in the SAO. The PPM north gives the J2000 positions and proper motions for $181\,731$ stars north of a declination $-2.5$ degrees and brighter than 10.5 mag, although a small sample of fainter stars is included. The published mean error of positions at epoch J1990 and proper motions are respectively 0.27 arcsec and 4.3 mas/year. The PPM south covers the rest of the celestial sphere and comprises $197\,179$ stars with an astrometric precision of 0.11 arcsec for the positions at J1990 and 3.0 mas/year for the proper motions up to a magnitude of 10.5, with few fainter stars. Both catalogues are constructed to represent as closely as possible the reference frame defined by the FK5. The southern part has been complemented in 1994 (Röser et al. 1994) with an additional set of $90\,000$ stars of comparable quality. A preliminary comparison based on Hipparcos provisional results was performed by Lindegren (Lindegren et al., 1995) which focused primarily on the zonal differences rather than on the system difference.

Although the PPM is on the average less accurate than the FK5, it contains a much larger number of objects over a wider range of magnitudes. Nearly $110\,000$ of these stars have been observed by Hipparcos and $95\,000$ are kept after the double stars and the solutions with identified problems in the Hipparcos Catalogue are filtered out. Compared to PPM the Hipparcos proper motions can be considered as error-free. Because the PPM has been tied up to the FK5 system, the PPM Catalogue is also perfectly suited to investigate the systematic difference in spin between the ICRS and the FK5 system. As a result of the different properties between the northern and southern hemisphere, three analyses have been carried out with respectively the northern hemisphere, the southern hemisphere and the complete catalogue. In each case the solution follows from a weighted least-squares fitting of the spin components to the proper motions differences. The overall results for all the $95\,000$ stars are shown in Table 2 for $\omega_x,\ \omega_y,\ \omega_z$. They differ significantly from the direct comparison to the FK5 without being completely out of track. As before one must not confuse the standard errors left in the post-fit residuals, which are very small here because of the large number of sources, with the actual uncertainty in the default of inertiality of the FK5 system. The major source of error in the latter, is precisely the remaining rotation rate of the Hipparcos system with respect to the ICRS, estimated to be of the order of 0.25 mas/yr. With $\omega_y = 0.83$ mas/yr, the effect on the precession constant amounts to $-2.2 \pm 0.6$ mas/yr, still smaller than the expected -3 mas/yr, but almost within one standard error of the VLBI and LLR value.

**Table 2.** Systematic rate difference between the Hipparcos and the PPM Catalogues.

| | Spin(mas/yr) | | |
|---|---|---|---|
| | | $\sigma_{\text{HIP}}$ | $\sigma_{\text{ICRS}}$ |
| $\omega_x$: | -0.68 | 0.03 | 0.25 |
| $\omega_y$: | 0.83 | 0.03 | 0.25 |
| $\omega_z$: | 0.25 | 0.03 | 0.25 |

Thanks to the large number of stars in this comparison, one can also search for a magnitude equation in the rotational difference. We have performed a similar analysis over bin 0.5 mag wide, independently for the northern, southern hemisphere and for the complete data set and determined the spin components in each of the three cases. The results are plotted in Figs. 5–7. In the plots the dashed lines correspond to the analysis of the complete sky within each magnitude bin and lies more or less exactly in the middle between the results based only on the northern and southern stars. The bulk of the data set comprises stars between

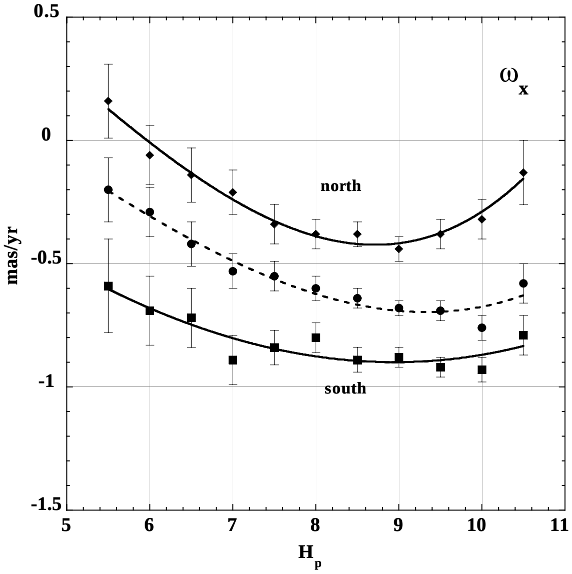


**Fig. 5.** Global spin (x-component) between the PPM and the Hipparcos proper motions per bin of 0.5 magnitude. The solid lines are for the northern and southern hemisphere and the dashed line for the full sky.

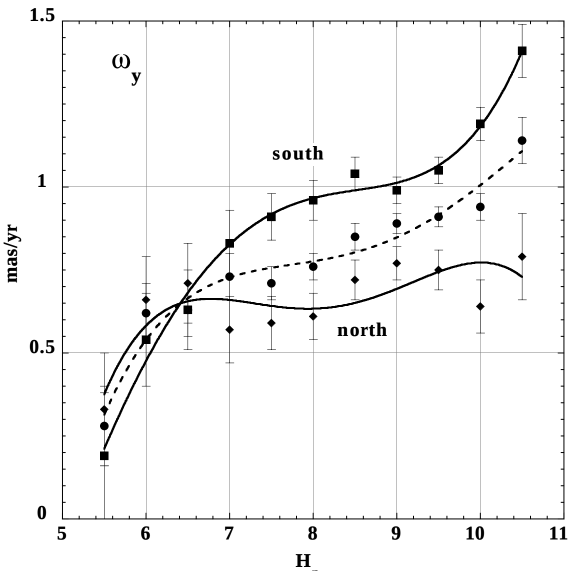


**Fig. 6.** Global spin (y-component) between the PPM and the Hipparcos proper motions per bin of 0.5 magnitude. The solid lines are for the northern and southern hemisphere and the dashed line for the full sky.

8.5 and 9.5 magnitude, which agrees with the overall results of Table 2. The bright end, which includes all the FK5 stars agrees very well also with the spin found in Table 1 for the FK5 alone, or better with the analysis restricted to the stars brighter than $V = 4.9$ (Sect. 4). It happens that the rate of change of $\omega$ with the magnitude at the bright end is the strongest for $\omega_y$ (Fig. 6), while the curves are rather flat for the $x$ and $y$ components, as anticipated from the FK5 analysis.

The difference between the northern and southern hemisphere is striking and may question the meaning, if not the validity, of the whole analysis. There is a known quality difference between the two hemispheres in the PPM individual proper motions, but this does not imply a systematic difference in the system itself. However the curves obtained for the individual hemispheres should be taken with caution because the spin signal on the celestial sphere is, to a great accuracy, orthogonal to any other zonal distortion over the full sky. Very likely this is not the case over a single hemisphere, at least for the distortions with the largest scales. Since no other distortion has been included in the analysis, part of these distortions have a non zero projection on the spin components and add to the spin itself. With this simple model there is no way to tell by what fraction the spin is polluted when the analysis is restricted to a limited coverage of the sky. In this respect, the spin determined with stars more or less regularly distributed over the complete sky is much more reliable with realistic error bars, even though the zonal effects remain undetermined. In short, the orthogonality between the systematic spin and the zonal terms allows the procedure to work properly blindly.

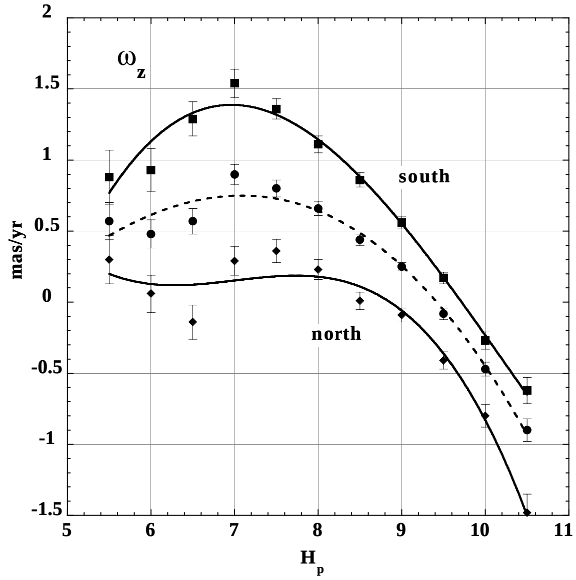


**Fig. 7.** Global spin (z-component) between the PPM and the Hipparcos proper motions per bin of 0.5 magnitude. The solid lines are for the northern and southern hemisphere and the dashed line for the full sky.

The variation of the spin components with the star magnitude is however a well established fact and indicates that the heterogeneity of the population prevents from giving a plain answer to the question of the rotation of the FK5 system (to the extent it can be materialized by the PPM proper motions at every magnitude) with respect to the ICRS. The large variations of the spin components with the stellar magnitudes obtained with the PPM helps understand the difficulty to reconcile the correction of the precession constant with the remaining rotation of the FK5 system determined from the FK5 proper motions alone, that is to say with a small sample of very bright stars. At the milliarcsecond/yr level, there is probably no hope to do much better in the interpretation of the spin rate between the two systems.

## 5. Conclusion

In this paper we have primarily determined the overall differences between the FK5 system, as determined by the FK5 Catalogue (positions and proper motions) or the PPM (proper motions only), and the Hipparcos Catalogue, that is to say the best optical counterpart of the ICRF. Convenient two-dimensional tables are provided to implement these global and zonal corrections to secondary Catalogues linked to the FK5. Obviously, when possible, the best solution remains the reprocessing of the data using the Hipparcos or Tycho stars as references. The lack of inertiality of the FK5 of about -3 mas/yr is confirmed, but to a lesser accuracy than the determinations based on the VLBI or LLR observations.

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## Appendix

The following tables give the position and proper motion differences between the FK5 and Hipparcos, smoothed over constant areas of 230 square degrees. The tables can be linearly interpolated between data entries in both directions.

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**Table 3.** Difference in right ascension ($\Delta\alpha\cos\delta$) between the FK5 Catalogue and Hipparcos at 1991.25. The differences FK5-Hipparcos are in mas and smoothed out over each cell of about 230 square degrees.

| | Right Ascension | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| sin $\delta$ | **10** | **30** | **50** | **70** | **90** | **110** | **130** | **150** | **170** | **190** | **210** | **230** | **250** | **270** | **290** | **310** | **330** | **350** |
| **0.9** | -44.7 | -45.2 | -26.4 | -2.1 | -8.7 | -37.7 | -28.1 | 15.5 | 27.6 | -3.5 | -42.6 | -55.7 | -56.9 | -91.3 | -97.3 | -35.3 | -0.3 | -22.0 |
| **0.7** | 25.9 | -12.3 | -33.1 | -27.2 | -25.9 | -43.9 | -37.8 | -4.5 | 10.8 | 4.1 | -5.0 | 4.7 | 18.9 | -1.9 | -23.3 | -5.1 | 22.0 | 38.5 |
| **0.5** | 27.7 | -19.6 | -43.7 | -38.2 | -33.5 | -42.9 | -36.0 | -10.6 | -2.3 | -4.3 | 10.0 | 32.0 | 37.8 | 24.7 | 14.4 | 17.3 | 30.5 | 46.5 |
| **0.3** | 6.7 | -38.5 | -53.4 | -46.3 | -43.5 | -38.3 | -18.0 | -1.0 | -6.5 | -11.1 | 18.0 | 47.8 | 37.5 | 23.8 | 32.5 | 41.2 | 44.1 | 39.8 |
| **0.1** | 21.2 | -21.3 | -41.4 | -47.4 | -51.4 | -38.0 | -7.0 | 12.8 | 15.2 | 22.4 | 48.4 | 67.2 | 52.5 | 41.8 | 51.6 | 60.0 | 63.4 | 55.0 |
| **-0.1** | 49.6 | 17.0 | -17.0 | -50.4 | -69.2 | -58.3 | -24.1 | 10.6 | 29.7 | 42.5 | 55.6 | 65.0 | 66.5 | 61.1 | 56.5 | 59.9 | 66.3 | 66.6 |
| **-0.3** | 66.2 | 44.8 | 14.4 | -31.4 | -68.3 | -63.0 | -19.6 | 31.9 | 54.3 | 45.0 | 38.4 | 56.0 | 76.4 | 65.3 | 49.7 | 65.9 | 82.4 | 79.3 |
| **-0.5** | 65.2 | 65.1 | 58.7 | 25.2 | -16.4 | -16.3 | 32.9 | 82.2 | 90.7 | 60.3 | 41.8 | 66.1 | 86.7 | 58.6 | 32.6 | 64.8 | 97.9 | 84.0 |
| **-0.7** | 23.9 | 47.2 | 49.2 | 27.3 | -0.7 | 4.1 | 49.9 | 70.9 | 39.4 | 0.5 | 1.6 | 38.4 | 53.6 | 12.1 | -27.4 | -2.1 | 32.4 | 23.6 |
| **-0.9** | -72.1 | -48.8 | -56.0 | -68.2 | -76.1 | -68.0 | -30.4 | -33.7 | -117.0 | -169.5 | -133.2 | -82.7 | -74.6 | -102.7 | -130.1 | -124.9 | -111.9 | -102.9 |

**Table 4.** Difference in declination between the FK5 Catalogue and Hipparcos at 1991.25. The differences FK5-Hipparcos are in mas and smoothed out over each cell of about 230 square degrees.

| | Right Ascension | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| sin $\delta$ | **10** | **30** | **50** | **70** | **90** | **110** | **130** | **150** | **170** | **190** | **210** | **230** | **250** | **270** | **290** | **310** | **330** | **350** |
| **0.9** | 11.5 | 9.2 | 12.4 | 11.4 | 31.2 | 40.0 | 19.1 | -14.5 | -36.8 | -34.6 | -24.0 | -6.5 | 7.1 | 8.6 | 16.1 | 18.8 | 23.7 | 28.0 |
| **0.7** | 9.3 | 15.0 | 17.9 | -3.5 | -17.0 | -13.3 | -17.0 | -18.2 | -17.4 | -15.4 | -1.7 | 16.5 | 35.8 | 38.5 | 29.2 | 21.3 | 20.4 | 18.5 |
| **0.5** | -11.2 | 0.6 | 4.4 | -31.3 | -67.3 | -70.7 | -63.6 | -53.4 | -51.6 | -58.0 | -50.4 | -25.1 | 8.9 | 20.5 | 10.4 | 1.6 | 1.2 | -3.2 |
| **0.3** | -53.0 | -40.9 | -26.3 | -56.8 | -91.0 | -93.2 | -86.9 | -80.2 | -83.1 | -100.9 | -96.7 | -56.6 | -22.6 | -15.1 | -19.0 | -26.3 | -26.6 | -35.3 |
| **0.1** | -78.8 | -72.3 | -50.3 | -64.5 | -93.0 | -98.0 | -99.6 | -104.3 | -103.4 | -105.4 | -92.5 | -53.6 | -26.5 | -22.1 | -22.6 | -33.0 | -42.9 | -56.3 |
| **-0.1** | -55.3 | -57.9 | -42.3 | -56.8 | -89.8 | -100.8 | -110.2 | -124.7 | -111.2 | -86.7 | -73.1 | -52.2 | -25.8 | -16.6 | -19.3 | -27.2 | -32.4 | -36.7 |
| **-0.3** | -38.3 | -36.9 | -32.1 | -61.2 | -90.5 | -92.9 | -107.2 | -121.6 | -97.2 | -72.0 | -70.8 | -64.4 | -45.6 | -32.7 | -29.5 | -33.3 | -32.2 | -28.1 |
| **-0.5** | -71.4 | -58.7 | -55.3 | -87.2 | -114.3 | -112.3 | -122.9 | -143.9 | -128.0 | -97.7 | -82.6 | -80.8 | -78.1 | -63.3 | -48.6 | -48.3 | -55.9 | -64.9 |
| **-0.7** | -85.7 | -70.9 | -72.4 | -95.8 | -123.4 | -134.1 | -147.7 | -166.9 | -144.3 | -95.3 | -66.9 | -76.2 | -88.2 | -72.7 | -54.5 | -53.2 | -73.8 | -91.7 |
| **-0.9** | -15.0 | 1.8 | -18.4 | -37.7 | -47.6 | -57.1 | -82.8 | -71.3 | -11.1 | 33.3 | 26.4 | -22.9 | -51.0 | -34.7 | -15.6 | -27.5 | -67.8 | -67.1 |

**Table 5.** Difference in proper motion in right ascension ($\Delta\mu_\alpha\cos\delta$) between the FK5 Catalogue and Hipparcos. The differences FK5-Hipparcos are in mas/yr and smoothed out over each cell of about 230 square degrees.

| | Right Ascension | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| sin $\delta$ | **10** | **30** | **50** | **70** | **90** | **110** | **130** | **150** | **170** | **190** | **210** | **230** | **250** | **270** | **290** | **310** | **330** | **350** |
| **0.9** | -0.3 | -0.9 | -1.5 | -1.2 | -1.1 | -1.3 | -0.8 | 0.3 | 0.3 | -0.3 | -0.7 | -0.5 | -0.3 | -0.9 | -0.9 | -0.4 | -0.7 | -0.7 |
| **0.7** | 0.9 | 0.4 | -0.1 | -0.6 | -1.0 | -0.9 | -0.5 | -0.2 | 0.0 | 0.1 | 0.1 | 0.2 | 0.7 | 0.7 | 0.3 | 0.2 | 0.5 | 0.9 |
| **0.5** | 0.9 | 0.2 | -0.5 | -0.9 | -1.2 | -1.2 | -0.8 | -0.4 | -0.2 | -0.1 | 0.2 | 0.5 | 0.8 | 1.0 | 0.8 | 0.5 | 0.8 | 1.2 |
| **0.3** | 0.7 | -0.0 | -0.7 | -0.7 | -0.8 | -0.8 | -0.5 | -0.0 | 0.0 | -0.0 | 0.3 | 0.7 | 0.8 | 1.0 | 1.1 | 1.0 | 1.2 | 1.2 |
| **0.1** | 1.4 | 0.9 | 0.1 | -0.1 | -0.2 | -0.3 | 0.0 | 0.4 | 0.5 | 0.6 | 0.9 | 1.4 | 1.4 | 1.5 | 1.6 | 1.7 | 2.0 | 1.9 |
| **-0.1** | 2.0 | 1.6 | 0.7 | 0.0 | -0.2 | -0.3 | -0.1 | 0.3 | 0.5 | 0.5 | 0.9 | 1.5 | 1.8 | 1.8 | 1.8 | 2.0 | 2.1 | 2.1 |
| **-0.3** | 2.2 | 2.0 | 1.6 | 0.9 | 0.3 | 0.2 | 0.3 | 0.8 | 1.1 | 0.6 | 0.3 | 0.9 | 1.6 | 1.6 | 1.4 | 1.8 | 2.1 | 2.1 |
| **-0.5** | 2.5 | 2.8 | 2.9 | 2.6 | 1.9 | 1.6 | 2.0 | 2.5 | 2.3 | 1.3 | 0.4 | 0.7 | 1.6 | 1.3 | 0.7 | 1.5 | 2.4 | 2.4 |
| **-0.7** | 1.5 | 2.3 | 2.6 | 2.4 | 1.8 | 1.4 | 2.1 | 2.4 | 1.3 | 0.2 | -0.4 | -0.2 | 0.4 | -0.2 | -1.1 | -0.3 | 0.9 | 1.0 |
| **-0.9** | -2.0 | -1.1 | -0.6 | -0.7 | -1.2 | -1.7 | -1.2 | -1.6 | -3.5 | -4.4 | -3.6 | -2.9 | -2.8 | -3.3 | -3.8 | -3.3 | -2.5 | -2.3 |

**Table 6.** Difference in proper motion in declination ($\Delta\mu_\delta$) between the FK5 Catalogue and Hipparcos. The differences FK5-Hipparcos are in mas/yr and smoothed out over each cell of about 230 square degrees.

| | Right Ascension | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin\delta$ | **10** | **30** | **50** | **70** | **90** | **110** | **130** | **150** | **170** | **190** | **210** | **230** | **250** | **270** | **290** | **310** | **330** | **350** |
| **0.9** | -0.1 | -0.3 | -0.5 | -0.7 | -0.3 | 0.3 | 0.5 | 0.6 | 0.7 | 1.0 | 1.4 | 1.3 | 1.0 | 1.2 | 1.5 | 0.9 | 0.3 | 0.2 |
| **0.7** | -0.3 | -0.4 | -0.2 | -0.1 | -0.0 | 0.3 | 0.7 | 0.8 | 0.9 | 1.0 | 1.0 | 0.8 | 0.9 | 1.2 | 1.3 | 0.8 | 0.3 | 0.0 |
| **0.5** | -0.7 | -1.0 | -0.6 | -0.3 | -0.2 | -0.1 | 0.3 | 0.5 | 0.5 | 0.1 | -0.2 | -0.1 | 0.5 | 1.0 | 0.9 | 0.4 | 0.0 | -0.2 |
| **0.3** | -1.0 | -1.3 | -0.9 | -0.6 | -0.5 | -0.1 | 0.2 | 0.4 | 0.1 | -0.6 | -0.8 | -0.4 | 0.2 | 0.6 | 0.5 | -0.0 | -0.4 | -0.5 |
| **0.1** | -1.1 | -1.0 | -0.6 | -0.5 | -0.7 | -0.2 | 0.1 | 0.1 | -0.1 | -0.4 | -0.4 | -0.0 | 0.4 | 0.6 | 0.5 | 0.0 | -0.6 | -0.8 |
| **-0.1** | -0.3 | -0.2 | -0.3 | -0.7 | -0.8 | -0.6 | -0.4 | -0.3 | 0.0 | 0.3 | 0.3 | 0.4 | 0.7 | 0.8 | 0.8 | 0.4 | -0.2 | -0.4 |
| **-0.3** | 0.0 | -0.0 | -0.7 | -1.0 | -0.8 | -0.7 | -0.6 | -0.3 | 0.5 | 0.9 | 0.9 | 0.9 | 0.9 | 0.7 | 0.6 | 0.4 | 0.1 | -0.1 |
| **-0.5** | -1.3 | -1.3 | -1.5 | -1.6 | -1.5 | -1.2 | -1.2 | -0.9 | 0.0 | 0.4 | 0.4 | 0.5 | 0.3 | -0.0 | -0.4 | -0.5 | -0.8 | -1.2 |
| **-0.7** | -2.6 | -2.3 | -2.2 | -1.9 | -1.9 | -2.0 | -2.2 | -1.9 | -0.8 | -0.2 | -0.1 | -0.1 | -0.4 | -0.7 | -1.0 | -1.4 | -2.0 | -2.5 |
| **-0.9** | -1.4 | -1.8 | -2.2 | -1.4 | -0.3 | -0.4 | -1.2 | -0.6 | 1.0 | 1.7 | 1.4 | 0.6 | -0.3 | -0.5 | -0.3 | -0.7 | -1.5 | -1.6 |