

Quantifying substructure in galaxy clusters

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Abstract. Substructure in galaxy clusters can be quantified with the robust Δ statistics (Dressler and Shectman 1988) which uses velocity kinematics and sky projected positions. We test its sensitivity using dissipationless numerical simulations of cluster formation. As in recent observations, about 30% of the simulated clusters show substructure, but the exact percentage depends on the chosen limit for defining substructure, and a better discriminator is the distribution function of the Δ statistics. The Dressler-Shectman statistics correlate well with other subcluster indicators, but with large scatter due to its sensitivity to small infalling groups and projection effects.

Key words: galaxies: clusters: general – cosmology: observations – cosmology: theory

1. Introduction

During the last two decades, substructure was detected in a significant fraction of groups and clusters of galaxies, both as double or secondary maxima in the sky-projected galaxy distribution of clusters (cf. Baier 1979 and Geller & Beers 1982), and as deviations of spherical symmetry in X-ray contour maps (Forman et al. 1981). However, quantifying substructure in clusters is a non-trivial problem. Analyzing substructure indicators with numerical simulations, West, Oemler & Dekel (1988) concluded that many subclumps may be pure chance projections, and that the remaining abundance of substructure can barely discriminate between a wide range of cosmological scenarios. Projection effects are less pronounced in X-rays, and recent X-ray data have provided more reliable information on the abundance of substructure (cf. Mohr, Fabricant & Geller 1993 and Buote & Xu 1997).

In the optical spectral range, one gets better results by including velocity information for a large sample of cluster galaxies. Substructure was identified using subgroup velocity statistics in clusters (Dressler & Shectman 1988), finding velocity offsets of cD galaxies with respect to the major cluster (Bird 1994), employing the hierarchical tree algorithm (Serna & Gerbal 1996) and wavelet analysis (Girardi et al. 1997 and Pagliaro et al. 1999). Recently,

Dressler & Shectman's statistics were used by Zabludoff & Mulchaey (1998) for 6 poor groups, and by Solanes, Salvador-Solé & González-Casado (1998) for 67 rich clusters from the ENACS survey. These different analyses agreed in (30 – 40)% of clusters which showed statistically significant substructure. But it appears that this amount depends on the analysis method (Pinkney et al. 1996) and on the imposed reliability criterion.

The existence of substructure in clusters suggests that the clusters are young objects since loosely bound subgroups can survive only a few cluster crossing times, i.e., shorter than the Hubble time. In the framework of the hierarchical structure formation scenario, clusters grow mainly by mergers of smaller objects and by accretion; substructure indicates recent major mergers. Starting from these ideas and the early termination of growth of density perturbations in a low density universe, the abundance of substructure was suggested as an effective measure of the mean matter density in the universe (Richstone, Loeb & Turner 1992, Bartelmann, Ehlers & Schneider 1993, Kauffman & White 1993, and Lacey & Cole 1994). The interpretation of subclusters as recent mergers might explain also some of the properties of cD cluster galaxies, such as their orientation with respect to the environment (West 1994) and the peculiar velocity distribution in rich clusters (Merritt 1985).

The velocity kinematics as a signature of substructure traces the galaxy distribution, therefore we employed dissipationless simulations in a large cosmological environment. The cosmological models take COBE-normalized perturbation spectra in four cosmological scenarios. Earlier theoretical studies concentrated on the analysis of the density contrast of clusters and the density profile (Jing et al. 1995, Thomas et al. 1998). There, relatively small differences in the cluster properties are found when comparing cluster profiles at the same overdensity. This concerns mainly the properties of relaxed clusters. Here we study in particular clusters which represent deviations from an equilibrium. For quantifying substructure in X-ray profiles of clusters, hydrodynamic simulations of cluster formation have to be employed, and early attempts at this have already yielded first promising results (Cen 1997 and Valdarnini, Ghizzardi & Bonometto 1999).

The outline of the paper is as follows. First we analyze the method for identification of substructure in galaxy clusters selected from numerical simulations. Then we apply the algorithm

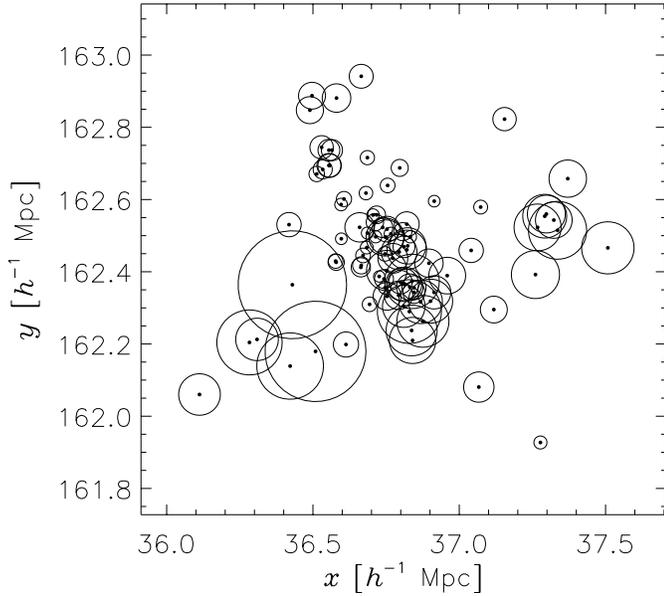


Fig. 1. Bubble plot for a particle group containing substructure.

to different cosmological models. In Sect. 4 we compare the substructure measure with other methods and with observational data. We conclude with a discussion of our results.

2. Method

The Dressler & Shectman (1998) statistics evaluate the velocity kinematics of galaxy groups identified in sky projected clusters. To be specific, one takes a number of neighbours N_{nn} from each galaxy in the projection, determines the mean velocity \bar{v}_{local} and velocity dispersion $\sigma_{v,local}$ of the subsample, and compares this with the mean velocity \bar{v} and velocity dispersion σ_v of the whole group,

$$\delta_i^2 = \frac{N_{nn}}{\sigma_v^2} [(\bar{v}_{local} - \bar{v})^2 + (\sigma_{v,local} - \sigma_v)^2]. \quad (1)$$

A measure of the amount of clumpiness in the cluster is the sum of the individual positive δ_i over all cluster galaxies N ,

$$\Delta = \sum_{i=1}^N \delta_i, \quad (2)$$

which is called the delta-deviation. The Δ -deviation is large for groups with kinematically distinct subgroups. Fig. 1 illustrates the statistics for a simulated typical dark matter cluster selected with a linking length of 0.2 times the mean interparticle spacing. Around each point a circle is plotted with radius proportional to $\exp(\delta_i)$. The cluster has a pronounced centre indicated by the central particles in the figure decorated with the small circles. To the right and above from the centre there are some subclumps which do not represent especially important subgroups with decoupled kinematics, but most probably they are small satellites slowly falling onto the group centre. To the left, there are some particles with large δ_i that contribute mostly to the cumulative Δ -deviation. The cluster in Fig. 1 represents

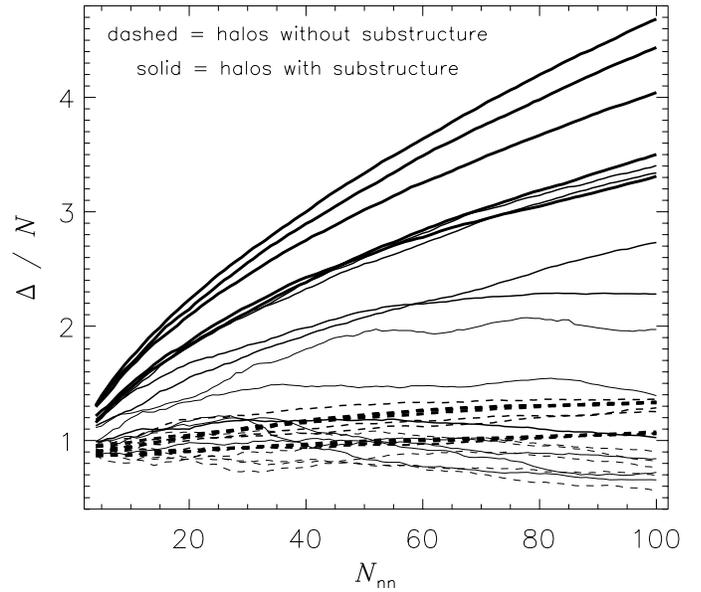


Fig. 2. Variation of Δ/N with the number of neighbours N_{nn} . The thickness of lines is proportional to the mass.

an example for marginally reliable substructure, we base our later statistical analysis on clusters with more pronounced substructure. An obvious advantage of the method is that no *a priori* selections or assumptions about the positions of subclumps have to be imposed.

In order to test and calibrate the statistics, we have compared five clusters of masses of about $10^{15} h^{-1} M_{\odot}$ (1000 particles), $5 \cdot 10^{14} h^{-1} M_{\odot}$ (500 particles), and $2 \cdot 10^{14} h^{-1} M_{\odot}$ (200 particles) in a standard CDM simulation. For each mass we study clusters both with and without obvious substructure as decided by eye from the 3-dimensional distribution.

The Δ -deviation statistics depend on the number of cluster galaxies with measured redshift N that can be included in the analysis, and on the number of neighbours in tested subgroups N_{nn} . If one assumes a random distribution of velocities one expects values $\Delta \propto N$ (Dressler & Shectman 1988), and for galaxy clusters with substructure, $\Delta \geq N$ (Pinkney et al. 1996). Therefore we normalize Δ with the total number of galaxies investigated, and in Fig. 2 we plotted Δ/N against the number of neighbours N_{nn} in subgroups. The selected groups with substructures (solid lines) show increasing values if the clusters have a large mass. Only the poor clusters lead to a poor discrimination of substructure. The curves suggest the use of a rather high value for the number of neighbours, while Pinkney et al. (1996) propose $N_{nn} = N^{1/2}$. The maximum for low mass objects (more than 200 particles) lies at about 25 neighbours which we therefore took as a reasonable value for the number of nearest neighbours in the following analysis. We took a fixed number in order to be independent of the sampling rate of velocities. A similar plot of Δ/N over the number of objects in clusters randomly selected demonstrates a very stable discrimination of clusters with substructure from those that are in equilibrium, and Δ/N remains almost constant for values $N \geq 100$. Ideally

Table 1. Cosmological scenarios and percentage of substructure. The box size L is given in $h^{-1}\text{Mpc}$, and the particle mass m_p in units of $10^{11}h^{-1}M_\odot$.

	Ω_0	h	σ_8	L	m_p	$P(\Delta/N \geq 1.4)$
SCDM1	1.0	0.5	1.18	200	11	33%
SCDM2	1.0	0.5	0.53	200	11	43%
ΛCDM	0.3	0.7	1.00	280	9	27%
OCDM	0.5	0.7	0.96	280	15	30%

this would be the number of required redshifts to be measured for analyzing clusters for substructure.

3. Application to cosmological simulations

We apply the substructure statistics to a set of cosmological models that are currently under investigation. Three of the models are COBE normalized according to the prescription of Bunn & White (1997) where we assume pure adiabatic perturbations and a baryon content of $\Omega_b h^2 = 1.3 \cdot 10^{-3}$. The standard CDM model (SCDM1) is taken as a reference model despite its enhanced power at cluster scales. In particular it produces a too high cluster abundance. As more realistic alternatives we take an OCDM model with $\Omega_0 = 0.5$ and a Λ CDM model with $\Omega_0 = 0.3$ and a cosmological constant to provide spatial flatness (i.e. $\Omega_{\Lambda,0} = 0.7$). Both models are promising since they lead nearly to the observed cluster abundance (Eke, Cole & Frenk 1996), and they represent models which reproduce the observed superclustering of galaxies (Doroshkevich et al. 1999). In addition to these three models we have used the output of the SCDM1 model at a redshift $z = 1.0$ to ensure the correct cluster abundance (SCDM2). The model parameters are summarized in Table 1. In particular it is shown that the simulations are running in boxes that are identical in physical size L , thereby making small differences in the mass resolution m_p despite the different density parameters. All models were run using the adaptive P³M code of Couchman (1991) with 128^3 particles. An analysis of the virialization of clusters in these simulations can be found in a previous paper (Knebe & Müller 1999).

As explained in the introduction, we expect differences in the abundance of substructure for different cosmologies, and therefore in the probability distribution of the delta-deviation. In a low-density universe (open or flat), structure formation ceases at earlier times compared to the SCDM models. This means that clusters in low-density universes should show less substructure since they formed earlier and therefore had more time to virialize. We always obtained strongly varying delta-deviations, therefore, we only analyze its abundance distribution. In Fig. 3 we show the cumulative probability distributions of the delta-deviations Δ/N of friends-of-friends clusters selected with dimensionless linking length (in terms of the mean interparticle separation) $ll = 0.2, 0.17,$ and 0.16 for SCDM, OCDM, and Λ CDM, respectively. We show these distributions for a fixed number density of simulated clusters $n_{cl} = 10^{-5}h^3\text{Mpc}^{-3}$.

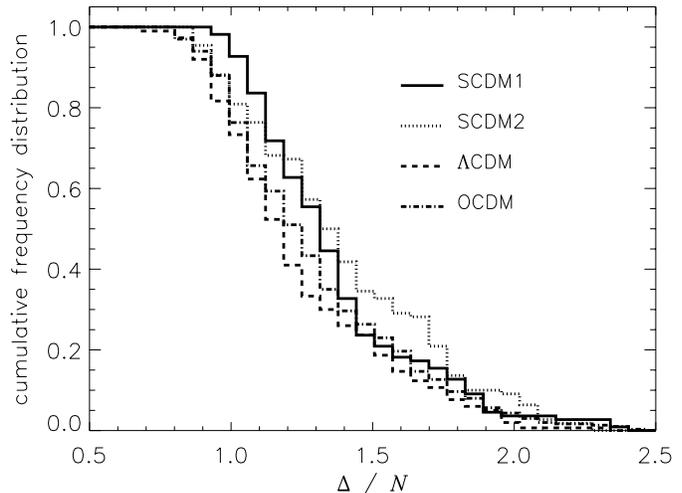


Fig. 3. Cumulative probability distribution of Δ for different cosmological models.

The distributions show differences which underline the cited expectation. In particular, both SCDM simulations have higher probabilities of substructure than the low density models. Especially the SCDM2 model shows a large number of clusters with $\Delta/N \geq 1.4$. On the other hand, the low density models Λ CDM and OCDM have a similar distribution of the delta-deviation, with Λ CDM lying below the OCDM model in agreement with the expectation due to the lower density parameter. To quantify the results, the probability of substructure is given in Table 1 as the percentage of clusters with $\Delta/N > 1.4$. We find that about 30% of the investigated objects show a significant level of substructure. This coincides with the observed probability of substructure in recent studies using the delta-deviation (Dressler & Shectman 1988, Zabludoff & Mulchaey 1998, Solanes, Salvado-Solé & González-Casado 1999). To check the significance, a Kolmogorov-Smirnov test was employed that is summarized in Table 2. The first column gives the maximum distance of the compared distributions, whereas the second column is the probability (in percent) that the models can be distinguished by the abundance of substructure as quantified by the delta-deviation (100 % means ‘different’ and 0 % means ‘identical’). The values in Table 2 show that the probability distributions for the SCDM models differ from that for the Λ CDM and OCDM models. On the other hand, the low density models with and without a cosmological constant are very similar, i.e., they cannot be discriminated. For all four models, the abundance of substructure depends more on the imposed limit in the delta-deviation than on the difference between models. Some degree of substructure is typical for all simulated clusters.

As we have recently shown in simulations (Knebe & Müller 1999), unvirialized particle groups can be identified with recent or ongoing mergers which could be a possible explanation for substructure. For this reason we have separated ‘virialized’ and ‘unvirialized’ clusters. The corresponding differential distribution of the delta-deviation in the SCDM1 model is shown in Fig. 4. Unvirialized clusters clearly have more substructure

Table 2. Kolmogorov-Smirnov (KS) test for the cumulative distributions. First column for each redshift gives maximum distance D , and second column the significance level.

Simulations	difference	reliability
SCDM1 vs. SCDM2	0.14	89 %
SCDM1 vs. ΛCDM	0.24	96 %
SCDM1 vs. OCDM	0.18	95 %
SCDM2 vs. ΛCDM	0.27	99 %
SCDM2 vs. OCDM	0.17	97 %
ΛCDM vs. OCDM	0.11	48 %

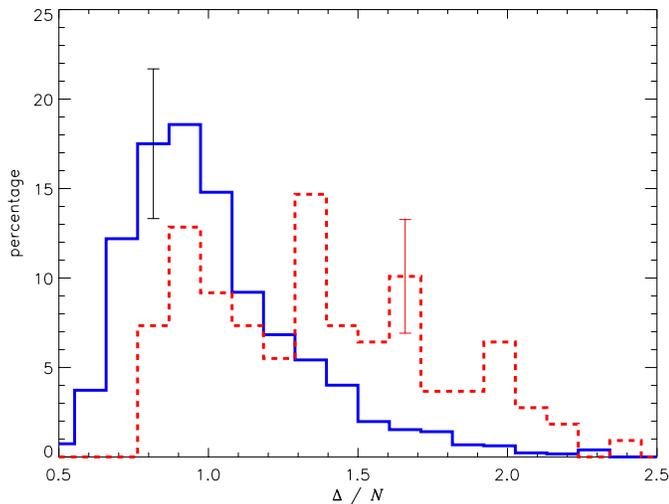


Fig. 4. Probability distribution of Δ for the SCDM1 model distinguishing virialized (solid) and unvirialized (dashed) particle groups (with Poisson error bars).

than relaxed systems. Substructure is therefore a clear sign of incomplete relaxation. Note that the distributions of virialized and unvirialized groups are separately normalized, but the number of unvirialized groups is always much smaller. The same behaviour is also found for the other models.

4. Comparison with other indicators of substructure

For testing the delta-deviation against other substructure indicators, we have also employed the friends-of-friends algorithm with a smaller linking length. Similar to the hierarchical tree (Serna & Gerbal 1996, Klypin et al. 1999), we determine the mass fraction of the two most massive subgroups to produce multiplicity statistics. To this aim, we build FOF clusters with half of the linking length of the original analysis. Then the total mass M of each particle group of the first analysis is decomposed into

$$M = m_1 + m_2 + m_{\text{rem}} \quad (3)$$

where m_1 and m_2 are the masses of the two most massive subclumps. The remaining mass m_{rem} consists of all the constituents lying in low density parts of the cluster, probably con-

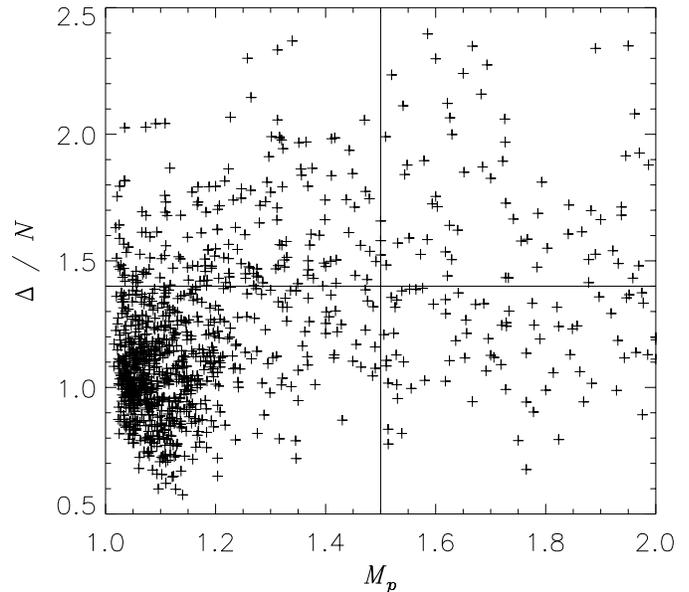


Fig. 5. Correlation for Δ/N vs. multiplicity M_p in the SCDM1 model. The lines $\Delta/N = 1.4$ and $M_p = 1.5$ show the adopted criteria for substructure.

taining mainly satellites recently accreted. We define the multiplicity of each cluster to be

$$M_p = \frac{m_1 + m_2}{m_1}. \quad (4)$$

For $M_p \approx 1$ we do not resolve substructure because the second most massive subgroup does not contribute to the total mass of the object. But for $M_p \approx 2$ both subclumps are of comparable mass and therefore we have a cluster with a possible double structure. A probable origin is a big merger of almost equal-mass progenitors.

Fig. 5 shows the correlation of Δ/N and M_p for the SCDM1 simulation. The other models look similar. The lines separate regions where both statistics indicate substructure. One notes a weak correlation with a wide scatter and a large number of outliers. The points in the upper left part show high values of the delta-deviation for which the multiplicity test shows that the subgroups are not very massive. So, they are probably small satellites falling onto the main cluster as seen, e.g., in Fig. 1. The lower right part shows clusters with subclumps of nearly equal masses, but no velocity deviations. A possible explanation are projection effects.

Another interesting point is the relationship of the remaining mass m_{rem} to the mass of the two most massive subclumps m_1 and m_2 . If m_{rem} is not negligible, we expect a loosely bound cluster with a large portion of accreted material which may be formed recently. In Fig. 6 we show the correlation between m_{rest}/m_1 and the multiplicity M_p for the SCDM1 simulation. The straight line gives the equality between $m_1 + m_2$ and m_{rem} . Above it we find loosely bound clusters which are obviously exceptions (about 13%). Below this line, the influence of recently accreted material is small. The loosely bound clusters lie mainly in the range where the multiplicity distribution

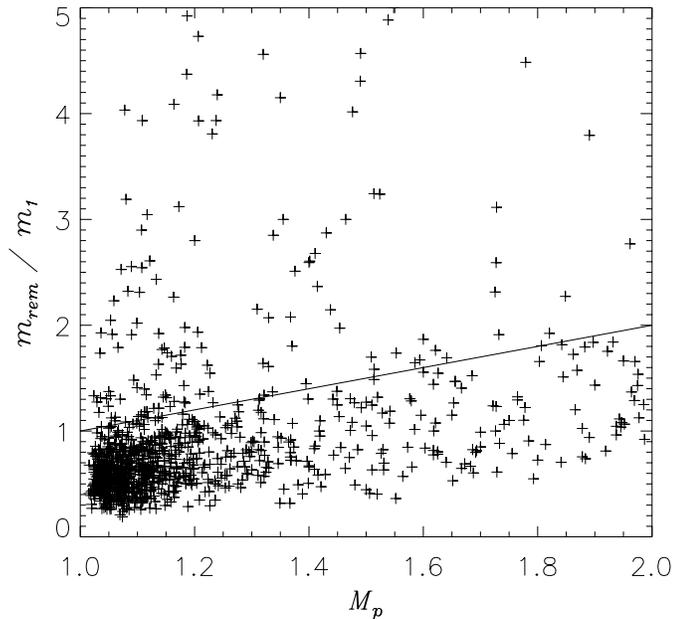


Fig. 6. Influence of the rest mass m_{rest} on the multiplicity M_p in the SCDM1 model. The straight line gives the equality between the masses of two biggest clumps and the rest.

indicates no significant substructure. This means that a steady accretion of material onto galaxy clusters tends to destroy substructure. Also we checked that a large accretion leads mostly to low values of Δ/N .

In Fig. 7 we show the dependence of identified substructure on the cluster mass M in the SCDM1 model. We find particle groups with and without substructure over the whole mass range, but the scatter for light clusters is much larger. When restricting to higher mass objects, the percentage of groups with substructure increases, and it leads to a value of about 30%, as in observed cluster samples (note that above we took a fixed cluster density which are taken from the same high mass range).

5. Conclusions

We quantified substructure in simulated clusters of galaxies using the delta-deviation statistics proposed by Dressler & Shectman (1988). Towards this aim, we studied the scaling of the statistics with the multiplicity of the total cluster and with the multiplicity of subgroups. Even with an optimal choice of both parameters which requires a large number of observed redshifts, we get a broad distribution for the statistics. The amount of subclustering depends on the chosen criterion, and we can reasonably reproduce the amount of substructure found recently in galaxy clusters redshift surveys. The chosen criterion ($\Delta/N > 1.4$) is similar to that used in observations, but the scatter is high, and reasonable changes may lead to substructure in the range of (20 – 50)%. The difference between different cosmologies is significant and a measure of the mean matter density of the universe. Among the models studied, Λ CDM shows the smallest percentage, and SCDM2 the highest percentage of subclustering. We recommend the cumulative distribution

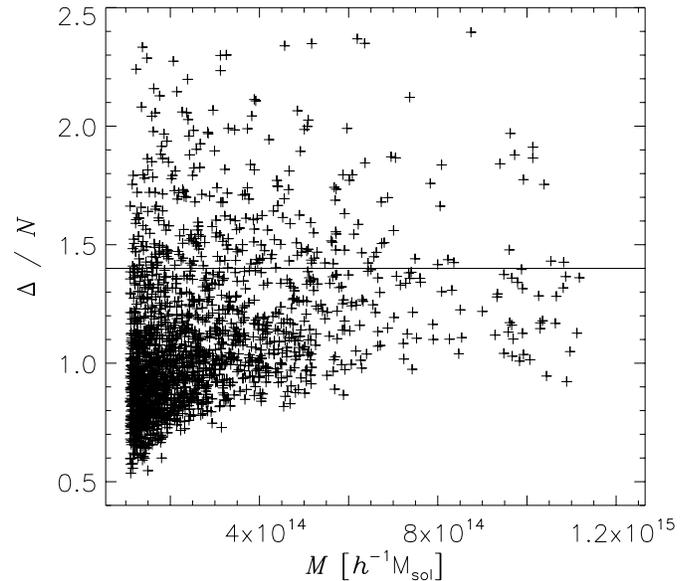


Fig. 7. Substructure dependence on mass of the galaxy cluster in the SCDM1 model, where the line $\Delta/N = 1.4$ gives the adopted substructure criterion.

of the delta-deviation as a natural quantifier of substructure. The differences in its distribution requires large catalogues of galaxy clusters, each with about 50 redshift measurements. On the other hand, it is shown that subclustering is a typical property of cluster formation in hierarchical theories of structure formation. In recent hydrodynamical simulations, a high percentage of substructure (in 4 of 10 clusters) were found in a low density Λ CDM model (Eke, Navarro & Frenk 1998). Therefore, quantifying substructure with a distribution function of a substructure indicator, as done here with dark matter simulations, seems to be a prerequisite for discriminating cosmological models. We suspect this is a general characteristic for different substructure indicators.

We compared the velocity kinematics as an indicator for subclusters with the subgroups found as friends-of-friends groups with half of the linking length. There is a weak correlation, i.e., both statistics define similar structures, but there is large scatter. On the one hand, projection effects influence the delta-deviation, and on the other hand, high values of the delta-deviation can be produced by small groups with separate velocity kinematics. These effects are often stronger than the differences between different cosmological scenarios. This may be the reason for the large differences in the literature on the amount of substructure in galaxy clusters.

It was shown that the delta-deviation is most sensitive to recent big mergers, and that big mergers are more typical for the high mass clusters. Furthermore, they occur more often in high-density than in low-density models.

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