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The asymmetries in radio source structures

II. Generalized kinematical model

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Abstract. This paper introduces the general formulation of a kinematical model which allows us to detect and calculate the asymmetries in the brightness distribution of double radio sources. It is shown that the asymmetry differs from that caused by pure Doppler effects, and a method of approximation of the intrinsic asymmetries is given. Two kinds of temporal evolution (i.e. power law and exponential law) are considered, providing two different types of asymmetry. Apart from the usual interpretation of the mathematical rules of asymmetries, a nonstandard interpretation of the parameters of the kinematical model is suggested. Finally, this paper argues that a considerable part of the observed asymmetry can be caused by surrounding matter and that the parameters of the model can be used to approximately determine the inhomogeneity of the surroundings deduced from a single map.

Key words: radio continuum: galaxies – galaxies: active – galaxies: jets – galaxies: quasars: general

1. Introduction

We can observe only the luminous part of the matter content in a structure. Hence, in the description of asymmetry, we look for differences between positions and brightness of corresponding elements on both sides of the core (cf. Best et al. 1995, Bridle et al. 1994). Many attempts have been made to determine the nature of the asymmetries observed in double extragalactic radio sources (e.g. Tribble 1992, Gopal-Krishna & Wiita 1996, Appl et al. 1996)

The results of hydrodynamic numerical modeling of a radio structure give us much information about the physical mechanisms of its formation and confinement (Hardee et al. 1992, 1998, Komissarov & Falle 1997). The physical conditions inside the jet such as density, pressure, strength and structure of the magnetic field, velocity distribution etc. can be investigated at a level of detail not accessible from observations. At present, a comparison of the results of hydrodynamic simulations with observational data is frequently only qualitative and we cannot obtain a sufficient number of fits for a statistical investigation. For this reason, complete samples of radio structures are investigated, using phenomenological models which are perhaps oversimplified but which allow us to perform various statistical investigations.

Observations provide us with a two-dimensional projection of the three dimensional double extragalactic radio source (hereafter called DERS). The asymmetry can be caused by various physical mechanisms (Bridle 1991, Roland et al. 1992, Woltjer 1996). At present, strong anisotropy in the core of the radio source is well established (Antonucci 1993, Zensus 1997). That means that orientation effects have to be important. This idea leads to the simplest unification scheme (Barthel 1989) but seems to be insufficient in order to explain the variety of all radio structures (for a review see Urry & Padovani 1995). The asymmetries which we observe in radio structures on kpc scales can be due to Doppler beaming and a time delay effect (Rees 1967, Ryle & Longair 1967). The observational support for the mildly relativistic motions on kpc scales was given by Biretta et al. (1995; see also Muxlow & Wilkinson 1991, Dennett-Thorpe et al. 1997, Hardcastle et al. 1999), and hot spot velocity $\sim 0.2c$ was detected in young DERS (Owsianik & Conway 1998, Owsianik et al. 1998). The asymmetries can also be interpreted as a result of different distances to different parts of the structure (Garington & Conway 1991, Tribble 1992), causing different parts of the structure to be viewed at different epochs of their intrinsic evolution and seen through different screens. The time delay effects and Doppler beaming depend on the orientation of the structure with respect to the observer's line of sight and dominate the observed asymmetry in radio structures (e.g. Readhead 1990). However, McCarthy et al. (1991) and other researchers (Best et al. 1995, Wardle & Aaron 1997) conclude that radio structures possess significant intrinsic asymmetry.

If we assume that, in the rest frame of the central engine (AGN), energy is released symmetrically on both sides, then two identical beams are created (cf. Bridle et al. 1994). The asymmetrical appearance of the structure is determined by the source orientation with respect to the observer's position, as well as by true (i.e. intrinsic) differences between them. Those may be: different velocities, different starting times, or different evolution of the jets (e.g. induced by their different environments – that is the forces acting on a jet). In the framework of a kinematic model presented in this paper we are able to describe the intrinsic asymmetry in DERS corrected for orientation effects.

The main aim of this paper is the presentation of the mathematical model of asymmetry observed in DERS. For a detailed description of the asymmetries we need a formula for the brightness dependence on time for a plasma volume ejected from the central engine. This important relation and details of the model are discussed in Sect. 2. In Sect. 3, the simplest interpretation of the model is given. The results of the model fitting for 65 real structures and a discussion are presented in Sect. 4. Sect. 5 presents the possible interpretation of the model parameters, and Sect. 6 summarizes the results.

2. The model

For the purpose of this work we adopt the Lagrangian method of a description of fluid motions (Müller 1997, Lucy 1977, Potter 1973). In this formulation the fluid is described as a set of fluid particles defined as consisting of the fluid contained within an infinitesimal volume, i.e. their linear dimensions are negligible (Milne-Thomson 1950). Thus we can treat a fluid particle as a geometrical point (hereafter plasma elements) when we discuss its movement.

The basic assumption made for the model is that the plasma outflow from the central source (Begelman et al. 1984) is described by a sequence of infinitesimally small plasma elements (the volumes of radio–emitting matter). It is also assumed that these elements come out of the central source in such a way that they may be unambiguously labelled (starting from the element no. 1) Since the model is intended to describe a double structure, exactly the same geometry is used in the case of the so-called counter-jet. The first of these outgoing elements is also labeled as no. 1. Consequently, we call elements with the same number on each side of the central engine 'twins'.

Laing (1993) suggested that the initial speeds of all kiloparsec scale jets are very similar, and all differences in morphology and luminosity depend on the jet power and its environment. We shall further assume that the energies released in both jets are equal, i.e. the numbers of plasma elements are equal, and their velocities relative to the central source are also exactly the same.

By postulating the existence of twin plasma elements on both sides of a central engine, and having their equations of motion and temporal evolution of brightness, we can find and compare twin fields on both sides of DERS with a precise mathematical method. The essence of the method used for calculations of the model parameters consists in interchanging the plasma elements of a twin pair between two opposite sides of an extended region of DERS (Ryś 1994 – Paper I). Performing this interchange we make a suitable modification of the positions and the flux in agreement with the adopted model of asymmetry. The model parameters are adjusted in such a way as to obtain the best possible reproduction of the original picture of brightness distribution (see Paper I for more details). Therefore we need a mathematical formula which would allow us to perform such an analysis.

2.1. Description of the plasma movement

A wide class of radio-source models (Kaiser & Alexander 1997, Baryshev & Teerikorpi 1995, Best et.al. 1995, Gopal-Krishna & Wiita 1987) have common physical assumptions which are seldom expressed, i.e.:

- There exists an analytical (time dependent) function describing the change of brightness of plasma elements.
- There exists an analytical (time dependent) function describing the change of position of plasma elements. This function can be approximated via Taylor series.

Let the distance of the plasma element from the central engine be described in the observer's rest frame by a time dependent function $r(t)$. If this function is sufficiently regular it can be expanded into the Taylor series:

$$r(t) = r(t_0) + \frac{dr}{dt} \cdot (t - t_0) + \frac{1}{2}\frac{d^2r}{dt^2} \cdot (t - t_0)^2 + \ldots \tag{1}$$

and using similar approximation for the distance of their twin from the central source in the counter-jet:

$$\bar{r}(t) = \bar{r}(t_0) + \frac{d\bar{r}}{dt} \cdot (t - t_0) + \frac{1}{2}\frac{d^2\bar{r}}{dt^2} \cdot (t - t_0)^2 + \ldots \tag{2}$$

In the following calculations the bar above a symbol denotes that the symbol concerns the counter jet. The model intends to be kinematic, therefore we omit higher than first order terms in the expansion. Denoting $(dr/dt) = V_{\rm obs}$ and $(d\bar{r}/dt) = \bar{V}_{\rm obs}$ we obtain,

$$r(t) - r(t_0) = V_{\rm obs} \cdot (t - t_0) \tag{3}$$

$$\bar{r}(t) - \bar{r}(t_0) = \bar{V}_{\rm obs} \cdot (t - t_0) \tag{4}$$

The limitation of the linear form chosen here is justified by its mathematical simplicity, which allows a one–to–one correspondence between plasma elements and their twins. Because in a real radio structure jets are decelerated, we have the possibility of another physical interpretation of the model parameters (see Sect. 5). In the case of a simple power law equation of motion (e.g. Gopal-Krishna & Wiita 1991), we should avoid their approximation by the above expansion, because such functions have no well defined values of the derivatives for $t = 0$ (see bellow). In that case we can simply divide the equations of motion for the twins, obtaining a model which mimics Doppler effects, as in the paper by Wardle & Aaron (1997).

By combining Eqs. 3 and 4 we eliminate the term $(t - t_0)$. Hence, we obtain the dependence between the positions of the twins:

$$\bar{r}(t) - \bar{r}(t_0) = \frac{\bar{V}_{\rm obs}}{V_{\rm obs}} \cdot (r(t) - r(t_0)) \tag{5}$$

This rule takes the simplest form if we choose $t_0 = 0$ demanding that $r(t_0) = 0$. We obtain:

$$\bar{r}_{\rm obs} = \frac{\bar{V}_{\rm obs}}{V_{\rm obs}} \cdot r_{\rm obs} + \bar{r}_{\rm obs}(0) \tag{6}$$

where index 'obs' denotes observer's frame of reference (with well defined time $t = t_{\rm obs}$). In the observer's rest frame the velocities of the plasma elements obey the equations (Rybicki & Lightman 1979):

$$V_{\rm obs} = \frac{V}{1 - \beta_x} = V \cdot D \qquad \bar{V}_{\rm obs} = \frac{V}{1 + \beta_x} = V \cdot \bar{D} \tag{7}$$

$V = \sqrt{\beta^2 - \beta_x^2}$, where β is the bulk velocity of plasma element in the reference frame of the central source, and β_x is the velocity component directed towards the observer (in units of the speed of light). We define the Doppler factor for each of the elements of the pair as $D = (1-\beta_x)^{-1}$ and $\bar{D} = (1+\beta_x)^{-1}$ respectively, and we find that $(\bar{V}_{\rm obs}/V_{\rm obs}) = (\bar{D}/D)$.

The internal asymmetry (i.e. $\bar{r}(0) \neq 0$) may be caused by a possible delay in the emerging of the "twin" element. Denoting the age of the plasma element in the jet by $t_{\rm obs}$ we can further assume that the age of its twin plasma element in the counter jet is $\bar{t}_{\rm obs} = t_{\rm obs} + \Delta t$ (in the observer's frame) and $\bar{r}_{\rm obs}(0) = \bar{V}_{\rm obs} \cdot \Delta t$. Therefore we can easily get formulae which allow to find $\bar{r}_{\rm obs}$ as a function of $r_{\rm obs}$ in the form:

$$\bar{r}_{\rm obs} = \frac{\bar{D}}{D}\left(r_{\rm obs} + \Delta t \cdot V_{\rm obs}\right) \tag{8}$$

The model is purely kinematic, i.e. V and D are constant, so we have: $V_{\rm obs} = r_{\rm obs}/t_{\rm obs} = r_{\rm max}/t_{\rm max}$, where $r_{\rm max}$ is the observed extension of the jet (i.e. position of the oldest plasma element) and $t_{\rm max}$ is the observed age of the structure (the age of oldest plasma element). We insert ($r_{\rm max}/t_{\rm max}$) instead of $V_{\rm obs}$ obtaining thus:

$$\bar{r}_{\rm obs} = \frac{\bar{D}}{D}\left(r_{\rm obs} + \frac{\Delta t \cdot r_{\rm max}}{t_{\rm max}}\right) \tag{9}$$

Because in practice we may not know any physical extension of the investigated structure, we have to use relative values in respect to total extension of the structure. We obtain such rules dividing both sides of Eq. (9) by $r_{\rm max}$ (which is the extension of the jet). Next we denote $\bar{D}/D = A$ as the ratio of the Doppler factors, $\Delta t/t_{\rm max} = B$, $r_{\rm obs}/r_{\rm max} = R$ and $\bar{r}_{\rm obs}/r_{\rm max} = \bar{R}$ obtaining:

$$\bar{R} = A \cdot (R + B), \tag{10}$$

The parameter B may be simply interpreted as representing the relative (to the age of the structure) time delay between arising of the twins. In order to compare the structures belonging to the opposite sides of the central source we can simply interchange the positions of elements in each pair according to Eq. (10).

2.2. Brightness evolution of the plasma element

When interchanging the positions of twin elements, we must take into account the changes in brightness due to their different ages – according to the adopted model of evolution, and corrected with respect to kinematical effects. We may write the properties of brightness asymmetry in a mathematical form adopting two simplest models of the evolution of the radio-flux emitted by a plasma volume (element).

In the literature (e.g. Pacholczyk 1970, Jackson 1975) one can find two different descriptions of the energy evolution in time, and hence the flux of radio waves emitted by the plasma element. These are the power law and exponential types of temporal dependencies. The details of physical mechanisms leading to those description of the flux evolution are discussed by Kardashev (1962).

We assume that all plasma elements evolve in the same way in their own frames of reference. However, kinematical effects change their observed brightness as seen in the observer's frame. In the following calculations f denotes the flux emitted by a plasma element on a jet side of the structure while $\bar{f}$ is the same quantity of its twin on the opposite side of the core.

As the first model we assume that $f(t) = f(0) \cdot t^{-\mu}$, where μ – is the index of the rate of decline. Because $t = D \cdot t_{\rm obs}$ and $\bar{t} = \bar{D} \cdot \bar{t}_{\rm obs}$ in the observer's rest frame we obtain the formula for flux of radio waves emitted by plasma element:

$$f_{\rm obs} = f(0) \cdot (D/\Gamma)^{\eta} \left(D \cdot t_{\rm obs}\right)^{-\mu}, \tag{11}$$

and similar formula for their twin:

$$\bar{f}_{\rm obs} = \bar{f}(0) \cdot (\bar{D}/\Gamma)^{\eta} \left(\bar{D} \cdot \bar{t}_{\rm obs}\right)^{-\mu} \tag{12}$$

where $\bar{t}_{\rm obs} = t_{\rm obs} + \Delta t, \Gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor, and $\eta = 3 + \alpha$. Hence, using Eqs. 3 and 4,

$$\begin{aligned}\bar{f}_{\rm obs} &= f_{\rm obs} \cdot A^{\eta}\left(1 + \frac{\Delta t}{t_{\rm obs}}\right)^{-\mu} \cdot A^{-\mu} = \\ &= f_{\rm obs} \cdot A^{\eta}\left(1 + \frac{B}{R}\right)^{-\mu} \cdot A^{-\mu}\end{aligned} \tag{13}$$

Another model widely considered in the literature (Nilson et al. 1993, Jackson 1973) concerns long temporal evolution of radio sources, in the form of exponential dependence on time $f(t) = f(0) \cdot \exp(-t/T)$, where T – is a characteristic time scale. The exponential evolution of brightness gives in the observer's frame:

$$f_{\rm obs} = f(0) \cdot (D/\Gamma)^{\eta} \cdot \exp(-t_{\rm obs} \cdot D/T) \tag{14}$$

and

$$\bar{f}_{\rm obs} = f(0) \cdot (\bar{D}/\Gamma)^{\eta} \cdot \exp(-\bar{t}_{\rm obs} \cdot \bar{D}/T) \tag{15}$$

Hence

$$\bar{f}_{\rm obs} = f_{\rm obs} \cdot A^{\eta} \exp\left(\frac{D \cdot t_{\rm obs} - \bar{D} \cdot \bar{t}_{\rm obs}}{T}\right), \tag{16}$$

where – according to Eq. 3 – we have adopted

$$\bar{f}_{\rm obs} = f_{\rm obs} \cdot A^{\eta} \exp\left(\frac{r_{\rm obs} - \bar{r}_{\rm obs}}{V \cdot T}\right), \tag{17}$$

Because V can be calculated as $V = r_{\rm max}/(D \cdot t_{\rm max})$, and denoting $\tau = T/(D \cdot t_{\rm max})$ we have:

$$\bar{f}_{\rm obs} = f_{\rm obs} \cdot A^{\eta} \exp\left(\frac{R - \bar{R}}{\tau}\right), \tag{18}$$

It is crucial to stress that these formulae for brightness (i.e. 13 and 18) are applicable also in the case when the plasma element starts to shine far away from the central source. It is simple to demonstrate this if we remember that $\bar{t}_{\rm obs} = t_{\rm obs} + \Delta t$, and we can add any interval of time to both sides of this equation – hence redefining t_0.

2.3. Final formulae

The formulas (10), (13) and (18) allow us to compare the structures on both sides of the central source by calculating parameters A, B and μ (or τ) simultaneously for all the twins observed in the structure. It is important to notice that we do not see any plasma elements separately on the map of the radio structure. In order to make the rules usable for the analysis of the map taken from observations, we can sum up the fluxes from all the plasma elements, having the same positions (in the observer's point of view) into the flux of map component (F) – such as is used in the description of brightness distribution of radio source maps (i.e. $F = \sum f_k$ and $\bar{F} = \sum \bar{f}_l$). This is possible due to linear dependence between the fluxes of twins ($f \sim \bar{f}$) in Eqs. 13 and 18. Therefore, we may investigate the asymmetries of DERS by comparing the structures seen on both sides of the core using the above equations by interchanging the positions and the fluxes of map component i.e., (R, F) instead of (R, f).

From the Eqs. (10), (13) and (18) one can readily obtain formulae which allow one both to interchange and to compare opposite parts of the structure on both sides of the central source. They also allow to take into account various corrections of purely kinematical origin (see also Sect. 5) in the form:

$$\begin{aligned}
\bar{R}^* &= A\,(R + B)\,, \\
\bar{F}^* &= F \cdot A^{\eta} \left(1 + \frac{B}{R}\right)^{-\mu} A^{-\mu} \\
R^* &= A^{-1}\left(\bar{R} - AB\right), \\
F^* &= \bar{F} \cdot A^{-\eta} \left(1 - \frac{AB}{\bar{R}}\right)^{-\mu} A^{\mu}
\end{aligned} \tag{19}$$

for the case of power evolution, and

$$\begin{aligned}
\bar{R}^* &= A\,(R + B) \\
\bar{F}^* &= F \cdot A^{\eta} \exp\left(\frac{1-A}{\tau} R - \frac{AB}{\tau}\right) \\
R^* &= A^{-1}\left(\bar{R} - AB\right) \\
F^* &= \bar{F} \cdot A^{-\eta} \exp\left(\frac{1-A^{-1}}{\tau} \bar{R} + \frac{B}{\tau}\right)
\end{aligned} \tag{20}$$

for the exponential one. An asterisk indicates positions and fluxes of map components in a new map which after convolution with instrumental profile could be compared with the original one. We propose to call the relationships (19) "the asymmetry generator" for a power law evolution and the relationships (20) "the asymmetry generator" for the exponential type of evolution.

3. The sketch explaining model properties

In its simplest interpretation the model described above can be called the flip–flop model with Doppler effects, or relativistic flip-flop model. By the pure Doppler model we mean that in which the asymmetries in brightness and distances are assumed to be produced by Doppler effects alone.

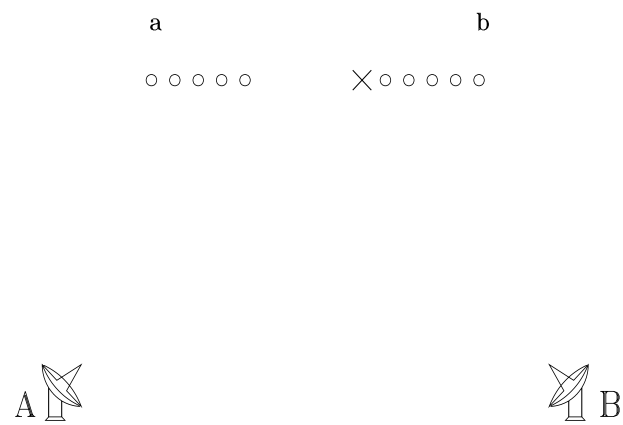

Fig. 1. Two observers are looking at the same structure but they are not seeing the same picture. The cross (×) denotes the position of the central source. The oldest plasma element in the structure is marked 'a' and 'b' is its twin (see text).

In the pure Doppler models (e.g. Best et al. 1995, Baryshev & Teerikorpi 1995) the structure of DERS is internally symmetrical and asymmetry is produced only by Doppler effects. The paper by Wills et al. (1978) introduced the flip-flop mechanism as a possible explanation of asymmetries in DERS. At present, the flip-flop hypothesis has been investigated in the paper of Icke et al. (1992), but the full cycle of such an event has never been observed in a real extragalactic radio source. So far all papers discuss orientation dependent effects or the flip–flop mechanism separately (e.g. Rudnick & Edgar 1984). We can combine these two mechanisms into a consistent scenario called the relativistic flip–flop mechanism i.e., we include the flip–flop effect into a pure orientation dependent model and investigate them jointly. One can check that in this case the rule for transformation of the structure possesses the same mathematical form as presented in Sect. 2.

The classical flip-flop mechanism switches the directions of matter/energy output of AGN. The ejections are possible only in one particular direction. When the energy outflow starts from the other side, the previous direction of outflow is stopped. The central engine operates as an effective "gun" shooting in one selected direction only.

Let us imagine two observers who look at the same structure from two different directions (see Fig. 1).

The 'observer A' sees 'element a' as brightened due to the Doppler effect at later stages of the evolution. It therefore appears to him older than its twin. This element appears also further apart from central engine than 'element b'. On the other hand 'b' (ejected later than 'a') is weakened by the Doppler effect and seen at earlier stages of its evolution. 'A' sees the structure with both mechanisms jointly increasing the distance of 'element a'.

The 'observer B' sees 'element a' as weakened by the Doppler effect at later stages of evolution only if time delay

Table 1. Properties of the models

	Model of flux decline	
	$f = f_0 \cdot t^{-\mu}$	$f = f_0 \cdot \exp(-t/T)$
Main asymm.	$A^{\eta} A^{-\mu}$	$A^{\eta} \exp(\frac{-AB}{\tau})$
Shape asymm.	$(1 - \frac{B}{R})^{-\mu}$	$\exp(\frac{1-A}{\tau} R)$
Structure type	more asymmetric in central region	more asymmetric in outer region

effect is smaller than the flip–flop switching period. The brightness of ‘b’ seen by ‘B’ is enlarged by the Doppler effect and ‘element b’ may extend further in distance from the central core than in the ‘element a’ case. ‘B’ observes that the change of distance ‘a’ and ‘b’ from the center caused by the Doppler effect is opposite to those caused by the flip-flop mechanism.

The evolution of emissivity of a plasma element determines how much flip-flop time delay balances the Doppler effect in brightness asymmetry of a structure. The differences between exponential and power law models are essential since they have different asymmetry generators.

In the power law case the formula for fluxes is inversely proportional to the distance from the core, multiplied by the factor B. Thus the non-zero value of B generates a distance dependent asymmetry. The ratio of the brightness of twin plasma elements is highest close to the core, decreasing with the distance from the core. We can say that the non-zero value of B generates asymmetry in the shape. In the exponential case, the term $(1-A)/\tau$ is multiplied by distance R and this term governs the shape asymmetry. Then we conclude that asymmetry grows with distance R. The power law model is more appropriate for sources with a stronger asymmetry near the center, whereas the exponential model gives a better fit for sources with more asymmetric external parts of the structure.

For a power law model the fluxes from each element on one side of a structure are multiplied by a term $A^{-\mu}$. From Eq. (20) we can also see that, for an exponential model, the flux emitted by each element on one side of the structure is multiplied by a term $\exp(-AB/\tau)$. In a particular example of a structure, the values of A, B and τ (or μ) are constant and those terms diminish (replace a part of) asymmetry described in a pure Doppler model by the term A^{η}. Those terms ($A^{-\mu}$ and $\exp(-AB/\tau)$) play the role of intrinsic asymmetry as discussed by Wardle & Aaron (1997). However, it should be noted that only in the case of the exponential model is a multiplicative factor connected with the intrinsic (in our model) asymmetry – B. Table 1 summarizes all these properties. In the following section we compare the results of modeling with both power law and exponential functions trying to determine which of them better describes the asymmetry observed in DERS.

4. Results for a sample of the structures

We applied the models discussed above to the description of the asymmetry in 65 radio structures selected from the survey by Machalski & Condon (1983a,b). The choice of this survey is determined by the fact that maps of the sources were archived with their models of radio brightness (the list of Clean Components). The method used for calculation of the model parameters does not allow a transformation of the one part of the structure into another for any map which is convolved with the instrumental profile. This is because when we perform the transformation (see rule (10)) with a convolved map, the result will appear as a map made with different instrumental profiles on both sides of the center.

The investigated sample contains 35 sources with optical identification (8 quasars, 25 galaxies, and 2 of unknown type). We use this identification to determine the position of the radio structure core. In order to search for differences in evolution, we divided the sample of sources into three formal classes of structures. From the analysis of the observed properties of 33 structures (with known or estimated redshift from optical identification) we deduced that there are 19 structures below critical power which divide the $FR1$ and $FR2$ class (Fanaroff & Riley 1974), i.e. with spectral power at 1.4 GHz lower than $10^{24.5}$ [W/Hz] (H=100 km s^{-1} Mpc^{-1}, q=0.5). Those 19 structures are marked on all figures by circles ($\circ$). All other sources with known redshift and spectral power above the $FR1/FR2$ division are marked on all figures by asterisks ($\star$). The sources without optical identification, or with no information on their redshift, are marked on the figures as ($+$). The complete list of the radio structures is given in Table 2.

For every DERS from the sample we adjusted the parameters of both asymmetry generators to obtain the best reproduction of the observed map. As a measure of the quality of the fit the parameter INC (incorrectness) defined by:

$$INC = \frac{\sum \mid F_i - F_i^* \mid}{\sum F_i}, \quad (21)$$

was used. F_i is a flux from i-th positions at the original map and F_i^* is a flux from the same position in a map of the structure reproduced by the model. The INC represents the percentage of the flux of original structure which was not correctly reproduced by the model. This quantity was minimized to obtain the best-fit values of A, B, τ, (or μ). All calculations were performed with a one dimensional map. In order to obtain comparable results for each structure in the sample, we take dimension of the gaussian instrumental profile ($HPBW$) 10 times smaller then angular extension of the investigated structure. For more details see Paper I.

4.1. Comparison of the two types of evolution

As the result of fitting we found that 35 sources have a better fit for the exponential rule of plasma evolution, while for 30 sources a better fit is obtained for a power-law function. This can be seen when we compare the value of incorrectness (with both models) obtained for best fitted parameters plotted in Fig. 2. Thus the first result is that a substantial fraction of sources fits better to the observational data when fast (i.e. exponential) evolution is assumed. This is in contrast to the existing literature, where

Table 2. The resultant parameters of the best fits.

$Name$	A_e	B_e	τ	A_p	B_p	μ
Q 0709+370	0.998	0.43	0.009	0.734	17.50	2.60
0710+346	0.931	3.50	0.024	0.787	0.03	8.00
Q 0749+379	0.635	7.56	0.126	0.598	10.15	7.14
0759+369	0.794	5.48	1.196	0.805	4.41	0.07
G 0803+488	0.901	20.53	0.434	0.800	15.89	2.80
0807+348	0.969	30.84	1.123	0.745	24.18	1.52
G 0809+328	0.729	6.00	0.911	0.750	4.82	0.37
G 0810+351	0.577	9.52	0.449	0.590	8.00	1.82
0810+370	0.642	4.44	0.071	0.569	1.98	8.23
G 0811+388	0.800	31.17	3.924	0.841	28.35	0.45
0822+345	0.902	3.85	0.994	0.921	4.88	0.20
G 0832+347	0.932	24.66	0.675	0.878	5.12	2.80
0834+369	0.998	0.60	0.112	0.928	1.00	2.21
0847+359	0.955	10.46	0.125	0.731	4.14	7.17
G 0852+493	0.952	1.55	0.465	0.930	0.46	1.17
G 0854+342	0.848	21.35	0.984	0.896	18.00	1.49
0904+488	0.658	3.07	0.176	0.516	12.58	3.60
G 0908+376	0.845	0.03	0.423	0.874	0.37	1.31
G 1015+491	0.955	8.19	0.827	0.453	1.10	2.80
1016+329	0.750	2.63	0.149	0.752	0.03	6.32
G 1020+486	0.905	20.57	0.361	0.611	3.35	4.89
G 1024+463	0.703	6.50	0.176	0.933	7.47	0.68
1042+481	0.775	12.18	0.174	0.607	0.82	6.20
1049+344	0.900	16.99	1.000	0.645	6.19	1.76
1049+488	0.713	7.71	0.226	0.696	0.04	4.40
G 1059+351	0.993	8.05	0.075	0.678	0.04	9.00
Q 1105+392	0.790	15.18	0.123	0.571	0.13	8.79
G 1125+325	0.578	25.12	0.228	0.379	0.07	5.45
G 1130+339	0.910	11.50	0.539	0.888	12.50	0.60
G 1140+491	0.472	6.78	0.122	0.517	7.44	6.65
G 1144+497	0.914	31.83	4.119	0.837	29.25	0.49
Q 1148+477	0.792	6.94	0.176	0.712	0.04	4.59
G 1158+345	0.673	0.93	0.096	0.573	11.20	11.15
1202+350	0.955	2.09	0.050	0.742	1.35	7.40
G 1204+353	0.884	19.77	0.822	0.816	17.23	1.44
1211+486	0.958	2.60	0.032	0.870	20.70	1.91
1234+371	0.741	2.02	0.175	0.713	30.48	0.84
Q 1237+353	0.916	2.73	0.725	0.860	0.03	2.80
G 1251+348	0.701	8.23	0.055	0.695	9.35	16.92
1257+383	0.919	11.42	0.426	0.784	18.49	2.00
G 1301+382	0.758	3.08	0.237	0.685	2.53	4.57
G 1323+370	0.808	8.04	0.276	0.711	0.04	4.20
G 1325+321	0.698	1.43	0.124	0.913	3.30	0.81
G 1339+472	0.724	4.31	0.275	0.667	1.20	4.09
1340+319	0.613	14.00	0.078	0.666	10.10	7.78
G 1340+353	0.796	1.07	0.095	0.786	0.29	7.32
1348+352	0.994	0.50	0.025	0.926	0.03	7.80
1401+387	0.980	1.02	0.075	0.937	0.52	8.09
G 1432+382	0.734	8.17	0.225	0.607	0.49	4.56
1436+340	0.904	18.00	0.306	0.836	23.98	0.67
1453+353	0.954	0.03	0.025	0.481	0.05	5.10
1502+339	0.727	17.04	0.882	0.643	23.16	1.07
1508+380	0.827	3.96	0.051	0.577	0.17	5.44
Q 1512+370	0.852	1.17	0.104	0.880	0.03	6.68
G 1539+350	0.905	3.00	0.021	0.779	5.58	2.54
G 1539+343	0.982	14.77	1.064	0.980	14.65	0.84
1542+35A	0.799	30.64	0.244	0.511	15.46	1.21
1542+35B	0.888	1.69	0.100	0.742	0.47	6.42
1547+386	0.598	4.47	0.075	0.337	0.07	6.01
1548+334	0.915	1.99	0.025	0.714	0.49	6.24
1559+345	0.928	1.99	0.025	0.831	0.48	6.41
G 1602+324	0.882	7.00	0.300	0.913	5.57	4.73
Q 1628+363	0.859	17.13	2.079	0.650	6.15	2.20
G 1633+374	0.975	10.25	1.010	0.979	12.07	0.21
1647+352	0.856	7.62	0.200	0.607	0.04	5.41

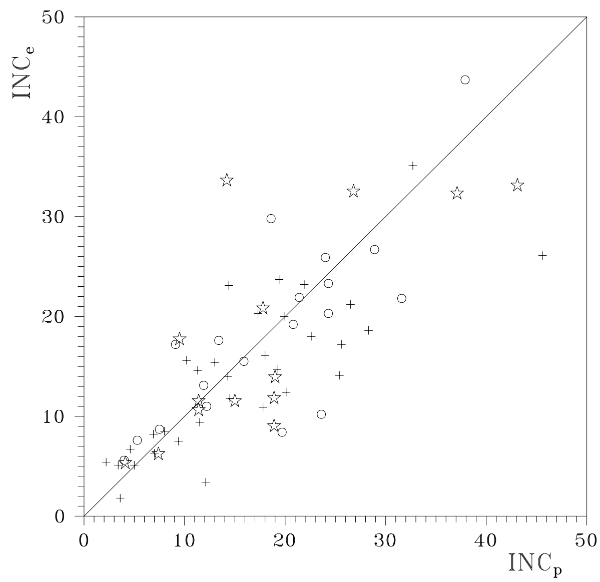

Fig. 2. Comparison of two models of brightness evolution. INC_e is an incorrectness parameter of the exponential model and INC_p is an incorrectness parameter of the power–law model. The structures with spectral power at 1.4GHz lower than $10^{24.5}$ [W/Hz] are marked by circles (○), above this limit by asterisks (⋆) and the sources with no information on their redshift are marked as (+). 35 sources have a better fit for the exponential model (i.e. lie below the dividing line).

power–law models are preferred (e.g. Wan & Daly 1998, Gopal-Krishna & Wiita 1991). However, we do not see any statistically important bias for stronger or weaker sources preferring one of the models.

4.2. Mean and median values of the parameters

For each source – i.e. with or without optical identification – the values of six parameters are determined. These are A_e, B_e, τ for the exponential model and A_p, B_p, μ for the power law model of evolution. Table 3 consists of mean and median values of parameters A_e, B_e, τ, A_p, B_p, μ.

The most frequently calculated parameter is the mean or median value of the velocity component directed towards the observer V_x. From the value of median A_e, A_p, we obtain median values of velocity component for the exponential model $V_x^e = 0.077 \pm 0.05$, and for the power law model: $V_x^p = 0.155 \pm 0.06$. From mean values of A_e, A_p, we have: $\langle V_x^e \rangle = 0.093 \pm 0.05$,

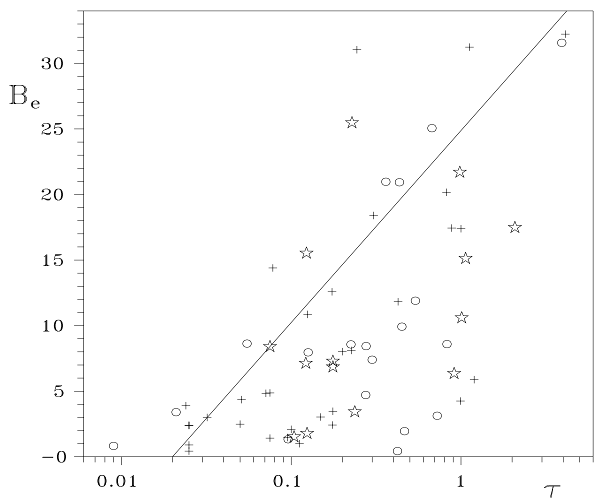

Fig. 3. The dependencies between the relative time delay parameter (B) and relative lifetime (τ) in the model with exponential evolution of plasma brightness. The solid line is a limitation for suggested dependence between parameters $B \sim \tau$.

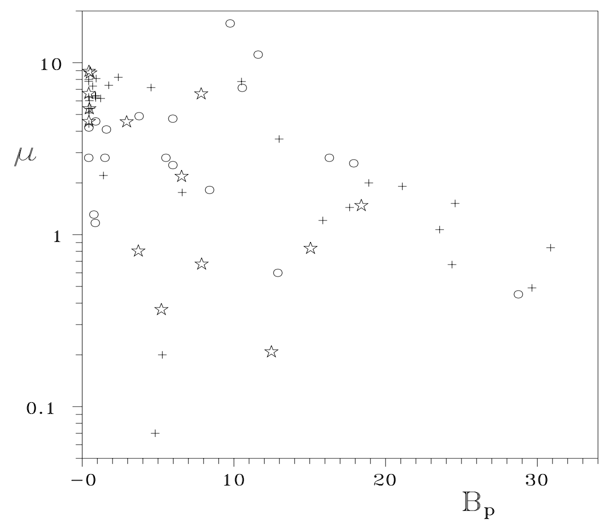

Fig. 4. The dependencies between the parameters of the model with power evolution. The B_p are 'flip-flop' parameters and μ – is the evolution rate. We can see a similarity between $B_p \sim \mu$ relation and $B_{\rm e} \sim \tau$ (in Fig. 3).

Table 3. Mean and median values of parameters

Parameter	Mean value	Median value
A_p	0.73 ± 0.02	0.74 ± 0.02
B_p	$9.2\% \pm 1.1\%$	$6.9\% \pm 1.1\%$
μ	-4.2 ± 0.4	-4.1 ± 0.4
$A_{\rm e}$	0.83 ± 0.02	0.86 ± 0.02
$B_{\rm e}$	$7.3\% \pm 1.2\%$	$4.2\% \pm 1.2\%$
τ	-0.54 ± 0.12	-0.21 ± 0.12

$\langle V_x^p \rangle = 0.148 \pm 0.06$. The values (mean and median) obtained with power law evolution are close to the results of authors arguing for $V_x \sim 0.15c$ (e.g. Bridle 1984, Best et.al. 1995), while the values obtained with exponential model are close to Scheuer's (1995) result arguing for $V_x < 0.1c$. Taking into account that the above discussed velocities are smaller than obtained in Paper I, we can state that inclusion of simple approximations to intrinsic asymmetries lead us to smaller velocities, but the most important influence on the values of velocity component V_x was induced by the adopted model of brightness evolution of plasma element.

4.3. The sequence of ages involved by models

If we adopt a single flip-flop model (Sect. 3) for all sources, i.e., the idea that all sources have the same physical values for the twin plasma element delay (Δt) and the characteristic time scale of plasma shining rate T, then we can expect a simple dependence between parameters B and τ.

In older structures we should observe more switching events and the $B_{\rm e}$ parameter should be smaller. We should keep in mind that parameter $B_{\rm e} = (\Delta t/t_{\rm max}) = (\Delta r/r_{\rm max})$ has its value expressed in units of a structure size or age (cf. Sect. 2.1). The sources with smaller values of relative lifetime τ contain plasma elements which are weaker (i.e. with smaller flux) than those from structures with larger $\tau \sim (T/t_{\rm max})$. The weakest elements were observed only in the oldest structures. Hence, we conclude that older structures have smaller values of $B_{\rm e}$ and τ than younger ones.

The parameters $B_{\rm e}$ and τ representing the 'flip–flop events' and relative lifetime are well correlated (correlation coefficient $\rho = 0.59 \pm^{.18}_{.27}$) as we can see in Fig. 3. Taking into account the value of the correlation coefficient I suggest a linear dependence $B_{\rm e} \sim \log(\tau)$. Such a dependence (or more precisely limitation) is shown by a dotted line in the Fig. 3, $B = -.17 + 8.54 \log(\tau)$ is an example of the evolutionary path for a single model.

In the considered sample there are no structures with the age estimated by other independent methods, hence we cannot mark any time scale on the axis. Preliminary calculations performed for few structures from Alexander & Leahy (1987) sample gave the confusing result (Ryś 1996) that spectral ageing is in disagreement with the sequence of ages predicted by our model. It might be explained using the results of Eilek (1996), Eilek & Arendt (1996) and Wiita & Gopal-Krishna (1990), which argued for strong influences of turbulence and diffusional processes on observed spectra of extended radio structures.

The correlations between the parameters of the power-law model can be interpreted in a similar way to those between parameters of the exponential model described above. In this model the parameter μ determines the speed of evolution and we expect that older structures in our model should possess larger value of μ. The correlation coefficient $\rho = 0.41 \pm^{.22}_{.32}$ for $B_p \sim \log(\mu)$ relation was obtained and one can suppose that in Fig. 4 there is no dependence but only a limitation.

4.4. The time delay parameters from both models

The generalized kinematical model gives us a better description of the asymmetry in brightness distribution of DERS than

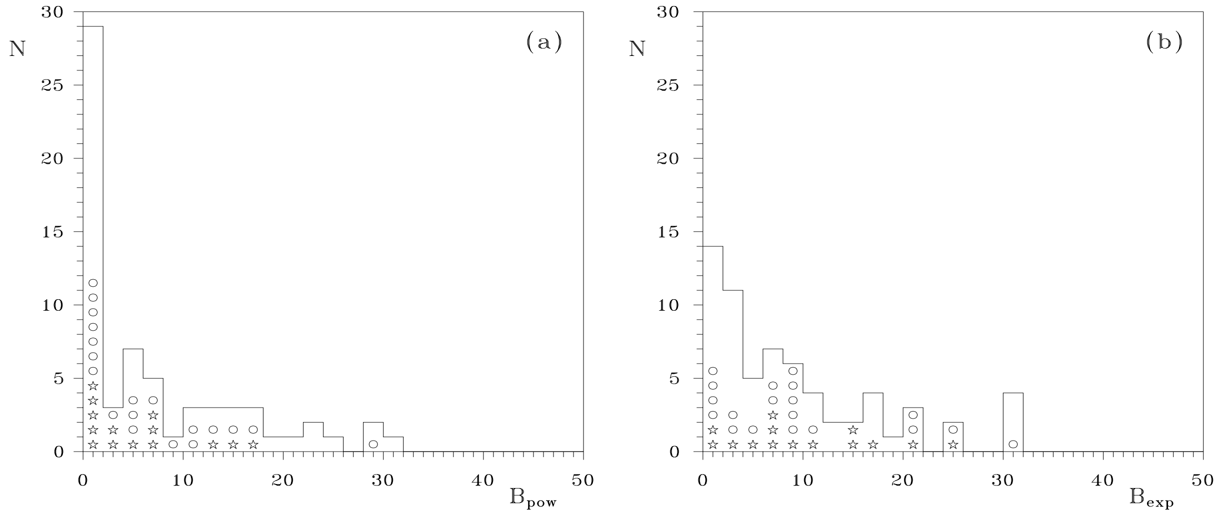

Fig. 5a and b. The counts N of the flip-flop parameter B_p (power model) for 65 sources of the sample (**a**) and the counts of the B_e obtained for the exponential model in (**b**). The values of both B parameters are expressed as a percent of the total extension of the structure. Fields not marked represent the sources for which redshift is unavailable.

simple models described in Paper I. In Fig. 5 we present the counts of sources in intervals of B. It was found that probably all structures of DERS need the non-zero value of additive type parameter B, but some of them have a very small value of B.

As mentioned earlier the values of B have been calculated as relative values with respect to the source size. When, during transformation of a radio structure, we change the value of B then the positions of map components in the transformed map move also (see Eq. 10). The obvious limit of importance for parameter B to be non-zero is the resolution in the map ($HPBW$ of Gaussian profile used in numerical calculations – equal to 10% of source size). For the exponential model more than 33% structures have a best-fit value of B_e greater than the used $HPBW$ of Gaussian profile (Fig. 5). The power law model gives us 30% sources with B_p greater than $HPBW$.

Therefore I conclude that we cannot neglect the flip-flop type parameters in the description of asymmetry of DERS for about 30% of sources. This conclusion is in agreement with the results of Arshakian & Longair (1999), who reported that 60% of FR2 radio galaxies and 75% of FR2 radio quasars are intrinsically symmetrical.

As we can see, the results of the present paper clearly demonstrate that the additive parameter (B, see Eq. 10) of the kinematic model is important if we want to describe the asymmetries in more detail. From the performed calculation it follows that the Doppler factor is not the only parameter responsible for brightness distribution asymmetry, however the parameter B, which has so far been interpreted as the time interval between switching of an active side of the central engine, could possess another interpretation, as discussed below.

5. Discussion of possible interpretation

The adopted asymmetry generator (Eqs. (20),(19)) may be interpreted in terms of physical ideas (models) for which we obtain the same mathematical formulas in the case of their linear approximation. The simple interpretation of a generalized kinematical model presented in Sect. 3 is, of course, not a unique interpretation.

Another natural interpretation is that the central engine is not a ‘perfect gun’, and we observe two similar streams emerging from central engine but one starts with a real and reasonable time delay. The most popular hypothesis is that of continuous and simultaneous outflow from the central source into two directions (Begelman et al. 1984). If we accept the result that more than 30% of sources have nonzero B parameters, we should introduce into the continuous supply model a significant delay between starting time of the jets (or twin plasma element) on both sides of the central source.

The separate class of interpretation of the model rules (Eqs. (20), (19)) comes out of the kinematical regime. It is hardly possible to estimate the influence of the external medium using only the equations of motion in the framework of the kinematical model, since they require an introduction of parameters representing deceleration, or in extreme conditions, a blast wave (Bowman 1994). The kinematical model is obtained by removing the deceleration terms (and higher order terms) of expansion (Eq. 1). We need to understand what this removal means from the viewpoint of the fitting procedure. If the removed terms posses non-zero mean value over the structure (in the sense of mean value of all plasma elements, weighted by brightness), then this value will change (or produce) the value of additive type parameter B determined during fitting procedure.

Most of the recent papers argue for the presence of the deceleration of the flows emerged from the central engine by an ambient medium (e.g. Bowman et al. 1996, Laing 1996, Bicknell 1995). Of course, the models with deceleration are more physical because they take into account the existence of matter surrounding the central engine and its (environmental) influence on the propagation of the jet. Such models could explain whether the flip-flop phenomenon is real (true) and indispensable for description of asymmetries in DERS, or whether it results from some simplification by describing the structure by kinematical model (see Sect. 4). In particular, models involving different types of confining media (e.g., Wan & Daly 1998, Gopal-Krishna & Wiita 1991) would lead to accelerations and/or decelerations upon crossing interfaces between interstellar and circumgalactic media.

Let us assume that the investigated structure has a non-zero value of deceleration and that it has zero value of time delay parameter B (it means that $r_{\rm obs}(t_0) = \bar{r}_{\rm obs}(t_0) = 0$). Then we can approximate parabolic motion by linear motion. If the deceleration is determined by the surrounding matter and this is different on both sides of the central engine we obtain different values of $\langle d^2R/dt^2 \rangle$ for the jet and for the counter-jet. This fact leads to a nonzero value of parameter B in our kinematical model.

Therefore we can state that: if the mean value (over life time of structure) of this term is different from zero, then it produces a non-zero value of parameter B,

$$\frac{1}{2}\left(\left\langle \frac{d^2R}{dt^2}(t-t_0)^2 \right\rangle - \left\langle \frac{d^2\bar{R}}{dt^2}(\bar{t}-t_0)^2 \right\rangle\right) = B \qquad (22)$$

where $\langle\rangle$ brackets denote the mean (average) value over the selected side of the structure. Parameter B describes differences between the mean value of deceleration on both sides of the central source (for example the influence of different densities).

Having the equation of motion, we can provide more detailed analysis of the obtained results and explain (try to understand) the influence of deceleration in more detail. For example, a hydrodynamic model of a jet with Navier-Stokes equation gives us (in Lagrangian formulation) the next term of the Taylor expansion in the form:

$$\frac{d^2R}{dt^2} = \frac{dV}{dt} = \frac{1}{\rho}\left(-\nabla p + \eta\nabla V + \left(\xi + \frac{\eta}{3}\right)\nabla\nabla V\right) \qquad (23)$$

Therefore the second possibility is that the $B \neq 0$ case would have an influence on the non-homogeneous external medium, i.e. a different environment on both sides of the central engine (see Roland et al. 1992 for short review). More detailed analysis of such possibilities (in preparation) should be performed similarly to the formalism introduced by Bicknell (1984).

An important possibility comes (also) from the special relativistic formulas for the case of decelerated movement of a plasma element. We should realize that a pair of plasma elements with the same initial velocity and the same constant deceleration (in their own rest frame) will be seen as having different velocities and different and non constant deceleration in the observer's rest frame (Rybicki & Lightman 1979, Ryś 1997). Hence, the time delay in the observer's rest frame can create the effects similar to the flip-flop case. However, we cannot exclude any other interpretation: – the value of parameters B may arise from the back flow effect (Leahy & Williams 1984), or existence of the cocoon (Begelman & Coffi 1989, Chyży 1997), or be caused by the motion of the host galaxy (Gopal-Krishna & Wiita 1996), or plasma elements may posses different boosting factor than pure Doppler factors of bulk motion, or have non-isotropic radiation field or magnetic field configuration (Tribble 1992).

This means that the generalized kinematical model presented here is a linear approximation for (every) physical phenomenon of outflow occurring in a real radio structure. Using such a model we would be able to describe more precisely the asymmetry in radio structures, revealing (detecting) some processes out of the kinematic regime. Having no more information than single (1.4 GHz -total power) map for each structure of the sample we were unable to decide which interpretation is physically reasonable. Therefore we have outlined only the general properties of the model and the main statistical behaviour of the model parameters in the sample, but without favouring any physical model. To estimate the true (real) influence of a possible processes, we ought to fit the simplest model of relativistic motion with the addition of a separate parameter describing deceleration, and perform fitting for structures from a better defined sample (in preparation).

6. Summary

In this paper, the general kinematical models are constructed and tested, exploiting the method of Paper I as a fitting procedure. The method allows us to determine simultaneously a few asymmetry parameters, taking information from the brightness distribution of the whole radio structure and could be used for investigations of asymmetries in both FR1 and FR2 types of structures. Two types of temporal evolution of radio wave flux emitted by plasma volume were applied, i.e.: the power law and the exponential law. For these cases we obtained two types of asymmetry in the brightness distribution of radio structures. The asymmetry of exponential type (i.e. stronger in outer regions of the structure) was found to occur in diffuse types of sources belonging to $FR1$ class. The power law models are more suitable for classical doubles ($FR2$) and have stronger asymmetry in the central region of a radio structure.

The model discussed here is the simplest extension of the pure orientation dependent model of asymmetry in the framework of the general kinematic approximation. The results of model fitting performed for both evolution models to 65 structures can be summarized as follows:

– More than 30% of the structures have brightness distribution asymmetry different from that caused only by Doppler effects.
– An inclusion of intrinsic asymmetry lead us to a smaller value of velocity component directed towards us, but the most important influence on obtained results involves the adopted model of temporal evolution of brightness.
– Taking into account that the deceleration processes does exist in an apparent radio structure, we can interpret the flip-flop type parameter of any kinematical model as a result of the deceleration process.

The mathematical rules describing the model have a form of general linear transformation, which can be produced as kinematic (linear) approximation for a wide class of models. All models of asymmetries in their linear approximation have the same mathematical form, but the physical interpretations need more information about structures than a single total power map.

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