Astron. Astrophys. 356, 612–618 (2000)

ASTRONOMY AND ASTROPHYSICS

Innermost stable circular orbits around strange stars and kHz QPOs in low-mass X-ray binaries

J.L. Zdunik[1], **P. Haensel**[1,2], **D. Gondek-Rosińska**[1], **and E. Gourgoulhon**[2]

[1] N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, 00-716 Warszawa, Poland
[2] Département d'Astrophysique Relativiste et de Cosmologie – UMR 8629 du CNRS, Observatoire de Paris, 92195 Meudon Cedex, France
(jlz@camk.edu.pl; haensel@camk.edu.pl; dorota@camk.edu.pl; Eric.Gourgoulhon@obspm.fr)

Received 23 December 1999 / Accepted 28 February 2000

Abstract. Exact calculations of innermost stable circular orbit (ISCO) around rotating strange stars are performed within the framework of general relativity. Equations of state (EOS) of strange quark matter based on the MIT Bag Model with massive strange quarks and lowest order QCD interactions, are used. The presence of a solid crust of normal matter on rotating, mass accreting strange stars in LMXBs is taken into account. It is found that, contrary to neutron stars, above some minimum mass (which for the considered equations of state ranged from $1.4\ \mathrm{M}_\odot$ to $1.6\ \mathrm{M}_\odot$) a gap always separates the ISCO and stellar surface, independently of the strange star rotation rate. For a given baryon mass of strange star, we calculate the ISCO frequency as function of stellar rotation frequency, from static to Keplerian configuration. For masses close to the maximum mass of static configurations the ISCO frequencies for static and Keplerian configurations are similar. However, for masses significantly lower than the maximum mass of static configurations, the minimum value of the ISCO frequency is reached in the Keplerian limit. Presence of a solid crust increases the ISCO frequency for the Keplerian configuration by about ten percent compared to that for a bare strange star of the same mass. For standard parameters of strange quark matter EOS, resulting in a maximum static mass of $1.8\ \mathrm{M}_\odot$, the ISCO frequency for $\gtrsim 1.4\ \mathrm{M}_\odot$ strange stars always exceeds 1.07 kHz (the upper QPO frequency reported for 4U 1820-30). We give an example of strange quark matter model, which yields maximum static mass of $2.3\ \mathrm{M}_\odot$, and for which the ISCO frequency of 1.07 kHz is allowed at stellar rotation rates 200-300 Hz, provided the strange star mass exceeds $2.2\ \mathrm{M}_\odot$. For this EOS even lower value $\nu_{\mathrm{ISCO}} \simeq 1$ kHz is reached near the Keplerian limit, for a broad range of stellar masses. While reproducing $\nu_{\mathrm{ISCO}} = 1.07$ kHz at slow rotation rates requires tuning of strange quark matter parameters, no such a tuning is required to reproduce orbital frequencies around strange stars equal to highest observed upper QPO frequencies.

Key words: dense matter – equation of state – gravitation – stars: neutron – X-rays: galaxies

Send offprint requests to: J.L. Zdunik

1. Introduction

Observations of quasi periodic oscillations (QPOs) in the X-ray fluxes from low-mass X-ray binaries (LMXB), which are believed to be due to the orbital motion of matter in an accretion disk, raised hopes concerning observational constraints on the equation of state (EOS) of matter at supranuclear densities (Kaaret et al. 1997; Kluźniak 1998; Zhang et al. 1998; Miller et al. 1998; Thampan et al. 1999; Schaab & Weigel 1999). General relativity predicts the existence of the marginally stable (MS) orbit, within which no stable circular motion is possible. This implies the existence of the innermost stable circular orbit (ISCO) around neutron stars. The frequency of the ISCO is an upper bound on the frequency of stable orbital motion around neutron stars. Whether the ISCO is separated from neutron star surface by a gap, or its radius coincides with stellar equatorial radius, depends on the star mass and on the EOS of neutron star matter. On the other hand, accreting neutron stars in LMXBs are expected to be rotating, and this influences both neutron star structure and the ISCO. Therefore, in order to attempt to use observed frequencies of QPOs to constrain the EOS of dense matter, one has to calculate the ISCO as a function of stellar mass and stellar rotation frequency. Such a procedure is based on the assumption that the observed upper QPO frequency is due to orbital motion, and that the effects of magnetic field, accretion, and radiation drag on the matter flow can be neglected.

A basic assumption of the present paper is that the frequency of the upper kHz QPO is the orbital frequency of the inner edge of an accretion disk surrounding the compact object, which will be identified with the ISCO. This is the leading interpretation of the QPOs. However, alternative models of the kHz QPOs were also proposed. In a model of Alpar & Yilmaz (1997) the kHz QPOs are explained in terms of wave packets of sound waves in the inner disk. In a series of papers, Titarchuk and collaborators propose a model in which the QPOs result from radial oscillations of the plasma in the boundary layer, i.e. in the region between the ISCO and stellar surface (see Titarchuk & Osherovich 1999 and references therein). These alternative models will not be considered in our study.

In the present paper we describe results of exact calculations of the ISCO, under the assumption that the compact object is not a neutron star, but a strange star. A strange star is composed of self-bound quark matter, which at zero pressure would constitute a real ground state of matter (strange matter), with energy per unit baryon number lower than that of ^{56}Fe crystal (Witten 1984; Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986; for a recent review of physics and astrophysics of strange matter, see Madsen 1999). Recently, strange stars were invoked by several authors in the context of modeling of observational properties of some X-ray and gamma-ray sources (Bombaci 1997; Cheng et al. 1998; Dai & Lu 1998; Li et al. 1999). First study of possibility of existence of strange stars in LMXBs exhibiting kHz QPOs was restricted to slow-rotation approximation for the ISCO, neglected the effect of rotation on the strange star structure, and used simplified EOS of strange matter, with massless, non-interacting quarks (Bulik et al. 1999). Very recently, the ISCOs around bare strange stars were calculated, assuming a simplified EOS of strange matter, for the limiting case of rotation at Keplerian frequency (Stergioulas et al. 1999). In both these studies the possible presence of the solid crust on the strange star surface was not taken into account.

In principle, a strange star could be covered by a thin crust of normal matter, a possibility which is particularly natural in the case of LMXB. The problem of formation and structure of a crust on an accreting strange star was studied by Haensel & Zdunik (1991) (see also Miralda-Escudé et al. 1990). Because of its low mass, typically $\lesssim 10^{-5}\ \mathrm{M}_\odot$, the effect of the crust on the exterior spacetime is negligible. However, it determines the location of the star surface, due to its finite thickness of $\sim 200-300$ m. The matter distribution within the strange core, relevant for the exterior metric of rotating strange star, is characterized by a very flat density profile: for a massive strange star, density at the stellar center is typically only 2-3 times larger than that at the outer edge of the strange core. This has to be contrasted with the density distribution within a massive neutron star, which decreases continuously from $\sim 10^{15}\ \mathrm{g\ cm^{-3}}$ at the center to a few $\mathrm{g\ cm^{-3}}$ at the surface. The differences between the density profiles of a strange star and a neutron star result from the basic difference in the EOS of their interiors. In the case of a rapidly rotating compact object (situation relevant to LMXB), the differences between matter distributions within neutron star and strange star may be expected to imply differences in the spacetime exterior to the compact object, and in particular, differences in the properties of the ISCO.

2. ISCOs around strange stars for standard MIT Bag Model of strange matter

Our EOS of strange matter, composed of massless u, d quarks, and massive s quarks, is based on the MIT Bag Model. It involves three basic parameters: the bag constant, B, the mass of the strange quarks, m_s, and the QCD coupling constant, α_c (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986). Our basic EOS corresponds to standard values of the Bag Model parameters for strange matter: $B = 56\ \mathrm{MeV/fm^3}$, $m_\mathrm{s} = 200\ \mathrm{MeV/c^2}$, and $\alpha_\mathrm{c} = 0.2$ (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986). This EOS of strange quark matter will be hereafter referred to as SQM1. It yields energy per unit baryon number at zero pressure $E_0 = 918.8\ \mathrm{MeV} < E(^{56}\mathrm{Fe}) = 930.4$ MeV. For the SQM1 EOS maximum allowable mass for static strange star models is $M_\mathrm{max}^\mathrm{stat} = 1.8\ \mathrm{M}_\odot$.

The general relativistic models of stationary rotating strange stars have been calculated by means of the multi-domain spectral method, developed recently by Bonazzola et al. (1998). Details of the calculation method, specifically adapted for rotating strange stars, may be found in Gourgoulhon et al. (1999). Having calculated a particular stationary rotating strange star model, and its exterior spacetime, we determine the frequency of a particle in stable circular orbit in the equatorial plane, $\nu_\mathrm{orb}(r)$, where r is the radial coordinate of the orbit. By testing the stability of orbital motion, we determine the radius of the innermost, marginally stable orbit, R_ms, and its frequency ν_ms (see, e.g., Datta et al. 1998, for the equations to be solved). Our numerical code calculating the ISCO has been successfully tested by comparing our results for the polytropic $\gamma = 2$ EOS with those obtained by Cook et al. (1994a). Let us notice that the high precision of our numerical method makes it particularly suitable for the determination of R_ms, which requires calculation of second derivatives of metric functions: these latter are better evaluated by the spectral method we employ than by means of finite differences.

No orbital motion is possible for $r < R_\mathrm{ms}$. The values of R_ms and ν_ms for particles corotating with strange star differ from those for counterrotating ones. In the present paper we restricted ourselves to the corotating case, relevant for the LMXB. We neglect the effect of magnetic field, accretion, and radiation drag on the location of the ISCO, which is justified for $B \lesssim 10^8$ G and $\dot{M} \ll \dot{M}_\mathrm{Edd}$.

Let us consider a strange star, rotating at a frequency ν_rot, with equatorial radius R_eq. If $R_\mathrm{ms} > R_\mathrm{eq}$, then stable orbits exist for $r > R_\mathrm{ms}$; the ISCO has then the radius R_ms and the frequency ν_ms, and there is a gap of width $R_\mathrm{ms} - R_\mathrm{eq}$ between the ISCO and the strange star surface. However, if $R_\mathrm{ms} < R_\mathrm{eq}$, then $R_\mathrm{ISCO} = R_\mathrm{eq}$, $\nu_\mathrm{ISCO} = \nu_\mathrm{orb}(R_\mathrm{eq})$; and the accretion disk extends then down to the strange star surface (or, more precisely, joins stellar surface via a boundary layer).

While the exterior spacetime is practically not influenced by the presence of a solid crust on the strange star surface, the value of R_eq is affected by it. Neutrons are absorbed by strange matter, and therefore the density at the bottom of the crust, $\rho_\mathrm{bott.cr.}$, cannot be higher than $\rho_\mathrm{n-drip} \simeq 4\times 10^{11}\ \mathrm{g\ cm^{-3}}$ (lower values of $\rho_\mathrm{bott.cr.}$ were discussed by Huang & Lu 1997). The equatorial thickness of the crust, t_eq, which we calculate, corresponds to $\rho_\mathrm{bott.cr.} = \rho_\mathrm{n-drip}$, and is therefore an upper bound on t_eq. At a fixed baryon number, rotation increases t_eq, as compared to the value for a static strange star, t_0. Dependence of t_eq on ν_rot is well described by a formula $t_\mathrm{eq}(\nu_\mathrm{rot}) = t_0\cdot[1+0.7(\nu_\mathrm{rot}/\nu_\mathrm{K})^2]$, where ν_K is the Keplerian (mass shedding) frequency of strange star. For rotating strange stars our formula for t_eq reproduces numerical results of Glendenning & Weber (1992) within better than 2% in all cases considered by these authors. It is obvious that the bare strange star rotating at Keplerian limit would be

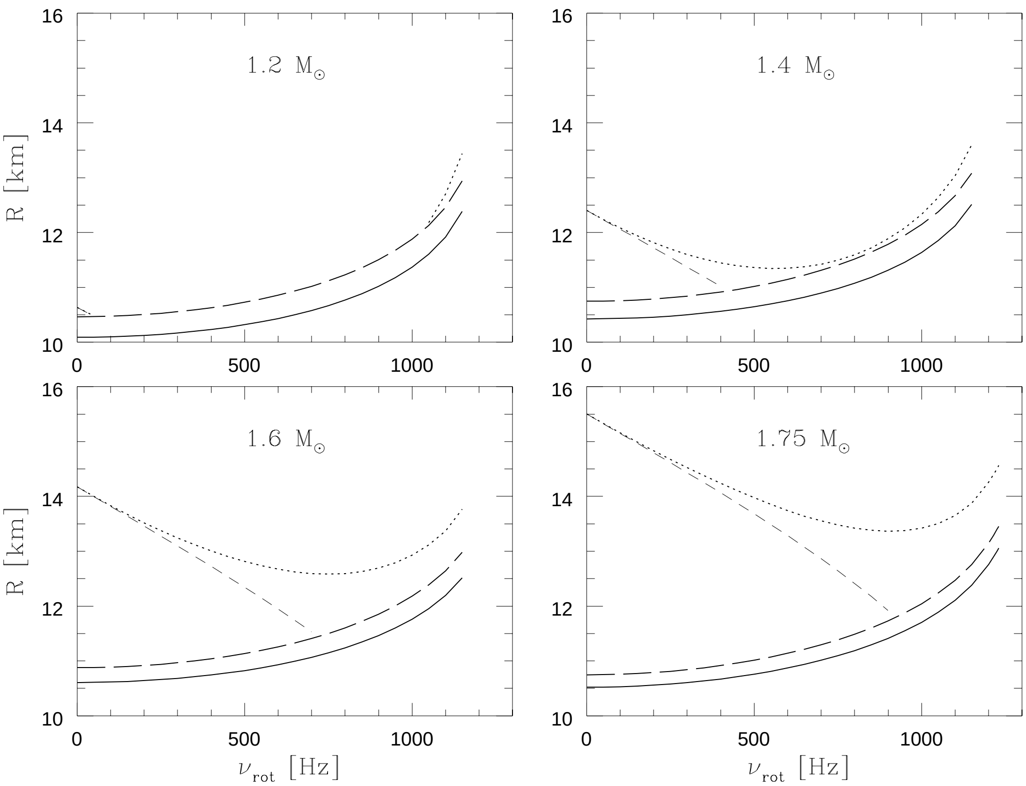

Fig. 1. The radius of the ISCO (dotted line), equatorial radius of a bare strange star (solid line), equatorial radius of a strange star with maximally thick solid crust (long-dashed line), versus rotation frequency of strange star. Thin short-dashed line corresponds to slow-rotation approximation of Kluźniak & Wagoner (1985). Calculations were performed for the SQM1 EOS of strange matter (see the text). Figures correspond to stellar models with fixed total baryon number, equal to that of a static star of gravitational mass of $1.2,\ 1.4,\ 1.6, 1.75\ \mathrm{M}_{\odot}$. Maximum mass for static strange stars is $1.8\ \mathrm{M}_{\odot}$

unstable if we added a crust of nonzero thickness due to the increase of the radius, leaving the mass and the angular momentum practically unaltered. Thus at a fixed baryon mass of rotating strange star, the presence of the crust implies a decrease of the Keplerian frequency. Knowing the dependence of the radius of the strange core and rotational frequency on the stellar angular momentum one can estimate the point of the Keplerian instability for a strange star with crust.

Let us consider first rotating strange stars for the SQM1 EOS, which corresponds to the "standard set" of the Bag Model parameters for strange matter. A sample of our results for sequences of rotating strange star models with fixed baryon number are presented in Fig. 1.

The form of Fig. 1 is analogous to that constructed by Miller et al. (1998) for neutron stars, and therefore is suitable for discussion of the differences between neutron stars and strange stars. For strange stars with static mass $M \gtrsim 1.4\ \mathrm{M}_{\odot}$, we have always $R_{\rm ms} > R_{\rm eq}$, for any $\nu_{\rm rot}$. So, for $M \gtrsim 1.4\ \mathrm{M}_{\odot}$ the gap between strange star surface (with or without solid crust) and the ISCO exists at any strange star rotation rate. Even for lower M, the gap, which disappears at moderate rotation rates, reappears at ~ 1 kHz frequency of rotation; this is visualized by the $M = 1.2\ \mathrm{M}_{\odot}$ case in Fig. 1. At $\nu_{\rm rot} = \nu_{\rm K}$, the ISCO is always separated from the strange star surface by a gap. Clearly, these features of the ISCO around strange stars are quite different from those obtained by Miller et al. (1998) for neutron stars with the FPS EOS (see their Fig. 1). Note that the existence of a gap ($R_{\rm ms} > R_{\rm eq}$) is expected to lead to a qualitatively different spectrum of X-ray radiation from LMXB, compared to the no-gap case (Kluźniak et al. 1990).

Constraints on EOS of dense matter, resulting from the kHz QPOs observations, were initially derived within the slow-rotation approximation, in which $R_{\rm ms} \simeq R_{\rm ms}^{\rm s.r.} = 6GM/c^2 \cdot [1-(2/3)^{3/2} j]$, $j \equiv Jc/GM^2$, and J is stellar angular momentum (Kluźniak & Wagoner 1985). However as pointed out by Shibata & Sasaki (1998), the mass quadrupole moment is as important as the angular momentum in determining the ISCO. Indeed as we can see in Fig. 1, slow rotation approximation yields $R_{\rm ms}^{\rm s.r.}$, which in the case of $M = 1.4\ \mathrm{M}_{\odot}$ diverges from exact $R_{\rm ms}$ for $\nu_{\rm rot} \gtrsim 500$ Hz. Moreover, for rotating strange stars $R_{\rm ms}^{\rm s.r.}$ leads always to disappearance of the gap at sufficiently high $\nu_{\rm rot}$, in contrast to exact calculation.

A quantity of particular interest in the context of the interpretation of observed kHz QPOs in LMXB, is the maximum frequency of the stable circular orbit at a given $\nu_{\rm rot}$, which we identify here with that of the ISCO. In principle, both $\nu_{\rm ISCO}$ and $\nu_{\rm rot}$ are observable (measurable) quantities, which can be thus used for confronting stellar models with observations. In Fig. 2 we present curves $\nu_{\rm ISCO}(\nu_{\rm rot})$ for the SQM1 EOS. As in Fig. 1, baryon masses are fixed along each curve, while labels correspond to gravitational mass of non-rotating strange star.

In all cases, displayed in Fig. 2, gap between stellar surface and ISCO exists, and therefore $\nu_{\rm ISCO} = \nu_{\rm ms}$. The dash-dotted line was calculated for a simplified EOS, with massless, non-interacting quarks ($m_{\rm s} = 0$, $\alpha_{\rm c} = 0$, $B = 56\ \mathrm{MeV/fm^3}$), hereafter referred to as SQM0 (such a type of the EOS of strange quark matter was used in Bulik et al. 1999). For $\nu_{\rm rot} \gtrsim 500$ Hz, neglecting strange quark mass (and, to a smaller extent, neglecting QCD interactions) leads to a rather severe underestimate of $\nu_{\rm ISCO}$ for rapidly rotating strange stars (by 200 Hz at

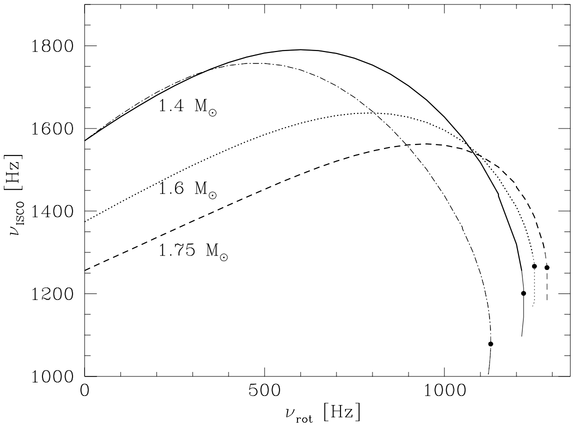

Fig. 2. Frequency of the ISCO versus the rotation frequency of strange star, for the same SQM1 EOS of strange matter. Dash-dotted line was obtained for a simplified SQM0 model of strange matter with massless, noninteracting quarks (see the text). Each curve corresponds to a fixed baryon mass, equal to that of a static strange star of gravitational mass indicated by a label. Along each curve, angular momentum increases from $J = 0$ (static configuration) to $J_{\rm max}$ (Keplerian limit). Filled circles correspond to Keplerian configurations of strange stars with crust. Segments below the filled circles can be reached only by the bare strange stars.

$\nu_{\rm rot} = 1$ kHz). As we stressed before, presence of the solid crust does not influence the space-time outside rotating strange strange star. However, solid crust decreases (by about 10%) the value of $J_{\rm max}$ of strange stars of a given baryon mass. Complete curves in Fig. 2 correspond to bare strange stars. Rotating configurations with crust terminate at filled dots, corresponding to the Keplerian limit in the presence of the crust. The effect of the presence of the crust on the value of $\nu_{\rm ISCO}$ at $\nu_{\rm K}$ turns out to be significant, which is due to the steepness of the $\nu_{\rm ISCO}(\nu_{\rm rot})$ curve for bare strange stars at $J \simeq J_{\rm max}$. For J approaching $J_{\rm max}$ bare strange star undergoes strong deformation with increasing J. This deformation in turn implies strong decrease of $\nu_{\rm ISCO}$ with increasing rotation frequency. Consequently, the values of $\nu_{\rm ISCO}$ for maximally rotating strange stars with crust is about hundred Hz higher than for bare strange stars. This effect increases with decreasing strange star mass. At fixed B, the effect is stronger for the EOS which produces less compact strange stars of a given mass. Therefore, it is strongest for the SQM0 EOS with massless, noninteracting quarks, where maximally rotating configurations of $1.4\ {\rm M}_\odot$ with crust have the ISCO frequency of 1.1 kHz, to be compared with less than 1 kHz for maximally rotating bare strange stars.

The problem of an appropriate parametrization of the one-parameter family of rotating strange stars with fixed baryon mass deserves a comment. These configurations may be labeled by the value of the total angular momentum J, which changes from $J = 0$ in the static case to $J_{\rm max}$ at the Keplerian limit. As one can see in Fig. 2, for bare strange stars, Keplerian configuration is not that with maximum $\nu_{\rm rot}$. The reason is that for very rapidly rotating strange stars the increase of the total angular momentum results mainly in the oblateness of the configurations leading to the significant increase of the equatorial radius without an increase of $\nu_{\rm rot}$ (or even with a decrease of $\nu_{\rm rot}$ very close to $J_{\rm max}$). As a consequence at fixed baryon mass the Keplerian configurations is reached not due to the increase of $\nu_{\rm rot}$ but because of the increase of the equatorial radius related to the deformation of the star. It is worth noticing that the difference between $\nu_{\rm rot,max}$ and $\nu_{\rm K}$ is of the order of one percent. The existence of this difference implies that for $J \simeq J_{\rm max}$ it is in principle possible to spin up the strange star by the angular momentum loss. Such a situation was previously discussed in the case of supramassive neutron stars (Cook et al. 1994b) and supramassive strange stars (Gourgoulhon et al. 1999).

3. Confronting the standard MIT Bag Model of strange matter with QPO observations

Let us pass now to the confrontation of our results for strange stars with observations of the QPOs. Nearly twenty LMXBs, exhibiting QPOs, have been observed (van der Klis 2000). The upper-peak frequency, $\nu_{\rm QPO}^{\rm u.p.}$, is usually interpreted as the frequency of the orbital motion around a neutron star. The most general observational constraint on a neutron star in LMXB is thus $\nu_{\rm QPO}^{\rm u.p.} \leq \nu_{\rm ISCO}$. Highest observed $\nu_{\rm QPO}^{\rm u.p.}$ is 1329 ± 4 Hz in 4U 0614+09 (van Straaten et al. 2000). Condition $\nu_{\rm ISCO} \geq 1.33$ kHz is satisfied by nearly all strange star models displayed in Fig. 2 (except those rotating very close to the Keplerian frequency and slowly rotating maximum mass model). In particular for the spin frequency of the star $\nu_{\rm spin} = 312$ Hz (Ford et al. 1997; van Straaten et al. 2000) all stellar configurations for SQM1 model of strange matter are allowed.

For neutron stars, condition $\nu_{\rm ISCO} \geq 1.2$ kHz (considered by Thampan et al. 1999 as the highest $\nu_{\rm QPO}^{\rm u.p.}$) eliminates stellar masses below some limit, ranging from $0.6\ {\rm M}_\odot$ for stiff EOS to $1.4\ {\rm M}_\odot$ for soft EOS (Thampan et al. 1999). In this case the innermost allowed orbit is defined by the radius of the star and corresponds to the Keplerian frequency at the surface $\nu_{\rm K}$. This conclusion would be stronger in the case of $\nu_{\rm QPO}^{\rm u.p.} = 1.33$ kHz excluding the softest EOS and shifting the above mass limits to a little higher values (see Fig. 1. in the paper by Thampan et al. 1999). Such a constraint does not apply to bare strange stars, for which in the limit of $M \ll {\rm M}_\odot$ one gets $\nu_{\rm K} \simeq (G\rho_{\rm sm}/3\pi)^{1/2} = 0.841 \cdot (\rho_{\rm sm,14})^{1/2}$ kHz, where $\rho_{\rm sm,14}$ is the density of strange matter at zero pressure, in the units of $10^{14}\ {\rm g/cm^3}$. For reasonably high values of $\rho_{\rm sm}$, in particular for those considered in the present paper, one gets $\nu_{\rm K} > 1.33$ kHz for low-mass, slowly rotating bare strange stars. In the case of strange stars with crust, condition $\nu_{\rm K} > 1.33$ kHz turns out to be violated for $M \lesssim 0.4\ {\rm M}_\odot$.

The behavior of QPOs in 4U 1820-30 has been interpreted as evidence for $\nu_{\rm QPO}^{\rm u.p.} = \nu_{\rm ms}$ in this LMXB (Kaaret et al. 1999, and references therein). Accepting such an interpretation of $\nu_{\rm QPO}^{\rm u.p} = 1.07$ kHz implies strong constraints on neutron star model, and therefore, on the neutron star EOS (Kluźniak 1998; Miller et al. 1998). Only a few existing EOS of neutron star matter allow simultaneously for $R_{\rm eq} < R_{\rm ms}$ and $\nu_{\rm ms} = 1.07$ kHz. It is clear from Fig. 2 that SQM1 models of strange stars cannot

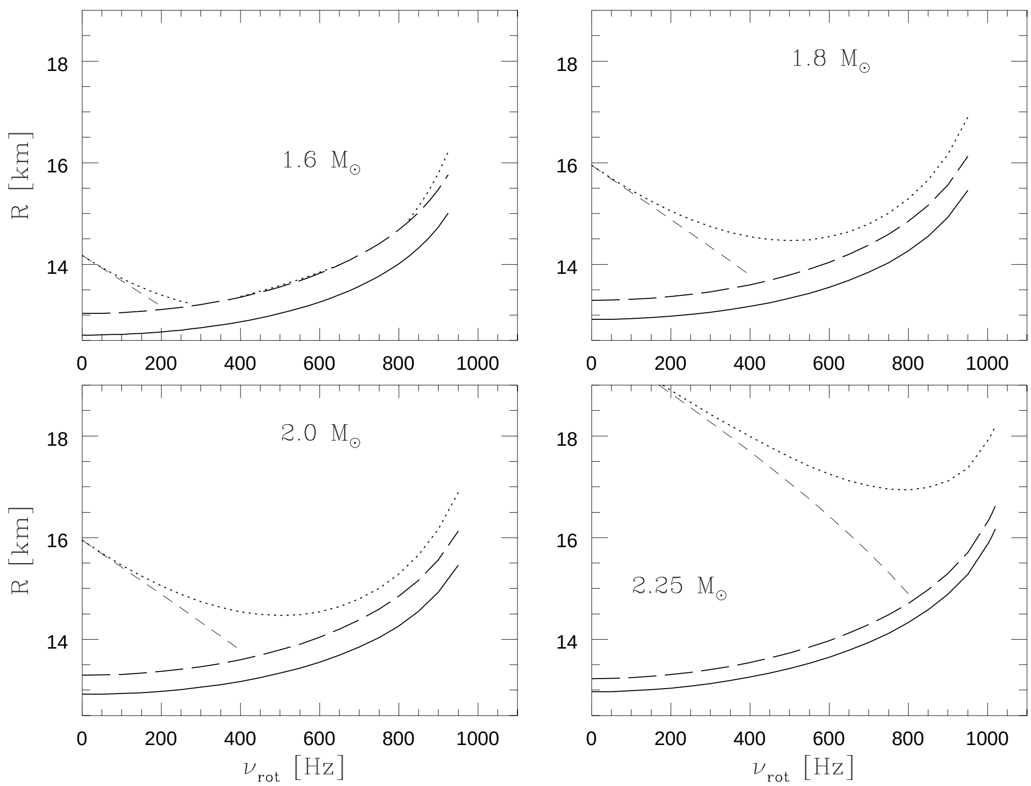

Fig. 3. The radius of the ISCO (dotted line), equatorial radius of bare strange star (solid line), equatorial radius of strange with a solid crust (long-dashed line), versus rotation frequency of strange star, for the SQM2 EOS of strange quark matter (see the text). Thin short-dashed line corresponds to slow-rotation approximation of Kluźniak & Wagoner (1985). Figures correspond to stellar models with fixed total baryon number, equal to that of a static star of the gravitational mass of $1.6,\ 1.8,\ 2.0,\ 2.25\ \mathrm{M}_\odot$. Maximum mass of static strange stars is $2.3\ \mathrm{M}_\odot$

give $\nu_{\rm ISCO}$ as low as 1.07 kHz at slow rotation rates. In the case of the SQM0 EOS one is able to get such low $\nu_{\rm ISCO}$ for bare strange stars of $M \lesssim 1.4\ \mathrm{M}_\odot$ rotating close to Keplerian frequency; we confirm in this way result of Stergioulas et al. (1999). However, as we see in Fig. 2, passing to an EOS which at the same value of B includes effects of strange quark mass and of lowest order QCD interaction increases the values of $\nu_{\rm ISCO}$ of bare strange stars at high rotation rates to such extent, that the value of 1.07 kHz cannot be reproduced.The presence of solid crust on rotating strange stars described by the "standard strange matter EOS" SQM1 excludes $\nu_{\rm ISCO}(\nu_{\rm K})$ lower than 1.2 kHz. In the case of the simplest SQM0 EOS the value of $\nu_{\rm ISCO}(\nu_{\rm K})$ is increased by the presence of the crust a little above 1.07 kHz. Generally, the presence of the crust on rotating strange star with standard strange matter EOS, such as SQM1 (or SQM0) excludes possibility of getting $\nu_{\rm ISCO}$ as low as 1.07 kHz for any possible rotation rates.

4. MIT Bag Model of strange matter consistent with QPO observations

In order to get $\nu_{\rm ISCO}$ as low as 1.07 kHz at slow and moderate rotation rates, one has to consider a specific set of the MIT Bag Model parameters, characterized by significantly lower values of both B and $m_{\rm s}$, and higher value of $\alpha_{\rm c}$, than those characteristic of the SQM1 model. In this way one is able to increase significantly the value of $M^{\rm stat}_{\rm max}$, and get ISCO frequencies as low as 1 kHz at slow rotation rates. For such a choice of EOS it is also relatively easy to get $\nu_{\rm ISCO} \lesssim 1$ kHz for a broad range of masses of configurations rotating close to the Keplerian limit (see below). An example of such an EOS, hereafter referred to as the SQM2, was obtained assuming $B = 40\ \mathrm{MeV/fm^3}$, $m_{\rm s} = 100\ \mathrm{MeV/c^2}$, and $\alpha_{\rm c} = 0.6$. At zero pressure, the SQM2 model yields energy per unit baryon number $E_0 = 874.2$ MeV. Let us stress that despite the relatively low value of B, the standard condition that neutrons do not fuse (coagulate) spontaneously into strangelets (droplets of quark matter), is satisfied by this model. Maximum mass of static strange stars for the SQM2 EOS is $2.3\ \mathrm{M}_\odot$.

Our SQM2 model is a rather extreme one, as far as the values of the B, $m_{\rm s}$, and $\alpha_{\rm c}$ parameters are concerned. Canonical value of B, resulting from fitting hadronic masses, is $59\ \mathrm{MeV/fm^3}$ (De Grand et al. 1975), significantly higher than $B = 40\ \mathrm{MeV/fm^3}$ used in the SQM2 model. On the other hand, $m_{\rm s} = 100\ \mathrm{MeV/c^2}$ of the SQM2 model is on the lower side of usually considered $m_{\rm s}$ values (Farhi & Jaffe 1984; Madsen 1999). Finally, $\alpha_{\rm c} = 0.6$ is on the upper side of the interval of the $\alpha_{\rm c}$ values considered in the strange matter calculations (Farhi & Jaffe 1984).

Our results for the SQM2 EOS, analogous to those displayed in Fig. 1 and Fig. 2 for the SQM1 EOS, are shown in Fig. 3 and Fig. 4. The main differences between these models can be explained by the scaling laws with the bag constant, discussed in Sect. 5. The features of $R_{\rm ms}$ and radius of the rotating strange star of given mass for SQM1 model corresponds to the star SQM2 with the mass larger by the factor $\sim (B_1/B_2)^{1/2}$. As one can see in Fig. 4, slowly rotating strange stars can have ISCO frequencies as low as $1-1.1$ kHz, provided their mass is sufficiently high, $M \simeq 2.2 - 2.3\ \mathrm{M}_\odot$ just because maximum allowable mass for static strange stars is sufficiently high. Moreover, for

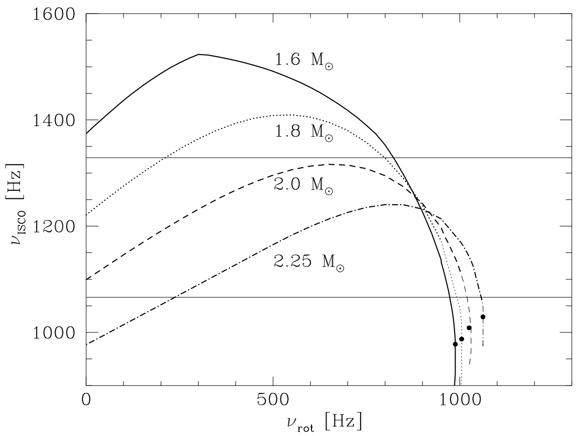

Fig. 4. Frequency of ISCO versus rotation frequency of strange star for the SQM2 EOS of strange matter (see the text). Lines correspond to stellar models with fixed total baryon number, equal to that of a static star of the gravitational mass of $1.6,\ 1.8,\ 2.0,\ 2.25\ \mathrm{M}_{\odot}$, Lower thin horizontal line: upper-peak frequency observed in 4U 1820-30, 1.07 kHz. Upper horizontal line: maximum upper-peak frequency observed in the QPOs in LMXBs, 1.33 kHz. Keplerian configurations for strange stars with crust are indicated by filled circles. Segments of curves below filled circles correspond to bare strange stars. Maximum mass for static strange stars is 2.3 $\mathrm{M}_{\odot}$.

bare strange stars, the ISCO frequency below 1 kHz can also be reached for very rapid rotation close to the Keplerian limit. Less massive is bare strange star, lower is the ISCO frequency reached at the Keplerian limit. The presence of the solid crust makes the window (subset) of rapidly rotating configurations allowing for $\nu_{\rm ISCO} = 1.07$ kHz significantly narrower. These configurations are very close to the Keplerian ones. Notice that the SQM2 EOS is simultaneously consistent with $\nu_{\rm ISCO} \geq 1.33$ kHz, provided the strange star mass $M \lesssim 1.8\ \mathrm{M}_{\odot}$.

5. Discussion and conclusion

The features of ISCOs around rapidly rotating strange stars, described in the present paper for a particular choice of strange matter EOS, are actually generic. The MIT Bag Model EOS of strange quark matter depends on B, $m_{\rm s}$, and $\alpha_{\rm c}$ in a way, which implies specific scaling properties with respect to change of B (Haensel et al. 1986; Zdunik & Haensel 1990). As a consequence, the global parameters of rotating strange stars scale with some power of B, which allows one to determine the values of M, $R_{\rm eq}$, $R_{\rm ms}$, etc., for B, $\alpha_{\rm c}$, and $m_{\rm s}$, from those calculated for B_0, $\alpha_{\rm c}$ and strange quark mass $m_{\rm s}(B_0/B)^{1/4}$. All length-type quantities (stellar radius, thickness of the crust and radius of the ISCO) scale as $B^{-1/2}$, and all frequencies ($\nu_{\rm ISCO}$, $\nu_{\rm rot}$) scale as $B^{1/2}$, e.g., $\nu_{\rm ISCO}[B] = \nu_{\rm ISCO}[B_0] \cdot (B/B_0)^{1/2}$ and $R_{\rm ms}[B] = R_{\rm ms}[B_0] \cdot (B/B_0)^{-1/2}$. Thus, for other values of B the patterns of lines in Figs. 1-4 do not change, provided one rescales the axes and stellar masses.

Our calculations show that the properties of the ISCOs around strange stars differ from those around neutron stars. A generic property is the existence of the gap between the ISCO and the stellar surface, for both slowly and rapidly rotating strange stars.

The highest observed QPO frequency of 1.33 kHz in 4U 0614+91 can be easily interpreted as an orbital frequency around strange star based on the standard SQM1 EOS of strange matter, with no significant constraint on strange star mass and rotation rate. In the case of the SQM2 EOS, the orbital origin of the 1.33 kHz QPO implies $M \lesssim 1.8\ \mathrm{M}_{\odot}$ at rotation frequencies ~ 300 Hz, while frequencies close to the mass shedding limit are excluded.

The value of the ISCO frequency at the Keplerian limit is significantly influenced by the presence of a crust on the strange star surface, which increases this frequency by about hundred Hz compared to the value for a bare strange star of the same mass. As one expects the presence of a crust on a strange star in a LMXB, we conclude that only slowly rotating strange stars with mass above 2.2 $\mathrm{M}_{\odot}$ seem to be consistent with $\nu_{\rm ISCO} = 1.07$ kHz. This excludes EOS of strange matter corresponding to the standard bag model parameters, and can be satisfied only by choosing a set of parameters quite different from the standard one.

The numerical results discussed in the present paper show that consistency of the ISCOs around slowly rotating strange stars with orbital-motion interpretation of QPOs in LMXBs can be achieved only with a substantial tuning of the MIT Bag Model parameters of strange matter. Our SQM2 EOS is a result of such a tuning. For this EOS, the condition $\nu_{\rm ISCO} \simeq 1$ kHz is satisfied not only for slowly rotating massive models with $M \gtrsim 2.2\,\mathrm{M}_{\odot}$, but also for a broad range of masses of configurations close to the Keplerian limit. In contrast to $\nu_{\rm ISCO}$ at low rotation rate, which decreases with increasing baryon mass, the ISCO frequency at the Keplerian limit decreases with decreasing baryon mass of rotating strange star.

Acknowledgements. During his stay at DARC, Observatoire de Paris, P. Haensel was supported by the PAST professorship of French MENRT. This research was partially supported by the KBN grants No. 2P03D.014.13, 2P03D.021.17. The numerical calculations have been performed on computers purchased thanks to a special grant from the SPM and SDU departments of CNRS. We are very grateful to the referee, P. Kaaret, for helpful comments and suggestions, which influenced the final version of the present paper.

References

Alcock C., Farhi C.E., Olinto A., 1986, ApJ 310, 261
Alpar M.A., Yilmaz A., 1997, New Astronomy 2, 225
Bombaci I., 1997, Phys. Rev. C 55, 1587
Bonazzola S., Gourgoulhon E., Marck J.A., 1998, Phys. Rev. D 58, 104020
Bulik T., Gondek-Rosińska D., Kluźniak W., 1999, A&A 344, L71
Cheng K.S., Dai Z.G., Wei D.M., Lu T., 1998, Sci 280, 407
Cook G.B., Shapiro S.L., Teukolsky S.A., 1994a, ApJ 422, 227
Cook G.B., Shapiro S.L., Teukolsky S.A., 1994b, ApJ 424, 823
Dai Z.G., Lu T., 1998, Phys. Rev. Lett. 81, 4301
Datta D., Thampan A.V., Bombaci I., 1998, A&A 334, 943
De Grand T., Jaffe R.L., Johnson K., Kiskis J., 1975, Phys. Rev. D 12, 2060

Farhi E., Jaffe R.L., 1984, Phys. Rev. D 30, 2379
Ford E.C., Kaaret P., Tavani M., et al., 1997, ApJ 475, L123
Glendenning N.K., Weber F., 1992, ApJ 400, 647
Gourgoulhon E., Haensel P., Livine R., et al., 1999, A&A 349, 851
Haensel P., Zdunik J.L., 1991, Nucl. Phys. B (Proc. Suppl.) 24B, 139
Haensel P., Zdunik J.L., Schaeffer R., 1986, A&A 160, 121
Huang Y.F., Lu T., 1997, A&A 325, 189
Kaaret P., Ford E.C., Chen K., 1997, ApJ 480, L27
Kaaret P., Piraino S., Bloser P.F., et al., 1999, ApJ 520, L37
Kluźniak W., 1998, ApJ 509, L37
Kluźniak W., Wagoner R.V., 1985, ApJ 297, 548
Kluźniak W., Michelson P., Wagoner R.V., 1990, ApJ 358, 538
Li X.-D., Bombaci I., Dey M., Dey J., van den Heuvel E.P.J., 1999, Phys. Rev. Lett. 83, 3776
Madsen J., 1999, In: Cleymans J. (ed.) Hadrons in Dense Matter and Hadrosynthesis. Lecture Notes in Physics, Springer-Verlag, p. 162
Miller M.C., Lamb F.K., Cook G.B., 1998, ApJ 509, 793
Miralda-Escudé J., Haensel P., Paczyński B., 1990, ApJ 362, 572
Schaab C., Weigel M.K., 1999, MNRAS 308, 718
Shibata M., Sasaki M., 1998, Phys. Rev D 58, 104011
Stergioulas N., Kluźniak W., Bulik T., 1999, A&A 352, L116
Thampan A.V., Bhattacharya D., Datta D., 1999, MNRAS 302, L69
Titarchuk L., Osherovich V., 1999, ApJ 518, L95
van der Klis M., 2000, to appear in: The Proceedings of the Third William Fairbank Meeting, Rome June 29-July 4, 1998, (astro-ph/9812392)
van Straaten S., Ford E., van der Klis M., Méndez M., Kareet P., 2000, (astro-ph/0001480)
Witten E., 1984, Phys. Rev. D 30, 272
Zdunik J.L., Haensel P., 1990, Phys. Rev. D 42, 710
Zhang W., Strohmayer T.E., Swank J.H., 1998, ApJ 482, L167