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LETTER

Letter to the Editor

Suppression of thermonuclear reactions in dense plasmas instead of Salpeter's enhancement

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Abstract. A kinetic description of collective plasma effects for the thermonuclear reaction rates based on the generalization of modern plasma fluctuation approach shows that, in contrast to Salpeter's enhancement, a decrease of the thermonuclear reaction rates is present which is larger for larger nuclei charge. By numerical evaluation of the thermonuclear reaction rates in the solar interior it is found that the decrease approaches a factor $1/2$ for reactions with Be nuclei, and this could be relevant for the problem of solar neutrino deficit.

Key words: nuclear reactions, nucleosynthesis, abundances – plasmas – Sun: interior – dense matter

1. Introduction

The motivation for developing a full kinetic description of nuclear reactions in dense plasmas is given by the following points:

1. The widely used Salpeter's enhancement of nuclear reaction rates by screening in dense plasma (Salpeter 1954) is based on the *assumption* that the problem of nuclear reaction rates in *many-particle* systems can by solved by finding an effective screened potential describing the quantum tunneling through the Coulomb potential barrier of *two* reacting nuclei. This assumption should be checked.
2. The statistical approach (Brown & Sawyer 1997) in which the static screening of two reacting nuclei was obtained *assumes* that the bare nuclei have thermal distributions, while in the plasma thermal distribution is established for dressed nuclei.
3. The test particle approach, leading to dynamic screening of nuclear reactions (Carraro et al. 1988) is in contradiction with the results (Brown & Sawyer 1997; Gruzinov 1998) corresponding to static screening, although the velocity of two reacting nuclei is super-thermal. In a kinetic description the results should be valid for arbitrary particle distributions including that describing bulk plasma particles and an extra test particle. Therefore the result of the kinetic description will be in full agreement with the test particle approach. It is expected (Shaviv & Shaviv 1999) that using a fully kinetic description of nuclear reactions in plasmas one can resolve the existing controversy - dynamic versus static screening (Carraro et al. 1988; Brown & Sawyer 1997; Gruzinov 1998).
4. It was noticed that plasma fluctuations can affect the nuclear reaction rates (Shaviv & Shaviv 1999,2000).
5. Although the main application of the results is related to thermal plasmas the kinetic description for non-thermal distributions can make the physics more transparent, since in kinetic description the individual processes are not hidden by balances occurring in thermal equilibrium.

2. Fluctuations in presence of nuclear reactions

The modern kinetic plasma approach in absence of nuclear reactions is based on the concept of fluctuations in non-equilibrium plasmas (see (Tsytovich 1995)). The fluctuations create a screening cloud around a single particle, and they screen the binary particle interactions. The screening in binary collisions according to the Landau-Balescu collision integral is always dynamic. The screening results in a self-energy for each particle when the particles are apart; it is always dynamic. These effects can be obtained for the simplest case of non-relativistic plasmas from the procedure of averaging over fluctuations through the equations for microscopic particle distributions containing only electrostatic potentials:

$$\frac{\partial f_i(\mathbf{p})}{\partial t} + \mathbf{v}\cdot\frac{\partial f_i(\mathbf{p})}{\partial \mathbf{r}} - Z_i e(\nabla\delta\phi)\cdot\frac{\partial f_i(\mathbf{p})}{\partial \mathbf{p}} = 0 \qquad (1)$$

where $f_i(\mathbf{p}) \equiv f_i(\mathbf{r},\mathbf{p},t)$ is the microscopic distribution function of nuclei/particle i, which will be normalized here so that $\int f_i(\mathbf{p})d\mathbf{p}/(2\pi)^3 = n_i$, n_i being the nuclei number density, Z_i the atomic number, and $\delta\phi$ the electrostatic fluctuating potential.

According to the formalism of the kinetic theory of plasma fluctuations (Tsytovich 1995, Klimontovich 1997) one writes the distribution function for nuclei of species i in the form $f_i(\mathbf{r},\mathbf{p},t) = \Phi_i(\mathbf{p}) + \delta f_i(\mathbf{r},\mathbf{p},t)$ such that $\Phi_i(\mathbf{p}) \equiv \langle f_i(\mathbf{r},\mathbf{p},t)\rangle$ is the distribution function of "dressed" particles, i.e., particles surrounded by their screening clouds, brackets $\langle ...\rangle$ denoting the average with respect to fluctuations, and $\delta f_i(\mathbf{r},\mathbf{p},t)$ being the fluctuating part of the distribution function for which $\langle \delta f_i(\mathbf{r}, \mathbf{p}, t)\rangle = 0$. By averaging (1) and subtracting the average equation from (1) one obtains two equations for the average, and for the fluctuating part of the distribution function. Assuming weak fluctuations, based on small parameter $1/N_\mathrm{d}$ (see below) one finds the known Landau-Balescu collisional integral for the average particle distribution (Tsytovich 1995), as well as the emission and scattering processes. All effects describe the processes for screened (dressed) particles (Ginzburg & Tsytovich 1994).

The expression for the self-energy E_i^s per dressed particle can be found in the form (not given explicitly previously): $E_i^s = \int d\mathbf{p} E_i^s(\mathbf{v})\Phi_i(\mathbf{p})/n_i(2\pi)^3$;

$$E_i^s(\mathbf{v}) = -\frac{(Z_i e)^2}{4\pi^2}\int \frac{d\mathbf{k}}{k^2} Re\left[\omega^2 \frac{\partial}{\partial\omega}\frac{1}{\omega\epsilon_{\mathbf{k},\omega}}\right]_{\omega=\mathbf{k}\cdot\mathbf{v}} + const. \tag{2}$$

It contains the averaged distribution function both explicitly and through the dielectric function for electrostatic disturbances $\epsilon_{\mathbf{k},\omega}$ in expression (2). This expression is determined up to a constant which should be found separately.

Relevant for the nuclear reaction is the dependence of the self-energy of individual nuclei (2) on nucleon velocity. Although the energy (2) is not the interaction energy it should be taken into account in conservation laws, and the problem is to find whether it can affect the nuclear reaction rates.

Fluctuations produce the particle dressing. The time needed for a particle to dress is much longer than the characteristic time for a nuclear reaction (the time scale of fluctuations is determined by the time taken by a thermal particle to cross the Debye sphere and it is much longer than the time of tunneling through the Coulomb barrier relevant to nuclear reactions). To evaluate the average reaction rates for dressed particles, the averaging over fluctuations should be performed in final expressions, while in the existing approach the opposite procedure is used assuming that the screening is the same during reactions and is equal to the average screening. The nuclear reaction rates are sensitive to the dependence on the relative energies of the reacting nuclei, and different values of fluctuating potential in the screening will affect the reaction rates differently. The resulting rates might not (and in fact they do not) recover the Debye screening.

To generalize the existing fluctuation approach by including nuclear reactions it is necessary to introduce a reaction rate probability $w_{ij}(\mathbf{p}, \mathbf{p}')$, for a nuclear reaction between nuclei of species i and j, with momentum $\mathbf{p}$ and $\mathbf{p}'$, respectively, which describes the nuclear reaction in presence of an arbitrary external fluctuating potential $\delta\phi$. This probability itself should be found by separate consideration. The starting equation for generalizing (1) is then:

$$\frac{\partial f_i(\mathbf{p})}{\partial t} + \mathbf{v}\cdot\frac{\partial f_i(\mathbf{p})}{\partial \mathbf{r}} - Z_i e(\nabla\delta\phi)\cdot\frac{\partial f_i(\mathbf{p})}{\partial \mathbf{p}} = - \int \frac{d\mathbf{p}'}{(2\pi)^3} w_{ij}(\mathbf{p}, \mathbf{p}') f_i(\mathbf{p}) f_j(\mathbf{p}') \tag{3}$$

In the same manner one finds from (3) the system of two equations for the average and the fluctuating part of the particle distribution function. The rate of change in the dressed nuclei distribution function, averaged over fluctuations, describes the nuclear reaction rate in the plasma. It is known that only dressed plasma particles can be in thermal equilibrium and their distribution is then the Maxwell distribution (this can be seen explicitly from Landau-Balescu collision integral). The concept of dressed plasma particles, which is directly related to the concept of macroscopic mass renormalization (Tsytovich 1962), is central also to the theory of emission and scattering in plasmas, the basic concepts of which are related to transition radiation and transition scattering (Ginzburg & Tsytovich 1994).

For a single plasma nucleon, the effect of fluctuations is that of screening the nucleon's own field, thus forming the self-energy cloud. When two nuclei approach each other and undergo a nuclear reaction, the effect of fluctuations is twofold: screening of the interaction, and generation of the self-energy of the two nuclei; the two effects occur simultaneously. The physical problem is the interference of these inseparable fluctuations and their net effect on nuclear reaction rates.

Thus, the general fluctuation approach can be formulated using (3). The renormalization of the distribution function related to the self-energy (2) can be used to find the constant in (2) and the effect it has on nuclear reaction rates.

The final explicit results can be found applying some assumptions although several of those are related to natural small parameters:

1. The fluctuations are assumed to be weak in the sense that an expansion in the fluctuating potential can be used. In Salpeter's simple approach it corresponds to weak screening. In the fluctuation approach the reaction rate between the nuclei i and the nuclei j in the presence of plasma, R_{ij}, is expressed through the reaction rate of bare nuclei $R_{ij}^{(0)}$ by $R_{ij} = (1 + \Lambda_{ij})R_{ij}^{(0)}$ with the correction $\Lambda_{ij} \ll 1$. For the case of Salpeter's enhancement $\Lambda_{ij} = \Lambda_{ij}^{(\mathrm{S})}$ where $\Lambda_{ij}^{(\mathrm{S})} = \frac{Z_i Z_j e^2}{dT}$, d being the Debye radius and T being the plasma temperature.
2. The number of particles in the Debye sphere N_d is large or $\frac{1}{N_\mathrm{d}} \ll 1$; $N_\mathrm{d} = 4\pi n d^3/3$ (d is the Debye radius). This small parameter can be used to find explicitly the fluctuations δf^C, leading to binary collisions in the absence of nuclear reactions.
3. The nuclear reaction rate is much smaller than the one determined by the fluctuations. The fluctuations can be expressed in the form $\delta f = \delta f^\mathrm{C} + \delta f^\mathrm{N}$ with δf^N induced by nuclear reactions. The small parameter is determined by $\delta f^\mathrm{N} \ll \delta f^\mathrm{C}$. The perturbation approach in this small parameter in (3) leads to the equation describing both the influence of nuclear reactions on binary collisions and the influence of fluctuations related to Coulomb collisions on nuclear reactions. By integrating of the averaged equations over all momenta of reacting nuclei one excludes the effects which conserve the number of particles such as described by δf^N and one obtains the change in reaction rates of dressed particles.

4. The fluctuation time is much larger than the characteristic time of crossing the potential barrier, and the spatial length of the fluctuations is much larger than the characteristic nucleon size (the distance between the turning points in crossing the potential barrier).

For these conditions the quantum theory of quasi-classical tunneling leads to the expression for the probability of nuclear reaction as a function of fluctuating potential

$$w_{ij} = w_{ij}^{(0)}(E_{\rm r} - e(Z_i + Z_j)\delta\phi) \quad (4)$$

where $E_{\rm r}$ is the relative energy of two nuclei and $w_{ij}^{(0)}$ is the probability in the absence of a fluctuating potential. In this approximation, the fluctuating potential is almost constant during tunneling, and the constant shift in energy described by (4) is similar to that used in (Salpeter 1954).

3. Cancellation of static corrections

The weak correction expansion of the probability (4) in the fluctuating potential is determined by two first order terms in the right hand side of the equation for Φ_i (obtained from (3) by the averaging procedure) and yields

$$\begin{aligned} & w_{ij}^{(0)} \left\langle \exp\left(\frac{(Z_i + Z_j)e\delta\phi}{T}\right) f_i f_j \right\rangle \\ & \approx w_{ij}^{(0)} \Bigg[\Phi_i\Phi_j + \langle \delta f_i \delta f_j \rangle \\ & \quad + \frac{(Z_i+Z_j)e}{T}\left(\langle\delta\phi\delta f_i\rangle \Phi_j + \langle\delta\phi\delta f_j\rangle \Phi_i\right) \\ & \quad + \frac{(Z_i+Z_j)^2e^2}{2T^2}\left\langle(\delta\phi)^2\right\rangle \Phi_i\Phi_j \Bigg] \end{aligned} \quad (5)$$

The relevant averages $\langle \delta f_i \delta f_j\rangle$, $\langle\delta\phi\delta f_i\rangle$, $\langle\delta\phi\delta f_j\rangle$ and $\langle(\delta\phi)^2\rangle$ entering (2) are obtained from the fluctuation theory used for the derivation of the binary collision integral (see (Tsytovich 1965)). Although these fluctuations are time-dependent dynamic fluctuations, several final expressions contain the static dielectric permittivity (this is found by integration with respect to frequencies using the fluctuation-dissipation theorem).

Terms proportional to Z_iZ_j are related to screening of the interaction and the correspondent corrections will be denoted as $\Lambda_{ij}^{\rm int}$. For these corrections, in the case one does not account for the fluctuations related with particle distributions (counting only the last term in (5)), thus only accounting for the effect of fluctuations on probability of nuclear reactions, one gets Salpeter's static corrections. The same result is obtained in the opposite case, i.e. on considering only the fluctuations of distribution function (the part of second term in (5) proportional to Z_iZ_j). One gets then again Salpeter's static corrections. This result coincides with the one indicated previously in (Gruzinov 1998) for static fluctuations, while here the fluctuations describe dynamically screened particles (this term gives the result coinciding with (Gruzinov 1998), this coincidence seems to be occasional). The interference of these fluctuations (the other two terms in (5) proportional to Z_iZ_j) was not previously considered in any form, and this interference causes the cancellation of the whole correction due to the screening of the interactions. The corresponding expression for correction $\Lambda_{ij}^{\rm int}$ is then

$$\begin{aligned} \Lambda_{ij}^{\rm int} &= \int \frac{e^2 d\mathbf{k}}{2\pi^2 T k^2}\left(1 - \frac{1}{\epsilon_{\mathbf{k},0}}\right) \\ & \quad \times \left\{\frac{1}{2}(2Z_iZ_j) + Z_iZ_j - (2Z_iZ_j)\right\} \\ &= \Lambda_{ij}^{(S)} + \Lambda_{ij}^{(S)} - 2\Lambda_{ij}^{(S)} = 0 \end{aligned} \quad (6)$$

with $\epsilon_{\mathbf{k},\mathbf{0}}$ being the static limit of the plasma dielectric permittivity $\epsilon_{\mathbf{k},\omega}$. The interference *cancels Salpeter's correction exactly yielding a zero net result for the interaction term corrections*. This result is obtained by collecting all Z_iZ_j terms in (5). All these terms contain only the static dielectric permittivity.

There exist also the self-energy terms in expansion (5). One finds that the part containing the static dielectric function $\epsilon_{\mathbf{k},0}$ also cancels; however such a cancellation is independent of the one occurring for screening of interactions. To treat the self-energy (2) one needs, as usual in kinetic theory, to renormalize the distribution function. The physical meaning of such a renormalization is related to the fact that only the particles with their self-clouds are stationary and can be described by thermal distributions. By using the renormalization one will finally deal with the usual distribution functions not containing the particle self-energy (2). The renormalization operated by a perturbation method also leads to an expression for the free energy per particle, $F_i(\mathbf{v})$, related to the self-energy (2): $F_i = \int F_i(\mathbf{v})\Phi_i d\mathbf{p}/n_i(2\pi)^3$ where

$$F_i(\mathbf{v}) = -\frac{4\pi Z_i^2 e^2}{(2\pi)^3}\int\frac{d\mathbf{k}}{k^2}\left[\omega^2\frac{\partial}{\partial\omega}\frac{1}{\omega}\left(\frac{1}{\epsilon_{\mathbf{k},\omega}} - \frac{1}{\epsilon_{\mathbf{k},0}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}} \quad (7)$$

This expression has been derived directly (Tsytovich & Bonatici 2000) by renormalization procedure in the limit of thermal equilibrium (no additional assumptions except smallness of $1/N_d$ have been made). It gives an explicit value for the constant in expression (2). The free energy is described by an expression which accounts for dynamic screening, and is referred to the energy of the particle which is at rest. Thus with increasing particle velocity *the free energy decreases*. One can interpret the constant in free energy as the energy stored in a heath bath (see (Bruggen & Gouch 1997)). It depends on thermal particle distribution entering directly, and through the dielectric permittivity, in (7). The renormalization procedure is valid for non-equilibrium systems and describes the effects of self-energy for the case where the free energy cannot by introduced. Upon collecting all terms containing the static screening from both fluctuations and renormalization, one finds complete cancellation of the interaction energy corrections, Eq. (6), and the static part of self-energy corrections, $\Lambda_{ij}^{\rm self,0}$, i.e.,

$$\begin{aligned} \Lambda_{ij}^{\rm int} + \Lambda_{ij}^{\rm self,0} &= \frac{e^2}{2\pi^2 T}\int\frac{d\mathbf{k}}{k^2}\left(1 - \frac{1}{\epsilon_{\mathbf{k},0}}\right) \\ & \times\left\{Z_iZ_j + \frac{1}{2}(Z_i^2 + Z_j^2) - (Z_i+Z_j)^2 + \frac{1}{2}(Z_i+Z_j)^2\right\} \\ &= 0. \end{aligned} \quad (8)$$

This cancellation resolves the controversy of dynamic and static screening since the remaining free energy terms are related with dynamic screening only and can be obtained from test particle approach

4. Net effect of influence of plasma on the rates of nuclear reactions

As for the nonzero contributions one obtains, to lowest significant order in $T/E^{\rm G}_{ij}$ ($\ll 1$), $E^{\rm G}_{ij}$ being the Gamow energy, for the reaction between the nuclei i and j (see (Tsytovich Bornatici 2000) where also the next order terms are calculated),

$$\Lambda_{ij} = \frac{e^2}{4\pi^2 T W^{(0)}} \int d\mathbf{p} d\mathbf{p}' w^{(0)}_{ij}(\mathbf{p},\mathbf{p}')\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}') \times \int \frac{d\mathbf{k}}{k^2}\left\{ Z_i^2 \left[\omega^2 \frac{\partial}{\partial\omega}\frac{1}{\omega}\left(1-\frac{1}{\epsilon_{\mathbf{k},\omega}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}} + Z_j^2 \left[\omega^2 \frac{\partial}{\partial\omega}\frac{1}{\omega}\left(1-\frac{1}{\epsilon_{\mathbf{k},\omega}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}'}\right\} \quad (9)$$

where

$$W^{(0)} = \int d\mathbf{p} d\mathbf{p}' w^{(0)}_{ij}(\mathbf{p},\mathbf{p}')\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}'). \quad (10)$$

Assuming the relative energy of the reacting nuclei as peaked at the Gamow energy, one can carry out the integration with respect to the relative energy by taking the corrections at the Gamow energy peak. Thus, one is left with the integration with respect to the center of mass motion and gets, for thermal distributions,

$$\Lambda_{ij} = -\frac{e^2}{2\sqrt{\pi} T d}\int_{-1}^{1} dx \int_{-1}^{1} dz \int_0^\infty dy y^2 \exp(-y^2) \times \left\{ Z_i^2 \frac{\sum_\alpha \frac{1}{d_\alpha^2}(2s^2_{\alpha,i}W(s_{\alpha,i})+1)}{\sqrt{\left(\sum_\alpha \frac{1}{d_\alpha^2}W(s_{\alpha,i})\right)\left(\sum_\alpha \frac{1}{d_\alpha^2}\right)}} + Z_j^2 \frac{\sum_\alpha \frac{1}{d_\alpha^2}(2s^2_{\alpha,j}W(s_{\alpha,j})+1)}{\sqrt{\left(\sum_\alpha \frac{1}{d_\alpha^2}W(s_{\alpha,j})\right)\left(\sum_\alpha \frac{1}{d_\alpha^2}\right)}} \right\} \quad (11)$$

where the sum over α includes both electrons and all ion species of the plasma, $\alpha = \{e, i..j..\}$; $W(s) = 1 + s\exp(-s^2)\left(i\sqrt{\pi} - 2\int_0^s \exp(t^2)dt\right)$ is the plasma dispersion function, x and z are, respectively, the cosine of the angle between $\mathbf{k}$ and the center of mass velocity $\mathbf{V}$, and between $\mathbf{k}$ and the relative velocity $\mathbf{v}_{\rm r} = \mathbf{v} - \mathbf{v}'$; $s_{\alpha,i} = \sqrt{m_\alpha/(m_i+m_j)}\left(yx + \lambda_{ij} z m_j/(m_i+m_j)\right)$; $s_{\alpha,j} = \sqrt{m_\alpha/(m_i+m_j)}\left(yx - \lambda_{ij} z m_i/(m_i+m_j)\right)$ where y is the normalized velocity of the center of mass, λ_{ij} is the normalized Gamow velocity and d is the total Debye radius, $y = V/\sqrt{2T/(m_i+m_j)}$; $\lambda_{ij} = v^{\rm G}_{ij}\sqrt{m_i+m_j}/2T$; $E^{\rm G}_{ij} = (v^{\rm G}_{ij})^2 m_i m_j/2(m_i+m_j)$; $1/d^2 = \sum_\alpha (1/d_\alpha^2)$.

By separating the contribution of electrons from the contributions of ions and by using the quasi-neutrality condition, the result (11) can be rewritten in a form containing only the sum over the ions, and be expressed in terms of their relative mass abundances $X_{j'} = m_{j'}n_{j'}/\sum_{j'} m_{j'}n_{j'}$

Table 1. Thermonuclear reactions relevant to the solar plasma. The numerical results refer to the normalized Gamow velocity, λ_{ij}, the corrections to the thermonuclear reaction rates due to plasma fluctuations, Λ_{ij}, Salpeter's enhancement, $\Lambda^{\rm S}_{ij}$, the factor by which the present reaction rates should be divided, F_{ij}, and the percentage deviation, $F_{ij} - 1$

Reaction	λ_{ij}	Λ_{ij}	$\Lambda^{\rm S}_{ij}$	F_{ij}	$F_{ij}-1$
$\rm p + p$	4.280	-0.0510	$+0.05$	1.106	10.6%
$\rm p+\,^2H$	4.757	-0.0514	$+0.05$	1.107	10.7%
$\rm ^3He+\,^3He$	8.150	-0.186	$+0.20$	1.223	22.3%
$\rm ^3He+\,^4He$	8.420	-0.190	$+0.2$	1.223	22.3%
$\rm ^7Li + p$	10.234	-0.268	$+0.15$	1.571	57.1%
$\rm ^7Be + p$	11.264	-0.458	$+0.2$	2.087	108.7%
$\rm ^7Be + e$	....	-0.567	$+0.2$	2.166	116.6%

5. Numerical results for the fusion reactions in the solar interior

The standard solar model gives the abundances of different ions for the central part of solar interior (Bahcall 1989). The following values for mass abundances are used in numerical computations: $X_{\rm H} = 0.3411, X_{\rm He} = 0.6387, X_{\rm C} = 0.00003, X_{\rm N} = 0.0063, X_{\rm O} = 0.0085$. The temperature and the density are needed only to calculate e^2/dT. A temperature $T = 1.5{\rm keV}$ and a density $n = 5\,10^{25}{\rm cm}^{-3}$ yields $e^2/Td = 0.05$, which amounts to Salpeter's corrections of 5% for the $\rm p, p$ reaction. The values of the parameter λ_{ij} are calculated from the values of Gamow energies for the most important reactions in the hydrogen chain. For each reaction the values of Λ_{ij} has been calculated by using (11). The corrections due to the terms next order in parameter $T/E^{\rm G}_{ij}$ are added to the values obtained from expresion (11). Their contribution vary (first to last line of the Table 1) from 10% to 2% to the value obtained from (11). In addition, it is calculated the Salpeter enhancement factor $\Lambda^{\rm S}_{ij}$ from the standard Salpeter's theory for static screening, along with the factor $F_{ij} = (1+\Lambda^{\rm S}_{ij})/(1+\Lambda_{ij})$ by which one should divide the reaction rates previously found on the basis of Salpeter's enhancement factor to obtain the rates according to the present approach. Table 1 summarizes the result of the numerical analysis.

For the $\rm ^7Be{+}e$ reaction, electrons are attracted by the nuclei and there is no Coulomb barrier. The electron wave function is changed in the vicinity of the nuclei and differs from a plane wave function valid for large distances from the nuclei. This gives an enhancement of the capture rate of electrons which was first calculated in (Bahcall 1962). The corrections given in Table 1 are relative to the expression taking into account Bahcall's enhancement factor. This factor was included both in the numerator and in the denominator of expression (9) (see expression (10)). Presence of this factor in the electron capture from the bound states changes the result by less than 0.1%. Although the electrons are attracted the cancellation of static contributions appears also due to possibility of using the expansion in the fluctuating potential since the energy of electrons when they react is much larger than their thermal energy. The last line of Table 1 was calculated by using necessary modifications in the expression (11).

The results of the last two lines of Table 1 are only marginally consistent with the approximation of weak influence of fluctuations on the rates of nuclear reactions (in fact, 0.5 is not a small number). Calculation of the next order terms in Λ_{ij} is desirable but need rather big analytical and computational work. Estimates give corrections of the order of 25% to the already calculated value Λ_{ij}. It is also desirable to find the next order corrections in the parameter $1/N_{\rm d}$ since in the solar interior $1/N_{\rm d} \approx 1/5$, but this is an even more complicated problem since fluctuations due to simple Coulomb interactions have not yet been derived properly for the terms of next order in $1/N_{\rm d}$. The corrections obtained here are larger for higher Z of the nuclei.

The decrease of the rate due to plasma fluctuations has to do with the fact that for overcoming the Coulomb barrier the nuclei should have a large kinetic energy, but the free energy (7) decreases with increasing nucleon velocity, with the result that the efficiency of barrier penetration will be reduced.

With reference to the solar neutrino problem (Kirsten 1999), the significant decrease of the rate of both proton and electron capture by $^{7}\mathrm{Be}$ shown in Table 1 is the most relevant result.

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