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Strong Stokes V asymmetries of photospheric spectral lines: What can they tell us about the magnetic field structure?

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Abstract. In an attempt to identify the mechanism responsible for the extremely asymmetric Stokes V profiles which were recently observed we analyzed several simple atmospheric configurations with separated layers of mass flow and magnetic field. We found that under appropriate conditions the models are capable of producing the observed one-lobe profiles.

Key words: line: formation – line: profiles – polarization – radiative transfer – Sun: magnetic fields

1. Introduction

The properties of polarized light are fully described by the four Stokes parameters I, V, Q, and U which, in turn, are determined by the atmosphere's thermal and magnetic structure. Thus a comparison of observed with computed Stokes parameters as function of wavelength ('Stokes profiles') allows to draw conclusions regarding the structure of the atmosphere.

Stokes V profiles of many photospheric lines arising from solar network and plage regions measured with low spatial resolution with the Fourier Transform Spectrometer (FTS) at NSO/Kitt Peak revealed systematic asymmetries between the areas and amplitudes of the two lobes of the Stokes V profiles (Stenflo et al. 1984; for an overview, see Solanki 1993). Such asymmetries arise if gradients of the line-of-sight components of both magnetic field and flow velocity are present (Illing et al. 1975). The absence of a significant wavelength shift of the Stokes V zero crossing in the FTS data (Solanki 1986) may be interpreted as due to spatially separated regions of magnetic field and flow along the line of sight. Such a configuration occurs at the periphery of a vertically oriented magnetic flux tube: the field lines fan out with height owing to the decreasing external gas pressure forming a 'magnetic canopy', i.e., a layer of more or less inclined magnetic field on top of a field-free region. If the flux tube is surrounded by a convective downdraft, a vertical ray penetrating the canopy and the deeper layers encounters strong gradients where the field strength drops rather abruptly to zero and, assuming the velocity *within* the magnetic region to be insignificant, the velocity rises abruptly at the same position. Grossmann-Doerth et al. (1988) have shown that the Stokes V profile of such a ray is significantly asymmetric with no zero-crossing shift.

Stokes V profiles calculated on the basis of flux tube models with external down-flow agree with the FTS data as regards sign and magnitude of area asymmetry. However, oscillatory motions within the flux tube are additionally required in order to reproduce the observed ratio between area and amplitude asymmetries (Grossmann-Doerth et al. 1989; Solanki 1989). The presence of significant motions *within* the magnetized plasma has also been inferred from the broad wings of observed Stokes V profiles (Solanki 1986). Moreover, such motions normally occur in numerical MHD simulations of flux tubes (Steiner et al. 1998). These results have been corroborated by polarimetric observations with spatial resolution of about $1''$ (Grossmann-Doerth et al. 1996; Martínez Pillet et al. 1997), which yielded Stokes V profiles with wide ranges of asymmetry and zero-crossing shift values. As a consequence, the absence of a Stokes V zero-crossing shift in the FTS data has to be explained as an averaging effect.

Recently, Sigwarth et al. (1999; see also Sigwarth 1999) obtained Stokes profiles with about $0.8''$ spatial resolution from a 'quiet' region and from a region of emerging magnetic flux. The authors found a significant fraction of Stokes V profiles with such strong asymmetry that they consist of virtually one lobe only (the extreme case of 'Type-1 Profiles' reported by Sanchez-Almeida et al. 1996). Is it possible that such profiles are formed by the 'canopy mechanism' as described above or are additional effects required, like the superposition of profiles arising from mixed polarity in the resolution element (Rüedi et al. 1992)? Or is magnetic micro-structuring of the atmosphere a prerequisite to the formation of such 'abnormal' Stokes profiles, as claimed by Sánchez Almeida & Lites (1999)? We have studied a number of simple models in an attempt to resolve this problem.

Sect. 2 summarizes the observational evidence for extremely asymmetric Stokes V profiles. In Sect. 3 we investigate the conditions for the formation of one-lobe profiles on the basis of calculated Stokes V profiles for various idealized configurations

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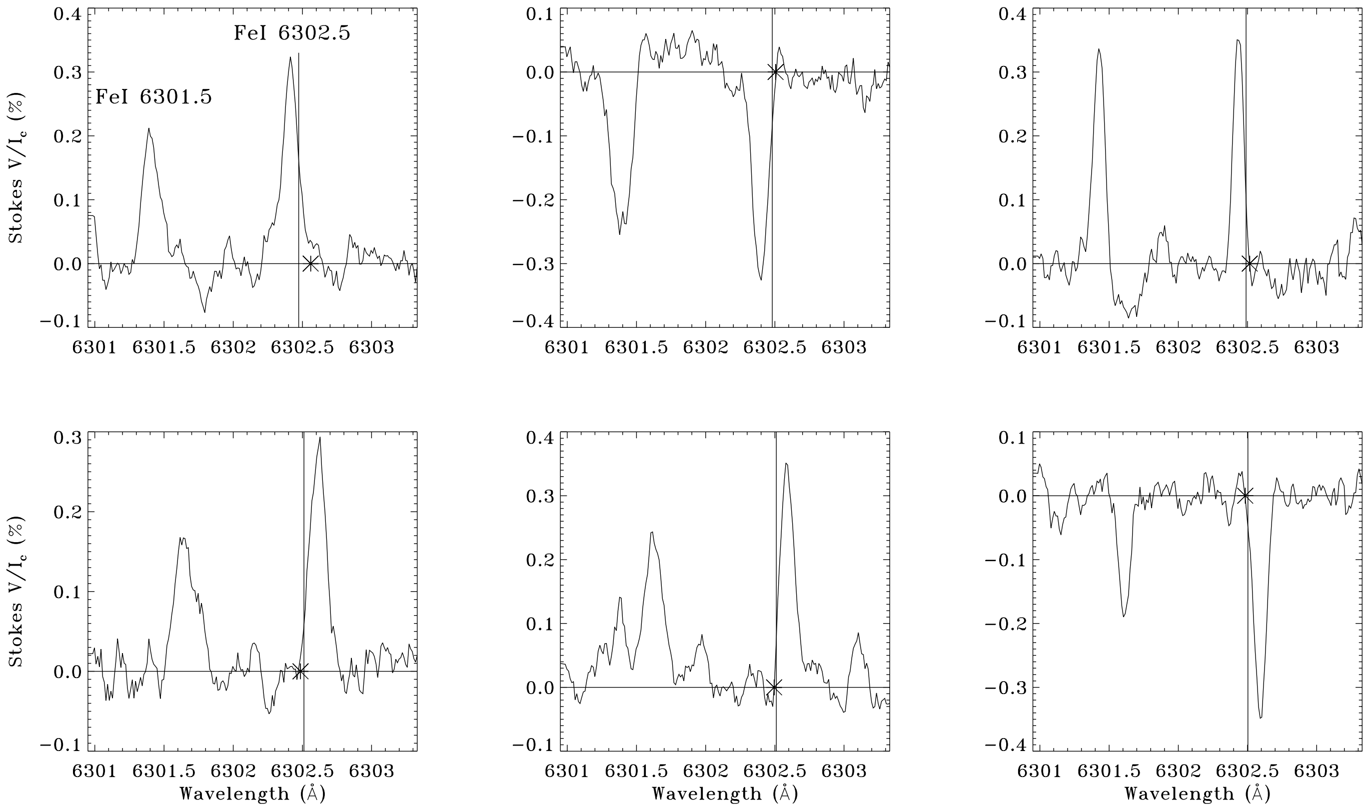

Fig. 1. Examples of observed one-lobe Stokes V profiles. Profiles with blue lobe only are shown in the upper, those with red lobe only in the lower row. Vertical solid lines mark the wavelength of the center of Stokes I and stars mark the Stokes V zero-crossing position (determined by extrapolation).

with discontinuities in magnetic field and velocity. We discuss the results in Sect. 4 and present our conclusions in Sect. 5.

2. Observations

The observational data we refer to were obtained in 1996 with the HAO/NSO Advanced Stokes Polarimeter (ASP; Elmore et al. 1992) at the NSO/Sacramento Peak Richard B. Dunn Solar Telescope. The data consist of high resolution Stokes I, Q, U, and V spectra of Fe I 6301.5 Å and Fe I 6302.5 Å from quiet regions at disc center. The RMS noise of Stokes V/I_c in the continuum is $< 3 \cdot 10^{-4}$, so that it is possible to analyze very weak signals with amplitudes down to 0.15%. A detailed description of the observations as well as an account of data reduction and analysis is given in Sigwarth (1999) and Sigwarth et al. (1999). About 10% of all analyzed Stokes V profiles of FeI 6302.5 have an ‘abnormal’ shape that deviates strongly from the nearly antisymmetric form found in low-resolution observations.

About 35% (1051 spectra) of the abnormal Stokes V profiles are of the *one-lobe* type, i.e., they have virtually one lobe only. In 835 cases the lobe is at the blue side (*blue-only* type) and in 216 cases at the red side (*red-only* type) of the profile with the opposite lobe virtually absent. Fig. 1 shows a few Stokes V profiles of this type. Most of the one-lobe Stokes V profiles are not accompanied by significant linear polarization (Stokes Q and U), so that the magnetic field at these positions is most likely not strongly inclined with respect to the vertical.

3. Synthetic Stokes profiles emerging from configurations with discontinuous spatial variations of magnetic field and velocity

One-lobe Stokes V profiles comprise about 3% of the total number of measured profiles in our data set. Thus they cannot be dismissed as extremely rare exceptions. On the other hand, neither simplified models of vertical magnetic flux tubes (e.g., Grossmann-Doerth et al. 1989) nor numerical MHD simulations of magnetic flux slabs (e.g. Steiner et al. 1998) yield such extremely asymmetric, let alone one-lobe Stokes V profiles as averages over the whole magnetic structure. We have therefore considered several simplified configurations in order to investigate under which conditions highly asymmetric Stokes V profiles could be formed. Having learned that gradual variations of magnetic field and velocity do not give rise to very large asymmetries we restricted our analysis to models with discontinuous variations of both these quantities.

3.1. Magnetic canopy

This model consists of a quiet sun atmosphere divided into an upper part in which a magnetic field is present but no bulk ve-

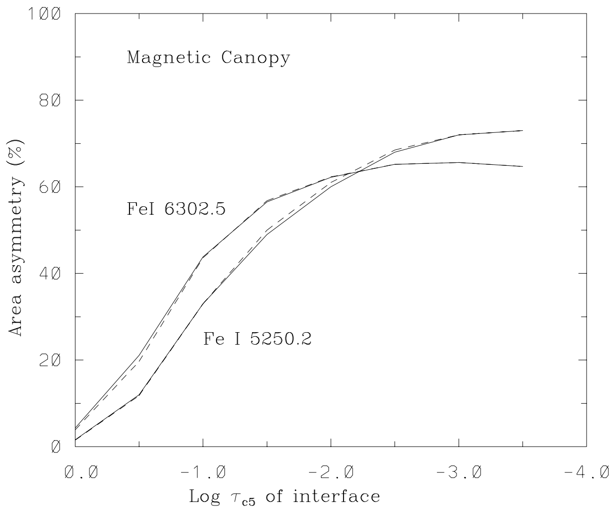

Fig. 2. Stokes V area asymmetry of the spectral lines Fe I 5250.2 and Fe I 6302.5 emerging from a magnetic canopy configuration as function of interface optical depth for 1500 G and 3 km s^{-1}. Solid and dashed lines represent 30° and 60° field inclination with respect to the vertical, respectively.

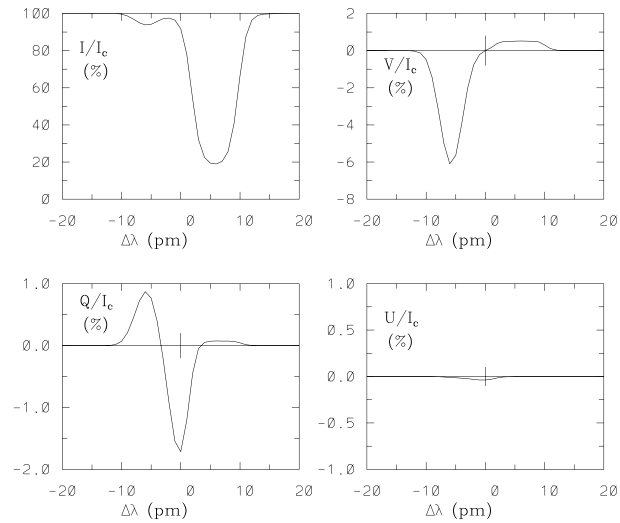

Fig. 3. Stokes parameters of Fe I 5250.2 emerging from a quiet sun magnetic canopy configuration with 1500 G, 30°, 3.3 km s^{-1} and the interface at $\log \tau_{c5} = -3.5$. The Stokes V area asymmetry is 71.3 %.

locity and a lower part with no magnetic field but a finite velocity. Both magnetic field strength and velocity are uniform in their respective regions. We computed the Stokes parameters of the spectral lines Fe I 5250.2 and Fe I 6302.5 for rays emerging from this atmosphere with a magnetic field strength of 1500 G and a velocity of 3 km s^{-1} (positive velocity corresponds to down-flow). These values were chosen in order to achieve approximate equality of Doppler- and Zeeman shift, thereby maximizing the asymmetry (Grossmann-Doerth et al. 1988). The area asymmetry of Stokes V was determined as function of position of the interface. The results are shown in Fig. 2; solid and dashed lines represent magnetic field inclinations with respect to the vertical of 30° and 60°, respectively. We use the common definition of the Stokes V asymmetry, i.e. $\delta a = (A_{V,blue} - A_{V,red})/(A_{V,blue} + A_{V,red})$ where A_V is the (absolute) value of the (blue or red) Stokes V amplitude or lobe area depending on whether amplitude or area asymmetry is meant. Note that the behaviour of both lines is quite similar and the inclination angle of the field has almost no influence on the asymmetry.

The Stokes V area asymmetry increases with interface height up to values around $\log \tau_{c5} = -3.5$ where it reaches about 71 % for Fe I 5250 and 67 % for Fe I 6302 (τ_{c5} is the optical depth of the continuous radiation at 5000 Å). Fig. 3 shows the four Stokes parameters of the Fe I 5250.2 line for the case of maximum asymmetry.

In order to understand the rapid rise of the Stokes V asymmetry with height of canopy interface we analyzed the Unno-Rachkovsky equations for a canopy configuration with *vertical* magnetic field because then the equations decouple and allow a simple solution for the right- and left-hand circularly polarized radiation and thus for Stokes V. This analysis, valid for a standard quiet solar atmosphere, is presented in the Appendix. It shows that, when growing, the asymmetry is increasingly governed by the intensity difference of the (unpolarized) radiation entering the magnetic layer at the wavelength positions of the centers of both Zeeman shifted absorption coefficients. The difference is growing with interface height since the wavelength position of the Doppler shifted absorption line formed in the non-magnetic layer coincides with one of the Zeeman components and, therefore, the asymmetry increases with decreasing line intensity. This also explains why the limiting asymmetry of Fe I 6302 is smaller than that of Fe I 5250: the former line is not as strong as the latter, so in the case of Fe I 6302 the intensity difference is smaller than in the case of the other line. It should be mentioned though, that – owing to the decreasing thickness of the magnetic layer – the amplitude of the remaining lobe of Stokes V decreases from about 40 % to a few percent in the same range in which the asymmetry increases from a few percent to more than 70 %.

So far we have considered Stokes V profiles for single rays corresponding to infinite spatial resolution. In a real observation features much smaller than about 1″ cannot be resolved. Hence, our computational results should be spatially averaged in order to make them comparable with observations. We assume the (inclined) canopy interface to run parallel to the magnetic field. Fig. 4 shows the average Stokes V profile of Fe I 5250.2, i.e. the arithmetic mean of 30 rays placed equidistantly in $\log \tau_{c5}$ from -4 to -0.5 of the canopy interface, corresponding to a horizontal interval of about 300 km. Note that the amplitude of the averaged Stokes V profile no longer is small but neither is the asymmetry still very high.

We also investigated a similar configuration, consisting of a low magnetic layer superposed by a non-magnetic region. We found the properties of Stokes V to be similar to those of the magnetic canopy: The asymmetry grows with decreasing thickness of the magnetic layer , i.e. with increasing depth of the interface position; it reaches values of about 80 % in the

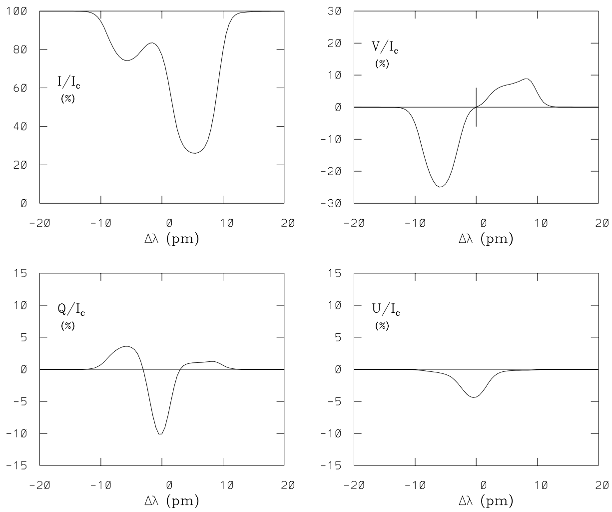

Fig. 4. Spatially averaged Stokes parameters of Fe I 5250.2 emerging from a quiet sun magnetic canopy configuration with 1500 G, 30°, 3 $\mathrm{km\,s^{-1}}$. The interface position varied linearly from -4 to -0.5 of $\log \tau_{c5}$. The Stokes V area asymmetry is 40 %.

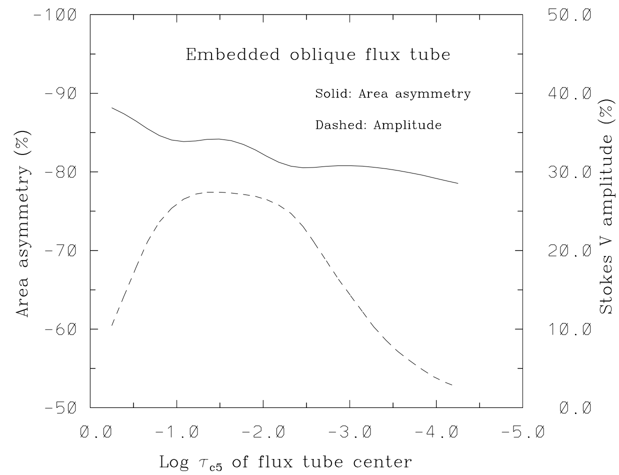

Fig. 5. Stokes V area asymmetry and amplitude of Fe I 5250.2 for a ray penetrating an embedded magnetic flux tube as function of optical depth of flux tube center. Quiet sun atmosphere with 1500 G, 30°, 3 $\mathrm{km\,s^{-1}}$ and tube width 0.5 in $\log \tau_{c5}$.

limit when the magnetic layer, this time deep in the atmosphere, becomes very thin. This configuration may represent the initial phase of magnetic flux emergence. However, in view of the short duration of this phase we do not believe that this configuration is frequently observed so for the sake of brevity we omit the details of this analysis.

3.2. *Embedded magnetic flux tube*

Next let us consider a magnetic flux tube of finite width embedded in a quiet solar atmosphere and inclined with respect to the vertical. We assume a velocity to be present *within* the tube and no flow in the external atmosphere. The flow is necessarily along the magnetic field, for us only the (vertical) line-of-sight component is relevant. Both magnetic field strength and velocity within the flux tube are supposed to be uniform. It should be noted, though, that since we are dealing with 2D-configurations this flux tube is, strictly speaking, a flux *sheet*. The configuration differs from those previously considered by the fact that *three* regions contribute to the resulting Stokes parameters: The region below the sheet, the sheet itself and the region above the sheet. We computed the profiles of the Stokes parameters of a single ray emerging from the atmosphere for a set of flux tube positions. Fig. 5 shows both area asymmetry and amplitude of Stokes V of Fe I 5250.2 for 1500 G, 30° and 3 $\mathrm{km\,s^{-1}}$ as function of optical depth of the center of the tube, whose thickness was chosen 0.5 in $\log \tau_{c5}$. It will be noted that unlike in the previously studied configurations the area asymmetry is fairly independent of the tube position and stays at high values. The amplitude of the intact (blue) lobe of Stokes V, on the other hand, varies greatly with tube position: its value is small if the tube is high up or deep down in the atmosphere and it assumes large values if the tube is located between -1 and -2 of $\log \tau_{c5}$. This behaviour is easily understood. If high up or deep down the

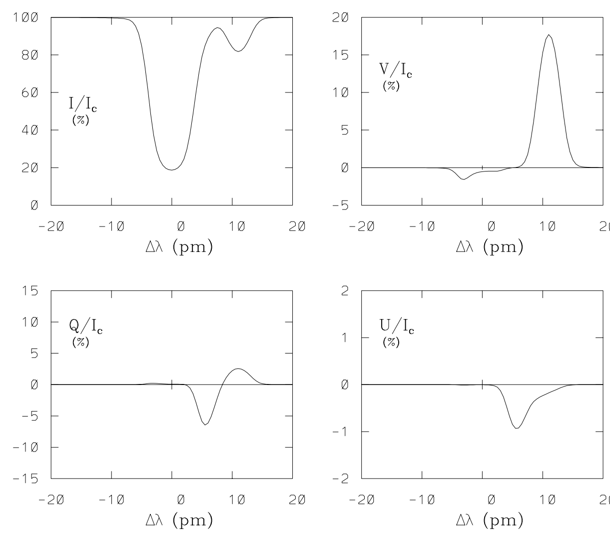

Fig. 6. Spatial averages of Stokes parameters of Fe I 5250.2 of 30 rays penetrating an embedded magnetic flux tube with 1500 G, 30°, 3 $\mathrm{km\,s^{-1}}$ and tube width 0.5 in $\log \tau_{c5}$. The intersection depth of the rays with the upper surface of the flux tube varied linearly from -0.5 to -4.5 of $\log \tau_{c5}$. The Stokes V area asymmetry is -84 % and the zero-crossing shift corresponds to 2.3 $\mathrm{km\,s^{-1}}$.

flux tube corresponds to the configurations discussed before, i.e. thin canopy or deeply embedded magnetic slab. In both cases the amplitude is small because these regions contribute only marginally to the spectral line. The amplitude is fairly large if the flux tube is located in a height range where the contribution function is large.

Again we have to average spatially in order to get a result that could be compared with observations. In Fig. 6 the arithmetic mean of the Stokes parameters of 30 rays are plotted which penetrate the flux tube in the range -4.5 to -0.5 of $\log \tau_{c5}$, equally spaced in $\log \tau_{c5}$ of the sheet's upper interface and corresponding to a horizotal interval of about 300 km. Note that, unlike

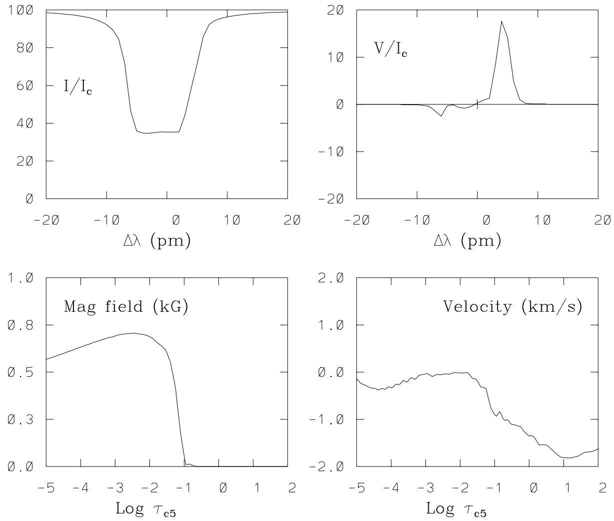

Fig. 7. Stokes I and Stokes V of Fe I 5250.2 of a ray penetrating the canopy area of a numerically simulated (non-stationary) magnetic flux slab at a chosen position and time (upper two panels). The lower two panels show magnetic field strength and velocity as function of $\log \tau_{c5}$. The area asymmetry of Stokes V is -75 %; the value is negative because the velocity below the canopy is negative (up-flow).

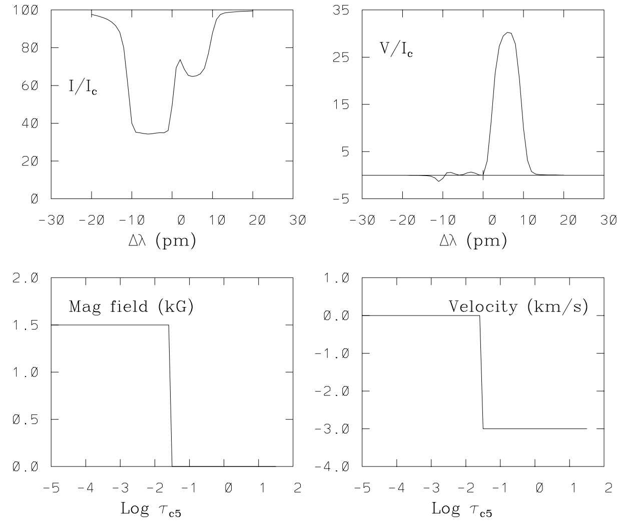

Fig. 9. Stokes I and Stokes V of Fe I 5250.2 emerging from a canopy configuration inclined by 30° and the interface at $\log \tau_{c5} = -1.5$. The lower panels show magnetic field strength and bulk velocity as function of $\log \tau_{c5}$. The thermodynamic structure of the atmosphere is identical with that of the magnetic flux slab model shown in Fig. 7. The Stokes V area asymmetry is -97%.

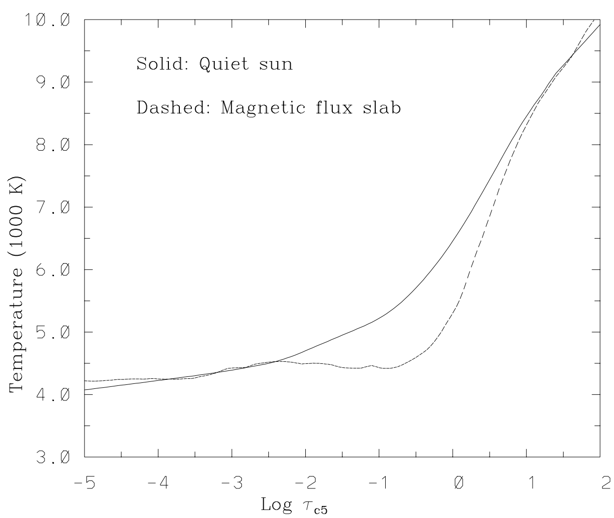

Fig. 8. Run of temperature with $\log \tau_{c5}$ for a quiet sun model and for a ray penetrating the canopy region of a computer simulated (non-stationary) magnetic flux slab at a chosen position and time.

the canopy configuration, the embedded flux tube, even when spatially averaged, gives rise to highly asymmetric Stokes V profiles with fairly large amplitudes.

3.3. The role of the temperature

The results shown above indicate that extremely asymmetric Stokes V profiles arise in a configuration with superposed magnetic and non-magnetic layers provided the contribution of the magnetic layer to the line formation is small, which normally means that its optical thickness must be small. However, in the course of the analysis of magnetic slabs produced by a numerical MHD simulation, we found highly asymmetric Stokes V profiles arising from configurations which seem to contradict these ideas. An example is shown in Fig. 7 where Stokes I and Stokes V of Fe I 5250.2 are plotted together with the run of flow velocity and magnetic field with optical depth. It is seen that we deal with a canopy situation with almost no flow velocity in the upper layer where a magnetic field of about 500 G is present and a field-free region with flow below. The fact that the flow direction is upward instead of downward as one would normally expect is of no relevance in the present context. So we would expect the Stokes V profile to be asymmetric but not as highly asymmetric as 75 % because the interface between both layers is located at about $\log \tau_{c5} = -1.5$. At that level Fig. 2 would predict an asymmetry of about 35 %, only half the actual value. The discrepancy may be resolved when considering the atmospheric temperature profile. The data of Fig. 2 were obtained with an atmospheric model of the quiet sun while the Stokes V profile of Fig. 7 was formed in a magnetic slab atmosphere produced by a numerical simulation. Fig. 8 shows the run of temperature with depth for both atmospheres. Note that unlike in the quiet sun the temperature in the magnetic slab remains almost constant down to about $\log \tau_{c5} = -0.5$ where it begins to rise rapidly. Thus we would expect the spectral line to be already fully developed at this depth and therefore, according to the deliberations of Sect. 3.1, the maximum asymmetry is already reached for that interface position.

The validity of our explanation is demonstrated by Fig. 9 which shows in the upper panels Stokes I and Stokes V of Fe I 5250.2 emerging from a canopy configuration identical to those analyzed in Sect. 3.1 with the interface located at $\log \tau_{c5} = -1.5$. According to Fig. 2 we should expect an asymmetry of about 50 %. Instead we obtained a true one-lobe

Stokes V profile with 97 % area asymmetry solely because we replaced the quiet sun model atmosphere by the slab temperature profile shown in Fig. 8. This example demonstrates that an 'abnormal' temperature profile may enhance the asymmetry of a Stokes V profile considerably allowing a large amplitude at the same time.

Encouraged by this result we repeated the computation of the average Stokes parameters for a magnetic canopy whose results are shown in Fig. 4, replacing the quiet sun temperature profile by the abnormal profile shown in Fig. 8. The result is shown in Fig. 10. Comparing Fig. 4 with Fig. 10 it will be seen that a different temperature profile may more than double the area asymmetry while there is no significant reduction of amplitude relative to the local continuum intensity.

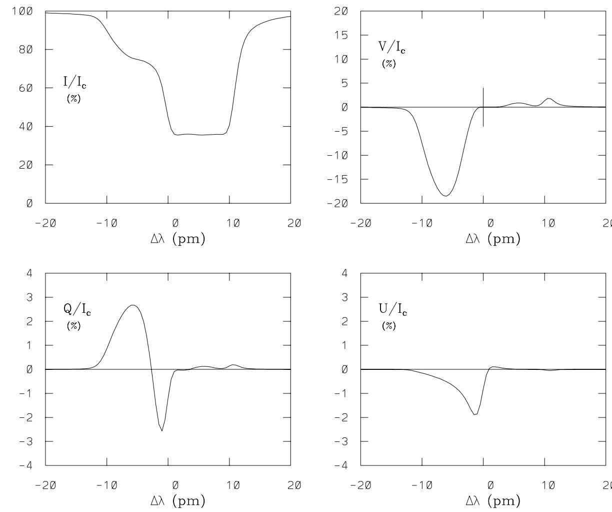

Fig. 10. Spatially averaged Stokes parameters of Fe I 5250.2 emerging from a quiet sun magnetic canopy configuration. Same as Fig. 4 except for the temperature: the quiet sun temperature profile was replaced by the abnormal profile shown in Fig. 8. The Stokes V area asymmetry is 86 %.

4. Discussion

As we have seen in the preceding section, highly asymmetric Stokes V profiles may arise from superposed atmospheric layers with different magnetic and flow properties. The conditions, however, for which large asymmetries and significant amplitudes are obtained are somewhat complex.

Canopy configurations produce very large asymmetries only for unrealistic spatial resolution; the amplitudes of these Stokes V profiles are invariably very small. When spatially averaged to a degree which would make the computed profiles comparable to observations, the Stokes V amplitudes become larger but the asymmetry values fall far below those of a one-lobe type. These statements, however, are only true if the run of temperature with depth corresponds to a quiet sun atmospheric model. A different temperature profile as, for example, that shown in Fig. 8 enhances the asymmetry to a degree which allows the canopy configuration to produce a one-lobe profile.

Embedded flux tubes are more promising because they produce very large asymmetries together with fairly large amplitudes even when spatially averaged *and* with a quiet sun temperature profile.

A vertical magnetic flux tube fans out with height, so that in some small region of the resulting canopy the condition for extreme asymmetry is met. However, for spatially unresolved tubes, the contribution of the central part of the tube dominates the aggregate Stokes V profile, which therefore is only moderately asymmetric. Strong internal velocities and shocks may deform the Stokes V profile considerably but do not lead to one-lobe profiles (Steiner et al. 1998). Interpreting the observed one-lobe profiles by superposed layers therefore requires either very extended canopies, which may form at the periphery of network patches, or inclined flux tubes, perhaps loops with an internal flow. Note, however, that the virtual absence of significant signals in Stokes Q and U indicates that the field cannot be strongly inclined ($\theta \lesssim 30°$ according to our calculations).

Two alternative possibilities come to mind for the origin of extremely asymmetric Stokes V profiles: Mixed magnetic polarity in the resolution element and magnetic micro-structuring of the atmosphere.

Mixed polarity: Superposition of (say, two) modestly asymmetric Stokes V profiles arising from patches of opposite magnetic polarity within the resolution element can lead to strongly asymmetric profiles if the values of internal velocity and Stokes V amplitude are nearly equal (e.g., Rüedi et al. 1992). We would then expect a whole spectrum of strongly asymmetric profiles, including double-humped profiles with two lobes of the same sign. One-lobe profiles would then represent a very special case, because one pair of corresponding lobes of the two superposed profiles has to cancel nearly exactly. We should therefore expect more double-humped than one-lobe profiles if this mechanism were a dominant source of one-lobe profiles. This is not the case, however, because we found 1051 one-lobe profiles in our data set but only 227 double-humped ones. On the other hand, abandoning the assumption of statistically uncorrelated components, it is possible to design small magnetic loops with appropriate velocity fields which, when their foot point regions are spatially averaged, will give rise to one-lobe Stokes V profiles. However, evidence against this scenario, and against the mixed polarity concept in general, is derived from the fact that a similar data set from a region of emerging magnetic flux with much more mixed polarity than the quiet sun data does show a *smaller* fraction of one-lobe profiles (Sigwarth 1999).

Magnetic micro-structuring: Sánchez Almeida et al. (1996) have shown that an atmosphere consisting of an ensemble of optically thin magnetic features embedded in a field-free gas with relative velocities between magnetic and non-magnetic components yields a large variety of asymmetric Stokes profiles (presumably including one-lobe profiles), depending on the choice of model parameters. Such an ensemble may be regarded as generalization of our canopy and flux-tube models insofar as now many interfaces contribute to the total asymmetry, each with a small amount.

5. Conclusions

A recent polarimetric observation of the quiet sun revealed the existence of extremely asymmetric Stokes V profiles. In a search for a mechanism which would reproduce these results we investigated several models where magnetic field and flow velocity are spatially separated with strong gradients at the interface. Gradual variations do not give rise to very high asymmetry. We found that all analyzed models may produce high asymmetries provided that the magnetic region is much smaller in thickness than the non-magnetic one. When spatially averaged, however, only the embedded flux tube model remains capable to produce extreme asymmetry – as long as the temperature profile is similar to that of the quiet sun. Altering the temperature profile may enhance the Stokes V asymmetry considerably. With an appropriate temperature distribution, a spatially averaged magnetic canopy configuration may also produce one-lobe Stokes profiles.

Principally, there are at least two more possible mechanisms which could produce extremely asymmetric Stokes V profiles.

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Appendix A: Stokes V asymmetry in a canopy configuration with vertical magnetic field

In order to understand the behaviour of the Stokes V asymmetry of a photospheric spectral line in a canopy configuration as function of interface height we shall consider the case of a purely vertical magnetic field in the canopy region because then the radiation transport equations of right- and left-circularly polarized light decouple and become identical to the transport equation of unpolarized light (Landi Degl'Innocenti 1992) and may easily be solved. A 'normal' Zeeman effect is assumed.

Let t be the optical depth of the continuous radiation at the frequency of the spectral line, t_1 the value of t at the interface of the canopy with the field-free atmosphere below, I_L and I_R the intensities of the emerging left- and right-circularly polarized radiation, respectively, κ_L and κ_R the absorption coefficients of the respective polarized radiation, $\gamma_p = \kappa_p/\kappa_c$ the ratio of the opacity of polarized radiation to that of the continuous radiation and I_1 the outward directed radiation intensity at the interface. Assuming LTE with $B(t)$ the Planck function, the intensity of the polarized light emerging from the atmosphere for a given wavelength is then represented by

$$2\, I_p(0) = I_1\, e^{-\tau_1} + F(T, \gamma_p) \tag{A.1}$$

with

$$F(T, \gamma_p) = \int_0^{t_1} (1 + \gamma_p)\, B(t)\, e^{-\Delta\tau}\, dt$$

where

$$\tau_1(t_1) = \int_0^{t_1} (1 + \gamma_p)\, dt \quad \text{and} \quad \Delta\tau(t) = \int_0^{t} (1 + \gamma_p)\, dt' \,.$$

The subscript p refers to left- or right-circularly polarized light, i.e. it represents either L or R. Strictly speaking, τ_1 and $\Delta\tau(t)$ should also have the index p; we omit it for better readability.

To derive Stokes V and its asymmetry we shall consider two wavelength points $\lambda^- = \lambda_0 - \Delta\lambda$ and $\lambda^+ = \lambda_0 + \Delta\lambda$, where λ_0 is the wavelength at center of the spectral line. Denoting by the superscripts $-$ and + these wavelength positions we have the four intensities of the polarized radiation emerging from the atmosphere:

$$2\, I_R^+ = I_1^+\, e^{-\tau_{1R}^+} + F_R^+ \;\; ; \; 2\, I_L^+ = I_1^+\, e^{-\tau_{1L}^+} + F_L^+$$

$$2\, I_R^- = I_1^-\, e^{-\tau_{1R}^-} + F_R^- \;\; ; \; 2\, I_L^- = I_1^-\, e^{-\tau_{1L}^-} + F_L^- .$$

The Stokes V parameters at both wavelengths are $V^+ = I_R^+ - I_L^+$ and $V^- = I_R^- - I_L^-$. Because of the symmetry of the Zeeman effect $\kappa_R^+ = \kappa_L^-$ and $\kappa_L^+ = \kappa_R^-$ and therefore $F_R^+ = F_L^-$, $F_R^- = F_L^+$, $\tau_{1R}^+ = \tau_{1L}^-$ and $\tau_{1R}^- = \tau_{1L}^+$. Hence

$$V^- + V^+ = \; 0.5\, (I_1^- - I_1^+)\, (e^{-\tau_{1R}^-} - e^{-\tau_{1L}^-}) \,.$$

Denoting by α the intensity ratio $\alpha = I_1^+/I_1^-$ we obtain

$$V^- + V^+ = \; 0.5\, I_1^-\, (1 - \alpha)\, (e^{-\tau_{1R}^-} - e^{-\tau_{1L}^-})$$

and

$$V^- - V^+ = \; 0.5\, I_1^-\, (1 + \alpha)\, (e^{-\tau_{1R}^-} - e^{-\tau_{1L}^-}) + F_R^- - F_R^+ .$$

For the Stokes V amplitude asymmetry, defined as $\delta a = (V^- + V^+)/(V^- - V^+)$, we thus obtain

$$\delta a = \frac{I_1^-\, (1 - \alpha)\, (e^{-\tau_{1R}^-} - e^{-\tau_{1L}^-})}{I_1^-\, (1 + \alpha)\, (e^{-\tau_{1R}^-} - e^{-\tau_{1L}^-}) + 2F_R^- - 2F_R^+} \,.$$

Assuming the magnetic field to point downwards, $V^- = I_R^- - I_L^-$ is negative, hence $\kappa_R^- > \kappa_L^-$ and $\kappa_R^+ < \kappa_L^+$. Let us further assume that the Zeeman shift is large compared to the width of the spectral line. Then we have $\kappa_R^- = \kappa_L^+ = 0.5\, \kappa_l$ and $\kappa_L^- = \kappa_R^+ \approx 0$ where κ_l denotes the line absorption coefficient. Thus we obtain:

$$\delta a = \frac{1 - \alpha}{1 + \alpha + R(t_1)} \tag{A.2}$$

with

$$R(t_1) = \frac{2\, (F_R^+ - F_R^-)}{I_{c1}\, e^{t_1}\, (1 - e^{-0.5 t_{l1}})}$$

where t_{l1} denotes the line optical depth and I_{c1} the outward directed continuous intensity at the interface. Since $\beta = (1 - \alpha)/(1 + \alpha)$ increases with decreasing α and α decreases with t_1, the asymmetry increases with decreasing t_1 as long as the fraction in (A.2) is dominated by β. To assess the role of $R(t_1)$ let us consider its numerator :

$$F_R^+ - F_R^- = \int_0^{t_1} B(t)\, e^{-t} [1 - (1 + \frac{\kappa_l}{2\kappa_c}) e^{-0.5 t_l}]\, dt$$

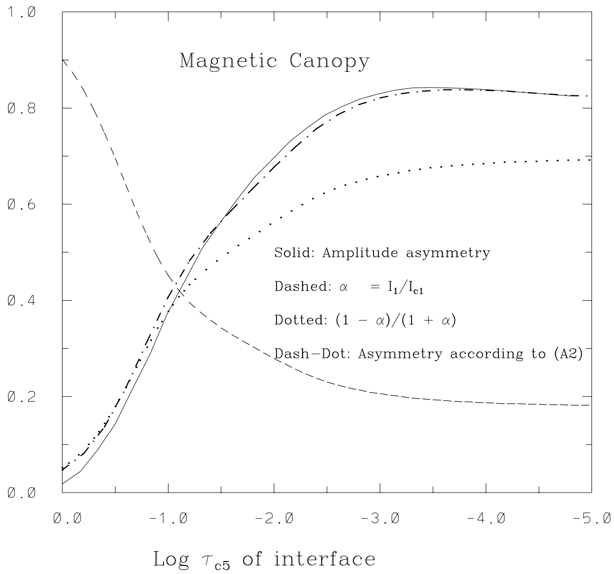

Fig. A.1. Stokes V amplitude asymmetry of Fe I 5250 emerging from a quiet sun canopy configuration with 1500 G and 3.3 km s^{-1} as function of τ_{c5} of canopy interface. Also plotted are $\alpha = I_1/I_{c1}$, $\beta = (1-\alpha)/(1+\alpha)$ and the asymmetry obtained by the evaluation of (A.2).

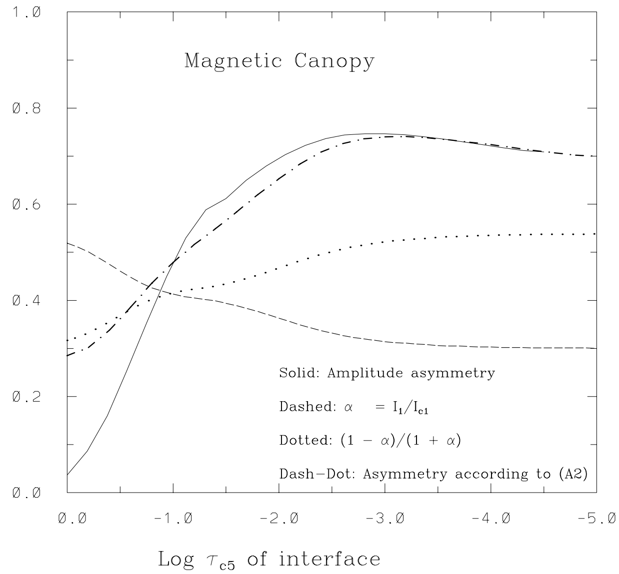

Fig. A.2. The same quantities as in Fig. A.1, for the line Fe I 6302.

which is positive for large and negative for small values of t_1. Since the denominator of $R(t)$ is always positive the sign of $R(t)$ is equal to the sign of the numerator. Thus the asymmetry as function of t_1 is smaller than β for large values of t_1; it becomes equal to β at some intermediate depth whence δa becomes larger than β. The limit of R(t) for vanishing t_1 is:

$$\lim_{t_1\to 0} R(t_1) = -2\,\frac{B(0)}{I_c(0)}\,,$$

hence

$$\lim_{t_1\to 0} \delta a = \frac{1-\alpha(0)}{1+\alpha(0)-2\,B(0)/I_c(0)}\,. \tag{A.3}$$

If Zeeman and Doppler shift are equal, $\alpha(t)$ represents the relative intensity of the spectral line at its center. Further, in a standard model of the quiet solar atmosphere $B(0)/I_c(0) \simeq 0.1$, hence

$$\lim_{t_1\to 0} \delta a \simeq \frac{1-\alpha(0)}{1+\alpha(0)-0.2}\,.$$

Fig. A.1 shows α, β and δa as function of $\log\tau_{c5}$ for the line Fe I 5250.2 and a model of the quiet solar atmosphere, Fig. A.2 the same for the line Fe I 6302. For the Fe I 5250 line the point where β and δa become equal is at about $t_1 = 0.1$ while for the Fe I 6302 line the crossing occurs much deeper in the atmosphere, close to $t_1 = 1$. In both cases the prediction of (A.3) is exactly realized: for the line Fe I 5250.2 $\alpha(0) = 0.182$, $B(0)/I_c(0) = 0.361/3.80 = 0.0947$, hence $(1-\alpha(0))/(1+\alpha(0)-2B(0)/I_c(0)) = 0.824$; as can be seen from Fig. A.1 the limit of the amplitude asymmetry is 82 %. Similarly for the line Fe I 6302, $\alpha(0) = 0.30$, $B(0)/I_c(0) = 0.446/2.97 = 0.15$, hence $(1-\alpha(0))/(1+\alpha(0)-2B(0)/I_c(0)) = 0.70$; Fig. A.2 shows that the limit of the asymmetry is 70 %.

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