

# Angular size in “quintessence” cosmology

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**Abstract.** We investigate the influence of an exotic fluid component (“quintessence”) on the angular size-redshift relation for distant extragalactic sources. Particular emphasis is given for the redshift  $z_m$  at which the angular size takes its minimal value. We derive an analytical closed form which determines how  $z_m$  depends on the parameter of the equation of state describing the exotic component. The results for a flat model dominated by a “quintessence” are compared in detail with the ones for the standard open model dominated by cold dark matter. Some consequences of systematic evolutionary effects on the values of  $z_m$  are also briefly discussed. It is argued that the critical redshift, for all practical purposes, may completely be removed if such effects are taken into account.

**Key words:** cosmology: theory

## 1. Introduction

Recent data from SNe Ia have provided strong evidence for an expanding Universe speeding up, rather than slowing down (Riess et al. 1998; Perlmutter et al. 1998). These observational evidences have stimulated great interest in a more general class of cosmological models driven by nonrelativistic matter and a “quintessence” component, i.e., an exotic fluid with an arbitrary equation of state  $p_x = \omega_x \rho_x$  ( $\omega_x \geq -1$ ), which probably dominates the bulk of matter in the observed Universe. Examples of these models include the evolving scalar field (Ratra & Peebles 1988; Frieman et al. 1998; Caldwell et al. 1998), the smooth noninteracting component (xCDM) (Turner & White 1997; Chiba et al. 1997), and still the frustrated network of topological defects in which  $\omega_x = -\frac{n}{3}$ , being  $n$  the dimension of the defect (Spergel & Pen 1997). Some observational aspects of these models have extensively been analyzed in the literature. For example, Waga & Miceli (1999), combining statistics of gravitational lenses and SNe Ia data have found  $\omega_x < -0.7$  (68% cl) for a spatially flat Universe. Efstathiou (1999), by using high- $z$  Type Ia supernovae and cosmic microwave background anisotropies, has found  $\omega_x < -0.6$  ( $2\sigma$ ) if the Universe is assumed to be spatially flat, or  $\omega_x < -0.4$  ( $2\sigma$ ) for universes of

arbitrary spatial curvature. Perlmutter et al. (1999) constrained  $\omega_x < -0.6$  (95% cl) using large-scale structure and SNe Ia in a spatially flat geometry. However, although carefully investigated in many of their theoretical and observational aspects, the influence of a “quintessence” component in some kinematic tests like the angular size-redshift relation still remains to be analyzed. In principle, the lensing effect of the expanding Universe may provide strong limits on the free parameter describing this exotic component. Therefore, it is interesting to explore how uncertainties in distance measures of extragalactic objects and their underlying evolutionary effects may alter the standard cold dark matter results.

On the other hand, the existing angular size data for distant objects are until nowadays somewhat controversial, specially because they involve at least two kinds of observational difficulties. First, any high redshift object may have a wide range of proper sizes, and, second, evolutionary and selection effects probably are not negligible. Indeed, the  $\Theta(z)$  relation for some extended sources samples seems to be quite incompatible with the predictions of the standard FRW model when the latter effects are not taken into account (Kapahi 1987; 1989). There have also been some claims that the best fit model for the observed distribution of high redshifts extended objects is provided by the standard Einstein-de Sitter universe ( $q_0 = \frac{1}{2}$ ,  $\Omega_\Lambda = 0$ ) with no significant evolution (Buchalter et al. 1998). However, all these results are in contradiction with the recent observations from type Ia supernovae. Indeed, such data seem to rule out world models filled only by baryonic matter, and more generally, any model with positive deceleration parameter. The same happens with the corresponding bounds using the ages of old high redshift galaxies (Dunlop et al. 1996; Krauss 1997; Alcaniz & Lima 1999).

The case for compact radio sources is also of great interest. These objects seem to be less sensitive to evolutionary effects since they are short-lived ( $\sim 10^3$  yr) and much smaller than their host galaxy. Initially, the data from a sample of 82 objects gave remarkable support for the Einstein-de Sitter Universe (Kellerman 1993). However, some analysis suggest that, although compatible with an Einstein-de Sitter Universe, the Kellerman data cannot rule out a significant part of the  $\Omega_M - \Omega_\Lambda$  plane (Kayser 1995). Some authors have also argued that models where  $\Theta(z)$  diminishes and after a given  $z$  remains constant

may also provide a good fit to Kellerman’s data. In particular, by analysing a subset of 59 compact sources within the same sample, Dabrowski et al. (1995) found that no useful bounds on the value of the deceleration parameter  $q_o$  can be derived. Indeed, even considering that Euclidean angular sizes ( $\Theta \sim z^{-1}$ ) are excluded at 99% confidence level, and that the data are consistent with  $q_o = 1/2$ , they apparently do not rule out higher values of the deceleration parameter (Stephanas & Saha 1995). More recently, based in a more complete sample of data, which include the ones originally obtained by Kellermann, it was argued that the  $\Theta(z)$  relation may be consistent with any model of the FRW class with deceleration parameter  $\leq 0.5$  (Gurvits et al. 1999).

In this context, we discuss the influence of a “quintessence” component (Q-model) on the angular size-redshift relation. Particular emphasis is given for the critical redshift at which the angular size of an extragalactic source takes its minimal value. In the limiting case ( $\omega_x = -1$ ), the results previously derived by Krauss & Schramm (1993) for a flat universe with cosmological constant ( $\Lambda$ CDM) are recovered. For comparison, we also consider the case of an open model dominated by nonrelativistic matter (OM).

## 2. Angular size and “quintessence”

Let us now consider the FRW line element ( $c = 1$ )

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)] \quad , \quad (1)$$

where  $\chi$ ,  $\theta$ , and  $\phi$  are dimensionless comoving coordinates,  $R(t)$  is the scale factor, and  $S_k(\chi)$  depends on the curvature parameter ( $k = 0, \pm 1$ ). The later function is defined by one of the following forms:  $S_k(\chi) = \sinh(\chi)$ ,  $\chi$ ,  $\sin\chi$ , respectively, for open, flat and closed Universes.

In this background, the angular size-redshift relation for a rod of intrinsic length  $D$  is easily obtained by integrating the spatial part of the above expression for  $\chi$  and  $\phi$  fixed. One finds

$$\theta(z) = \frac{D(1+z)}{R_o S_k(\chi)} \quad . \quad (2)$$

The dimensionless coordinate  $\chi$  is given by

$$\chi(z) = \frac{1}{H_o R_o} \int_{(1+z)^{-1}}^1 \frac{dx}{xE(x)} \quad , \quad (3)$$

where  $x = \frac{R(t)}{R_o} = (1+z)^{-1}$  is a convenient integration variable. For flat Q-models, the dimensionless function  $E(x)$  takes the following form

$$E_Q(x) = \left[ (1 - \Omega_x)x^{-1} + \Omega_x x^{-(1+3\omega_x)} \right]^{\frac{1}{2}} \quad , \quad (4)$$

where  $\Omega_x = \frac{8\pi G\rho_x}{3H_o^2}$  is the present day density parameter associated with the “quintessence” component. Observe that the flat constraint condition,  $\Omega_M + \Omega_x = 1$ , where  $\Omega_M = \frac{8\pi G\rho_M}{3H_o^2}$  has been explicitly used in the derivation of (4).

Before proceed further, it is interesting to make explicit the connection with some special cases already established in the

literature. If  $\Omega_x = 0$ , or still if  $\Omega_x = 1$  and  $\omega_x = 0$ , one obtains from (2)-(4) the angular diameter expression of the Einstein-de Sitter universe (Sandage 1988)

$$\Theta(z) = \frac{DH_o(1+z)^{\frac{3}{2}}}{2 \left[ (1+z)^{\frac{1}{2}} - 1 \right]} \quad . \quad (5)$$

If  $\omega_x = -1$ , the Q-model reduces to a  $\Lambda$ CDM universe, the details of which has been analysed by Krauss & Schramm (1993). In particular, if the pair  $(\Omega_x, \omega_x) = (1, -1)$ , the angular diameter of this Q-model is the same of a flat universe with a pure cosmological constant, namely

$$\Theta(z) = \frac{DH_o(1+z)}{z} \quad . \quad (6)$$

We recall that expression (5) yields a well-known result that the angular diameter in Einstein-de Sitter model has a minimum at  $z_m = 5/4$  (Hoyle 1959), whereas (6) shows us that the extreme Q-model,  $(\Omega_x, \omega_x) = (1, -1)$ , has no minimum at all ( $z_m = \infty$ ). Indeed, for any expanding FRW type cosmology, the typical behavior of the angular size relation is the existence of a critical redshift greater than the above Einstein-de Sitter value. We also observe that a new analytical result is obtained by taking  $\omega_x = -1/3$  for arbitrary values of  $\Omega_x$ . From (2)-(4) one finds

$$\Theta(z) = \frac{DH_o(1+z)}{2\sqrt{\Omega_x}} \left\{ \ln[\sqrt{\alpha} + \sqrt{\sqrt{\alpha} + 1} - \frac{\sqrt{\alpha}}{\sqrt{(1+z)}} - \sqrt{\frac{\sqrt{\alpha}}{\sqrt{(1+z)}} + 1}] \right\}^{-1} \quad . \quad (7)$$

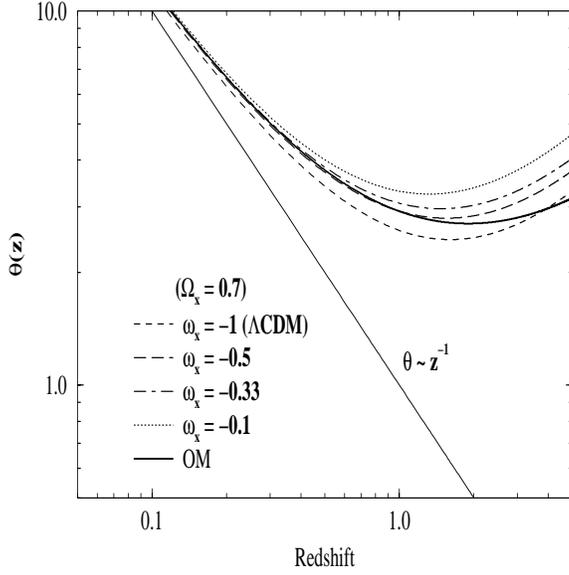
where  $\alpha = \frac{\Omega_x}{1-\Omega_x}$ .

The expression (2) for  $\Theta(z)$  cannot be written in simple analytical form, unless the pair of parameters  $(\Omega_x, \omega_x)$  take the above mentioned values. For generic cases, the results can be obtained only by numerical treatment.

In Fig. 1 we show a log-log plot of angular size versus redshift for flat Q-models with  $\Omega_x = 0.7$  and some selected values of  $\omega_x$ . For comparison we have also considered the standard OM cosmology ( $\Omega_M = 0.3$ ). As can be seen there, for all values of  $\omega_x$ , the angular size initially decreases with increasing  $z$ , reaches its minimum value at a given  $z_m$ , and eventually begins to increase for fainter magnitudes. Note also that the standard OM behavior may be interpreted as an intermediary case between  $\Lambda$ CDM ( $\omega_x = -1$ ) and a Q-model with  $\omega_x \leq -0.5$ , though its critical redshift is displaced to higher values.

## 3. The critical redshift

As widely known, the existence of a critical redshift  $z_m$  on the angular size-redshift relation may qualitatively be understood in terms of an expanding space. The light observed today from a source at high  $z$  was emitted when the object was closer. The relevant aspect here is how this effect may be quantified in terms of the  $\omega_x$  parameter. To analyze the sensivity of the critical redshift to “quintessence”, we adopt here an approach different of the one applied by Krauss & Schramm (1993) to the case of a flat  $\Lambda$ CDM universe.



**Fig. 1.** Angular diameter for flat Q-models. Thick solid curve is the prediction for an open universe ( $\Omega_M = 0.3$ ) with null cosmological constant.

The redshift  $z_m$  at which the angular size takes its minimal value is the one cancelling out the derivative of  $\Theta$  with respect to  $z$ . Hence, from (2) we have the condition

$$S_k(\chi_m) = (1 + z_m)S'_k(\chi_m) \quad , \quad (8)$$

where  $S'_k(\chi) = \frac{\partial S_k}{\partial \chi} \frac{\partial \chi}{\partial z}$ , a prime denotes differentiation with respect to  $z$  and by definition  $\chi_m = \chi(z_m)$ . Observe also that (3) can readily be differentiated yielding

$$(1 + z_m)\chi'_m = (R_o H_o)^{-1} S_Q(\Omega_x, \omega_x, z_m) \quad , \quad (9)$$

where

$$S_Q(\Omega_x, \omega_x, z_m) = [(1 - \Omega_x)(1 + z_m) + \Omega_x(1 + z_m)^{1+3\omega_x}]^{\frac{1}{2}} \quad . \quad (10)$$

Now, combining Eqs. (8)-(10), we find

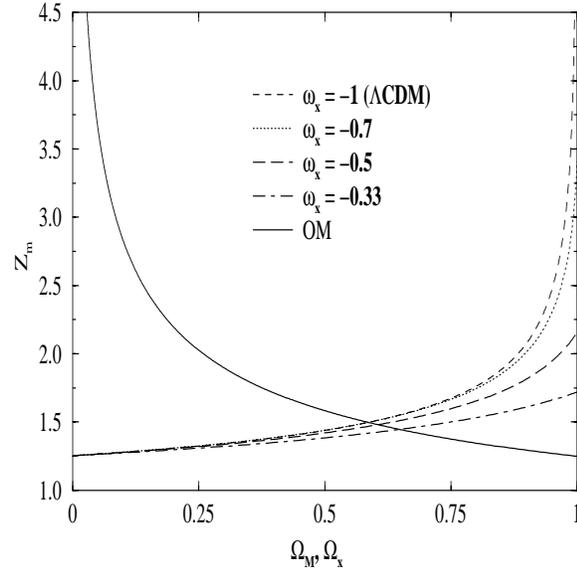
$$\int_{(1+z_m)^{-1}}^1 \frac{dx}{xE_Q(x)} = S_Q(\Omega_x, \omega_x, z_m) \quad . \quad (11)$$

The meaning of the above equation is self evident. It represents an implicit integro-algebraic equation for the critical redshift  $z_m$  as a function of the parameters defining the flat Q-models. In general, this expression cannot be solved in closed analytical form for  $z_m$ . However, by taking the limit  $\Omega_x = 0$  in (11), the value  $z_m = 1.25$  is readily obtained as should be expected. The interesting point here is that (11) is quite convenient for a numerical treatment. A similar equation can also be derived for an open cold dark matter universe (OM). We find

$$\Delta^{-1} \tanh \left[ \Delta \int_{(1+z_m)^{-1}}^1 \frac{dx}{xE_{OM}(x)} \right] = F_{OM}(\Omega_M, z_m), \quad (12)$$

where  $\Delta = (1 - \Omega_M)^{\frac{1}{2}}$  and the functions  $E_{OM}, F_{OM}(\Omega_M, z_m)$  are given by

$$E_{OM}(x) = [1 - \Omega_M + \Omega_M x^{-1}]^{\frac{1}{2}} \quad (13)$$



**Fig. 2.** Critical redshift  $z_m$  for flat Q-models as a function of  $\Omega_x$  and some selected values of  $\omega_x$ . The solid line is the prediction of the open cold dark matter (OM) cosmology.

$$F_{OM}(\Omega_M, z_m) = [1 - \Omega_M + \Omega_M(1 + z_m)]^{\frac{1}{2}} \quad . \quad (14)$$

In Fig. 2 we show the diagrams of  $z_m$  as a function of the density parameter  $\Omega_x$ , and some selected values of  $\omega_x$  (at this point the reader should compare our results with the alternative numerical method developed by Krauss & Schramm (1993) for a  $\Lambda$ CDM universe). Note that Eq. (12) has also been used to plot the case for the open universes (solid line). In the former case, the curves show us clearly that all the Q-models belongs to the same class, which contains the case of a pure cosmological constant. The smallest value of the critical redshift is exactly the one given by Einstein-de Sitter universe ( $\Omega_x = 0$ ). This value is pushed to the right direction, that is, for any value of  $\omega_x$  it is displaced to higher redshifts as the  $\Omega_x$  parameter increases. For instance, consider that  $\omega_x = -0.5$ . By taking  $\Omega_x = 0.5$  and  $\Omega_x = 0.8$ , we find  $z_m = 1.42$  and  $z_m = 1.65$ , respectively. In the opposite extreme ( $\Omega_x \rightarrow 1$ ) the critical redshift is finite unless the parameter of the equation of state take the extreme value for a pure  $\Lambda$  ( $\omega_x = -1$ ). Note also that for a given value of  $\Omega_x$ , the minimum is also displaced for higher redshifts when the  $\omega_x$  parameter diminishes. However, we see that the effect is small if the density parameter  $\Omega_x$  is low, say, smaller than 0.2, since the curves are nearly flat and practically coincident below this limit. This coincidence is even more surprising for  $\omega_x \leq -0.7$ . Note that open models driven by cold dark matter (OM) affects strongly the angular size, however, in a somewhat different manner as compared to what happens in a generic Q-model (see Fig. 2). In particular, if the density parameters are small, say,  $\Omega_M, \Omega_x$ , smaller than 1/3, the critical redshift is much bigger in the former than in the later. This is easy to understand physically, because at this limit the contribution of the quintessence is small, leading to results close to the flat Einstein-de Sitter universe.

**Table 1.** Critical redshift  $z_m$  for some selected values of the parameter  $\omega_x$ . We also quote the results for an open cold dark matter model (OM). For  $\omega_x = -1$ , the results of the flat Q-models are the same of a flat  $\Lambda$ CDM universe previously derived by Krauss & Schramm (1993) using a different method.

$\Omega_M$	OM ( $z_m$ )	$\omega_x = -1$ ( $z_m$ )	$\omega_x = -0.7$ ( $z_m$ )	$\omega_x = -0.5$ ( $z_m$ )	$\omega_x = -0.33$ ( $z_m$ )
1.0	1.25	1.25	1.25	1.25	1.25
0.9	1.29	1.28	1.28	1.27	1.27
0.8	1.34	1.31	1.31	1.30	1.29
0.7	1.41	1.34	1.34	1.33	1.32
0.6	1.48	1.38	1.38	1.37	1.35
0.5	1.58	1.44	1.44	1.42	1.38
0.4	1.71	1.50	1.50	1.47	1.42
0.3	1.89	1.60	1.60	1.55	1.47
0.2	2.20	1.76	1.74	1.65	1.53
0.1	2.82	2.08	2.0	1.81	1.61
0.0	$\infty$	$\infty$	3.27	2.16	1.72

For a large class of Q-models considered in this letter, the critical redshifts  $z_m$  are displayed in Table 1. The OM results have also been quoted for comparison. The third column with  $\omega_x = -1$  corresponds to a flat  $\Lambda$ CDM model (see Krauss & Schramm 1993). From these results one arrives to an inevitable conclusion: even neglecting evolution, the redshift at which the angular size is minimal cannot alone discriminate between world models since different scenarios may provide the same  $z_m$  values. In particular, for the observationally favoured open universe ( $\Omega_M = 0.3$ ) we find  $z_m = 1.89$ , a value that may also be obtained for Q-models having  $0.85 \leq \Omega_x \leq 0.93$  and  $-1 \leq \omega_x \leq -0.5$ . However, if the angular diameter results are combined with other tests, some interesting cosmological constraints may be obtained. For instance, at galactic scales, the observed COBE normalized pattern of density fluctuations is more difficult to fit within a low-density open universe than in Q-models (Caldwell et al. 1998). Another real possibility is that the universe is actually in an accelerated expansion state ( $q_o < 0$ ), as indicated recently by measurements using Type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1998). In this case, any model of the standard FRW class is ruled out regardless of its curvature parameter. However, the bidimensional parameter space ( $\Omega_x, \omega_x$ ) is still large enough to accommodate Q-models predicting both an accelerated expansion (if  $q_o < -\frac{1}{3\Omega_x}$ ), and high values of the critical redshift, say, close to the values of  $z_m$  given by the open models.

The same analytical procedure developed here may be applied when evolutionary and/or selection effects due to a linear size-redshift or to a luminosity-redshift dependence are taken into account<sup>1</sup>. As widely believed, a plausible way of standing for such effects is to consider that the intrinsic linear size has a similar dependence on the redshift as the coordinate dependence, i.e.,  $D = D_o(1+z)^c$ , being  $c < 0$

<sup>1</sup> For a more detailed discussion on these effects see Buchalter et al. 1998

(Ubachukwu 1995; Buchalter et al. 1998). In this case, Eq. (11) is still valid but the function  $S_Q(\Omega_x, \omega_x, z_m)$  must be divided by a factor  $(1+c)$ . The displacement of  $z_m$  relative to the case with no evolution ( $c = 0$ ) due to the effects above mentioned may be unexpectedly large. For example, if one takes  $c = -0.8$  as found by Buchalter et al. 1998, the redshift of the minimum angular size for the Einstein-de Sitter case ( $\Omega_x = 0$ ) moves from  $z_m = 1.25$  to  $z_m = 11.25$ . In this way, the minimal is clearly removed for all practical purposes. This result may be a possible explanation why the data of Gurvits et al. (1999), although apparently in agreement with the Einstein-de Sitter universe, do not show clear evidence for a minimal angular size close to  $z = 1.25$ , as should be expected for this model. This sort of effect is even greater when an additional “quintessence” component is also considered. In the same vein, since evolution is not forbidden from any principle, we stress that constraints from angular size redshift relation should be taken with some caution.

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