Astron. Astrophys. 357, 809–815 (2000)

ASTRONOMY AND ASTROPHYSICS

Non radial motions and the shapes and the abundance of clusters of galaxies

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Received 4 January 1999 / Accepted 4 January 2000

Abstract. We study the effect of non-radial motions on the mass function, the *velocity dispersion function* (hereafter VDF) and on the shape of clusters of galaxies using the model introduced in Del Popolo & Gambera (1998a,b, 1999). The mass function of clusters, obtained using the quoted model, is compared with the statistical data by Bahcall & Cen (1992, 1993) and Girardi et al. (1998), while the VDF is compared with the Center for Astrophysics (hereafter CfA) data by Zabludoff et al. (1993) for local clusters and those of Mazure et al. (1996) and Fadda et al. (1996). In both cases the model predictions are in good agreement with the observational data showing once more how non-radial motions can reduce many of the discrepancies between Cold Dark Matter (hereafter CDM) model predictions and observational data. Finally we study the effect of non-radial motions on the intrinsic shape of clusters of galaxies showing that non-radial motions produce clusters less elongated with respect to CDM model in agreement with de Theije et al. (1995, 1997) results.

Key words: cosmology: theory – cosmology: large-scale structure of Universe – galaxies: formation

1. Introduction

At its appearance the CDM model contributed to obtain a better understanding of the origin and evolution of the large scale structure in the Universe (White et al. 1987; Frenk et al. 1988; Efstathiou 1990; Ostriker 1993). The principal assumptions of the *standard* CDM (SCDM) model (see also Liddle & Lyth 1993) are:

- a flat Universe dominated by weakly interacting elementary particles having low velocity dispersion at early times. The barionic content is determined by the standard big bang nucleosynthesis model (Kernan & Sarkar 1996; Steigman 1996; Olive 1997; Dolgov 1997);
- critical matter density;
- expansion rate given by $h = 0.5$;
- a scale invariant and adiabatic spectrum with a spectral index, $n \equiv 1$;
- the condition required by observations, that the fluctuations in galaxy distribution, $(\delta\rho/\rho)_{\rm g}$, are larger than the fluctuations in the mass distribution, $(\delta\rho/\rho)_\rho$ by a factor $b > 1$.

If this last assumption is not introduced, the pairwise velocity dispersion is larger then that deduced from observations and the galaxy correlation function is steeper than that observed (Davis et al. 1985). After the great success of the model in the 80's, a closer inspection of the model has shown a series of deficiencies, namely:

- the strong clustering of rich clusters of galaxies, $\xi_{\rm cc}(r) \simeq (r/25h^{-1}{\rm Mpc})^{-2}$, far in excess of CDM predictions (Bahcall & Soneira 1983);
- the overproduction of clusters abundance. Clusters abundance is a useful test for models of galaxy formation. This is connected to three relevant parameters: the mass function, the VDF and the temperature function. Using N-body simulations, Jing et al. (1994) studied the mass function of rich clusters at $z = 0$ for the CDM model concluding, if the density spectrum is normalized to the Cosmic Background Explorer (hereafter COBE) (Smoot et al. 1992) quadrupole $Q_{COBE} = 6.0 \times 10^{-6}$, that the mass function is higher than the observed one by Bahcall & Cen (1992, 1993). Bartlett & Silk (1993) come to a similar conclusion using the Press-Schechter (1974) formula. They found that the CDM model with the COBE normalization produces a temperature function of clusters higher than that given by the observations by Edge et al. (1990) and by Henry & Arnaud (1991);
- the conflict between the normalization of the spectrum of the perturbation which is required by different types of observations; in fact, the normalization obtained from COBE data (Smoot et al. 1992) on scales of the order of 10^3 Mpc requires $\sigma_8 = 0.95 \pm 0.2$, where σ_8 is the rms value of $\frac{\delta M}{M}$ in a sphere of $8h^{-1}$Mpc. Normalization on scales $10 \div 50$Mpc obtained from QDOT (Kaiser et al. 1991) and POTENT (Dekel et al. 1992) requires that σ_8 is in the $0.7 \div 1.1$, range which is compatible with COBE normalization while the observations of the pairwise velocity dispersion of galaxies on scales $r \leq 3$ Mpc seem to require $\sigma_8 \leq 0.5$.

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– the incorrect scale dependence of the galaxy correlation function, $\xi(r)$, on scales $10 \div 100\, h^{-1}\mathrm{Mpc}$, having $\xi(r)$ too little power on the large scales compared to the power on smaller scales (Maddox et al. 1990; Saunders et al. 1991; Lahav et al. 1989; Peacock 1991; Peacock & Nicholson 1991); the APM survey (Maddox et al. 1990), giving the galaxy angular correlation function, the 1.2 Jy IRAS power spectrum, the QDOT survey (Saunders et al. 1991), X-ray observations (Lahav et al. 1989) and radio observations (Peacock 1991; Peacock & Nicholson 1991) agree with the quoted conclusion. As shown in studies of galaxy clustering on large scales (Maddox et al. 1990; Efstathiou et al. 1990b; Saunders et al. 1991) the measured rms fluctuations within spheres of radius $20h^{-1}$Mpc have value $2 \div 3$ times larger than that predicted by the CDM model.

Several alternative models have been proposed in order to solve the quoted problems (Peebles 1984; Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Bond et al. 1988; Schaefer et al. 1989; Holtzman 1989; Efstathiou et al. 1990a; Turner 1991; Schaefer 1991; White et al. 1993a; Shaefer & Shafi 1993; Holtzman & Primack 1993; Bower et al. 1993). Most of them propose in some way a modification of the primeval spectrum of perturbations. In two previous papers (Del Popolo & Gambera 1998a; 1999) we showed how, starting from a CDM spectrum and taking into account non-radial motions, at least the problem of the clustering of clusters of galaxies (first point above) and the problem of the X-ray temperature (second problem quoted above) can be considerably reduced.

In this paper we extend the model to two other tests of the abundance of clusters: the mass function and the VDF, permitting to estimate the expected number density of clusters within a given range in mass and velocity, respectively. We also study the effect of non-radial motions on the shapes of galaxy clusters. In recent papers (de Theije et al. 1995; de Theije et al. 1997) it has been shown that clusters of galaxies are more nearly spherical and more centrally condensed than the predictions of CDM models with $\Omega = 1$.

As we shall see, non-radial motions have the effect to produce more spherical clusters and are able to reconcile the CDM with $\Omega = 1$ predictions on clusters elongations with observations.

In Sect. 2 we shall use the same model introduced by Del Popolo & Gambera (1998a,b; 1999) to take into account non-radial motions, arising from the tidal interaction of the protoclusters with the neighbouring protostructures, and we shall compare the mass function calculated using the CDM model, taking into account non-radial motions, with the observed mass function obtained by Bahcall & Cen (1992, 1993) and Girardi et al. (1998). In Sect. 3 we repeat the calculation for the VDF and compare the theoretical VDF with the CfA data by Zabludoff et al. (1993) and with the data by Mazure et al. (1996) and Fadda et al. (1996). In Sect. 4 we study the effect of non-radial motions on the ellipticity of clusters and finally in Sect. 5 we give our conclusions.

2. Non-radial motions and the mass function

One of the most important constraints that a model for large-scale structure must overcome is that of predicting the correct number density of clusters of galaxies. This constraint is crucial for several reasons. According to the gravitational instability scenario, galaxies and clusters form where the density contrast, δ, is large enough so that the surrounding matter can separate from the general expansion and collapse. Consequently the abundance of collapsed objects depends on the amplitude of the density perturbations. In the CDM model these latter follow a Gaussian probability distribution and their amplitude on a scale R is defined by $\sigma(R)$, the r.m.s. value of δ, which is related to the power spectrum, $P(k)$. In hierarchical models of structure formation, like CDM, $\sigma(R)$ decreases with increasing scale, R, and consequently the density contrast required to form large objects, like clusters of galaxies, rarely occurs. The present abundance of clusters is then extremely sensitive to a small change in the spectrum, $P(k)$. Moreover, the rate of clusters evolution is strictly connected to the density parameter, Ω_0. Then, clusters abundance and its evolution are a probe of Ω_0 and $P(k)$ and can be used to put some constraints on them.

The abundance of clusters of galaxies, together with the mass distributions in galaxy halos and in rich clusters of galaxies, the peculiar motions of galaxies, the spatial structure of the microwave background radiation is one of the most readily accessible observables which probes the mass distribution directly.

The most accurate way of assessing the cluster abundance is via numerical simulations. However, there is an excellent analytic alternative, Press & Schechter's theory (Press & Schechter 1974; Bond et al. 1991). Brainerd & Villumsen (1992) studied the CDM halo mass function using a hierarchical particle mesh code. From this last work it results that the Press & Schechter formula fits the results of the simulation up to a mass of 10 times the characteristic 1σ fluctuation mass, M_*, being $M_* \simeq 10^{15} b^{-6/(n_l+3)} M_\odot$, where b is the bias parameter and n_l is the local slope of the power spectrum. For the case of critical-density universes, N-body simulations (Lacey & Cole 1994) have been shown to be in extremely well agreement with Press & Schecter's theory.

Press-Schechter's theory states that the fraction of mass in gravitationally bound systems larger than a mass, M, is given by the fraction of space in which the linearly evolved density field, smoothed on the mass scale M, exceeds a threshold $\delta_{\rm c}$:

$$F(> M) = \frac{1}{2} erfc \left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(R_f, z)} \right) \tag{1}$$

The fraction of the mass density in non-linear objects of mass M to M+d M is given by differentiating Eq. (1) with respect to mass:

$$n(M,z)dM = -\left(\frac{2}{\pi}\right)^{1/2} \frac{\rho_{\rm b}}{M} \cdot \frac{\delta_{\rm c}}{\sigma^2} \exp(-\frac{\delta_{c^2}}{2\sigma^2}) \frac{{\rm d}\sigma}{{\rm d}M} {\rm d}M \tag{2}$$

where $\rho_{\rm b}$ is the comoving background density, $R_{\rm f}$ is the comoving linear scale associated with M, $R_f = \left(\frac{3M}{4\pi\rho_{\rm b}}\right)^{1/3}$. Press-

Schechter's result predicts that only half of the mass of the Universe ends up in virialized objects but in particular cases this problem can be solved (Peacock & Heavens 1990; Cole 1991; Blanchard et al. 1992).

The mass variance present in Eq. (1) can be obtained once a spectrum, $P(k)$, is fixed:

$$\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k k^2 P(k) W^2(kR) \quad (3)$$

where $W(kR)$ is a top-hat smoothing function:

$$W(kR) = \frac{3}{(kR)^3}\left(\sin kR - kR\cos kR\right) \quad (4)$$

and the power spectrum $P(k) = Ak^n T^2(k)$ is fixed giving the transfer function $T(k)$:

$$T(k) = \frac{[\ln(1+18k\sqrt{a})]}{18\sqrt{a}} \cdot [1 + 1.2k^{1/2} - 27k + + 347(1-\sqrt{a}/5)k^{3/2} - 18(1-0.32a^2)k^2]^{-2} \quad (5)$$

(Klypin et al. 1993), where A is the normalization constant, $a = (1+z)^{-1}$ is the expansion parameter and k is the wavenumber measured in units of Mpc^{-1}. This spectrum is valid for $k < 30\mathrm{Mpc}^{-1}$ and $z < 25$. The accuracy (maximum deviation) of the spectrum is 5%. It is more accurate than Holtzman's (1989) spectrum, used by Jing et al. (1994) and Bartlett & Silk (1993) to calculate the mass function and the X-ray temperature function of clusters, respectively. The spectrum is lower by 20% on cluster mass scales than Holtzman's (1989). The spectrum was normalized to the COBE quadrupole $Q_2 = 17\mu$ K, corresponding to $\sigma_8 = 0.66$. As shown by Bartlett & Silk (1993) the X-ray distribution function, obtained using a standard CDM spectrum, over-produces the clusters abundances data obtained by Henry & Arnaud (1991) and by Edge et al. (1990). This has lead some authors (White et al. 1993b) to cite the cluster abundance as one of the strongest pieces of evidence against the standard CDM model when the model is normalized so as to reproduce the microwave background anisotropies as seen by the COBE satellite (Bennett et al. 1996; Banday et al. 1996; Górsky et al. 1996; Hinshaw et al. 1996).

The discrepancy can be reduced taking into account the non-radial motions that originate when a cluster reaches the non-linear regime as follows.

A fundamental role in Press-Schechter's theory is played by the value of $\delta_{\rm c}$. This value is quite dependent on the choice of smoothing window used to obtain the dispersion (Lacey & Cole 1994). Using a top-hat window function $\delta_{\rm c} = 1.7\pm0.1$ while for a Gaussian window the threshold is significantly lower. In a non-spherical context the situation is more complicated. Considering the collapse along all the three axes the threshold is higher, whereas the collapse along the first axis (pancake formation) or the first two axes (filament formation) corresponds to a lower threshold (Monaco 1995). The threshold, $\delta_{\rm c}$, does not depend on the background cosmology.

As shown by Del Popolo & Gambera (1998a; 1999), if non-radial motions are taken into account, the threshold $\delta_{\rm c}$ is not constant but is function of mass, M (Del Popolo & Gambera 1998a; 1999):

$$\delta_{\rm c}(\nu) = \delta_{\rm co}\left[1 + \int_{r_{\rm i}}^{r_{\rm ta}} \frac{r_{\rm ta}L^2\cdot \mathrm{d}r}{GM^3r^3}\right] \quad (6)$$

where $\delta_{\rm co} = 1.68$ is the critical threshold for a spherical model, $r_{\rm i}$ is the initial radius, $r_{\rm ta}$ is the turn-around radius and L the angular momentum. In terms of the Hubble constant, H_0, the density parameter at current epoch, Ω_0, the expansion parameter a and the mean fractional density excess inside a shell of a given radius, $\overline{\delta}$ Eq. (6) can be written as (Del Popolo & Gambera 1998a; 1999):

$$\delta_{\rm c}(\nu) = \delta_{\rm co}\left[1 + \frac{8G^2}{\Omega_o^3 H_0^6 r_{\rm i}^{10}\overline{\delta}(1+\overline{\delta})^3}\int_{a_{\rm min}}^{a_{\rm max}} \frac{L^2\cdot \mathrm{d}a}{a^3}\right] \quad (7)$$

where a is the expansion parameter, and $a_{\rm min}$ its value corresponding to $r_{\rm i}$. The mass dependence of the threshold parameter, $\delta_{\rm c}(\nu)$, was obtained in the same way sketched in Del Popolo & Gambera (1999): we calculated the binding radius, $r_{\rm b}$, of the shell using Hoffmann & Shaham's criterion (1985):

$$T_{\rm c}(r,\nu) \le t_0 \quad (8)$$

where $T_{\rm c}(r,\nu)$ is the calculated time of collapse of a shell and $t_{\rm o}$ is the Hubble time. We found a relation between ν and M through the equation $M = 4\pi\rho_{\rm b}r_{\rm b}^3/3$. We so obtained $\delta_{\rm c}[\nu(M)]$. We obtained the total specific angular momentum, $h(r,\nu) = L(r,\nu)/M_{\rm sh}$, acquired during expansion, in the same way sketched in Del Popolo & Gambera (1998a; 1999). Taking account that $\delta_{\rm c}$ depends on M Eq. (2) becomes:

$$n(M,z)\mathrm{d}M = \left(\frac{2}{\pi}\right)^{1/2} \frac{\rho}{M} \frac{\exp(-\frac{\delta_{\rm c}^2)}{2\sigma^2}}{\sigma^2}\left(\sigma\frac{\mathrm{d}\delta}{\mathrm{d}M} - \delta_{\rm c}\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right)\mathrm{d}M \quad (9)$$

The result of the calculation is shown in Fig. 1. Here, the mass function of clusters, derived using a CDM model with $\Omega_0 = 1$, $h = 1/2$ normalized to $Q_{\rm COBE} = 17\mu$ K and taking into account non-radial motions (solid line), is compared with the statistical data by Bahcall & Cen (1992, 1993) (full dots) and with that of Girardi et al. (1998) (open squares) and with a pure CDM model with $\Omega_0 = 1$, $h = 1/2$ (dashed line). Bahcall & Cen (1992, 1993) estimated the cluster mass function using optical data (richness, velocities, luminosity function of galaxies in clusters) as well as X-ray data (temperature distribution function of clusters). Groups poorer than Abell clusters have also been included thus extending the mass function to lower masses than the richer Abell clusters. Girardi et al. (1998) data are obtained from a sample of 152 nearby ($z \le 0.15$) Abell-ACO clusters. As shown, the CDM model that does not take account of the non-radial motions over-produces the clusters abundance. The result is in agreement with the study of the mass function in the SCDM model by Jing & Fang (1994) and by Bahcall & Cen (1992, 1993). Even with a lower normalization, CDM cannot reproduce the cluster abundance as stressed by Bartlett & Silk (1993), on the contrary a reduction of normalization produces a too steep mass function (Bahcall & Cen 1993; Bartlett

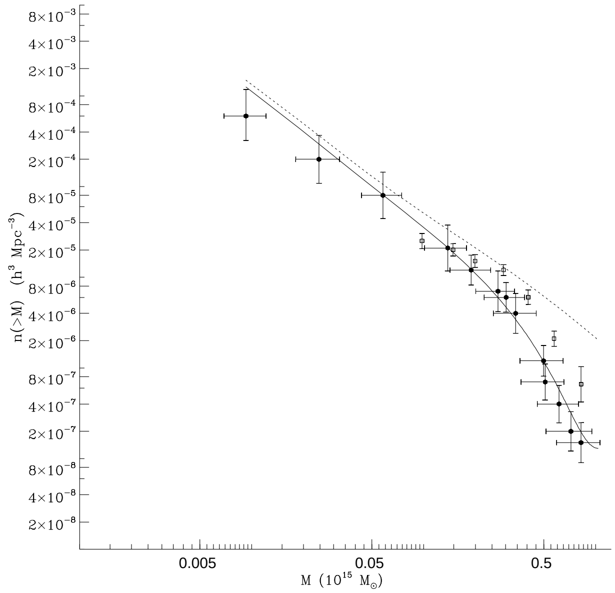

Fig. 1. Cumulative mass function calculated using a CDM model without taking into account non-radial motions (dashed line) and taking account non-radial motions (solid line) compared with Bahcall & Cen (1992, 1993) data (full dots) and with that of Girardi et al. (1998) (open squares).

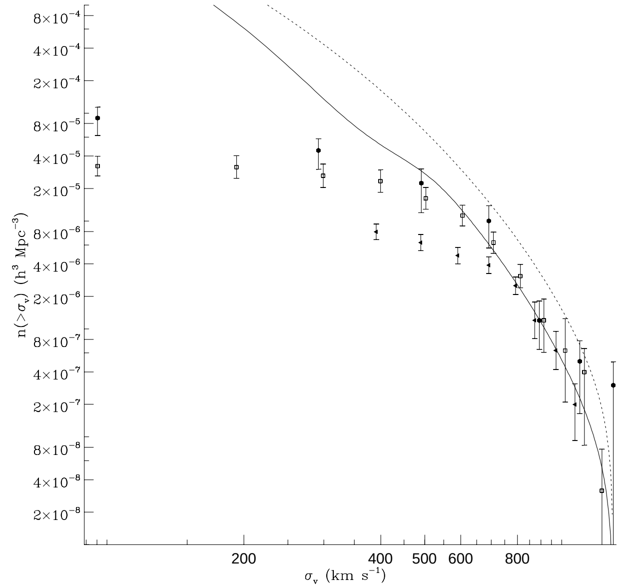

Fig. 2. Cumulative VDF calculated using a CDM model without taking into account non-radial motions (dashed line) and taking into account non-radial motions (solid line) compared with Zabludoff et al. (1993) data (full dots) and with those by Mazure et al.(1996) (full triangles) for $R \geq 1$ clusters and Fadda et al. (1996) (open squares) for $R \geq -1$ clusters. The theoretical curves are obtained using a $\sigma_{\rm v}$-M relation with zero scatter.

1997). The introduction of non-radial motions (solid line) reduces remarkably the abundance of clusters with the result that the model predictions are in good agreement with the observational data. This result confirms what found in Del Popolo & Gambera (1999) showing how a mass dependent threshold, $\delta_{\rm c}(M)$, (dependence caused by the developing of non-radial motions) can solve several of SCDM discrepancies with observations.

3. Non-radial motions and the velocity dispersion function

The VDF is defined in a similar way to the mass function, namely it is the number of objects per unit volume with velocity dispersion larger than $\sigma_{\rm v}$. Since the velocity dispersion $\sigma_{\rm v}$ can be observed directly (on the contrary, mass measurement is usually model dependent), VDF provides a good test of theoretical models. Observed $\sigma_{\rm v}$ comes from the measurement of galaxy redshift. The VDF can be calculated starting from the mass function:

$$n(\sigma_{\rm v}) = n(M)\frac{{\rm d}M}{{\rm d}\sigma_{\rm v}} \tag{10}$$

The cumulative VDF can be obtained integrating Eq. (10):

$$n(>\sigma_{\rm v}) = \int_{\sigma_{\rm v}}^{\infty} n(\sigma_{\rm v'}){\rm d}\sigma_{\rm v'} \tag{11}$$

In order to use Eq. (10) to calculate the VDF we need a relation between the velocity dispersion, $\sigma_{\rm v}$, and mass, M. This can be obtained in several ways. The typical virial temperature may be written as:

$$kT_{\rm vir} = \frac{\alpha G\mu m_{\rm H}}{3}\frac{M_{\rm vir}}{R_{\rm vir}} \tag{12}$$

where α is a factor of order unity. Evrard (1989) found in N-body simulations of a CDM model that typical clusters had $\alpha \simeq 0.8$. The molecular weight, μ, corresponding to a fully ionized gas with primordial abundances is $\simeq 0.6$ and $m_{\rm H}$ is the proton mass. $M_{\rm vir}$ and $R_{\rm vir}$ are respectively the virial mass and virial radius, which are connected by:

$$M_{\rm vir} = 178\frac{4\pi}{3}\rho_{\rm b}R_{\rm vir}^3 \tag{13}$$

Eq. (12) has been tested in several N-body simulations. These numerical simulations give (Evrard et al. 1996):

$$T_{\rm vir} = (6.8h^{2/3}{\rm keV})M^{2/3} \tag{14}$$

The relation is so good that Evrard (1997) uses it as a primary mass indicator for observed clusters when considering the baryon fraction over an ensemble of clusters. Using:

$$\frac{3}{2}kT_{\rm vir} = \frac{1}{2}\mu m_{\rm H}\sigma_v^2 \tag{15}$$

(see Thomas & Couchman 1992; Evrard 1990) togheter with Eq. (14) we find the necessary relation between σ_v and M:

$$\sigma_{\rm v} = 824{\rm km/s}\left(\frac{hM}{10^{15}M_{\odot}}\right)^{1/3} \tag{16}$$

(Evrard 1989; Lilje 1990). In Fig. 2 we compare the VDF obtained from a CDM model taking account of non-radial motions (solid line) with the CfA data by Zabludoff et al. (1993) (full dots) based on their survey of $R \geq 1$ Abell clusters within $z \leq 0.05$ and with the data by Mazure et al. (1996) (full triangles) and Fadda et al. (1996) (open squares). Mazure et al.

(1996) data are obtained from a volume-limited sample of 128 $R_{\rm ACO} \geq 1$ clusters while that of Fadda et al. (1996) are obtained from a sample of 172 nearby galaxy clusters ($z \leq 0.15$). We also plot the VDF obtained from a CDM model without non-radial motions (dashed line). The SCDM model predicts more clusters than the CfA observation except at $\sigma_{\rm v} \simeq 1100 {\rm km/s}$. As reported by Jing & Fang (1994) the SCDM model can be rejected at a very high confidence level ($> 6\sigma$). The reduction of the normalization reduces the formation of clusters, thus resolving the problem of too many clusters, but leads to a deficit at $\sigma_v \simeq 1100 {\rm km/s}$. When non-radial motions are taken into account (solid line) we obtain a good agreement between theoretical predictions and observations. Both CDM and CDM with non-radial motions predict more clusters of low velocity dispersion ($\sigma_v \leq 300 {\rm km/s}$) than the observation. This discrepancy is not significant because the data at $\sigma_{\rm v} \leq 300 {\rm km/s}$ could be seriuosly underestimated because:

- as Zabludoff et al. (1993) stressed, their calculations of group velocity dispersions $\leq 300 {\rm km/s}$ are often underestimates;
- Zabludoff et al. (1993) measure $\sigma_{\rm v}$ only for CfA groups with ≥ 5 group members while for $\sigma_{\rm v} \leq 300 {\rm km/s}$ a fraction of groups could have less than 5 members.

Also Fadda et al. (1996) cannot draw firm conclusions about their incompleteness level, and hence about the behaviour of the σ distribution for $\sigma \leq 650 {\rm km/s}$, while Mazure et al. (1996) has a supposed completeness limit of $\sigma \simeq 800 {\rm km/s}$. A comparison between Eq. (16) and N-body simulations of clusters of galaxies in a CDM model shows that Eq. (16) holds with a rms scatter of $\simeq 10\%$ (Evrard 1989; Lilje 1990). To take into account the scatter in Eq. (14) and Eq. (16) it should be necessary to convolve $n(\sigma_{\rm v})$ with a Gaussian having dispersion of $10\% \div 20\%$ (see Lilje 1990). The result of this operation is that the abundance of clusters with high velocity dispersion depends on the assumed value of the rms scatter: a larger scatter implies a larger $n(\sigma_{\rm v})$. The result of this effect is the worsening of the problem of clusters abundance over-production in the CDM model without non-radial motions.

4. Non-radial motions and the shape of clusters

Most clusters, like elliptical galaxies, are not spherical and their shape is not due to rotation (Rood et al. 1972; Gregory & Tifft 1976; Dressler 1981). The perturbations that gave rise to the formation of clusters of galaxies are alike to have been initially aspherical (Barrow & Silk 1981; Peacock & Heavens 1985; Bardeen et al. 1986) and asphericities are then amplified during gravitational collapse (Lin et al. 1965; Icke 1973; Barrow & Silk 1981). The elongations are probably due to a velocity anisotropy of the galaxies (Aarseth & Binney 1978) and according to Binney & Silk (1979) and to Salvador-Solé & Solanes (1993) the elongation of clusters originates in the tidal distortion by neighboring protoclusters. In particular Salvador-Solé & Solanes (1993) found that the main distorsion on a cluster is produced by the nearest neighboring cluster having more than 45 galaxies and the same model can explain the alignement between neighboring clusters (Binggeli 1982; Oort 1983; Rhee & Katgert 1987; Plionis 1993) and that between clusters and their first ranked galaxy (Carter & Metcalfe 1980; Dressler 1981; Binggeli 1982; Rhee & Katgert 1987; Tucker & Peterson 1988; van Kampen & Rhee 1990; Lambas et al. 1990; West 1994). Clusters elongations and alignement could be also explained by means of Zeldovich's (1978) "pancakes" theory of cluster formation but this top-down formation model is probably ruled out for several well known reasons (Peebles 1993).

The observational information on the distribution of clusters shapes is sometimes conflicting. Rhee et al. (1989) found that most clusters are nearly spherical with ellipticities distribution having a peak at $\epsilon \simeq 0.15$ while Carter & Metcalfe (1980), Binggeli (1982), Plionis et al. (1991) found that clusters are more elongated with the peak of the ellipicities distribution at $\epsilon \simeq 0.5$. More recently de Theije et al. (1995), de Theije et al. (1997) re-analyzed the data studied by Rhee et al. (1989) and that by Plionis et al. (1991) concluding that:

a) richer clusters are intrinsically more spherical than poorer ones;
b) the projected elongations of clusters are consistent with a prolate distribution with clusters ellipticity distribution having a peak at $\epsilon \simeq 0.4$ and extending to $\epsilon = 0.8$;
c) in a $\Omega_0 = 1$ CDM scenario clusters tend to be less spherical than those in a $\Omega_0 = 0.2$ universe and are too elongated with respect to real observed clusters.

To study the effect of non-radial motions on the shape of clusters we shall use a model introduced by Binney & Silk (1979). In that paper they showed that tidal interactions between protoclusters and the neighbouring protostructures should yield prolate shapes (before virialization) with an axial ratio of protostructures of $\simeq 0.5$, the typical value found in clusters. After virialization the pre-existing elongation is damped and the axial ratio leads to values of about $0.7 \div 0.8$, that are higher with respect to observations. As observed by Salvador-Solé & Solanes (1993) this last discrepancy can be removed taking into account that tidal interaction keeps going on after virialization and that on average the damping of elongations due to violent relaxation is eliminated by its growth after virialization. Then this growth restores a value of ϵ near the one that clusters had before virialization.

According to the quoted Binney & Silk (1979) model, an initially spherical protostructure (e.g. a protocluster) of mass M having at distance $r(t)$ from its centre a series of similar protostructure of mass M' shall be distorted. In order to calculate the distortion we must write the equation of motion of a particle at position R relative to the centre of M (assumed as origin of coordinates). Supposing that the effective perturbing mass is less by a factor $\Delta = (\rho - \rho_{\rm b})/\rho_{\rm b}$ than its true mass M', in the limit $|r| \gg |R|$ we have:

$$\ddot{R} = -\frac{GM}{R^3} R + \frac{G\Delta M'}{r^3} \left(3 \frac{Rr}{r^2} r - R \right) \quad (17)$$

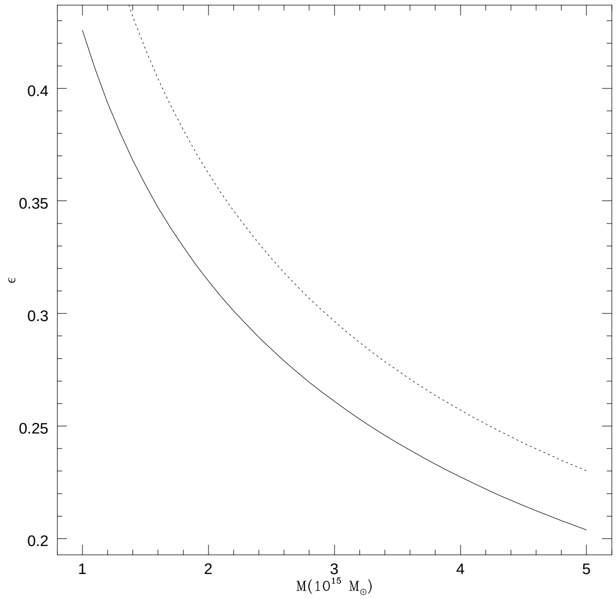

Fig. 3. Ellipticity, ϵ, of clusters versus mass, M. The dashed and solid lines represent ϵ for a CDM without and with non-radial motions, respectively.

If the tidal interaction is treated as first order, writing $R(t) = R_0 + R_1$ with $\ddot{R_0} = -\frac{GM}{R_0^3} R_0$ and $R_0 = R_m a(t)$, it is possible to show that the component of R_1 parallel to R_m is:

$$R_1(x) = \frac{\mu}{2}\left[\frac{3(r_m R_m)^2/r_{\rm m}^2 - R_{\rm m}^2}{R_{\rm m}}\right] G(x, x_1) \tag{18}$$

where $x = 2a$ and x_1 is the value of x at which the perturbation was switched on. The term in square parentheses reduces to $2|R_{\rm m}|$, if R_m is parallel or antiparallel to $\mathbf{r}_{\rm m}$, and to -$|R_{\rm m}|$ when they are orthogonal. Then the initially (at time given by $x = x_1$) spherical density enhancement M becomes a prolate spheroid having ellipticity:

$$\epsilon = (1 - b/a) = \frac{3}{2}\mu G(x, x_1)/(1 + \mu G) \simeq \frac{3}{2}\langle\mu^2\rangle^{1/2} G(2, x_1) \tag{19}$$

where $G(2, x_1)$ is defined in the quoted paper (see Eq. 13c) and $\langle\mu^2\rangle^{1/2}$ is given by (Binney & Silk 1979):

$$\langle\mu^2\rangle^{1/2} = \left[\int\int \mu^2(M, M', r) N(M', r) {\rm d}r {\rm d}M'\right]^{1/2} \tag{20}$$

where

$$\mu = \frac{\pi^2}{8}\left(\frac{R_{\rm m}}{r_{\rm m}}\right)^3 \frac{M'}{M}$$
$$R_{\rm m} = \left(\frac{3M}{4\pi\rho_{\rm m}}\right)^{1/3}$$
$$r_{\rm m} = r(t_{\rm m})$$

being $t_{\rm m}$ the time of maximum expansion, $r_{\rm m} = r(t_{\rm m})$, and $N(M', r){\rm d}r$ is the number of condensations of mass M' lying between r and $r + {\rm d}r$ from M. We calculated this quantity using the Press-Schechter's theory (see Sect. 2):

$$N(M', r) = 4\pi r^2 n(M') \tag{21}$$

To calculate $n(M')$ in the case of CDM without non-radial motions we used Eq. (2) while for a CDM with non-radial motions we used Eq. (9). With the previous definitions $\langle\mu^2\rangle^{1/2}$ is given by:

$$\langle\mu^2\rangle^{1/2} = \frac{1}{9\rho_{\rm b}^{1/2}}\left[\int \frac{N(M', r){\rm d}M'}{M + M'}\right]^{1/2} \tag{22}$$

In Fig. 3 we show the shape of ϵ as a function of mass, M.

As shown ϵ declines with mass in agreement with the above quoted point a): richer clusters are more spherical than poorer clusters. The physical reason for this may be that regions of higher density turn around earlier from Hubble flow than lower density regions (de Theije et al. 1995; Ryden 1995). The fundamental point in which we are interested is the effect of non-radial motions on ϵ. In a CDM model that takes into account non-radial motions (solid line) ϵ is smaller than in the simple CDM (dashed line). This is in agreement with the point c): non-radial motions reduce the elongation of clusters. For a cluster of $10^{15} M_\odot$ we get a value of $\epsilon \simeq 0.5$, if non-radial motions are excluded, while $\epsilon \simeq 0.43$ when non-radial motions are taken into account. Increasing the mass, as expected, clusters tend to become more and more spherical. Finally Binney & Silk (1979) model predicts that even a spherical density enhancement M soon becomes a prolate spheroid in agreement with point b) and also with Salvador-Solé & Solanes (1993) result.

5. Conclusions

In this paper, using the model introduced by Del Popolo & Gambera (1998a; 1999), we have studied how non-radial motions change the mass function, the VDF and the shape of clusters of galaxies. We compared the theoretical mass function obtained from the CDM model taking into account non-radial motions with the experimental data by Bahcall & Cen (1992, 1993) and Girardi et al. (1998). The VDF, calculated similarly to the mass function, was compared with the CfA data by Zabloudoff et al. (1993) and those of Mazure et al. (1996) and Fadda et al. (1996). Taking account of non-radial motions we obtained a notheworthy reduction of the discrepancies between the CDM predicted mass function, the VDF and the observations. Non-radial motions are also able to change the shape of clusters of galaxies reducing their elongations with respect to the prediction of the SCDM model. This last result is in agreement with recent studies of the shapes of clusters by de Theije et al. (1995, 1997).

Acknowledgements. We are grateful to an unknown referee for his helpful suggestions and to Prof. E. Recami and Prof. E. Spedicato for stimulating discussions while this work was in process.

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