Astron. Astrophys. 357, 968–976 (2000)

ASTRONOMY
AND
ASTROPHYSICS

Deconfinement transition in rotating compact stars

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Received 1 February 2000 / Accepted 15 February 2000

Abstract. Using the formalism of general relativity for axially symmetric gravitational fields and their sources - rotating compact stars - a perturbation theory with respect to angular velocity is developed and physical quantities such as mass, shape, momentum of inertia and total energy of the star are defined. The change of the internal structure of the star due to rotation has been investigated and the different contributions to the moment of inertia have been evaluated separately. Numerical solutions have been performed using a two-flavor model equation of state describing the deconfinement phase transition as constrained by the conservation of total baryon number and electric charge. During the spin down evolution of the rotating neutron star, below critical values of angular velocity a quark matter core can appear which might be detected as a characteristic signal in the pulsar timing. Within the spin-down scenario due to magnetic dipole radiation it is shown that the deviation of the braking index from $n = 3$ could signal not only the occurrence but also the size of a quark core in the pulsar. A new scenario is proposed where, due to mass accretion onto the rapidly rotating compact star, a spin-down to spin-up transition might signal a deconfinement transition in its interior.

Key words: dense matter – stars: evolution – stars: interiors – stars: neutron – stars: rotation

1. Introduction

In many recent astrophysical applications of the theory of dense matter it is necessary to investigate the properties of rapidly rotating compact objects within general relativity theory. The reason for this development is the hope that changes in the internal structure of the dense matter, e.g. during phase transitions, could have observable consequences for the dynamics of the rotational behavior of these objects. Particular examples are the observations of glitches and postglitch relaxation in pulsars which are discussed as signals for superfluidity in nuclear matter (Pines & Ali Alpar 1992) and the suggestion that the braking index is remarkably enhanced when a quark matter core occurs in the centre of a pulsar during its spin-down evolution (Glendenning et al. 1997). Further constraints for the nuclear equation of state come from the observation of quasi-periodic brightness oscillations (QPO's) in low-mass X-ray binaries which entail mass and radius limits for rapidly rotating neutron stars, see Lamb et al. (1998), Li et al. (1999).

The problem of rotation in the general relativity theory was and remains one of the central and complicated problems (Glendenning 1997). Compared to the modern methods of numerical solutions to this problem (Friedman et al. 1986; Salgado et al. 1994) the method of perturbation theory (Hartle 1967; Hartle & Thorne 1968; Sedrakian & Chubarian 1968) is a physically transparent and systematic approach to the solution of the problem of stationary gravitational fields and their sources. This approach has been applied successfully in general relativity as well as in alternative theories of gravitation (Grigorian & Chubarian 1985).

From a practical point of view in the definition of the integral characteristics of the astrophysical objects, it is important to analyze the asymptotical expansion of the metric tensor at large distances from the stars, to be able to compare the results with observational data. One can of course introduce the physical parameters of the configuration using the symmetry properties of the object and the gravitational field by expressing them in terms of conserved quantities. In this work we use the definition of the J_z projection of the angular momentum of the nonspherical rotating star (z is the axis of the star's rotation and symmetry of the gravitational field) as a conserved integral of motion. It is a well known integral of the non diagonal element of the energy-momentum tensor in the frame of spherical coordinates.

Using the method of perturbation theory, we are going to calculate the total mass, angular momentum and shape deformation from the solution of the gravitational field equations in case of hydrodynamical, thermodynamical and chemical equilibrium for a given total baryon number and angular velocity Ω of the object. According to Sedrakian & Chubarian (1968) the expansion parameter is of the order of the ratio of the rotational and gravitational energy. It has been shown that, for compact objects in the stationary rotating regime without matter flux, the first two terms of the series expansion give a sufficiently good approximation (see also Glendenning 1997; Weber 1999).

The evolution of the rotating stars could have different origins and scenarii. Our aim in this work is to discuss possible

Send offprint requests to: D. Blaschke signals for a deconfinement phase transition during the evolution of a rotating compact object on the basis of solutions for the Ω dependence of the moment of inertia.

2. Self-consistent set of field equations for stationary rotating stars

2.1. Einstein equations for axial symmetry

The general form of the metric for an axial symmetric space-time manifold is

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 e^\mu [d\theta^2 + \sin^2\theta(d\phi + \omega dt)^2], \quad (1)$$

written in a spherical symmetric coordinate system in order to obtain the Schwarzschild solution as a limiting case. This line element is time-translational and axial-rotational invariant; all metric functions are dependent on the coordinate distance from the coordinate center r and azimuthal angle θ between the radius vector and the axis of symmetry.

Reversal symmetry of the time and polar angle ϕ require that all metric coefficients except ω must be even functions of the angular velocity

$$\Omega = \frac{d\phi}{dt} \quad (2)$$

of the star, the gravitational field of which is described by the Eq. (1). The physical characteristics of the rotating object depend on the centrifugal forces in the local inertial frame of the observer. In general relativity, due to the Lense-Thirring law, rotational effects are described by $\bar{\omega}$ the difference of the frame dragging frequency $-\omega$ and the angular velocity Ω

$$\bar{\omega} \equiv \Omega + \omega(r, \theta). \quad (3)$$

The energy momentum tensor of stellar matter can be approximated by the expression of the energy momentum tensor of an ideal fluid

$$T_\mu^\nu = (\varepsilon + p) u_\mu u^\nu - p\delta_\mu^\nu, \quad (4)$$

where u^μ is the 4-velocity of matter, p the pressure and ε the energy density.

We assume that the star due to high viscosity (ignoring the super-fluid component of the matter) rotates stationarily as a solid body with an angular velocity Ω that is independent of the spatial coordinates. The time scales for changes in the angular velocity which we will consider in our applications are well separated from the relaxation times at which hydrodynamical equilibrium is established, so that the assumption of a rigid rotator model is justified.

Therefore there are only two non-vanishing components of the velocity

$$\begin{aligned} u^\phi &= \Omega\, u^t, \\ u^t &= 1/\sqrt{e^\nu - r^2 e^\mu \bar{\omega}^2 sin^2\theta}. \end{aligned} \quad (5)$$

The equation of state which we will use for our investigation of the deconfinement transition in rotating compact stars will be introduced in Sect. 3. Once the energy-momentum tensor (4) is fixed by the choice of the equation of state for stellar matter, the unknown metric functions ν,λ, μ, $\bar{\omega}$ can be determined by the set of Einstein field equations for which we use the following four combinations.

There are three Einstein equations for the determination of the diagonal elements of the metric tensor

$$\begin{aligned} G_r^r - G_t^t &= 8\pi G(T_r^r - T_t^t), \\ G_\theta^\theta + G_\phi^\phi &= 8\pi G(T_\theta^\theta + T_\phi^\phi), \\ G_\theta^r &= 0, \end{aligned} \quad (6)$$

and one for the determination of the non diagonal element

$$G_\phi^t = 8\pi G T_\phi^t. \quad (7)$$

Here G is the gravitational constant and G_μ^ν the Einstein tensor.

We use also one equation for the hydrodynamical equilibrium (Euler equation)

$$H(r,\theta) \equiv \int \frac{dp'}{p' + \varepsilon'} = \frac{1}{2}\ln[u^t(r,\theta)] + \mathrm{const}, \quad (8)$$

where the gravitational enthalpy H thus introduced is a function of the energy and/or pressure distribution.

The parameters of the theory are the angular velocity of the rotation Ω and the central energy density $\varepsilon(0)$ of the star configuration.

2.2. Perturbative approach to the solution

The problem of the rotation can be solved iteratively by using a perturbation expansion of the metric tensor and the physical quantities in a Taylor series with respect to the angular velocity. As a small parameter for this expansion we use a dimensionless quantity which is the ratio of the rotational energy to the gravitational one for a homogeneous Newtonian star $E_{\rm rot}/E_{\rm grav} = (\Omega/\Omega_0)^2$, where $\Omega_0^2 = 4\pi G\rho(0)$ with the mass density $\rho(0)$ at the center of the star. This expansion gives sufficiently correct solutions already at $O(\Omega^2)$, since the expansion parameter is limited to values $\Omega/\Omega_0 \ll 1$ by the condition of mechanical stability of the rigid rotation. This can easily be seen by considering as an upper limit for attainable angular velocities the so called Kepler one $\Omega_K = \sqrt{GM/R_e^3}$ with M being the total mass and R_e the equatorial radius. For homogeneous Newtonian spherical stars $\Omega < \Omega_K = \Omega_0/\sqrt{3}$ which is fulfilled in particular also for the configurations with a deconfinement transition which we are going to discuss later in this paper, see Fig. 4.

The expansion of the metric tensor is given by

$$g_{\mu\nu}(r,\theta;\Omega) = \sum_{j=0}^{\infty} \left(\frac{\Omega}{\Omega_0}\right)^j g_{\mu\nu}^{(j)}(r,\theta). \quad (9)$$

According to the symmetries of the metric coefficients introduced in Eq. (1) we have even orders $j = 0, 2, ...$ for the diagonal elements[1]

$$
\begin{aligned}
e^{-\lambda(r,\theta;\Omega)} &= e^{-\lambda^{(0)}(r)}[1 + (\Omega/\Omega_0)^2 f(r,\theta)] + O(\Omega^4), \\
e^{\nu(r,\theta;\Omega)} &= e^{\nu^{(0)}(r)}[1 + (\Omega/\Omega_0)^2 \Phi(r,\theta)] + O(\Omega^4), \\
e^{\mu(r,\theta;\Omega)} &= r^2[1 + (\Omega/\Omega_0)^2 U(r,\theta)] + O(\Omega^4), \qquad (10)
\end{aligned}
$$

and odd orders only for the frame dragging frequency ω

$$
\omega(r,\theta;\Omega) = \frac{\Omega}{\Omega_0}\, q(r,\theta) + O(\Omega^3). \qquad (11)
$$

The distributions of pressure, energy density and "enthalpy" introduced in Eq. (8) are also included in the scheme of this perturbation expansion

$$
\begin{aligned}
p(r,\theta;\Omega) &= p^{(0)}(r) + (\Omega/\Omega_0)^2 p^{(2)}(r,\theta) + O(\Omega^4), \\
\varepsilon(r,\theta;\Omega) &= \varepsilon^{(0)}(r) + (\Omega/\Omega_0)^2 \varepsilon^{(2)}(r,\theta) + O(\Omega^4), \\
H(r,\theta;\Omega) &= H^{(0)}(r) + (\Omega/\Omega_0)^2 H^{(2)}(r,\theta) + O(\Omega^4). \qquad (12)
\end{aligned}
$$

All functions with superscript (0) denote the solution of the static configuration and therefore they are only functions of the distance from the center r, the others are the corrections corresponding to the rotation.

This series expansion allows one to transform the Einstein equations into a coupled set of equations for the coefficient functions which can be solved by recursion. At zeroth order we recover the nonlinear problem of the static spherically symmetric star configuration (Tolman-Oppenheimer-Volkoff equations), see the next Sect. 2.3. The first recursion step is to solve Eq. (7) in order to obtain the dragging frequency in $O(\Omega)$ and to define the moment of inertia for the spherically symmetric configuration. In Sect. 2.5 we will consider the second order contribution in the Ω- expansion (10), (12) where the $O(\Omega^2)$ corrections to the moment of inertia can be found. The next terms in the expansion which are of $O(\Omega^3)$ correspond to corrections of the frame dragging frequency and will be neglected since they go beyond the approximation scheme adopted in the present paper.

2.3. Zeroth order: Static spherically symmetric star models

The functions of the spherically symmetric solution in Eqs. (10) and (12) can be found from Eq. (6) and Eq. (8) in zeroth order of the Ω- expansion.

They obey the following equation (Tolman-Oppenheimer-Volkoff)

$$
\frac{dp^{(0)}(r)}{dr} = -G[p^{(0)}(r) + \varepsilon^{(0)}(r)]\frac{m(r) + 4\pi p^{(0)}(r) r^3}{r[r - 2Gm(r)]}, \qquad (13)
$$

where $m(r)$ is the distribution of accumulated mass

$$
m(r) = 4\pi \int_0^r \varepsilon^{(0)}(r') r'^2 dr' \qquad (14)
$$

within a sphere of radius r. For the gravitational potentials we have

$$
\lambda^{(0)}(r) = -\ln[1 - 2Gm(r)/r], \qquad (15)
$$

$$
\nu^{(0)}(r) = -\lambda^{(0)}(R_0) - 2G\int_r^{R_0} \frac{m(r') + 4\pi p^{(0)}(r') r'^3}{r'[r' - 2Gm(r')]} dr'. \qquad (16)
$$

R_0 is the spherical radius of the star, which is defined by $P^{(0)}(R_0) = 0$. The set of Eq. (13) and Eq. (14) fulfils the following conditions at the center of the configuration: $\varepsilon^{(0)}(0) = \varepsilon(0)$ and $m(0) = 0$. The central energy density $\varepsilon(0)$ is the parameter of the spherical configuration. The total mass of the spherically distributed matter in the selfconsistent gravitational field is $M_0(\varepsilon^{(0)}(0)) = m(R_0)$.

2.4. Moment of inertia

In the first order of the approximation we are solving Eq. (7), where the unknown function $q(r,\theta)$ defined by Eq. (11) is independent of the angular velocity. Using the static solutions Eqs. (13)-(15), and the representation of $q(r,\theta)$ by the series of the Legendre polynomials,

$$
q(r,\theta) = \sum_{m=0}^{\infty} q_m(r) \frac{dP_{m+1}(\cos\theta)}{d\cos\theta}, \qquad (17)
$$

we find the equations for the coefficients $q_m(r)$. It is proved that $q(r,\theta)$ is a function of the distance r only, i.e. that $q_m(r) = 0$ for $m > 0$, see Hartle (1967), Hartle & Thorne (1968), Sedrakian & Chubarian (1968).

Let us write down the equations for $\bar\omega(r) = \Omega(1 + q_0(r)/\Omega_0)$, which is more suitable for the solution of the resulting equation in first order

$$
\frac{1}{r^4}\frac{d}{dr}\left[r^4 j(r)\frac{d\bar\omega(r)}{dr}\right] + \frac{4}{r}\frac{dj(r)}{dr}\bar\omega(r) = 0, \qquad (18)
$$

which corresponds to Ref. Hartle (1967), where it was obtained using a different representation of the metric. Here, we use the notation $j(r) \equiv e^{-(\nu^{(0)}(r)+\lambda^{(0)}(r))/2}$, for which outside of configuration $j(r) = 1,\ r > R_0$.

By definition, the angular momentum of the star in the case of stationary rotation is a conserved quantity and can be expressed in invariant form

$$
J = \int T_\phi^t \sqrt{-g}\, dV, \qquad (19)
$$

where $\sqrt{-g}dV$ is the invariant volume and $g = \det||g_{\mu\nu}||$. For the case of slow rotation where the shape deformation of the rotating star can be neglected and using the definition of the moment of inertia $I_0(r) = J_0(r)/\Omega$ accumulated in the sphere with radius r, we obtain from Eq. (19)

$$
\frac{dI_0(r)}{dr} = \frac{8\pi}{3} r^4 [\varepsilon^{(0)}(r) + p^{(0)}(r)] e^{(-\nu^{(0)}(r)+\lambda^{(0)}(r))/2} \frac{\bar\omega(r)}{\Omega}. \qquad (20)
$$

[1] Notation corresponds to the works of Sedrakian & Chubarian (1968).

Using this equation one can reduce the second order differential equation (18) to the first order one

$$\frac{d\bar{\omega}(r)}{dr} = \frac{6GJ_0(r)}{r^4 j(r)}. \tag{21}$$

and solve (18) as a coupled set of first order differential equations, one for the moment of inertia (20) and the other (21) for the frame dragging frequency $\bar{\omega}(r)$.

This system of equations is valid inside and outside the matter distribution. In the center of the configuration $I_0(0) = 0$ and $\bar{\omega}(0) = \bar{\omega}_0$. The finite value $\bar{\omega}_0$ has to be defined such that the dragging frequency $\bar{\omega}(r)$ smoothly joins the outer solution

$$\bar{\omega}(r) = \Omega \left(1 - \frac{2GI_0}{r^3}\right). \tag{22}$$

at $r = R_0$, and approaches Ω in the limit $r \to \infty$. In the external solution (22) the constant $I_0 = I_0(R_0)$ is the total moment of inertia of the slowly rotating star and $J_0 = I_0\Omega$ is the corresponding angular momentum. In this order of approximation, I_0 is a function of the central energy density or the total baryon number only. Explicit dependences of the moment of inertia on the angular velocity occur in the second order of approximation.

2.5. Second order corrections to the moment of inertia

Due to the rotation in Ω^2-approximation the shape of the star is an ellipsoid, and each of the equal-pressure (isobar) surfaces in the star is an ellipsoid as well. All diagonal elements of the metric and energy-momentum tensors could be represented as a series expansion in Legendre polynomials

$$g^{(2)}_{\mu\nu}(r,\theta) = \sum_{l=0}^{\infty} (g_{\mu\nu})_l(r) P_l(\cos\theta). \tag{23}$$

It has been shown that the only non vanishing solutions obeying the continuity conditions on the surface are those with $l = 0, 2$.

The deformation of the isobaric surfaces due to the rotation can be parametrized by the shift $R(r,\theta) - r = \Delta(r,\theta)$ which describes the deviation from the spheric distribution as a function of the radius r in the given polar angle θ and is completely determined by

$$R(r,\theta) = r + \left(\frac{\Omega}{\Omega_0}\right)^2 [\Delta_0(r) + \Delta_2(r)P_2(\cos\theta)], \tag{24}$$

since the expansion coefficients of the deformation $\Delta_l(r)$ can be calculated from the pressure corrections

$$\Delta_l(r) = -\frac{p_l(r)}{dp^{(0)}(r)/dr}. \tag{25}$$

$l \in \{0, 2\}$ is the polynomial index in the angular expansion. The function $R(R_0, \theta)$ is the distance of the star surface from the center of the configuration in the direction with the angle θ to the polar axis. In particular, we can define the equatorial radius $R_e = R(R_0, \theta = \pi/2)$ and the polar radius $R_p = R(R_0, \theta = 0)$ and the eccentricity $\epsilon = \sqrt{1 - (R_p/R_e)^2}$.

The correction to the momentum of inertia $\Delta I(r) = I(r) - I_0(r)$ can be represented in the form

$$\Delta I = \Delta I_{\rm Redist.} + \Delta I_{\rm Shape} + \Delta I_{\rm Field} + \Delta I_{\rm Rotation}. \tag{26}$$

The first three contributions can be expressed by integrals of the angular averaged modifications of the matter distribution, the shape of the configuration and the gravitational fields, in the form

$$\Delta I_\alpha = \int_0^{I_0(R_0)} dI_0(r)[W_0^{(\alpha)}(r) - W_2^{(\alpha)}(r)/5], \tag{27}$$

where

$$W_l^{(\rm Field)}(r) = \left(\frac{\Omega}{\Omega_0}\right)^2 \{2U_l(r) - [f_l(r) + \Phi_l(r)]/2\}, \tag{28}$$

$$W_l^{(\rm Shape)}(r) = \left(\frac{\Omega}{\Omega_0}\right)^2 \frac{d\,\Delta_l(r)}{dr}, \tag{29}$$

$$W_l^{(\rm Redist.)}(r) = \left(\frac{\Omega}{\Omega_0}\right)^2 \frac{p_l(r) + \varepsilon_l(r)}{p^{(0)}(r) + \varepsilon^{(0)}(r)}, \tag{30}$$

respectively, which have to be determined from the Eq. (6) in second order approximation. The contribution of the change of the rotational energy to the moment of inertia

$$\Delta I_{\rm Rotation} = \frac{4}{5}\int_0^{I_0(R_0)} dI_0(r)\left[r^2\bar{\omega}^2(r)e^{-\nu_0(r)}\right]. \tag{31}$$

includes the frame dragging contribution. In this expansion we neglect the influence of the change of the frame dragging frequency, since it corresponds to the next order of the perturbative expansion $\sim O(\Omega^3)$. For a more detailed description of the solutions of the field equations in the $\sim O(\Omega^2)$ approximation we refer to the works of Hartle (1967), Hartle & Thorne (1968) as well as Sedrakian & Chubarian (1968).

3. Model EOS with deconfinement phase transition

For the investigation of the deconfinement phase transition expected to occur in neutron star matter at densities above the nuclear saturation density $n_0 = 0.16\ \text{fm}^{-3}$ several approaches to quark confinement dynamics have been discussed, see e.g. Blaschke et al. (1990), Blaschke et al. (1999), Drago et al. (1996) which lead to interesting conclusions for the properties of quark matter at high densities. Most of the approaches to quark deconfinement in neutron star matter, however, use a thermodynamical bag-model for the quark matter and employ a standard two-phase description of the equation of state (EOS) where the hadronic phase and the quark matter phase are modeled separately and the resulting EOS is obtained by imposing Gibbs' conditions for phase equilibrium with the constraint that baryon number as well as electric charge of the system are conserved (Glendenning 1992, 1997). Since the focus of our work is the elucidation of qualitative features of signals for a possible deconfinement transition in the pulsar timing, we will consider here such a rather standard, phenomenological model for an EOS with deconfinement transition.

The total pressure $p(\{\mu_i\}, T)$ as a thermodynamical potential is given by

$$p(\{\mu_i\}, T) = (1-\chi)p_H(\{\mu_h\}, T) + \chi p_Q(\{\mu_q\}, T) + p_L(\{\mu_l\}, T), \quad (32)$$

where

$$p_H(\{\mu_h\}, T) = \sum_{h=n,p} p_h^{id}(\mu_h^*, T; m_h^*) + \frac{1}{2}(g_\omega \bar{\omega}_0)^2 - \frac{1}{2}(g_\sigma \bar{\sigma})^2 \quad (33)$$

is the EOS of the relativistic $\sigma - \omega$ mean-field model (Walecka model) for nuclear matter (Walecka 1974; Kapusta 1989), where the masses and chemical potentials have to be renormalized by the mean-values of the $\sigma-$ and $\omega-$ fields $m_h^* = m_h - g_\sigma \bar{\sigma}$, $\mu_h^* = \mu_h - g_\omega \bar{\omega}_0$. The pressure for two-flavor quark matter within a bag model EOS is given by

$$p_Q(\{\mu_q\}, T) = \sum_{q=u,d} p_q^{id}(\mu_q, T; m_q) - B \quad (34)$$

where B denotes the phenomenological bag pressure that enforces quark confinement and the transition to nuclear matter at low densities. For our numerical analyses in the present work, we assume here a value of $B = 75\ \mathrm{MeV fm^{-3}}$ which allows us, e.g., to discuss a neutron star of 1.4 solar masses with an extended quark matter core.

In a neutron star, these phases of strongly interacting matter are in $\beta-$ equilibrium with electrons and muons which contribute to the pressure balance with

$$p_L(\{\mu_l\}, T) = \sum_{l=e^-,\mu^-} p_l^{id}(\mu_l, T; m_l). \quad (35)$$

In the above expressions $p_i^{id}(\mu_i, T; m_i) = p_i^-(\mu_i, T; m_i) + p_i^+(\mu_i, T; m_i)$ is the partial pressure of the Fermion species i as a sum of particle and antiparticle contributions defined by

$$p_i^{\pm}(\mu_i, T; m_i) = \gamma_i \int_0^\infty \frac{d^3k}{(2\pi)^3} T \ln[1 + \exp(x^{\pm}(k))], \quad (36)$$

where $x^{\pm}(k) = \sqrt{k^2 + m_i^2}/T \pm \mu/T$. All the other thermodynamic quantities of interest can be derived from the pressure (32) as, e.g., the partial densities of the species

$$n_j(\{\mu_i\}, T) = \frac{\partial p(\{\mu_i\}, T)}{\partial \mu_j}. \quad (37)$$

The chemical equilibrium due to the direct and inverse β-decay processes imposes additional constraints on the values of the chemical potentials of leptonic and baryonic species (Glendenning 1997; Sahakian 1995). Only two independent chemical potentials remain according to the corresponding two conserved charges of the system, the total baryon number N_B as well as electrical charge Q

$$n_B = \frac{N_B}{V} = (1-\chi)n_H(\{\mu_h\}, T) + \chi n_Q(\{\mu_q\}, T) \quad (38)$$

$$0 = \frac{Q}{V} = (1-\chi)q_H(\{\mu_h\}, T) + \chi q_Q(\{\mu_q\}, T) + q_L(\{\mu_l\}, T). \quad (39)$$

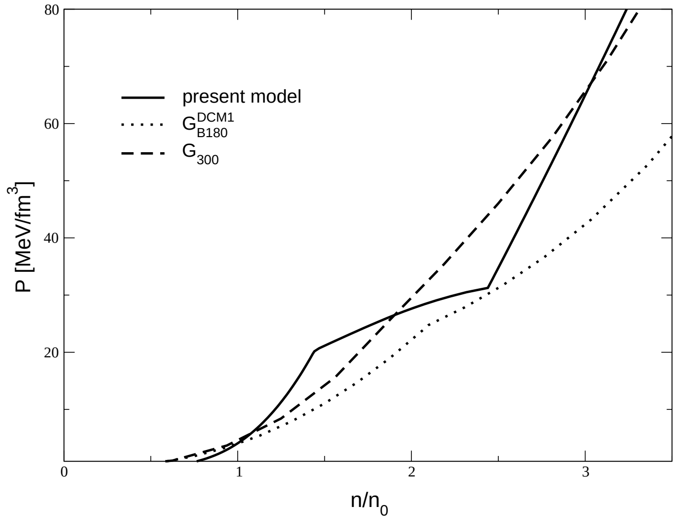

Fig. 1. Model equation of state for the pressure of hybrid star matter in β-equilibrium as a function of the baryon number density. The hadronic equation of state is a relativistic mean-field model ($\sigma - \omega$), the quark matter one is a two-flavor bag model with $B = 75$ MeV fm^{-3}. For comparison the relativistic mean-field EoS of Glendenning (1989) including ρ mesons, hyperons and muons (incompressibility $K = 300$ MeV, dotted line) and that of Glendenning (1992) including a deconfinement transition to three-flavor quark matter in a bag model with $B = 180$ MeV^4 (dashed line) are shown.

The deconfinement transition is obtained following the construction which obeys the global conservation laws and allows one to find the volume fraction of the quark matter phase $\chi = V_Q/V$ in the mixed phase where $p_H(\{\mu_h\}, T) = p_Q(\{\mu_q\}, T)$, so that at given n_B and T the total pressure under the conditions (38), (39) is a maximum, see Glendenning (1992, 1997).

In Fig. 1 we show the model EoS with deconfinement transition as described above. Note that in the density region of the phase transition there is a monotonous increase of the pressure which gives rise to an extended mixed phase region in the compact star after solution of the equations of hydrodynamic stability (13). For comparison, the relativistic mean-field EoS of Glendenning (1989) including ϱ mesons, hyperons and muons (incompressibility $K = 300$ MeV, dotted line) and that of Glendenning (1992) including a deconfinement transition to three-flavor quark matter in a bag model with $B = 180$ MeV^4 are shown, see also the monographs by Glendenning (1997) and Weber (1999).

In Fig. 2 we show the composition of the hybrid star matter as a function of the total baryon density at $T = 0$. Solving the Tolman-Oppenheimer-Volkoff equations (13)- (15) for the hydrodynamical equilibrium of static spherically symmetric relativistic stars with the above defined EOS, we find that a configuration at the stability limit could have a quark matter core with a radius as large as $\sim 75\%$ of the stars radius.

What implications this phase transition for rotating star configurations might have will be investigated in the next section by applying the method developed in Sect. 2 for the above EOS.

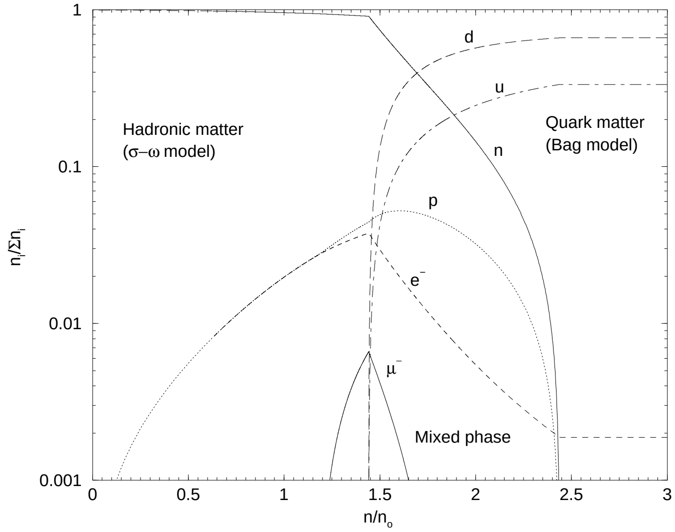

Fig. 2. Composition of hybrid star matter in β-equilibrium as a function of baryon number density for the model EoS used in this work.

4. Results and discussion

The results for the stability of rotating neutron star configurations with possible deconfinement phase transition according to the EOS described in the previous section are shown in Fig. 3, where the total baryon number, the total mass, and the moment of inertia are given as functions of the equatorial radius (left panels) and in dependence on the central baryon number density (right panels) for static stars (solid lines) as well as for stars rotating with the maximum angular velocity $\Omega_{\max}$ (dashed lines).

The dotted lines connect configurations with fixed total baryon numbers $N_B/N_\odot = 1.3, 1.55, 1.8, 2.14$ and it becomes apparent that the rotating configurations are less compact than the static ones. They have larger masses, radii and momenta of inertia at less central density such that for suitably chosen configurations a deconfinement transition in the interior can occur upon spin-down.

In order to demonstrate the consistency of our perturbative approach, we show in Fig. 4 the values of the expansion parameter for the maximally attainable rotation frequencies of stationary rotating objects (Ω_K/Ω_0) as a function of the central density characterizing the configuration.

In Fig. 5 we show the critical regions of the phase transition in the inner structure of the star configuration as well as the equatorial and polar radii in the plane of angular velocity Ω versus distance from the center of the star. It is obvious that with the increase of the angular velocity the star is deforming its shape. The maximal eccentricities of the configurations with $N_B = 1.3\,N_\odot, N_B = 1.55\,N_\odot$ and $N_B = 1.8\,N_\odot$ are $\epsilon(\Omega_K) = 0.7603$, $\epsilon(\Omega_K) = 0.7655$ and $\epsilon(\Omega_K) = 0.7659$, respectively. Due to the changes of the central density the quark core could disappear above a critical angular velocity.

In Fig. 6 we display the dependence of the moment of inertia on the angular velocity for configurations with the same total baryon number $N_B = 1.55\,N_\odot$ together with the different contributions to the total change of the moment of inertia. As it is shown the most important contributions come from the mass redistribution and the shape deformation. The relativistic contributions due to field and rotational energy are less important. In the same Fig. 6 we show the decrease of the spherical moment of inertia due to the decrease of the central density for high angular velocities which tends to partially compensate the further increase of the total moment of inertia for large Ω. There is no dramatic change in the slope of $I(\Omega)$ at $\Omega_{\rm crit} = 2.77$ kHz.

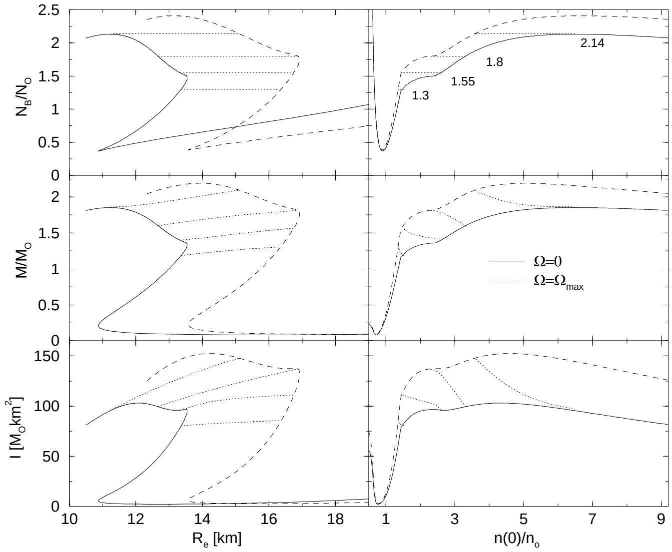

Fig. 3. Baryon number N, mass M, and moment of inertia I as a function of the equatorial radius (left panels) and the central density (right panels) for neutron star configurations with a deconfinement phase transition. The solid curves correspond to static configurations, the dashed ones to those with maximum angular velocity $\Omega_{\rm K}$. The lines between both extremal cases connect configurations with the same total baryon number $N_B/N_\odot = 1.3, 1.55, 1.8, 2.14$.

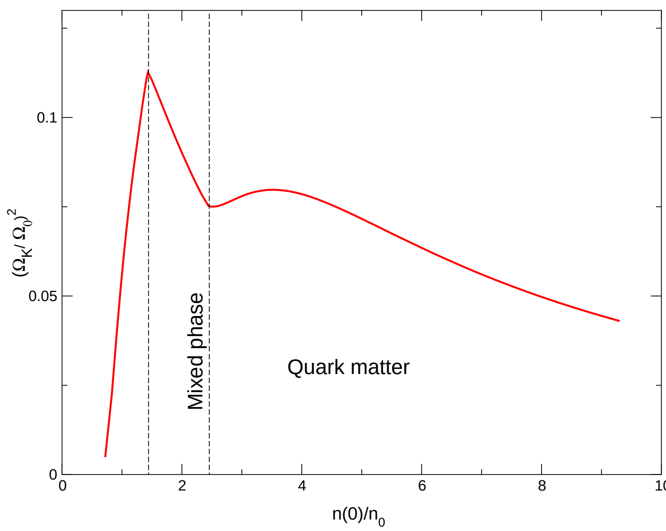

Fig. 4. Expansion parameter for the maximally attainable rotation frequencies of stationary rotating objects $(\Omega_K/\Omega_0)^2$ as a function of the central density characterizing the configuration. Vertical dashed lines indicate the density band for which a mixed phase occurs, see Fig. 2.

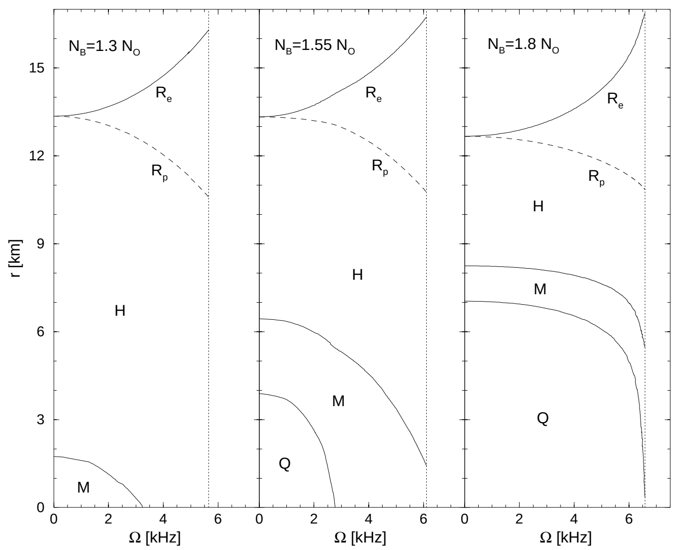

Fig. 5. Phase structure of rotating hybrid stars in equatorial direction in dependence of the angular velocity Ω for stars with different total baryon number: $N_B/N_\odot = 1.3, 1.55, 1.8$. Dahsed vertical lines indicate the maximum frequency Ω_K for stationary rotation.

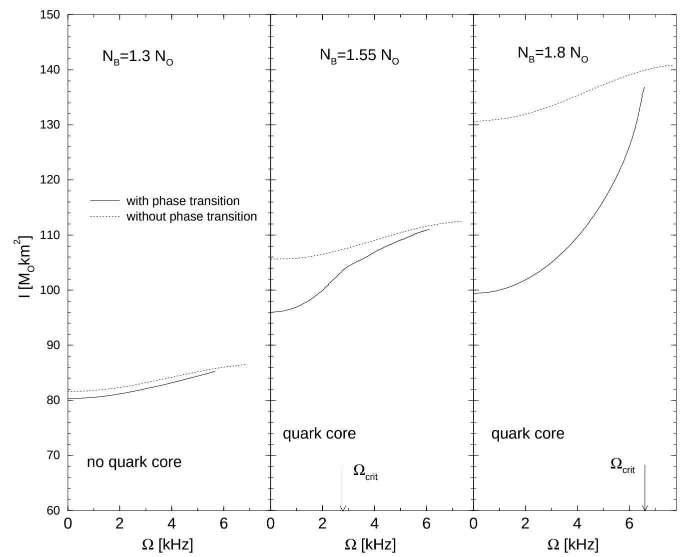

Fig. 7. Moment of inertia as a function of angular velocity with (solid lines) and without (dashed lines) deconfinement phase transition for fixed total baryon number $N_B/N_\odot = 1.3$ (left panel), $N_B/N_\odot = 1.55$ (middle panel), $N_B/N_\odot = 1.8$ (right panel).

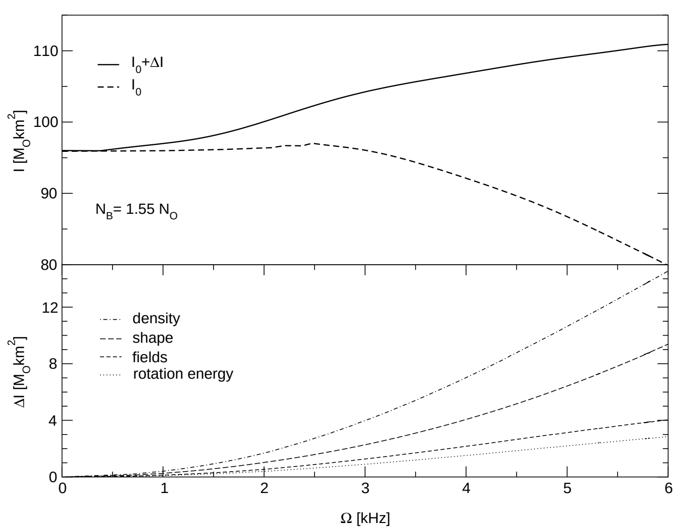

Fig. 6. Contributions to the dependence of the moment of inertia on the angular velocity.

Fig. 7 shows the dependence of the moment of inertia as a function of the angular velocity. It is demonstrated that the behavior of $I(\Omega)$ for a given total number of baryons N_B strongly depends on the presence of a pure quark matter core in the center of the star. If the core does already exist or it does not appear when the angular velocity increases up to the maximum value $\Omega_{\max}$ then the second order derivative of the moment of inertia $I(\Omega)$ does not change its sign. For the configuration with $N_B = 1.55\ N_\odot$ the critical value for the occurrence of the sign change is $\Omega_{\rm crit} = 2.77$ kHz while for $N_B = 1.8\ N_\odot$ it is close to $\Omega_K = 6.6$ kHz.

In order to point out possible observable consequences of such a characteristic behaviour of $I(\Omega, N_B)$ we consider two possible scenarios for changes in the pulsar timing: (A) dipole radiation and the resulting dependence of the braking index on the angular velocity as suggested by Glendenning et al. (1997) and (B) mass accretion onto rapidly rotating neutron stars.

4.1. Magnetic dipole radiation

Due to the energy loss by magnetic dipole radiation plus emission of electron-positron wind the star has to spin-down and the resulting change of the angular velocity can be parametrized by a power law

$$\frac{\dot{\Omega}}{\Omega} = -K\Omega^{n(\Omega)-1}, \tag{40}$$

where K is a constant and $n(\Omega)$ is the braking index

$$n(\Omega) = \frac{\ddot{\Omega}\,\Omega}{\dot{\Omega}^2} = 3 - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}, \tag{41}$$

where we used the notation $I' = (\partial I(\Omega, N_B)/\partial\Omega)|_{N_B=\rm const}$, with the corresponding definition of I'' (see also Glendenning et al. 1997; Grigorian et al. 1999).

In Fig. 8 we display the result for the braking index $n(\Omega)$ for a set of configurations with fixed total baryon numbers ranging from $N_B = 1.55\,N_\odot$ up to $N_B = 1.9\,N_\odot$, the region where during the spin-down evolution a quark matter core could occur for our model EOS. We observe that only for configurations within the interval of total baryon numbers $1.4 \le N_B/N_\odot \le 1.9$ a quark matter core occurs during the spin-down as a consequence of the increasing central density (see also Fig. 5) and the braking index shows variations. The critical angular velocity $\Omega_{\rm crit}(N_B)$ for the appearance of a quark matter core can be found from the minimum of the braking index Eq. (41). As can be seen from Fig. 8, all configurations with a quark matter core have braking indices $n(\Omega) < 3$ and braking indices significantly larger than 3

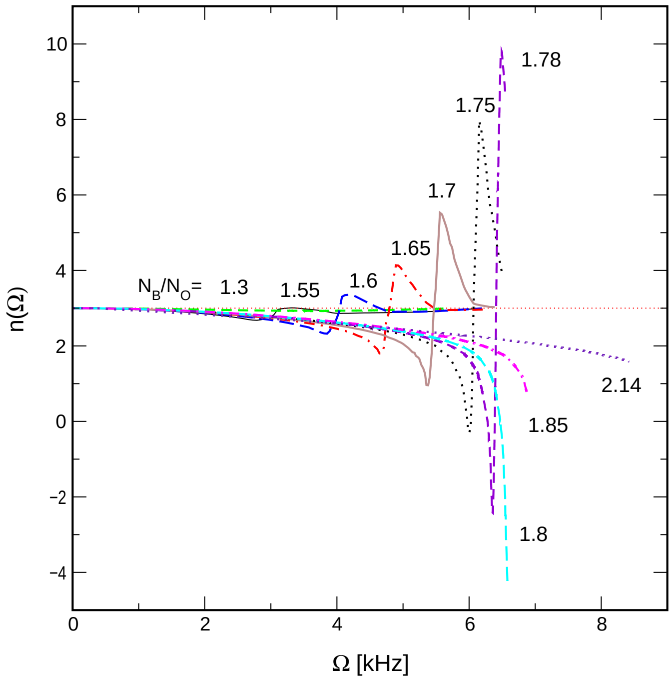

Fig. 8. Braking index due to dipole radiation from fastly rotating isolated pulsars as a function of the angular velocity. The minima of $n(\Omega)$ indicate the appearance/ disappearance of quark matter cores.

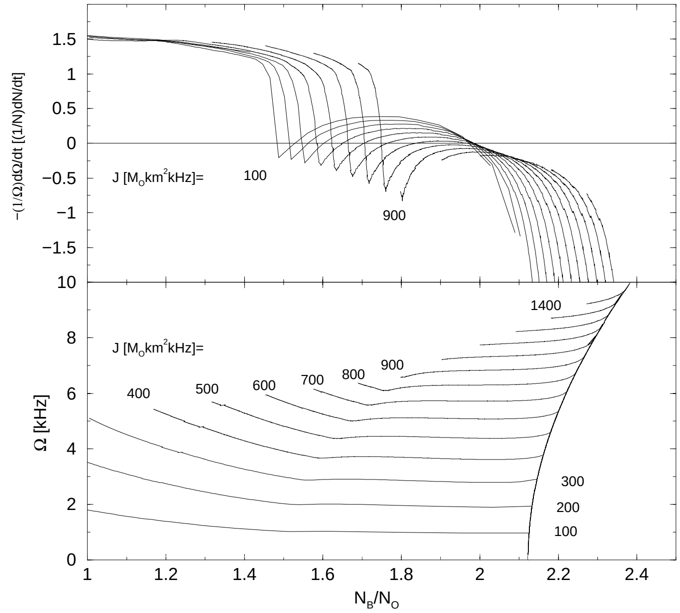

Fig. 9. Total baryon number dependence of the spin-down rate $\dot{\Omega}/\Omega$ in units of the baryon number accretion rate ($\dot{N}_B/N_B$) and the corresponding angular velocity (lower panel) for different (conserved) total angular momenta $J[\mathrm{M}_\odot \mathrm{km}^2\mathrm{kHz}] = 100, 200, \ldots, 1400$. The suggested signal for a deconfinement transition in rapidly rotating neutron stars with baryon number accretion is a transition from a spin-down to a spin-up regime, i.e. a zero in the spin-down rate.

can be considered as precursors of the deconfinement transition. The magnitude of the jump in $n(\Omega)$ during the transition to the quark core regime is a measure for the size of the quark core. It would even be sufficient to observe the maximum of the braking index $n_{\max}$ in order to infer not only the onset of deconfinement ($\Omega_{\max}$) but also the size of the quark core to be developed during further spin-down from the maximum deviation $\delta n = n_{\max} - 3$ of the braking index. For the model EOS we used a significant enhancement of the braking index which does only occur for pulsars with periods $P < 1.5$ ms (corresponding to $\Omega > 4$ kHz), which have not yet been observed in nature. Thus the signal seems to be a weak one for most of the possible candidate pulsars. However, this statement is model dependent since, e.g., for the model EOS used by Glendenning et al. (1997), which includes the strangeness degree of freedom, a more dramatic signal at lower spin frequencies has been reported. Therefore, a more complete investigation of the braking index for a set of realistic EOS should be performed.

4.2. Mass accretion

A higher spin-down rate than in isolated pulsars might be possible for rotating neutron stars with mass accretion. In that case, at high rotation frequency the angular momentum transfer from accreting matter and the influence of magnetic fields can then be neglected (Shapiro & Teukolsky 1983) so that the evolution of the angular velocity is determined by the dependence of the moment of inertia on the total mass, i.e. baryon number,

$$\frac{\dot{\Omega}}{\Omega} = -\left(\frac{N_B}{I}\frac{\mathrm{d}I}{\mathrm{d}N_B}\right)\bigg|_{J=\mathrm{const}}\frac{\dot{N}_B}{N_B}, \tag{42}$$

where $J = I\,\Omega = \mathrm{const}$ has been assumed.

In Fig. 9 we consider the change of the pulsar timing due to mass accretion with a constant accretion rate $\dot{N}_B/N_B$ for fixed total angular momentum as a function of the total baryon number. Here the change from spin-down to spin-up behaviour during the pulsar evolution signals the deconfinement transition. When the pulsar has developed a quark matter core then the change of the moment of inertia due to further mass accretion is negligible and has no longer influence on the pulsar timing. However, in real systems the transfer of angular momentum from the accreting matter can lead to a spin-up already. Then the transition to the quark matter core regime should be observable as an increase in the spin-up rate.

It will be interesting to investigate in the future whether e.g. low-mass X-ray binaries (LMXBs) with mass accretion might be discussed as possible candidates for rapidly rotating neutron stars for which consequences of the transition to the quark core regime due to mass accretion might be detected. Recently, quasi-periodic brightness oscillations (QPO's) with frequences up to ~ 1200 Hz have been observed (Lamb et al. 1998) which entail new mass and radius constraints for compact objects. Note in this context that the assumption of a deconfined matter interior of the compact star in some LMXBs as e.g. SAX J1808.4-3658

(Li et al. 1999) seems to be more consistent than that of an ordinary hadronic matter interior.

5. Conclusions

It has been demonstrated, through the example of the deconfinement transition from hadronic to quark matter, that the rotational characteristics of neutron stars (braking index, spin-down rate) are sensitive to changes of their inner structure and can thus be investigated in order to detect structural phase transitions.

The theoretical basis for the present work was a perturbation method for the solution of the Einstein equations for axial symmetry which allows us to calculate the contribution of different rotational effects to the change of the moment of inertia. This quantity can be used as a tool for the investigation of the changes in the rotation timing for different scenarios of the neutron star evolution.

The deviation of the braking index from the value $n = 3$ (magnetic dipole radiation) as a function of the angular velocity has been suggested as a possible signal for the deconfinement transition and the occurrence of a quark matter core in pulsars. We have reinvestigated this signal within our approach and could show that the magnitude of this deviation is correlated with the size of the quark core, since the influence of the Ae phase crust on such processes is negligible.

For neutron stars with mass accretion, we have suggested that under the assumption of total angular momentum conservation a flip from spin-down to spin-up behaviour signals the appearance of a quark matter core. A more detailed investigation is necessary to identify possible candidates of rotating compact objects with mass accretion (see e.g. Lamb et al. (1998)) for which the suggested deconfinement signal could be relevant.

Although the quantitative details of the reported deconfinement signals are quite model-dependent and might change when one uses more realistic equations of state, e.g. including the strangeness degree of freedom (Glendenning et al. 1997; Weber 1999), the relation between the magnitude of the effect and the size of the quark core which has been found here is expected to be model-independent and should be confirmed by subsequent studies.

Acknowledgements. We thank A. Drago, F. Weber and N. Glendenning for their comments on this work. D.B. acknowledges discussions during the workshop on "Understanding Deconfinement in QCD" at the ECT* in Trento. Our thanks go to R. Avakian, A. Sedrakian and D. Sedrakian for their stimulating interest in this work and for their useful remarks. The work of E.C. and H.G. was supported by the Volkswagen Stiftung under grant No. I/71 226. G.P. received support from the Deutscher Akademischer Austauschdienst (DAAD) and H.G. acknowledges a stipend from the Deutsche Forschungsgemeinschaft (DFG), grant no. 436 ARM 17/1/00.

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