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The response of a dwarf nova disc to real mass transfer variations

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Abstract. We present simulations of dwarf nova outbursts taking into account realistic variations of the mass loss rate from the secondary. The mass transfer variation has been derived from 20 years of visual monitoring and from X-ray observations covering various accretion states of the discless cataclysmic variable AM Herculis. We find that the outburst behaviour of a fictitious dwarf nova with the same system parameters as AM Her is strongly influenced by these variations of the mass loss rate. Depending on the mass loss rate, the disc produces either long outbursts, a cycle of one long outburst followed by two short outbursts, or only short outbursts. The course of the transfer rate dominates the shape of the outbursts because the mass accreted during an outburst cycle roughly equals the mass transferred from the secondary over the outburst interval. Only for less than 10% of the simulated time, when the mass transfer rate is nearly constant, the disc is in a quasi-stationary state during which it periodically repeats the same cycle of outbursts. Consequently, assuming that the secondary stars in non-magnetic CV's do not differ from those in magnetic ones, our simulation indicates that probably all dwarf novae are rarely in a stationary state and are constantly adjusting to the prevailing value of the mass transfer rate from the secondary.

Key words: accretion, accretion disks – stars: binaries: close – stars: individual: AM Her – stars: novae, cataclysmic variables

1. Introduction

Cataclysmic variable systems (CV's), in which a white dwarf accretes material from a Roche lobe filling secondary star (see Warner 1995 for an encyclopaedic review) can be broadly divided into three subclasses: in non-magnetic systems, the white dwarf accretes via an accretion disc; in the magnetic polars, the infalling matter couples directly onto the strong magnetic field of the white dwarf before it can build a disc and is funnelled to accretion region(s) near the magnetic pole(s) of the white dwarf; finally, in CV's containing a weekly magnetic white dwarf (intermediate polars), a partial disc may exist, with the mass flowing from the inner edge of the disrupted disc through magnetically funnelled accretion curtains onto the white dwarf.

Dwarf novae are non-magnetic cataclysmic variable stars which show characteristic eruptions with photometric amplitudes in the range of 2–8 magnitudes [1]. These eruptions typically last a few days to several weeks and recur quasi-periodically on timescales of weeks up to many years. They are generally thought to result from thermal instabilities associated with hydrogen ionization in the accretion disc (see Cannizzo 1993a for a review and Ludwig et al. 1994 for a detailed parameter study).

An important boundary condition of the disc-limit-cycle model is the mass transfer rate from the secondary to the accretion disc. In most studies of dwarf nova outbursts this mass transfer rate is kept constant (Cannizzo 1993b; Ludwig & Meyer 1997; Hameury et al. 1998, 1999). Duschl & Livio (1989) were the first to examine combined mass transfer and disc outbursts, though within the framework of the mass transfer instability model where individual and short-lived mass transfer events are capable of producing single outbursts. Smak (1991, 1999) discussed mass transfer variations in the context of the superoutburst phenomenon in dwarf novae of the SU UMa type and dwarf nova outbursts with enhanced mass transfer during outburst.

King & Cannizzo (1998) and Leach et al. (1999) tested how the accretion disc in a dwarf nova system behaves if the mass transfer from the secondary varies abruptly between different levels. They found that these mass transfer variations produce only subtle effects on normal dwarf novae, including variations in the outburst shape, and that the dwarf novae keep on having outbursts even if the transfer rate is reduced close to zero.

In spite of these efforts we are left with the question of what the real variations of the mass transfer rates from the secondaries in dwarf nova systems are and how they can influence the outburst behaviours of the accretion discs. Fortunately, nature provides an answer to the first question in the form of discless cataclysmic variables, the polars or AM Her systems. In these systems, the mass transfer rate can be estimated directly from the observations because there is no accretion disc acting as a mass buffer. Thus, we can use the long-term light curve of an AM Her system as a measure for the mass transfer variations in a fictitious but realistic dwarf nova system with the same system

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[1] Also intermediate polars are known to show outbursts in their partial discs; e.g. EX Hya, GK Per

parameters. In the present paper, we present the results of such a numerical experiment.

In the following section, we briefly review the equations for the viscous and thermal evolution of the accretion disc, discuss the numerical methods used in our code, and compare the results of two standard models with other fine mesh computations. Thereafter we derive the mass transfer rate in AM Her as a function of time, $\dot{M}_{\rm tr}(t)$. Finally, we apply this mass transfer rate to a fictitious dwarf nova and discuss the effects that can be observed in the outburst behaviour.

2. Finite element methods in the context of disc evolution

The classical equation describing the viscous evolution of the surface density Σ in a geometrically thin, axisymmetric accretion disc is obtained by combining the vertically averaged Navier-Stokes and mass-conservation equations:

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\Big(R^{\frac{1}{2}}\frac{\partial}{\partial R}(R^{\frac{1}{2}}\nu\Sigma)\Big) + \frac{\dot{M}_{\rm tr}(t,R)}{2\pi R} \quad (1)$$

where $\nu = (2/3)\alpha R_{\rm g} T/\mu\Omega$ is the kinematic viscosity, α denotes the viscosity parameter, $R_{\rm g}$ the gas constant and μ the mean molecular weight. The first term on the right hand side is the classical disc diffusion term and the second term describes the external mass-deposition. For the purposes of this study, we follow Cannizzo (1993b) and simply add the mass lost from the secondary $\dot{M}_{\rm tr}$(t) in a Gaussian distribution at a fixed radius near the outer edge.

Similarly, one can derive an energy equation (in the central temperature T:

$$\frac{\partial T}{\partial t} = \frac{2(H-C+J)}{c_{\rm p}\Sigma} - \frac{R_{\rm g}T}{\mu c_{\rm p}}\frac{1}{R}\frac{\partial(Rv_{\rm R})}{\partial R} - v_{\rm R}\frac{\partial T}{\partial R}, \quad (2)$$

where $c_{\rm p}$ denotes the specific heat,

$$v_{\rm R} = \frac{-3\nu}{R}\frac{\partial\log(\nu\Sigma r^{\frac{1}{2}})}{\partial\log R}$$

is the local radial flow velocity, $H = \frac{9}{8}\nu\Omega^2\Sigma$ represents viscous heating, $C = \sigma T_{\rm eff}^4$ is the radiative cooling and

$$J = \frac{3}{2}c_{\rm p}\nu\frac{\Sigma}{R}\frac{\partial}{\partial R}(R\frac{\partial T}{\partial R})$$

the radial energy flux carried by viscous processes (see Smak 1984; Mineshige & Osaki 1983; Mineshige 1986; Ichikawa & Osaki 1992 and Cannizzo 1993b, for discussions).

We solve Eqs. (1) and (2) using a combined Finite-Element / Finite-Difference algorithm (FE for the spatial part and FD for the time-evolution). Apart from our own work (Schreiber & Hessman 1998) the method of Finite Elements has not been used in this context. As this method proved to be extremely robust, it warrants a somewhat more detailed description.

The idea of FE is to divide the region of interest (the disc radii between $R_{\rm in}$ and $R_{\rm out}$) into $n-1$ elements and to expand the function $u(x)$ which is supposed to solve the differential equation with suitable functions $u(x) = \sum_{i=1}^{n} a_i\varphi_i(x)$ for every element. In order to get a continuous solution over all elements, the functions (φ) of every element have to be transformed to the so-called local basis functions N_i and the coefficients to the so-called nodes (c_i) before collected together (Gruber & Rappaz 1985, Schwarz 1991). To solve the differential equation, the function

$$u(x) = \sum_{k=1}^{n} c_i N_i(x) \quad (3)$$

has to meet the requirement formulated by Garlerkin: the integral of the residuum (which one gets by inserting Eq. (3) into the differential equation) weighted with the functions $N_j(x)(j = 1,...,n)$ has to vanish. This requirement, the interchange of integration and summation and partial integration lead to matrix-equations of the form

$$B\dot{c} + Ac = D, \quad (4)$$

with $A = (a_{ij}), B = b_{ij}$ and $D = d_i$ $(i = 1,..,n; j = 1,..,n$ for n nodes).

To solve the differential Eqs. (1) and (2), we have to fill A, B, D in the sense mentioned above and calculate c from Eq. (4). After transforming to the variables $X = 2R^{\frac{1}{2}}$ and $S = X\Sigma$ we derive for the surface density from Eq. (1):

$$\begin{aligned} a_{ij} &= \sum_{k=1}^{n}\nu_k\Big(\int\frac{12}{X^2}\frac{\partial N_i}{\partial X}N_k\frac{\partial N_j}{\partial X}dX \\ &\quad + \int\frac{12}{X^2}N_i\frac{\partial N_k}{\partial X}\frac{\partial N_j}{\partial X}dX\Big) \\ b_{ij} &= \int N_i N_j dX \\ d_i &= \sum_{k=1}^{n}\int\frac{2\dot{M}_{{\rm tr},k}N_k}{\pi X^2}N_j dX. \end{aligned}$$

Similarly it is easy to obtain for the central temperature Eq. (2):

$$\begin{aligned} a_{ij} &= \sum_{k=1}^{n}\Big(\int p_k^{(1)}\frac{\partial N_i}{\partial R}N_k\frac{\partial N_j}{\partial R}dR \\ &\quad + \int p_k^{(2)}N_k\frac{\partial N_i}{\partial R}N_j dR + \int p_k^{(3)}N_i N_k N_j dR\Big) \\ b_{ij} &= \int N_i N_j dR \\ d_j &= \sum_{k=1}^{n}\int p_k^{(4)}N_k N_j dR. \end{aligned}$$

The coefficients $p_k^{(i)}$ are given by

$$\begin{aligned} p_k^{(1)} &= 3c_{\rm p}\nu\Sigma \\ p_k^{(2)} &= \frac{3c_{\rm p}\nu\Sigma}{R} - v_{\rm R} \\ p_k^{(3)} &= \frac{R_{\rm g}}{\mu c_{\rm p}R}\frac{\partial Rv_{\rm R}}{\partial R} \\ p_k^{(4)} &= \frac{9/4\nu\Sigma\Omega^2 - \sigma T_{\rm eff}}{c_{\rm p}\Sigma}. \end{aligned}$$

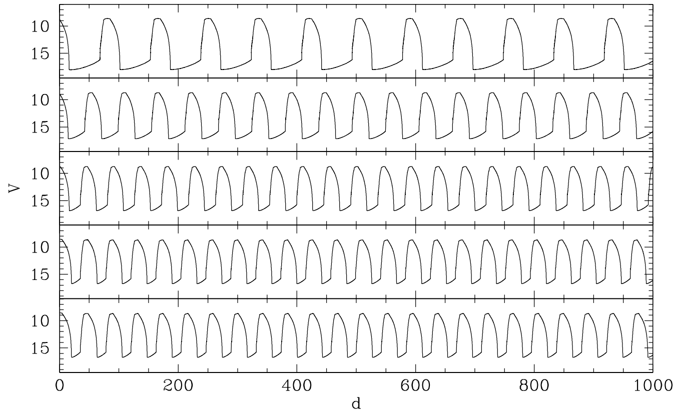

Fig. 1. Comparison with Cannizzo (1993b, his Fig. 6d). From top to bottom we used 40, 60, 80, 100 and 200 nodes. Convergence is obtained for 100 nodes.

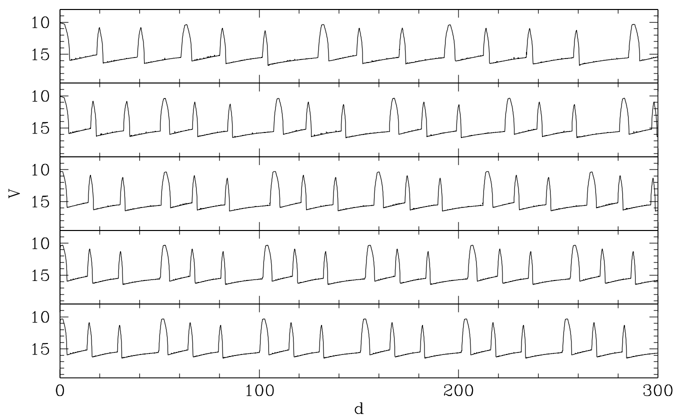

Fig. 2. Comparison with Ludwig & Meyer (1997). From top to bottom we again used 40, 60, 80,100 and 200 nodes. Our code accurately reproduces the sequence of one long outburst followed by two short outbursts.

The index k refers to the value of the quantities at node number k, which is equivalent to the radius R_k. Notice that we only used linear basis functions in this paper.

As mentioned above, Eq. (4) has to be solved to get $c(t+\Delta t)$ from $c(t)$. Using simple finite differences leads to

$$\begin{aligned} c(t+\Delta t) \;&=\; c(t)\frac{1}{2}\Delta t(B^{-1}Ac(t) \\ &\quad +B^{-1}Ac(t+\Delta t)(+B^{-1}D)). \end{aligned} \tag{5}$$

In order to test our code, we carried out two sets of calculations using the binary parameters and cooling functions from Cannizzo (1993b)

$$\begin{aligned} M_1 \;&=\; 1M_\odot,\, \alpha_\mathrm{h}=0.1,\, \alpha_\mathrm{c}=0.02, \\ R_\mathrm{in} \;&=\; 5.0\times10^8\mathrm{cm},\, R_\mathrm{out}=4.0\times10^{10}\mathrm{cm}, \\ \dot{M}_\mathrm{tr} \;&=\; 1.5\times10^9 M_\odot/\mathrm{yr} \end{aligned}$$

and Ludwig & Meyer (1997)

$$\begin{aligned} M_1 \;&=\; 0.63M_\odot,\, \alpha_h=0.2, \alpha_c=0.04, \\ R_\mathrm{in} \;&=\; 8.4\times10^8\mathrm{cm},\, R_\mathrm{out}=1.7\times10^{10}\mathrm{cm}, \\ \dot{M}_\mathrm{tr} \;&=\; 5\times10^{15}\mathrm{g/s}. \end{aligned}$$

The resulting light curves are shown in Figs. 1 and 2. Our code reproduces the sequence of only relatively long outbursts found by Cannizzo (1993b) for the parameter of SS Cygni as well as the sequence of one long outburst followed by two short outbursts found by Ludwig & Meyer (1997) to describe VW Hydri. The short outbursts arise when there is not enough mass stored in the disc and therefore the heating wave gets reflected before it has reached the outer edge of the disc.

We find that at least 100 nodes are necessary for long-term convergence. The outburst and quiescence duration decreases with an increasing number of nodes because – with finer zoning – the length of time spent on the viscous plateau becomes shorter (Cannizzo 1993b). This effect is smaller in Fig. 2 because the disc is smaller in this system.

We conclude that our FE-code produces results which are in excellent agreement with those of other fine-mesh computations.

3. The mass loss rate of the secondary star in AM Herculis

The strong magnetic field of the white dwarf primary in polars prevents the formation of an accretion disc. Without an accretion disc acting as a buffer for the transferred mass, the mass loss rate from the secondary equals the mass accretion rate on the white dwarf *at every moment*, $\dot{M}_\mathrm{acc} = \dot{M}_\mathrm{tr}$ (the free-fall time is $\lesssim 1$ h). As an observational consequence, any variation in the rate at which the secondary star loses mass through the L_1 point will result in a quasi-immediate change of the observed accretion luminosity. The brightest polar, AM Her, has been intensely monitored at optical wavelengths by observers of the AAVSO for more than 20 years and shows an irregular long-term variability, switching back and forth between high- and low states of accretion on timescales of days to months.

The problem of deriving the mass loss history of the secondary star is then equivalent to that of determining the accretion luminosity L_acc as a function of time. As the bulk of the accretion luminosity is emitted in the X-ray regime, a bolometric correction relating the densely monitored optical magnitude to the total luminosity has to be derived. This approach has been followed in detail by Hessman et al. (A&A, submitted) using X-ray observations obtained at multiple epochs. We summarize only briefly the results here. The accretion luminosity is computed from the observed accretion-induced flux F_acc as

$$L_\mathrm{acc}(t) = 4\pi d^2 F_\mathrm{acc}(t) \tag{6}$$

with $d = 90\,pc$ (Gänsicke et al. 1995). We further use $F_\mathrm{acc} \approx F_\mathrm{SX} + 3\times F_\mathrm{HX}$ with F_SX and F_HX the observed soft and hard X-ray fluxes respectively. The factor three accounts for the additional cyclotron radiation emitted from the accretion column and for the thermal reprocession of bremsstrahlung and cyclotron radiation intercepted by the white dwarf and emitted in the ultraviolet (Gänsicke et al. 1995). The mass loss (= transfer) rate is then

$$\dot{M}_\mathrm{tr}(t) = \frac{L_\mathrm{acc}(t)R_\mathrm{wd}}{GM_\mathrm{wd}} \tag{7}$$

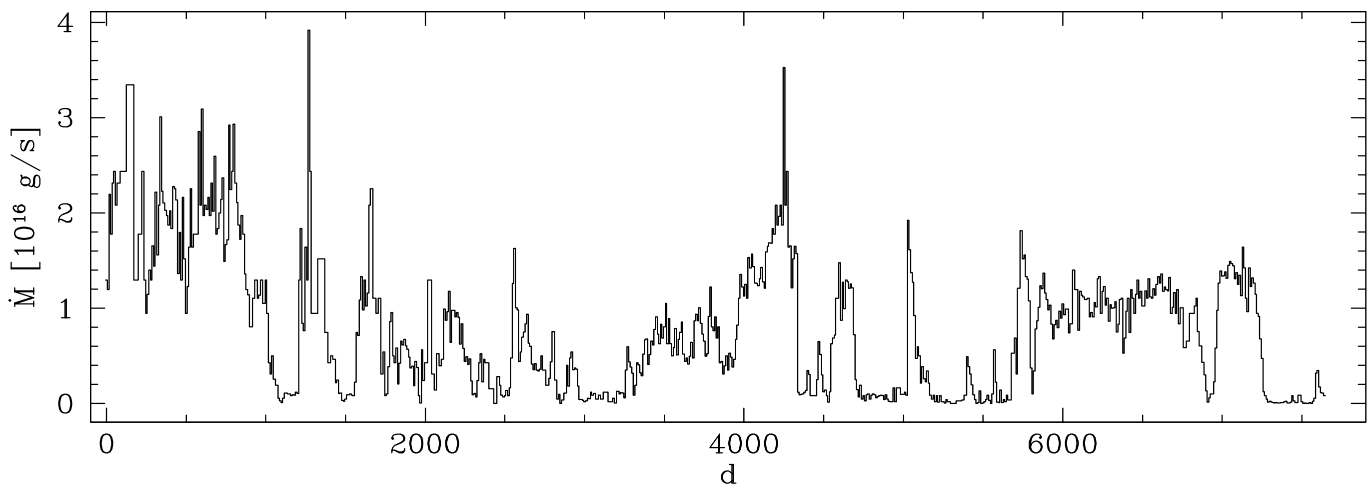

Fig. 3. The mass transfer rate in AM Her as a function of time.

where G is the gravitational constant and $R_{\rm wd}$ and $M_{\rm wd}$ are the white dwarf radius and mass, respectively. As the actual properties of the white dwarf in AM Her are still the subject of controversial discussions (Gänsicke et al. 1998, Cropper et al. 1999), we use the parameters of an average white dwarf, $M_{\rm wd} = 0.6\,M_\odot$ and $R_{\rm wd} = 8.4 \times 10^8\,{\rm cm}$. $\dot{M}_{\rm tr}(t)$ is shown in Fig. 3. The average value of the mass transfer rate in AM Her is $\dot{M}_{\rm av} = 7.88 \times 10^{15}{\rm g\,s^{-1}} = 1.24 \times 10^{-10} M_\odot {\rm yr}^{-1}$. The derived mass transfer rates of AM Her are in general agreement with results published in the literature. For example, Beuermann & Burwitz (1995) found transfer rates between 0.8 and $2.0 \times 10^{-10}\ M_\odot{\rm yr}^{-1}$ and Greeley et al. (1999) estimated a mass transfer rate of $2 \times 10^{16}\ {\rm g\,s^{-1}}$ for the high state of AM Her from far ultraviolet spectra.

4. Results

4.1. The fictitious dwarf nova

We devised a fictitious dwarf nova with a non-magnetic primary of mass $M_{\rm wd} = 0.6\,M_\odot$ and an orbital period of $P = 3.08\,{\rm hr}$, i.e. a non-magnetic twin of AM Her. For these binary parameters, we obtain $R_{\rm in} \simeq R_{\rm wd} = 8.4 \times 10^8$ cm for the inner disc radius and $R_{\rm out} = 2.2 \times 10^{10}$ cm for the outer edge of the disc. We then used our FE code to follow the structure of the accretion disc in our fictitious dwarf nova for 7000 d, applying the variable mass transfer rate $\dot{M}_{\rm tr}(t)$ derived above and standard viscosity parameters $\alpha_{\rm h} = 0.2$ and $\alpha_{\rm c} = 0.04$.

We show 500 day-long samples of our calculations in Figs. 4–8. In each figure, the top panel shows the mass transfer rate as a function of time (solid line) and the average mass transfer rate $\log(\dot{M}_{\rm av}[{\rm g\,s^{-1}}]) = 15.90$ (dotted line). The panel below displays the disc mass $M_{\rm disc}$ normalized with the averaged disc mass $\overline{M}_{\rm disc} = 1.64 \times 10^{23}{\rm g}$. The two lower panels display the light curves calculated with the varying mass transfer rate and the light curves calculated with the constant average mass transfer rate, respectively. For the constant average mass transfer rate the disc goes through a $\sim$ 60 day-long cycle including one long outburst followed by two short outbursts. The long outbursts are those in which the entire disc is transformed into the hot state while the short outbursts arise when the outward moving heating wave is reflected as a cooling wave before it has reached the outer edge of the disc.

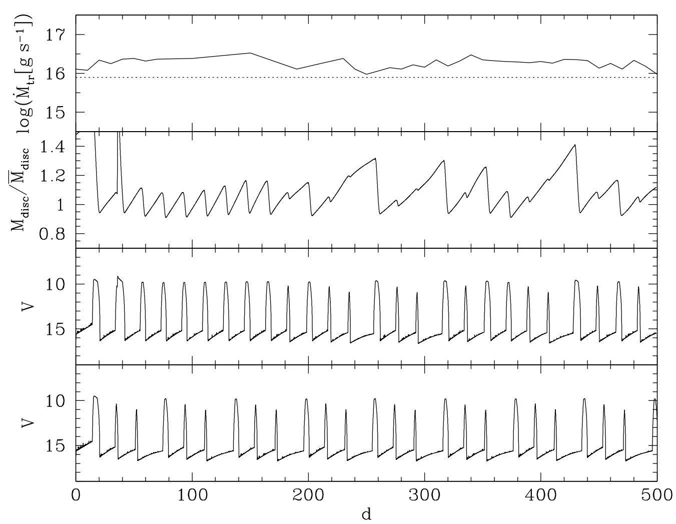

Fig. 4. From top to bottom as a function of time: the mass transfer rate (top, solid line), the averaged mass transfer rate (top, dotted line), the normalized disc mass and the light curves produced by the fictitious dwarf nova with the variable transfer rate and the averaged transfer rate adopted from AM Her.

Our numerical experiment clearly demonstrates that the outburst light curve of the fictitious system is strongly affected by the variations of the mass transfer rate. Even in the case of relatively small fluctuations effects on the outburst behaviour ($16.0 < \log(\dot{M}_{\rm tr}[{\rm g\,s^{-1}}]) < 16.4$) are clearly present in the light curve. In Fig. 4, the mass transfer rate is always high during the 500 days but the disc switches between an accretion state with only long outbursts and states where one or two short outbursts follow a long outburst. When the transfer rate decreases somewhat ($\sim$ day 200), the disc does not save enough mass to

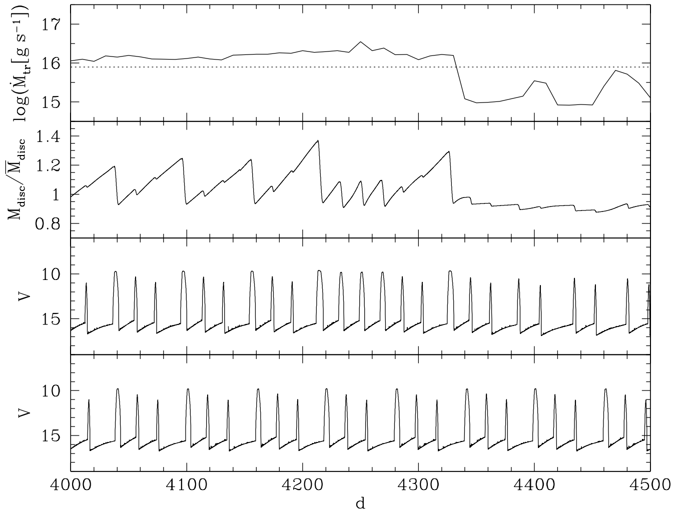

Fig. 5. The same as Fig. 4 but another snapshot of the simulation. The sharp decline of the mass transfer rate is immediately reproduced by the accretion disc.

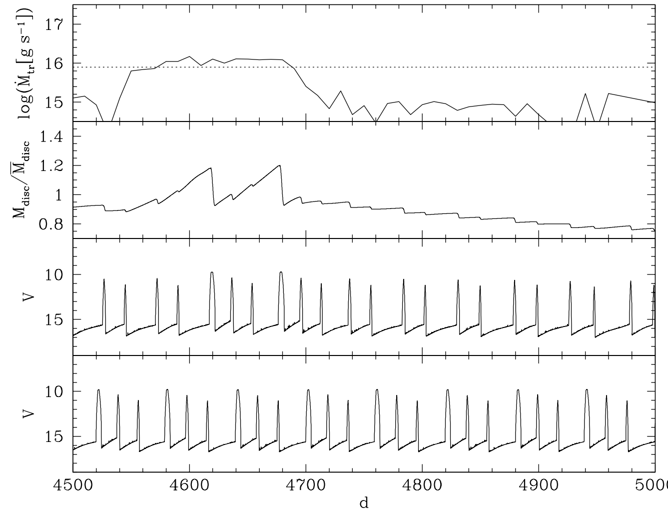

Fig. 6. The same as Fig. 4 but another snapshot of the simulation. Even the long period of low transfer rates does not stop the outburst activity of the disc.

create consecutive long outbursts until the mass transfer rate increases again (∼ day 350).

In addition to this effect, our experiment shows that a sharp decrease in the mass transfer rate instantaneously changes the outburst behaviour of the accretion disc. In Fig. 5 the disc first behaves as in Fig. 4 but when the transfer rate drops sharply (day 4340), the long outbursts immediately vanish and the duration of the quiescent phase increases somewhat.

Another remarkable point is that even during relatively long periods of very low transfer rates (Fig. 6, days 4700– 5000) the disc does not stop its outburst activity but produces only short outbursts with slowly decreasing amplitudes and increasing quiescence intervals. This confirms the findings of King & Cannizzo (1998).

Fig. 7 shows 500 days of our simulation in which the adopted mass transfer from the secondary varies strongly on a timescale of roughly 20 days. This is the most frequent case during our calculation but there are rare periods of nearly constant mass transfer. Fig. 8 shows the light curve of the fictitious system during 500 days in which the mass transfer rate nearly exactly equals its averaged value (top). As a result the light curves computed with the real mass transfer rate and the averaged value look equal.

In summary, one can say that the variations of the mass transfer rate leads the disc to switch between three states in which only long outbursts occur ($\log(\dot{M}_{\rm tr}) \geq 16.3$), one long outburst is followed by one or two short outbursts ($16.3 > \log(\dot{M}_{\rm tr}) > 15.7$), and only short outbursts occur ($\log(\dot{M}_{\rm tr}) \leq 15.7$).

In order to understand the described behaviour of the disc we take into account the viscous timescale $t_{\rm v} \sim R^2/\nu$ which gives an estimate of the timescale for a disc annulus to move a radial distance R. For the quiescent state $t_{\rm v,c}$ and the outburst state $t_{\rm v,h}$ and with $R = R_{\rm out}$ (where the mass transferred from the secondary is added to the disc) we obtain for the viscous timescale

$$t_{\rm v,c} \sim \frac{R^2}{\nu} \sim 2000\,{\rm d}, \tag{8}$$

$$t_{\rm v,h} \sim \frac{R^2}{\nu} \sim 15\,{\rm d}. \tag{9}$$

The mass added to the disc during quiescence is stored in the disc because it moves inward on the long timescale $t_{\rm v,c}$ whereas during an outburst even mass from outer regions can reach the white dwarf within a viscous timescale. This makes it possible that the mass accreted onto the white dwarf during a long outburst can be up to roughly one third of the disc mass ($\sim$ day 4220). Therefore the disc can relax to equilibrium with the mass transfer rate in only one outburst in the case of high transfer rates (Fig. 4).

The prompt response of the disc to the sharp decline in the mass transfer rate (Fig. 5) can be understood in the same way: due to the short viscous time ($t_{\rm v,h}$) the disc accretes a substantial fraction of the disc mass ($\sim 1/4 M_{\rm disc}$) during the last long outburst which immediately prevents long outbursts when the mass transfer rate becomes low ($\sim$ day 4340).

Finally, the long period of low transfer rates (Fig. 6) does not prevent outbursts because the mass accreted during the short outbursts is only a few percent of the disc mass. Therefore the disc relaxes to the mass transfer rate on a longer timescale.

4.2. The mass transfer and mass accretion rates

An important point in understanding the physics of accreting binaries is to know how far the outburst behaviour and hence the resulting light curves depend on real variations of the mass transfer rate.

To answer this question, we compare the averaged mass accretion rate onto the white dwarf with the mass transfer rate. Fig. 9 shows that the time in which the disc relaxes to an equilibrium with the mass transfer rate depends on the occurrence of long outbursts: when the accretion rate is averaged over 20 days

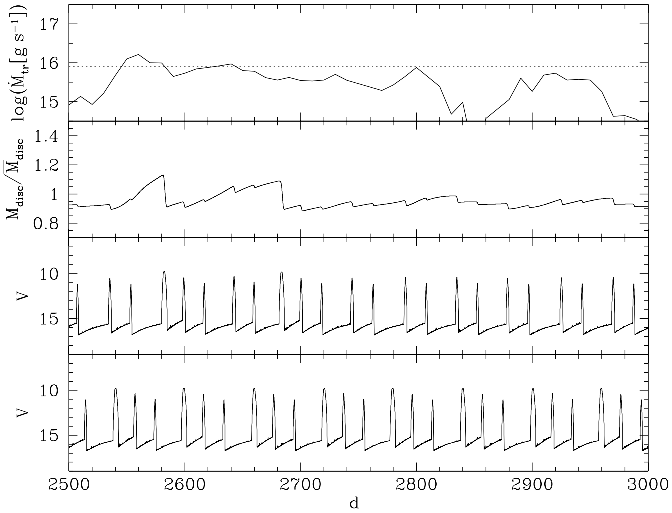

Fig. 7. The same as Fig. 4 but a snapshot of the simulation where the transfer rate strongly varies.

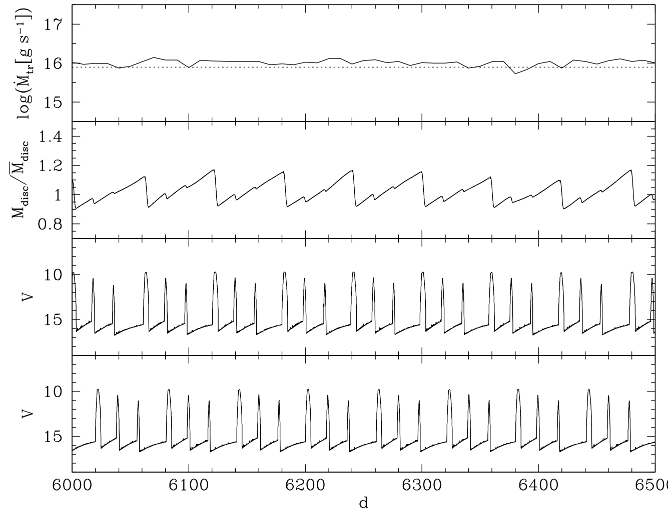

Fig. 8. The same as Fig. 4 but a snapshot of the simulation where the transfer rate nearly stays constant.

(dotted line in Fig. 9) the mass transfer and the accretion rate correspond roughly only during periods where only long outbursts occur (days 40 – 150, see also Fig. 4) but for the periods where the disc goes through a cycle of short and long outbursts the accretion rate has to be averaged over 60 days to match the mass transfer rate (days 150 – 500). If the mass transfer rate drops steeply and stays in a low–state (days 4700 – 5000 in Fig. 10, see also Fig. 6), the accretion rate needs more than 60 days to follow this behaviour, because the disc produces only short outbursts in which only a small percentage of the disc mass is involved.

In order to make this plausible we give a relaxation timescale $t_{\rm r}$ as the ratio of the viscous timescale in outburst with the relative mass fraction accreted during an outburst:

$$t_{\rm r} \sim t_{\rm v,h}\frac{M_{\rm disc}}{\Delta M_{\rm disc}} \tag{10}$$

For high mass transfer rates this timescale is around $70\,{\rm days}$ and so the correspondence of the averaged accretion rate and the mass transfer rate in Fig. 9. is not surprising. In the case of low transfer rates (Fig. 10, day 4700–5000) this timescale is longer ($\sim 300\,{\rm days}$) because only roughly 5 percent of the disc mass are accreted during an outburst.

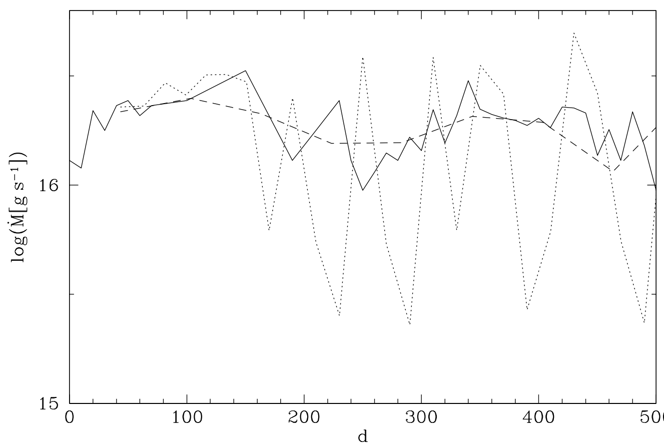

Fig. 9. The mass transfer rate (solid line) and the accretion rate onto the white dwarf as function of time. The accretion rate is averaged over 60 days (dashed line) and averaged over 20 days (dotted line).

4.3. Dependence on the primary mass

In the numerical experiment above, we have assumed an average white dwarf mass for the primary in AM Her. The literature holds a large spectrum of white dwarf mass estimates for AM Her, $M_{\rm wd} = 0.39\,M_\odot$ (Young & Schneider 1981), $M_{\rm wd} = 0.69\,M_\odot$ (Wu et al.1995), $M_{\rm wd} = 0.75\,M_\odot$ (Mukai & Charles 1987), $M_{\rm wd} = 0.91\,M_\odot$ (Mouchet 1993) and $M_{\rm wd} = 1.22\,M_\odot$ (Cropper et al. 1998). Based on the observed ultraviolet spectrum of AM Her and on its well-established distance, Gänsicke et al. (1998) estimated $R_{\rm wd} \approx 1.1\times10^9$ cm, and, using the Hamada & Salpeter (1961) mass-radius relation for carbon cores, $0.35\,M_\odot < M_{\rm wd} < 0.53\,M_\odot$. As the Hamada & Salpeter mass-radius relation is valid for cold white dwarfs, the finite temperature $\approx 20\,000\,{\rm K}$ of the white dwarf in AM Her would allow also somewhat higher masses, $M_{\rm wd} \approx 0.65 M_\odot$, which is very close to the *average* mass of field white dwarfs, $0.6 M_\odot$, that we used.

Even though we exclude a massive white dwarf based on the observational evidences, we repeated our simulation with $M_{\rm wd} = 1.0 M_\odot$ in order to test the sensitivity of our results.

In a first step, we recompute $\dot{M}_{\rm tr}(t)$ from Eq. (7) with $M_{\rm wd} = 1.0\,M_\odot$ and a corresponding $R_{\rm wd} = 5.4\times10^8$ cm. The resulting mean accretion rate is $3.0\times10^{15}{\rm g\,s^{-1}}$, a factor 2.6 lower than before. Then, we simulated once more 7000 days of disc evolution with the new $\dot{M}_{\rm tr}(t)$, $\alpha_{\rm h} = 0.1$, $\alpha_{\rm c} = 0.02$ and $R_{\rm out} = 2.8\times10^{10}\,{\rm cm}$.

In Fig. 11 we show 500 days of our calculation with $M_{\rm wd} = 1.0\,M_\odot$. The disc produces only short outbursts and the outburst cycle of four outbursts with decreasing amplitude is hardly

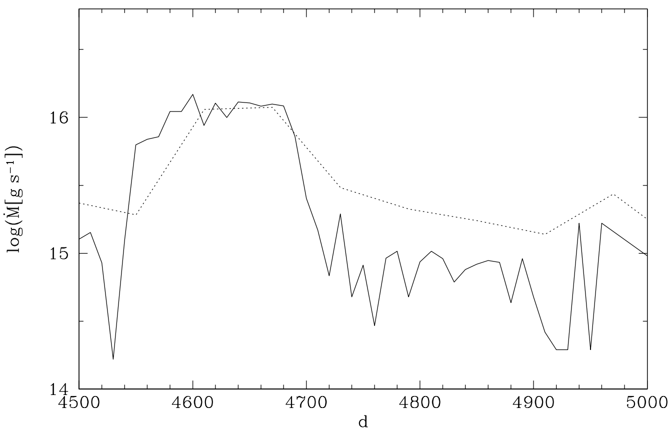

Fig. 10. The mass transfer rate (solid line) and the accretion rate onto the white dwarf as function of time. The accretion rate is averaged over 60 days (dotted line).

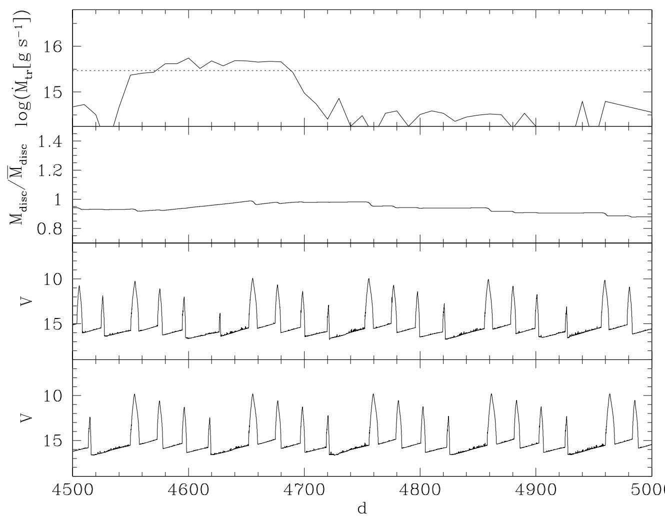

Fig. 11. The same as in Fig. 6 but assuming a more massive primary $M_{\rm wd} = 1.0\,M_{\odot}$

changed even by drastic variations of the mass transfer rate (day 4700).

The different responses that our fictitious dwarf novae with $0.6 M_{\odot}$ and $1.0 M_{\odot}$ white dwarfs show to the variable mass transfer rate are easy to understand: both the increased primary mass and the decreased radius of the white dwarf reduce – as mentioned above – the derived average mass transfer rate.

In addition, the outer disc radius of the fictitious system with $M_{\rm wd} = 1.0 M_{\odot}$ increases. The disc becomes more massive, i.e. $\overline{M}_{\rm disc} = 4.59 \times 10^{23}$g, because of its increased size.

Due to the reduced mass transfer rate and the increased disc size, the heating waves are able to reach the outer edge of the disc only during the first outburst of the outburst cycle and only in cases where the mass transfer rate is high ($\log\,\dot{M}_{\rm tr} \geq 16.1$). Even in this rare situation the disc stays only a few days in the hot state and accretes only a small fraction of the disc mass ($\sim 1/8\,M_{\rm disc}$). For lower transfer rates (the extremely more frequent case shown in Fig. 11) the heating front gets reflected before it has reached the outer edge of the disc. Hence, only a small percentage of the disc mass is involved in any outburst. Therefore, and due to the longer viscous timescale, the relaxation timescale given in Eq. (10) is always larger than a few years.

Summing up, the adopted primary mass and the average accretion rate play an important role on the influence that the variable mass transfer rate has on the outburst behaviour.

5. A note on the disc limit-cycle model

A number of new features have been added to the well known limit-cycle model (Meyer & Meyer-Hofmeister 1981; Smak 1982; Cannizzo et al. 1982). Hameury et al. (1998, 1999, 2000) have shown how the light curves change if the disc size is allowed to vary and what effects irradiation has on the outburst behaviour. Additionally these authors discussed other sources of uncertainties such as the tidal torque and the evaporation of the inner parts of the disc (Meyer & Meyer-Hofmeister 1994). They concluded that all these effects must be included in order to obtain meaningful physical information on e.g. the viscosity from the comparison of predicted and observed light curves. Another point of importance is the interaction of the accretion stream leaving the secondary star and the accretion disc. Schreiber & Hessman (1998) tested the influence of stream overflow on the disc evolution in dwarf novae. They found that significant stream overflow can lead to reversion of the inward-travelling cooling front and create an outward-travelling heating front. This behaviour would produce small dips in the light curve during the declining phase.

The model we used here contains neither irradiation nor allows it the disc radius to vary. The stream mass is simply deposited in a small Gaussian distribution near the outer edge of the disc. Therefore we do not attempt at present a comparison with observed light curves. Nevertheless, all calculated light curves with constant mass transfer from the secondary relax in a quasi-stationary outburst cycle (of one or more outbursts) which repeat periodically. Our model shows in a qualitative way that real mass transfer variations may have a dominant influence on the outburst behaviour at least in systems with relatively small discs and high mass transfer.

6. Discussion and conclusions

There is no reason to assume that the mass transfer variations of the secondary observed in AM Her are not present in non-magnetic systems. Our numerical experiment including realistic variations of the mass transfer rate in a dwarf nova system is, therefore, a significant step towards a better understanding of dwarf nova light curves, and, thereby, of the underlying disc limit cycle.

The light curve produced by our fictitious system switches between three states depending on the actual mass transfer rate. High transfer rates lead to only long outbursts where the entire disc is transformed into the hot state. If the transfer rate is near the average value the disc goes through a cycle of three outbursts, one long outburst followed by two short ones. Even long periods of low transfer rates do not force the disc to stop its outburst activity: long outbursts are suppressed and the duration of quiescence increases but the disc always produces short outbursts. From this follows that the low-states of VY Sculptoris stars (a subgroup of novalike variables) could not be caused by low transfer rates alone (see also Leach et al. 1999).

We find that in our fictitious system the mass accreted during an outburst cycle is dominated by the course of the mass transfer rate if the mass transfer rate varies significantly. The disc always relaxes to equilibrium with the mass input from the secondary. Thus, our experiment strongly supports King & Cannizzo's (1998) claim that dwarf nova accretion discs are probably never in a stationary state but are constantly adjusting to the prevailing value of $\dot{M}_{\rm tr}$. Only during periods where the mass transfer rate is nearly exactly constant the disc periodically repeat the quasi stationary outburst cycle. Such periods are rare but occur in AM Her.

The strong influence of the mass transfer rate on the outburst behaviour of the fictitious system clearly indicates that probably most (if not all) the deviations from periodic outburst cycles seen in the light curves of dwarf novae are caused by variations of the mass transfer rate.

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