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Spectrum of a random f-mode and the SOHO/MDI data

K. Murawski

Department of Environmental Physics, Technical University of Lublin, ul. Nadbystrzycka 40, 20-618 Lublin, Poland
(kamur@akropolis.pol.lublin.pl)

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Abstract. According to the SOHO/MDI observations (Duvall et al. 1998, Antia & Basu 1999) the solar *f*-mode frequency departs from the parabolic dispersion relation $\omega^2 = gk$. We propose to explain this behaviour in terms of a space- and time-dependent random flow that occurs in the convection zone. A time-dependent random flow speeds up the *f*-mode while the effect of a space-dependent random flow is to slow it down. A competition between these two effects brings a reduction of frequencies and a line-width increase at low l. Theoretical estimation of the frequency shift and line-width leads to a conclusion that for $l < 2000$ the results are consistent with the recent SOHO/MDI data.

Key words: convection – Sun: atmosphere – Sun: granulation – Sun: oscillations – turbulence

1. Introduction

The SOHO/MDI observations (Duvall et al. 1998, Antia & Basu 1999) of the solar *f*-mode provided an additional evidence that its frequency departs from the parabolic dispersion relation $\omega_0^2 = gk$. We apply a space- and time-dependent random flow in the convection zone and show that an interaction between this flow and the *f*-mode can explain for low spherical degree $l < 2000$ the results of the SOHO/MDI observations. In particular, a space-dependent random field, which was already discussed by Murawski & Roberts (1993) and Murawski et al. (1998), scatters coherent energy into incoherent energy by exciting random waves and results in attenuation and slowing down of the *f*-mode. In the case of a time-dependent random field, the energy can also be transferred from the turbulent field into the *f*-mode, leading to mode acceleration and amplification.

The paper is organized as follows. Sect. 2 presents the dispersion relation for the case of a space- and time-dependent random flow. The following section shows the results of numerical solutions of the dispersion relation. The final section contains conclusions.

2. Theory

Using a plane parallel one-dimensional model for the solar equilibrium, we consider the *f*-mode that propagates along an interface between two semi-infinite layers of perfect gas that is stratified under gravity $g = 274$ m/s^2. It is assumed that the gas is incompressible, free of a magnetic field and the oscillations are adiabatic. For adiabatic *f*-mode oscillations see Gabriel (1991). Moreover, we assume that at the equilibrium a weak random flow occurs in the convection zone only and that this flow is both space- and time-dependent. This flow satisfies Gaussian statistics and possesses characteristic spatial l_x and temporal l_t scales as well as its magnitude is σ. This model is described in details by Murawski & Roberts (1993). To obtain random velocity field correction to the parabolic dispersion relation we adopt the method of Howe (1971) which is a perturbative method that applies the binary collision (a weak random field) approximation. This model leads to the following dispersion relation:

$$\omega^2 - gk \; = \; \frac{4}{\pi^2} l_x l_t \sigma^2 \omega k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \times \frac{\hat{k}\hat{\omega} e^{-l_x^2(\hat{k}-k)^2} e^{-l_t^2(\hat{\omega}-\omega)^2}}{\hat{\omega}^2 - g\hat{k}} \, d\hat{k}\, d\hat{\omega}. \tag{1}$$

From this dispersion relation it follows that the dependence of the cyclic frequency ω on the wavevector k differs from the non-turbulent dispersion relation that is given by the left hand side of this equation.

Eq. (1) is more general than the corresponding relations used by Murawski & Roberts (1993) and Murawski et al. (1998). The generalization is based on an adoption of a space- and time-dependent random flow while Murawski & Roberts (1993) and Murawski et al. (1998) derived the dispersion relation in the case of a space-dependent flow only. We recover the limit discussed in the former studies in the case when $l_t \to \infty$. In particular, in the case of $l_t \to \infty$ ($l_x \to \infty$) a random medium becomes deterministic in time (space). Then, $l_x e^{-l_x^2 k^2} \to \delta(k)$ ($l_t e^{-l_t^2 \omega^2} \to \delta(\omega)$), where $\delta(a)$ is the Dirac's delta function of its argument a. In the limit of $l_t \to \infty$ ($l_x \to \infty$) dispersion relation (1) corresponds to a space-dependent (a time-dependent) flow.

With a use of the normalized horizontal wavenumber $K = kl_x$ and the normalized frequency $\Omega = \omega l_t$ Eq. (1) can be rewritten as

$$K - \frac{l_x}{gl_t^2}\Omega^2 \; = \; \frac{4\sigma^2}{\pi^2 g^2 l_t^2} K\Omega \Big[\pi\Omega + \sqrt{\pi} \frac{l_x}{gl_t^2}$$

$$\times \int_{-\infty}^{\infty} \hat{\Omega}^3 e^{-(\hat{\Omega}-\Omega)^2} Z\left(\frac{l_x}{g l_t^2}\hat{\Omega}^2 - K\right) d\hat{\Omega}\Big], \quad (2)$$

where $Z(\xi)$ is the plasma dispersion function (Fried & Conte 1961),

$$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-z^2}}{z-\xi}\, dz, \qquad Im(\xi) > 0. \quad (3)$$

3. Numerical results

We have solved Eq. (2) numerically for various values of the convective velocity variance, σ, the correlation length l_x, and the correlation time l_t and compared the results with the SOHO/MDI observational data. The integral in Eq. (2) has been computed by the use of the 8th point Gauss quadrature method.

In the following figures, the frequency difference $\Delta\nu \equiv \nu - \nu_0$ and the line-width Γ are displayed as a function of l. The quantity $\Delta\nu$ is obtained by determining $\nu = \omega/2\pi$ numerically from dispersion relation (2) and then subtracting the cyclic frequency $\nu_0 = \sqrt{gk}/2\pi$ that pertains in a static convection zone. The line-width Γ can be expressed through the imaginary part of the frequency, $\Gamma = -2\,Im(\omega)$ (e.g., Osaki 1990). The SOHO/MDI data (Duvall et al. 1998, Antia & Basu 1999) are shown for comparison purposes.

A random flow shifts the *f*-mode frequency. This shift is described by the real part of ν. In the case of a space-dependent flow, the *f*-mode spends more time propagating against the flow than with the flow. As a result, its effective speed and consequently frequency are reduced. A detailed explanation of this can be found in Murawski & Roberts (1993). As a consequence of scattering by turbulent flow, the energy of the *f*-mode is partially transformed between the turbulent and coherent fields. This phenomenon is associated with the imaginary part of the frequency, $Im(\omega)$. A negative (positive) imaginary part of the frequency represents the damping (amplification) of the coherent *f*-mode field due to scattering by the turbulent flow. The *f*-mode damping (amplification) is a result of the generation of the turbulent (coherent) field at the expense of the coherent (turbulent) field. It is generally believed (Pelinovsky, private communication) that a space-dependent (time-dependent) random field leads to wave damping (amplification). However, this scenario depends on the process of interaction between the incident wave, a scattered wave, and the so called inhomogeneous wave (Murawski & Pelinovsky 2000). This interaction, in the case of a space-dependent flow, can lead to wave amplification and frequency increase at low l.

Fig. 1 presents the frequency difference $\Delta\nu$ and the line-width Γ as functions of the angular degree l for the random flow with its magnitude $\sigma = 1.5\,\mathrm{km\,s^{-1}}$, the spatial scale $l_x = 1400\,\mathrm{km}$ (solid line), $l_x = 1000\,\mathrm{km}$ (dotted line) and $l_x = 600\,\mathrm{km}$ (dashed line). The correlation time l_t corresponds to the life-time (~ 10 min) of a granule and the correlation length l_x is associated with the spatial scale (200 km–2000 km) of granulation. A fit to the data of Antia & Basu (1999) is poor for $l > 1900$; the theoretical curve falls down while the observational data is shifted up.

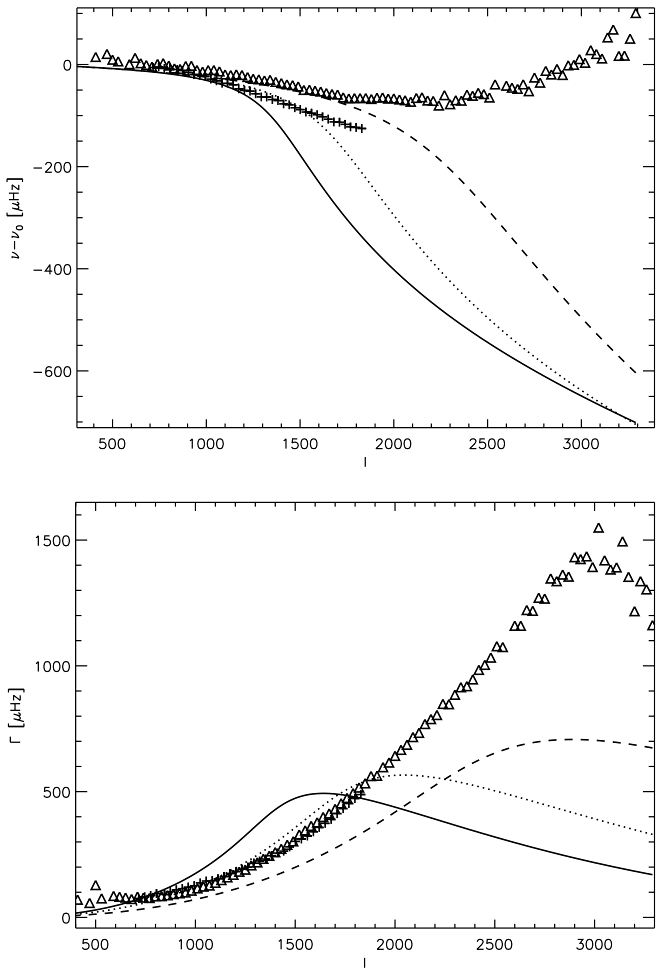

Fig. 1. Frequency difference $\nu - \nu_0$ and the line-width Γ as functions of the angular degree l for the case of random flow with $\sigma = 1.5\,\mathrm{km\,s^{-1}}$, $l_t = 600\,\mathrm{s}$, $l_x = 1400\,\mathrm{km}$ (solid line), $l_x = 1000\,\mathrm{km}$ (dotted line) and $l_x = 600\,\mathrm{km}$ (dashed line). The crosses and triangles correspond to the observational SOHO/MDI data of Duvall et al. (1998) and Antia & Basu (1999), respectively.

An agreement in Γ between the theoretical results and the observational data is good for $l < 1800$ for which the data of Duvall et al. (1998) and Antia & Basu (1999) are close to each other. For higher values of l, the theoretical curves exhibit maxima which occur at higher l for lower l_x (smaller granules). These maxima are associated with enhanced damping of the *f*-mode when its wavelength $\lambda \sim l_x$.

Fig. 2 displays a dependence of $\Delta\nu$ and Γ on the correlation time l_t. Three values of l_t have been chosen: $l_t = 1800\,\mathrm{s}$ that is equal to the turn-over time (solid line), $l_t = 300\,\mathrm{s}$ equal to the period of 5 min oscillations (dotted line), and $l_t = 100\,\mathrm{s}$ (dashed line). As a result of shorter time-scale l_t a frequency of the *f*-mode is lifted up. A good fit to the observational data is obtained in the case of $l_t = 1800\,\mathrm{s}$ (crosses) and $l_t = 300\,\mathrm{s}$ for $l < 1500$ (triangles). The dashed curve exhibits a cross-over, at which $\Delta\nu = 0$, at $l \simeq 1000$ which is close to the observational value $l = 800$ (Libbrecht et al. 1990, Fernandes et al. 1992).

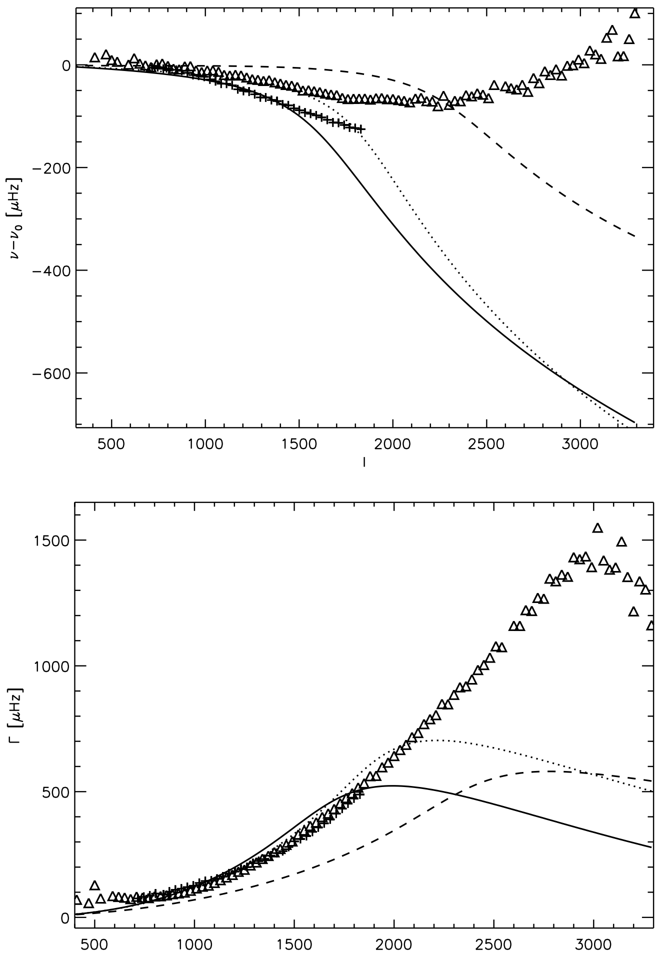

Fig. 2. Frequency difference $\nu - \nu_0$ and the line-width Γ as functions of the angular degree l for the case of random flow with $\sigma = 1.5\,\mathrm{km\,s^{-1}}$, $l_x = 1000\,\mathrm{km}$, $l_t = 1800\,\mathrm{s}$ (solid line), $l_t = 300\,\mathrm{s}$ (dotted line) and $l_t = 100\,\mathrm{s}$ (dashed line). The crosses and triangles correspond to the observational SOHO/MDI data of Duvall et al. (1998) and Antia & Basu (1999), respectively.

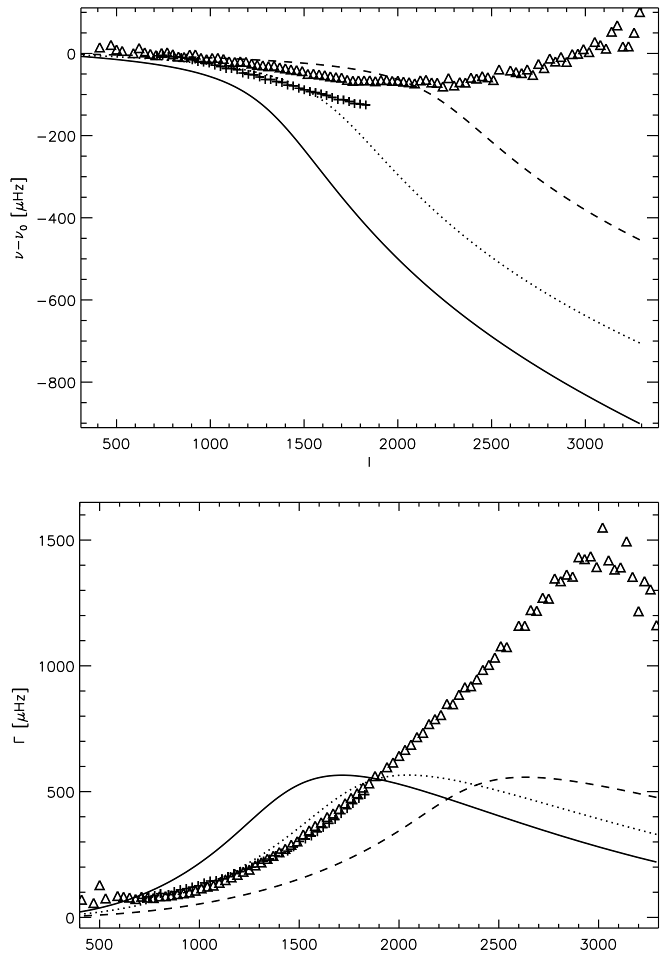

Fig. 3. Frequency difference $\nu - \nu_0$ and the line-width Γ as functions of the angular degree l for the case of random flow with $l_x = 1000\,\mathrm{km}$, $l_t = 600\,\mathrm{s}$, $\sigma = 2\,\mathrm{km\,s^{-1}}$ (solid line), $\sigma = 1.5\,\mathrm{km\,s^{-1}}$ (dotted line) and $\sigma = 1\,\mathrm{km\,s^{-1}}$ (dashed line). The crosses and triangles correspond to the observational SOHO/MDI data of Duvall et al. (1998) and Antia & Basu (1999), respectively.

The observational line-widths are well approximated in the range of $l < 1700$ by the solid and dotted lines. A shorter-lived flow ($l_t = 100\,\mathrm{s}$) shifts up the frequencies and decreases the line-width.

Fig. 3 shows the *f*-mode spectrum dependence on the strength of a random flow, σ. In agreement with the former results (e.g., Murawski & Roberts 1993) a stronger random flow decreases more significantly frequencies and increases the line-width of the *f*-mode.

4. Conclusions

We have applied an analytical perturbation technique (Howe 1971) and numerical methods to estimate the imaginary part and the frequency shift, and show that for $l < 2000$ the results are close to the properties of the *f*-mode obtained from the high-resolution SOHO/MDI data (Duvall et al. 1998). A poor fit at high l of the theoretical data to the SOHO/MDI data of Antia & Basu (1999) implies that either the theory needs some refinement, other physical processes come into play or the SOHO/MDI data requires verification. While all these effects can be blamed for the poor fit we limit ourself here to some criticism addressed to the theory.

The method of Howe (1971) suffers from a drawback that it concerns the case of a weak random field and only single values for σ, l_x and l_t can be applied simultaneously. The above presented results reveal that granulation of various strength, spatial and temporal scales influences differently a spectrum of the solar *f*-mode. In particular, from Fig. 1 it follows that at low l ($l < 1700$) the line-width Γ is higher for larger granules while at high l smaller granules contribute more to Γ. Therefore, we expect that averaging of the results over the range of these scales would lead to a better agreement between the theoretical and observational data. Indeed, such averaging over l_x leads to a significant improvement of the results in Γ in the case of a space-dependent flow (Mędrek & Murawski 2000).

Finally, we should admit that the model of the solar atmosphere that is discussed in this paper is very simple. For example, the solar surface is modeled as as a sharp interface between two incompressible fluids. Given the surface wave nature of the *f*-mode, such an approximation is acceptable in the small wavelength limit, otherwise the *f*-mode sees the vertical density stratification. The proper atmospheric stratification has been discussed by Murawski (2000). Moreover, the flow topology exerts an influence on the *f*-mode. In this paper the flow is horizontal. Murawski & Pelinovsky (2000) showed that a random flow associated with downdrafts can amplify the sound waves and a flow at the centers of granulation damps these waves. We expect that a similar scenario is also valid in the case of the *f*-mode. However, from an analytical point of view incorporation of these effects into a realistic model of the solar atmosphere is a formidable task, and so we are sympathetic with the assumptions we were driven to invoke.

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