Astron. Astrophys. 358, 425–427 (2000)

ASTRONOMY AND ASTROPHYSICS

*Research Note*

# The big bang as a higher-dimensional shock wave

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Received 18 October 1999 / Accepted 22 December 1999

**Abstract.** We give an exact solution of the five-dimensional field equations which describes a shock wave moving in time and the extra (Kaluza-Klein) coordinate. The matter in four-dimensional spacetime is a cosmology with good physical properties. The solution suggests to us that the 4D big bang was a 5D shock wave.

**Key words:** relativity – cosmology: cosmic microwave background – cosmology: theory – cosmology: early Universe

## 1. Introduction

The idea that the universe was created from nothing has a very long history. Some highlights in the scientific literature include the argument that the big bang was a transition from an earlier four-dimensional Minkowski space to a later space with standard Friedmann-Robertson-Walker properties (Bonnor 1960; Wesson 1985). It is also possible in principle that the big bang was a quantum tunneling event from nothing into 4D de Sitter space (Vilenkin 1982). These and other ideas connected with inflation can be put on a firmer basis if the manifold is extended from 4D to higher dimensions (for a review see Overduin & Wesson 1997). For example, it is well known that any solution of 4D general relativity can be embedded in a *flat* 10D space. In what follows, we will use the minimal extension from 4D (Einstein) space to 5D (Kaluza-Klein) space to argue that the 4D big bang may be the signature of a 5D blip or shock wave.

We will draw on recent results in three areas of higher-dimensional cosmology.

(a) We can take an empty 5D space that is in general curved and derive from it a matter-filled 4D space (Wesson & Ponce de Leon 1992; Coley 1994; Overduin & Wesson 1997). That such models have matter in 4D but are empty in 5D follows from new work on an old theorem of differential geometry due to Campbell (1926). Let us consider a solution of the 4D Einstein field equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is the energy-momentum tensor. (Here and elsewhere we use a choice of units to set the gravitational constant and the speed of light equal to unity; lowercase Greek letters run 0,123 and uppercase Latin letters run 0,123,4.) Then it can be shown that the 4D Einstein equations can be locally embedded in the field equations of 5D Kaluza-Klein theory *without* sources, which are given in terms of the Ricci tensor by $R_{AB} = 0$ (Romero et al. 1996; Lidsey et al. 1997; Wesson 1999). This is a very powerful theorem.

(b) We can take an empty and *flat* 5D space and embed in it matter-filled *curved* 4D spaces (Abolghasem et al. 1996; Liu & Wesson 1998; Wesson 1994). This means that the 4D big bang could be an artifact of a bad choice of 5D coordinates.

(c) We can study *wave-like* solutions of $R_{AB} = 0$ (Liu et al. 1993; Liu & Wesson 1994; Billyard & Wesson 1996; Sajko et al. 1998, 1999). Some of these have remarkable physical properties, and indicate that the big bang could have been a quantum transition from an oscillating to a growing (inflationary) mode.

In the next section, we will combine results from the above three areas to derive an exact solution in 5D which has good physical properties in 4D and implies a significant change in how we can view the big bang.

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## 2. A 5D shock wave and the 4D big bang

We choose coordinates $x^A = t, r\theta\phi, l$ with $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$, as usual. A wave in the $t/l-$plane should depend on $u \equiv t - l$. One such solution of $R_{AB} = 0$ is given by the following 5D line element:

$$dS^2 = b^2dt^2 - a^2\left(dr^2 + r^2d\Omega^2\right) - b^2dl^2 \quad (1)$$

$$a = (hu)^{\frac{1}{2+3\alpha}} \quad (2)$$

$$b = (hu)^{-\frac{1+3\alpha}{2(2+3\alpha)}} . \quad (3)$$

This may be confirmed either algebraically using the expanded form of the field Eq. s (Wesson 1999) or computationally using a fast computer package (Lake et al. 1995). The class (1)–(3)

depends on two constants, $h$ and $\alpha$. The first has physical dimensions of $L^{-1}$ and is related to Hubble's parameter (see below). The second is dimensionless and is related to the properties of matter associated with the solution. These can be evaluated using the regular technique, wherein $R_{AB} = 0$ is broken down to $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ with an induced or effective energy-momentum tensor that depends on the pressure $p$ and density $\rho$ of a cosmological perfect fluid (Overduin & Wesson 1997). There is an associated equation of state, and after some algebra we find

$$p = \alpha\rho \tag{4}$$

$$8\pi\rho = \frac{3h^2}{(2+3\alpha)^2} a^{-3(1+\alpha)}. \tag{5}$$

We see that $\alpha = 0$ corresponds to the late (dust) universe, and $\alpha = 1/3$ corresponds the the early (radiation) universe.

To elucidate the physical properties of the solution, it is instructive to change from the coordinate time $t$ to the proper time $T$. This is defined by $dT = b\, dt$, so

$$T = \frac{2}{3}\left(\frac{2+3\alpha}{1+\alpha}\right)\frac{1}{h}\,(hu)^{\frac{3}{2}\left(\frac{1+\alpha}{2+3\alpha}\right)}. \tag{6}$$

The 4D scale factor which determines the dynamics of the model by (2) and (6) is then

$$a(T) = \left[\frac{3}{2}\left(\frac{1+\alpha}{2+3\alpha}\right)hT\right]^{\frac{2}{3(1+\alpha)}}. \tag{7}$$

For $\alpha = 0$, $a(T) \propto T^{2/3}$ as in the standard (Einstein-de Sitter) dust model. For $\alpha = 1/3$, $a(T) \propto T^{1/2}$ as in the standard radiation model. The value of Hubble's parameter is given by

$$H \equiv \frac{1}{a}\frac{\partial a}{\partial T} = \frac{1}{a}\frac{\partial a}{\partial t}\frac{dt}{dT} = \frac{h}{(2+3\alpha)}\,(hu)^{-\frac{3}{2}\left(\frac{1+\alpha}{2+3\alpha}\right)} \tag{8}$$

$$= \frac{2}{3\,(1+\alpha)\,T}. \tag{9}$$

For $\alpha = 0$ and $1/3$, (9) shows that $H$ has its standard values in terms of the proper time. We can also convert the density (5) from $t$ to $T$ using (6), and find

$$8\pi\rho = \frac{4}{3}\frac{1}{(1+\alpha)^2}\frac{1}{T^2}. \tag{10}$$

For $\alpha = 0$ we have $\rho = 1/6\pi T^2$, and for $\alpha = 1/3$ we have $\rho = 3/32\pi T^2$, the standard FRW values. Thus, the 5D solution (1)–(3) contains 4D dynamics and 4D matter that are the same as in the standard 4D cosmologies for the late and early universe.

However, while the 5D approach does no violence to the 4D one, it adds significant insight. The big bang occurs in proper time at $T = 0$ by (10); but it occurs in coordinate time at $a = 0$ or $u = t - l = 0$ by (5) and (2). Now the field equations $R_{AB} = 0$ are fully covariant, so any choice of coordinates is valid. Therefore, we can interpret the physically-defined big bang either as a singularity in 4D or as a hypersurface $t = l$ in 5D. Both interpretations are mathematically valid, so the choice is to a certain extent philosophical. Our opinion is tipped by a closer examination of the solution (1)–(3) using a computer package (Lake et al. 1995). It shows that not only is $R_{AB} = 0$, but the Riemann-Christoffel tensor is $R_{ABCD} = 0$ also. This puts the solution (1)–(3) into the same mathematical class as others in the literature (Abolghasem et al. 1996; Liu & Wesson 1998; Wesson 1994). But this fact also puts the solution into a new physical class: it is a plane wave or soliton moving in a *flat* and empty 5D space. [In 5D, the group of coordinate transformations $x^A \to x^A(x^B)$ is wider that the 4D group $x^\alpha \to x^\alpha(x^\beta)$, so $x^4$-dependent transformations are mathematically equivalent in 5D but physically *non*-equivalent in 4D. In principle it is possible to find coordinate transformations between all metrics with $R_{ABCD} = 0$, but in practice the algebraic complexity involved makes the task presently impossible.] In other words, we can view the big bang either as a singularity in 4D, or as a *non*-singular event in 5D. In the latter interpretation, it is analogous to a 3D shock wave passing through a 2D surface.

## 3. Conclusion

We have given an exact solution (1)–(3) of the 5D field equations $R_{AB} = 0$ which when reduced to the 4D field equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ describes a cosmology with good physical properties (4), (5), (10) and good dynamics (7), (9). Kaluza-Klein gravity agrees with the classical tests of relativity in the solar system (Kalligas et al. 1995; Will 1992); and the cosmological solution gives back the same properties as the 4D Friedmann-Robertson-Walker models with flat space sections, so to this extent it is agreement with astrophysical data (Leonard & Lake 1995). However, the solution adds the insight that the singular 4D big bang may be viewed as a non-singular 5D shock wave.

The mere existence of solutions (1)–(3) raises fundamental questions about observational cosmology. Is the universe higher-dimensional? (This is implied by particle physics, and what we have done above can clearly be extended to 10D superstrings and 11D supergravity: see Overduin & Wesson 1997). If there are extra dimensions, then what coordinate system is practical cosmology using? (There is no big bang in a geometrical sense in 5D, but there is in a physical sense in 4D because of the choice of time; see Wesson 1994). It seems to us that these questions can be answered empirically. The best way appears to involve the 3 K microwave background radiation. In the conventional 4D view, this is thermalized in the big-bang fireball. In the higher-dimensional view, some other mechanism must operate, such as a variation of particle masses that leads to efficient Thomson scattering (Hoyle 1975). We need to look into the detailed physics and decide by observational data which is the best approach.

*Acknowledgements.* We thank A. Billyard and J. M. Overduin for comments, and NSERC for financial support.

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