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The physical foundations of stellar magnetic field diagnosis from polarimetric observations of hydrogen lines

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Abstract. We have completed a new study of the formation of hydrogen lines in dense magnetized plasmas. We have developed a global formalism including the effects of the magnetic field and of the motional electric field. The latter, resulting from the motion of the radiating hydrogen atom in the magnetic field, modifies the structure of the matrices appearing in the equation of transfer of the Stokes vector and introduces a coupling between microscopic ("intrinsic") and macroscopic (Doppler) contributions to the line profiles. We have used this new treatment of line formation to revisit the interpretation of photopolarimetric (Stokes I and V) observations of Balmer lines in terms of mean longitudinal magnetic field.

Key words: line: formation – polarization – radiative transfer – stars: atmospheres – stars: magnetic fields

1. Introduction

The vast majority of the stellar parameters that can be derived from observation (such as the effective temperature or surface gravity) are scalar, and hence fully characterized by a single number. In contrast the magnetic field is a vector, so its determination requires the measurement of three components. Furthermore, these components can often show large variations from point to point across the stellar disk. However, with current instrumentation, observations give access primarily to quantities averaged over the visible stellar hemisphere. Therefore, detection of magnetic fields can be achieved only provided that they have a sufficient large-scale organization. This is why most measurements achieved so far pertain to chemically peculiar A and B stars (commonly referred to collectively as Ap stars) and to white dwarfs.

In this study, we focus our attention on the determination of the (disk-integrated) component of the magnetic vector along the line of sight, i.e., the *mean longitudinal magnetic field* $\langle B_z \rangle$. This quantity is diagnosed from the observation of circular polarization in spectral lines. For the interpretation of the measured polarization signal in terms of the magnetic field, two methods have been extensively used. One, the *photographic technique* is based on the consideration of wavelength shifts of metal lines between opposite circular polarizations (Babcock 1947). The other frequently used approach is the *Balmer line photopolarimetry* (Angel & Landstreet 1970), in which the Stokes V signal in the wings of a Balmer line of hydrogen (most often Hβ) is compared with the derivative of the Stokes I profile at the same wavelength. One of the main interests of this photopolarimetric technique is that it can be applied in a number of cases where the photographic technique is mostly useless, such as white dwarfs, OB stars, and fast-rotating Ap stars. For detailed descriptions of the methods of stellar magnetic field diagnosis, see Mathys (1989). Note also that measurements of the mean longitudinal magnetic field repeated throughout the rotation cycle of a star can be exploited to derive some constraints on the (spatially unresolved) geometrical structure of the field.

For a number of Ap stars, the mean longitudinal magnetic field has been determined through application of both abovementioned methods, and in a fraction of those, discrepant $\langle B_z \rangle$ values have been obtained. Those differences vary from star to star, and sometimes even throughout the rotation cycle of a given star. Until now, it has not been possible to distinguish any systematic pattern in those discrepancies (e.g. correlations with other stellar parameters like rotation or temperature). In order to try to understand their origin, we have reconsidered the method of Balmer line photopolarimetry, as it is based on a model of hydrogen line formation which is not quite physically sound (Stehlé et al. 2000), and field measurements derived from it seem questionable (Brillant 1999).

After a brief description of the two field determination techniques (Sect. 2), we present the physics of the formation of polarized spectral lines of hydrogen in the presence of a uniform and constant magnetic field. We emphasize the contribution of the motional electric field (related to the motion of the radiating hydrogen atoms in the magnetic field) and we present the general form of the equation of transfer of polarized light which is applicable in the considered physical context. This is a vectorial equation. It involves 4×4 absorption and emission matrices, whose elements can be expressed in terms of generalized line profiles that we have computed (Sect. 3). Solving this transfer

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equation, we can compute synthetic local emergent profiles of the Hβ line in the various Stokes parameters, for a given vertical structure of the stellar atmosphere and a given geometry of the magnetic field. These emergent profiles are then integrated over the stellar disk. The measurement of the resulting synthetic profiles as in the photopolarimetric technique is simulated, and the result is interpreted in the same way as for actual observations, so as to derive a value of the longitudinal field which can be compared with the model input.

At this stage, we have not tried to model any specific magnetic star. Instead, we preferred to study as an illustrative example a hypothetical star with atmospheric parameters and a magnetic field structure representative of those generally encountered in Ap and Bp stars. On that basis, we discuss the physical meaning of the longitudinal field determinations achieved through Balmer line photopolarimetry and we draw preliminary conclusions on the diagnostic potential of this method.

2. Determination of the mean longitudinal magnetic field

In the photographic technique, spectra are recorded simultaneously in the two opposite circular polarizations on the same detector (originally a photographic plate, nowadays a CCD). The mean longitudinal magnetic field $\langle B_z \rangle$ is derived from the shift between the wavelength of the centre of gravity of the line in right circular polarization ($\langle \lambda_R \rangle$) and in left circular polarization ($\langle \lambda_L \rangle$) through application of the relation:

$$\langle \lambda_R \rangle - \langle \lambda_L \rangle = \bar{g}\, \frac{e\, \lambda_0^2}{2\, \pi\, m_{\rm e}\, c^2}\, \langle B_z \rangle\, . \qquad (1)$$

λ_0 is the nominal wavelength of the transition and $\bar{g}$ the effective Landé factor (Mathys 1991). Typically this approach is applied to metal lines observed in the visible range. Because the wavelengths of the centres of gravity in both polarizations are determined through integration over the whole line profile, this method is sometimes called an *integral* method.

The other method considered here, Balmer line photopolarimetry, rests on the observation with a photometer of the wings of a hydrogen line (in practice, one of the first members of the Balmer series) through an interference filter of typically 5 Å bandwidth. Light of right and left circular polarization is recorded in alternation through use of a variable retarder, and the flux difference between the two polarizations (that is, the signal in the Stokes parameter V), $\mathcal{F}_V$, is recorded. A scan through the line wing in unpolarized light is also carried out to determine its slope in the Stokes parameter I, $d\mathcal{F}_I/d\lambda$. The mean longitudinal field is derived from these two quantities through application of the relation (Landstreet 1982):

$$\mathcal{F}_V = -\frac{e\, \lambda_0^2}{4\, \pi\, m_e\, c^2}\, \langle B_z \rangle\, \frac{d\mathcal{F}_I}{d\lambda}\, . \qquad (2)$$

This relation is assumed to be valid both locally (substituting B_z for $\langle B_z \rangle$) and after averaging over the visible stellar hemisphere. Landstreet also argued that it does apply after convolution with the instrumental profile (in practice, the interferential filter used). Since this method relies on the consideration of the derivative of $\mathcal{F}_I$ with respect to the wavelength, it is sometimes called a *differential* method.

3. Hydrogen line formation in a magnetized plasma

The formalism that we have developed to treat the problem of formation of hydrogen lines in a magnetized plasma has been presented in detail elsewhere (Brillant et al. 1998). For the sake of clarity, let us summarize here the main points of that work.

To deal with the formation of hydrogen lines in the presence of a magnetic field, it is necessary to solve the transfer equation for polarized light. In that equation, the propagation of the Stokes vector S ($\mathsf{S} = (I, Q, U, V)^t$) is described in terms of the 4×4 opacity matrix χ and of the emissivity 4-vector ϵ. For simplicity, we restrict ourself to considering the case of a bound-bound transition i-f over an unpolarized continuum, and we neglect the redistribution effects. For radiation of angular frequency ω, the equation of transfer of the Stokes vector in a direction which makes an angle Θ with the direction Z of the normal to the atmosphere (supposed plane-parallel) is (Mathys 1982, 1985; Landi Degl'Innocenti 1983):

$$\cos\Theta \frac{d\mathsf{S}}{dZ} = -\chi\, \mathsf{S} + \epsilon\, , \qquad (3)$$

with

$$\begin{aligned} \chi &= \chi_{\rm line} + \chi_{\rm continuum}\, , \\ \epsilon &= \epsilon_{\rm line} + \epsilon_{\rm continuum}\, . \end{aligned} \qquad (4)$$

We assume that the Zeeman sublevels of the transition i-f are evenly populated. The populations of the levels involved in the optical transition are denoted N_i and N_f. We neglect the atomic coherence effects between these levels. We assume that the emission and absorption line profiles are identical. For the contribution of the line i-f, the line opacity matrix and emissivity vector can be written as:

$$\begin{aligned} \chi_{\rm line} &= (N_i - \tfrac{g_i}{g_f}\, N_f)\, \frac{2\, \pi^2\, \omega}{\hbar\, c}\, \mathsf{b} \\ \epsilon_{\rm line} &= N_f\, \frac{2\, \hbar\, \omega^3}{(2\, \pi\, c)^2}\, \mathsf{b}\, (1, 0, 0, 0)^t\, . \end{aligned} \qquad (5)$$

The absorption matrix b is defined by

$$\mathsf{b} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} . \qquad (6)$$

In the absence of any polarigenic mechanism, the matrix b is proportional to the unit matrix. In the specific case considered here, we give below the expression of the matrix elements η and ρ. For this purpose, we adopt the geometry shown in Fig. 1. We denote by $\boldsymbol{k}$ the direction of propagation of the radiation. $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are two mutually orthogonal reference directions of polarization. $\boldsymbol{e}_1$ lies in the plane defined by $\boldsymbol{B}$ and $\boldsymbol{k}$. A cartesian reference system xyz is defined such that the magnetic vector $\boldsymbol{B}$ is parallel to the z axis. The angle between $\boldsymbol{B}$ and $\boldsymbol{k}$ is denoted by θ. The velocity of the radiating hydrogen atom can

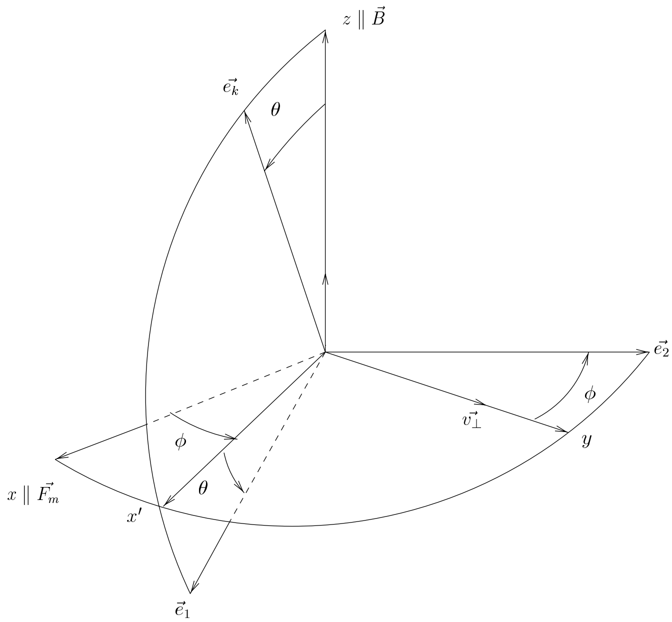

Fig. 1. Geometry adopted for the calculation of the polarized transfer matrices

be advantageously decomposed in a component parallel to the magnetic field ($\boldsymbol{v}_\parallel$) and a component perpendicular to $\boldsymbol{B}$ ($\boldsymbol{v}_\perp$). The angle between $\boldsymbol{v}_\perp$ and $\boldsymbol{e}_2$ is ϕ.

If the effect of the motional electric field $\boldsymbol{F}_m = \boldsymbol{v}_\perp \times \boldsymbol{B}$ is neglected, the system under study has the property of cylindrical symmetry about the direction of the magnetic field. The expression of the matrix b (Mathys 1985) reflects this symmetry:

$$\mathsf{b} \equiv \mathsf{b}_0 = \begin{pmatrix} C_2\,(I_{xx}+I_{yy}) + 2\,S_2\,I_{zz} & S_2\,(2\,I_{zz} - (I_{xx}+I_{yy})) & 0 & -C_1 \times (R_{xy}-R_{yx}) \\ S_2\,(2\,I_{zz} - (I_{xx}+I_{yy})) & C_2\,(I_{xx}+I_{yy}) + 2\,S_2\,I_{zz} & C_1 \times (I_{xy}-I_{yx}) & 0 \\ 0 & -C_1 \times (I_{xy}-I_{yx}) & C_2\,(I_{xx}+I_{yy}) + 2\,S_2\,I_{zz} & S_2\,(2\,R_{zz} - (R_{xx}+R_{yy})) \\ -C_1 \times (R_{xy}-R_{yx}) & 0 & -S_2\,(2\,R_{zz} - (R_{xx}+R_{yy})) & C_2\,(I_{xx}+I_{yy}) + 2\,S_2\,I_{zz} \end{pmatrix}, \tag{7}$$

with $I_{xx} = I_{yy}$, $R_{xx} = R_{yy}$, and

$$\begin{aligned} C_1 &= \cos\theta\,, \\ C_2 &= (1+\cos^2\theta)/2\,, \\ S_2 &= (\sin^2\theta)/2\,. \end{aligned} \tag{8}$$

In Eq. (7), I_{kl} and R_{kl} are, resp., the real and imaginary parts[1] of the generalized complex profile, J_{kl}. Indices k and l refer to the cartesian components (D_x, D_y, and D_z) of the dipole operator involved, at the microscopic scale, in the optical transition i-f.

[1] While this may appear counter-intuitive, I is used for the real part of the complex profile, and R for its imaginary part, for the sake of consistency with previous works (such as Mathys 1985). The notation finds its origin in early versions of the formalism where I did indeed appear as an imaginary part (see e.g. Eq. (3.11) of Mathys 1983).

For instance, in the ideal case of a non-degenerate line (which is not the case of hydrogen), the expression of the general complex profile J_{zz} would be:

$$\begin{aligned} J_{zz} &= I_{zz} + i\,R_{zz} \\ &= |<i|D_z|f>|^2 \left[\frac{1}{\pi}\,\frac{\gamma}{\Delta\omega^2+\gamma^2} + i\,\frac{\Delta\omega}{\Delta\omega^2+\gamma^2}\right] \end{aligned} \tag{9}$$

(for the more general case, see Brillant et al. 1998). If one only considers the scalar (diagonal) part of b, one notes that the emerging spectrum can be expressed in terms of the intensities emitted in directions parallel and perpendicular to the magnetic vector.

When we account for the contribution of the motional electric field, the cylindrical symmetry is broken (in particular, $I_{xx} \neq I_{yy}$ and $R_{xx} \neq R_{yy}$), and one can show that:

$$\mathsf{b} = \mathsf{b}_0 + \cos 2\phi\, \mathsf{b}_{cross}\,, \tag{10}$$

with

$$\mathsf{b}_{cross} = \begin{pmatrix} -S_2 \times (I_{xx}-I_{yy}) & C_2 \times (I_{xx}-I_{yy}) & 0 & 0 \\ C_2 \times (I_{xx}-I_{yy}) & -S_2 \times (I_{xx}-I_{yy}) & 0 & 0 \\ 0 & 0 & -S_2 \times (I_{xx}-I_{yy}) & C_2 \times (R_{xx}-R_{yy}) \\ 0 & 0 & -C_2 \times (R_{xx}-R_{yy}) & -S_2 \times (I_{xx}-I_{yy}) \end{pmatrix}. \tag{11}$$

One can see that, since the Doppler shift is proportional to $(v_\parallel\,\cos\theta + v_\perp\,\sin\theta\,\sin\phi)$, a correlation between b and the Doppler effect appears. This correlation results from both the explicit dependence of b on ϕ, and through the implicit dependence of its elements on $v_\perp$, at the microscopic level, from the Stark effect generated by the motional electric field. Accordingly, the average over the velocities of the radiating atoms cannot be carried out through the "standard" convolution of the "intrinsic" profile by the Doppler broadening function. However, appropriate convolutions can be used in the case where the effect of the motional electric field on the profiles is small, so that its contribution can be accounted for by a corrective term quadratic in $v_\perp$ (Brillant et al. 1998).

The generalized profiles of hydrogen require specific treatment. For a given value of the magnetic field strength, they have been computed by Brillant et al. (1998) taking into account the following microscopic effects:

- the Zeeman effect due to the magnetic field. The corresponding hamiltonian is denoted by H_z;
- the Stark effect generated by the motional electric field (H_m);
- the plasma Stark effect (H_s), which results from the interaction between the electric microfields due to the plasma ions ($\boldsymbol{E}_i$) and electrons ($\boldsymbol{E}_e$).

More explicitly, the relevant hamiltonians are:

$$\begin{aligned} H_z &= \hbar\,\omega_L\,L_z\,, \\ H_m &= -D_x\,F_m\,, \\ H_s &= -\boldsymbol{D}\cdot(\boldsymbol{E}_e + \boldsymbol{E}_i)\,. \end{aligned} \tag{12}$$

$\boldsymbol{D}$ is the electronic dipole operator, which is taken between two states with the same principal quantum number. The states are assumed to be degenerate; that is, fine structure effects are neglected. The Larmor frequency ω_L is defined by $\omega_L = eB/(2m_e c)$. The electronic and ionic microfields are random quantities, which in the far line wings yield scalar intensities proportional to $|\Delta\omega|^{-5/2}$ (Holtsmark's law). The reader is referred to Brillant et al. (1998) for the details of the profile calculation. The Stark effect is computed taking into account the ion dynamics effects using a combination of the Model Microfield Method (Brissaud & Frisch 1971; Frisch & Brissaud 1971; see also Stehlé & Hutcheon 1999) and of the Unified Theory for ions and electrons (Greene 1982). The elements of the matrix b are either symmetric or antisymmetric with respect to the line centre, and features are present in the line around $\pm\omega_L$.

4. Computation of synthetic line profiles and magnetic field determination

We aim to study the validity of Eq. (2), which has been widely used for the determination of magnetic fields from *photopolarimetric* observations of hydrogen Balmer lines.

As a case study, we have considered a hypothetical star characterized by parameters representative of a magnetic Bp star: $\log L/L_\odot = 2.78$, $T_{\rm eff} = 16400$ K, $M = 5.0\ M_\odot$, $v \sin i = 35\ {\rm km\,s^{-1}}$, $i = 43°$, metal abundance 10 times solar.[2] We have computed the corresponding model atmosphere, using Kurucz's ATLAS9 code (Kurucz 1985). In this step, we have implicitly assumed that the magnetic fields do not have any significant influence on the structure of the atmosphere (which is obtained for a null magnetic field).

For the magnetic field structure, we have adopted the collinear superposition of a dipole, a quadrupole, and an octupole, all centred at the centre of the star. At the magnetic pole, the contribution of each of these components is respectively $B_d = 16700$ G, $B_q = 9500$ G, and $B_o = -4000$ G. The angle β between the magnetic and rotation axes is set to 87°.

The emergent line profiles were synthesized by application of Feautrier's (1964) method, as adapted by Rees et al. (1989) for polarized light transfer. In this approach, the first-order differential equation of transfer is converted to a second-order equation. The method rests on the following assumptions:

- the continuum around the line is wavelength independent and unpolarized;
- scattering is treated in complete redistribution;
- the atmosphere is in steady state.

On the other hand, for the source function, we have used a Planck function. In a first step, calculations were performed without Doppler effect (which is likely to be unimportant in the line wings anyway) and neglecting the effect of the motional electric field.[3] The magnetic field has been assumed constant along a light path making an angle Θ with the normal to the surface of the (plane-parallel) atmosphere. We use the notation $\mu = \cos\Theta$.

The emergent flux in the Stokes parameters I, Q, U, and V is computed in each point (x, y) of a grid covering the stellar disk, using the local value $\Theta(x, y)$ of the angle between the line of sight and the normal to the atmosphere and for the local magnetic field $\boldsymbol{B}(x, y)$. The magnetic field, which is one of the inputs of the model, is known. The purpose of the exercise is to find out to what extent the intensity of its mean longitudinal component can be retrieved from the emergent line profiles, or more specifically from the fluxes $\langle\mathcal{F}_V\rangle_{\rm filter}$ and $\langle\mathcal{F}_I\rangle_{\rm filter}$ averaged over the passband of an interferential filter located at approximately 5 Å from the centre of the line Hβ, in its red and blue wings. Accordingly, we have computed averages over ideal square filters with 5 Å halfwidths of the fluxes integrated over the stellar disk. Note that the averages do not depend on which line wing is considered. From the integrated fluxes, by application of Eq. (2), we have calculated the value of the mean longitudinal magnetic field $\langle B_z\rangle_{\rm obs}$ that would have been derived from the observation of the considered line. We have studied the variation of that determination with stellar rotation.

If Eq. (2) is correct, the quantity derived through this "measurement" should be a weighted average over the visible stellar disk of the component of the magnetic vector along the line of sight:

$$\langle B_z\rangle_{\rm geo} = \frac{\int\int_{\rm disk} \epsilon_1(\mu(x,y))\, B_z(x,y)\, dx\, dy}{\int\int_{\rm disk} \epsilon_1(\mu(x,y))\, dx\, dy}\,. \tag{13}$$

As the field determination is based on a comparison of the observed Stokes V profile with the derivative of the observed Stokes I profile, $\epsilon_1(\mu)$ is a "new" limb-darkening factor, defined by:

$$\epsilon_1(\mu) \equiv \frac{dI(\mu)/d\lambda}{dI(\mu=1)/d\lambda}\,. \tag{14}$$

[2] These values, and the magnetic model below, are inspired by consideration of the star HD 175362, to which the hypothetical star studied here should bear some resemblance. However, we have by no means made any attempt to generate a realistic model of HD 175362.

[3] Taking the Lorentz and Doppler effects into account represents a considerable increase of complexity in the numerical calculation. Indeed, since both effects are correlated and cannot generally be uncoupled in the calculation, including them would require us to recompute the profiles of all the coefficients appearing in the transfer matrix for each line of sight on the stellar disk. Given that the present approach already represents a major step forward, with the first exact treatment of the polarized radiative transfer in a stellar hydrogen line (including proper disk integration), it was judged preferable to avoid too big a jump at once in complexity, so as to gain insight first into the consequences of a physical treatment of the line formation, without taking the risk of confusing the issues by introducing all the new contributions at once. In a second step, the Doppler and Lorentz effects will be incorporated in the calculations, but the results obtained so far neglecting them were deemed sufficiently novel and interesting by themselves so as to deserve to be published without the further delay that would result from awaiting the outcome of the full calculation (which is very time consuming).

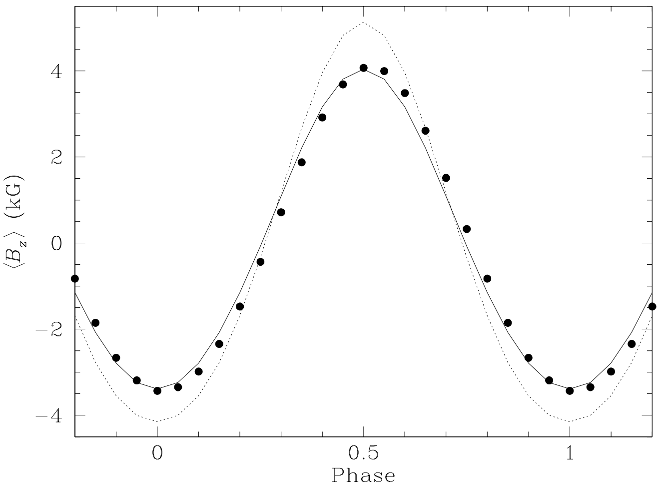

Fig. 2. Comparison between the mean longitudinal field derived for the considered model star through simulation of the Balmer line photopolarimetric technique, applying the physically correct equation (16) (*dots*), and the "input" field computed geometrically with Eq. (13), with $\epsilon_1(\mu)$ given by Eq. (15) (*solid line*) and with $\epsilon_1(\mu) = \mu$ (*dotted line*)

We have computed this limb-darkening factor for the selected model atmosphere. In the Hβ line wing, this weighting factor can be approximated by a quadratic relation:

$$\epsilon_1(\mu, 5\ \text{Å}) = 0.26 + 1.20\,\mu - 0.46\,\mu^2\,. \tag{15}$$

Using this pseudo-limb-darkening factor, we found that the value of the mean longitudinal magnetic field derived observationally, $\langle B_z \rangle_{\rm obs}$, is not equal to the model "input" value, that is the average $\langle B_z \rangle_{\rm geo}$ computed by application of Eq. (13). This inconsistency indicates that Eq. (2) is not correct. On the basis of arguments about the shape of the elementary profiles η_I and η_V appearing in the expression of the absorption matrix b (Eq. (6)), we have been able to show that Eq. (2) must be replaced in the line wings by the more accurate relation:

$$\mathcal{F}_V = -\frac{e\,\lambda_0^2}{5\,\pi\,m_e\,c^2}\,\langle B_z \rangle\,\frac{d\mathcal{F}_I}{d\lambda}\,. \tag{16}$$

The relation given by Eq. (2) would be valid for a microscopic profile with a lorentzian shape (Eq. (9)), which implicitly assumes that the states involved in the radiative transition keep their identity between consecutive collisional interactions with the surrounding plasma particles, i.e. for most of the time. In that case, the atomic states are, on average, little perturbed by the plasma. For the hydrogen atom in a plasma, the situation is different. The profile at large detunings is determined by very strong interactions, during which atomic states are strongly intermixed by Stark effect: this leads to the relation given in Eq. (16). A rigorous demonstration of this expression will be presented in a separate paper. The application of Eq. (16) to the determination of $\langle B_z \rangle_{\rm obs}$ yields a value in very satisfactory agreement with the input $\langle B_z \rangle_{\rm geo}$. This shows that the interpretation of the observed polarization in terms of magnetic field is well understood. This is illustrated in Fig. 2. The figure also shows the variation curve of $\langle B_z \rangle_{\rm geo}$ which would be obtained using for the limb-darkening factor $\epsilon_1(\mu) = \mu$, as assumed by Landstreet (1982) to establish Eq. (2). The match with the field values derived from Eq. (16) is significantly worse than when the "correct" limb-darkening factor is used. Note that this implies an even larger discrepancy with the longitudinal field that would be determined by application of Eq. (2).

5. Conclusion

We have presented a model of spectral synthesis in polarized light for a star with typical properties of a magnetic Ap-Bp star. This study rests on the full and accurate treatment of the radiative transfer and of the physics at the microscopic scale. We have shown how the motional electric field modifies the structure of the transfer matrices. We have confirmed that the relation given by Landstreet (1982) for the derivation of the mean longitudinal field from observations of circular polarization in the wings of Balmer lines of hydrogen (Eq. 2) yields the correct order of magnitude. However, this relation must be corrected by a factor of 4/5 to account for the physics of the Stark effect in the wings of hydrogen lines. This conclusion is valid for the interpretation of the polarization observed locally at a given point of the stellar surface. For a disk-integrated observation, one must furthermore use an appropriate limb-darkening factor, based on the derivative of the Stokes I profile with respect to the wavelength, and not on the Stokes I profile itself, as was done so far. The latter remark is valid only for the Balmer line *photopolarimetric* technique. The methods relying upon consideration of quantities integrated over a whole line width (such as the *photographic* technique of magnetic field diagnosis) must use a standard limb darkening. In other words, disk averaging in the photopolarimetric and photographic techniques involves different weighting functions: there is no reason to expect that they lead to the same results. The situation is further complicated by the inhomogeneous distribution of heavy elements on the surface of Ap stars. This tends to explain the discrepancies between the results of both approaches. In the future, it will be interesting to revisit the published values of the longitudinal magnetic field determined by the Balmer line photopolarimetry in the light of the improved treatment of the physics of line formation. But before doing so, it would be desirable to incorporate the Doppler and Lorentz effects in the line profile calculations, since their consideration may lead to a revision of some of the results derived here (e.g., the exact value of the "renormalization" factor arising from the elementary profiles may turn out to be somewhat different from 4/5). This additional step will have to be carried out before more definitive conclusions can be drawn.

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