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On the mass of moderately rotating strange stars in the MIT bag model and LMXBs

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Abstract. We compute the maximum mass of moderately rotating strange stars as a function of the strange quark mass, of the QCD coupling constant, α_c, and of the bag constant (vacuum energy density), B, in the MIT bag model of quark matter with lowest order quark-gluon interactions. For a fixed value of B, the maximum stellar mass depends only weakly on α_c, and is independent of this coupling in the limit of massless quarks. However, if it is the value of the chemical potential of quark matter at zero pressure which is held constant, for example at the value corresponding to the stability limit of nucleons against conversion to quark matter, the maximum mass of the strange star is higher by up to 25% for $\alpha_c = 0.6$, than for non-interacting quarks, and this may be relevant in the discussion of kHz QPO sources. The maximum mass of a non-rotating strange star could be sufficiently high to allow an orbital frequency as low as 1.0 kHz in the marginally stable orbit. However, for all $\alpha_c < 0.6$, the stellar mass cannot exceed $2.6 M_\odot$ at any rotational period ≥ 1.6 ms.

Key words: dense matter – equation of state – stars: binaries: general – X-rays: stars

1. Kilohertz QPOs and the mass of LMXBs

The discovery of kHz QPOs (Strohmayer et al. 1996; van der Klis et al. 1996) in several low-mass X-ray binaries (LMXBs) has renewed interest in the maximum mass of neutron stars, as its value limits the maximum observable orbital frequency (Kluźniak et al. 1990). The maximum masses for neutron stars modeled with various equations of state (e.o.s.) are well established (Arnett & Bowers 1977; Friedman et al. 1986; Cook et al. 1994; Salgado, et. al. 1994), and have been examined in the context of kHz QPOs (Kluźniak 1998). In principle, LMXBs could contain strange stars instead (Cheng and Dai 1996), and it has been asked whether the observed values of kHz QPOs are compatible with the masses of such stars (Bulik et al., 1999a,b).

The existence of strange stars would shed light on the issue of stability of quark matter. It has been suggested that a quark fluid, composed of roughly equal number of up, down and strange quarks, may be the ground state of bulk matter (Bodmer 1971; Witten 1984) and a detailed description of such strange matter has been given by Farhi & Jaffe (1984). This idea has not been universally accepted, and it was argued that for realistic values of the QCD coupling constant, the phase transition to quark matter would occur at densities too high to be of interest (e.g., Bethe et al. 1987).

Strange stars may be very hard to distinguish from neutron stars, particularly in LMXBs, as they may have a crust of normal matter (Alcock et al. 1986; Haensel et al. 1986). The crust will contribute little to the mass ($\leq 10^{-5} M_\odot$), but is expected to have a thickness sufficient to support nuclear burning, including helium flashes responsible for X-ray bursts, thus mimicking neutron stars. The heat released in conversion (at the bottom of the crust) of nuclei into strange matter is directed into the strange matter core (Miralda-Escudé et al. 1990; Haensel & Zdunik 1990). At any rate, steady release of energy from nuclear conversion is very difficult to distinguish from the gravitational binding energy released in steady accretion.

However, in young radio pulsars the crust thickness does make a difference – the crust in strange stars would be far too thin to allow the redistribution of angular momentum necessary to explain the glitch phenomenon (sudden spin-up of a pulsar), therefore glitching pulsars are though to be neutron stars and not quark stars (Alpar 1987). It is likely that the coalescence of two strange stars would lead to the contamination of our Galaxy with chunks of strange matter, and would thus preclude the formation of young neutron stars (Madsen 1988; Caldwell & Friedman 1991). These arguments make unlikely the presence of strange stars in the population of ordinary pulsars (including binary Hulse-Taylor type pulsars) or their accreting counterparts, the high-mass X-ray binaries. However, the presence of strange stars among the millisecond pulsars or LMXBs seems to be allowed (Kluźniak 1994; Cheng & Dai 1996). In these old systems, the stellar mass itself may yield clues as to the nature of the compact object.

We are led to consider the maximum mass of strange stars in the expectation that LMXB masses will become available in the near future, either through a better understanding of accretion phenomena, including the observed quasi-periodic oscillations (QPOs) in the X-ray light curve, or through classical optical

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determinations of binary parameters for newly discovered transient sources.

The maximum frequency of stable orbital motion is attained in the innermost (marginally) stable circular orbit (ISCO) allowed by general relativity, and if its value for some X-ray source were that of the observed maximum QPO frequency – ranging from 1.0 kHz to 1.2 kHz in twelve LMXBs – conclusions could be reached about the nature of the compact object. [For a review of QPOs see e.g. van der Klis (1998).]

The question, whether ISCO frequencies as low as the maximum observed QPO frequencies can be attained outside quark stars, has been answered in the affirmative for rapidly rotating strange stars, with an equation of state (e.o.s.) based on the MIT bag model of quark matter – strange stars rotating close to the equatorial mass-shedding limit can have ISCO frequencies below 1 kHz for masses as low as $1.4 M_{\odot}$ (Stergioulas et al. 1999). However, the same question has not yet been answered for moderately rotating strange stars, of periods $P \sim 3\,\mathrm{ms}$ or more, for which such ISCO frequencies would imply larger masses: $M = 2.2 M_{\odot}(1 + 0.75j)(1.0\,\mathrm{kHz}/f_{max})$, where f_{max} is the ISCO frequency, and $j \leq 0.3$ is the dimensionless angular momentum (Kluźniak et al. 1990; Kluźniak 1998).

There is a compelling reason to consider stars rotating at relatively low rates. The period of the compact star in the the transient LMXB SAX J 1808.4-3658 (Wijnands & van der Klis 1998), the first of possibly many such transients to be discovered, has been measured to be 2.5 ms. Whether or not kHz QPOs will be discovered in that source, its mass may eventually be determined by optical studies of the binary companion.

It has also been argued that the oscillations seen in some X-ray bursters imply a stellar rotational frequency of $\sim 300\,\mathrm{Hz}$ (Strohmayer et al. 1997). Finally, it is not yet clear whether strange stars can attain the equatorial mass-shedding limit because of the unusually high value of $T/W > 0.2$ calculated for their models – note that for Newtonian stars the secular instability to a bar-mode deformation sets in already at $0.1275 \leq T/W \leq 0.1375$ (Bonazzola et al. 1996). For a discussion of other modes, also in general relativity, see, e.g., Gourgoulhon et al. (1999), and references therein.

For all these reasons, we considered exact numerical models of strange stars rotating with frequencies up to 700 Hz, and found that, to within a few per cent, the maximum mass of such strange stars is the same as that of the static configurations. Therefore, we investigate the maximum mass of static (non-rotating) strange stars, which till now has been discussed in the context of QPOs only in the simplest case of ideal quark gas in the bag model (Bulik et al. 1999a). Here, we use the e.o.s. of interacting, massive quarks within the MIT bag model of self-bound quark matter.

2. Strange stars

The maximum mass of quark stars was first derived (Brecher & Caporaso 1976) for an unusually low value of the bag constant. Witten (1984) showed that the maximum mass of a static strange star is, within the MIT bag model (Farhi and Jaffe 1984),

$$M_{max} = 1.98 M_{\odot}(B/B_1)^{-1/2}, \tag{1}$$

where $B_1 = 58.9\,\mathrm{MeV{\cdot}fm^{-3}}$.

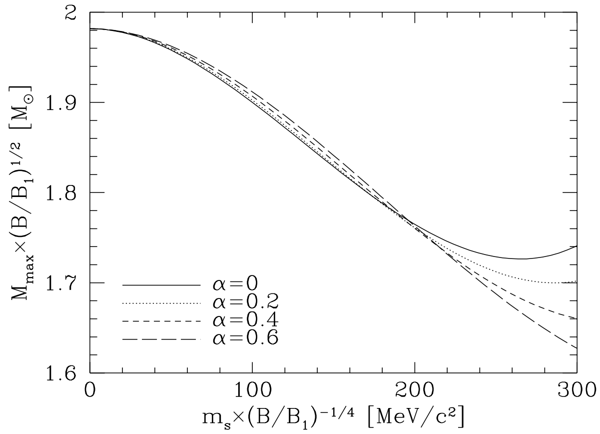

Fig. 1. The maximum mass of a strange star as a function of the strange quark mass, for various values of the QCD coupling constant. The stellar mass scales as the inverse square root of the bag constant, B, provided that the quark mass is scaled with $B^{1/4}$. Here, $B_1 = 58.9\,\mathrm{MeV\,fm^{-3}}$.

There seems to have been no systematic investigation of the *maximum* mass of the star as a function of the three basic parameters of quark matter in the MIT bag model: the mass of the strange quark, m_s, the bag constant B, and the strength of the QCD coupling constant, $\alpha_c = g^2/4\pi$. Detailed models of strange stars have been constructed and the structure of strange stars, including the mass–radius relationship has been discussed extensively in the literature (Alcock et al. 1986; Haensel et al. 1986; Glendenning 1989; Frieman & Olinto 1989; Prakash et al. 1990), however discussion of stellar parameters tended to concentrate on the maximum value of the bag constant, setting a *lower* bound on the maximum mass and an upper bound on the rotational frequency.

In this section, we discuss only static stellar models, constructed in general relativity by solving the TOV equations (Oppenheimer & Volkoff 1939). We neglect the crust, whose contribution to the maximum mass is quite minor for stars composed mostly of self-bound quark matter (Haensel et al. 1986). Following Farhi and Jaffe (1984), we take m_s and α_c to be renormalized at $q = 313$ MeV, and as a model for quark matter consider a "bag" of positive vacuum energy density, B, filled with quarks (of two massless flavors and one massive) having interactions through first order in α_c. The actual form of the thermodynamic expressions we use can be found in Haensel et al. (1986). As both the pressure and the density scale with the bag constant, TOV equations imply that the stellar mass and radius scale as $M \propto B^{-1/2}$, and $R \propto B^{-1/2}$ (Witten 1984), provided that the masses of the quarks scale as $m \propto B^{1/4}$.

In Fig. 1, we plot the rescaled value of the maximum stellar mass, $M(B/B_1)^{1/2}$, as a function of the rescaled strange quark mass, $m_s(B_1/B)^{1/4}$, for various values of the QCD coupling constant. Note that the highest value of M_{max}, the maximum mass of the strange star, is independent of α_c, if B is fixed, and is obtained for $m_s = 0$, i.e., for massless quarks. But in fact, the value of B is not known, and as its lower bound does depend on the value of α_c, the actual physical bounds on the maximum mass of a strange star will depend, through B, on the coupling constant. This is discussed in the next section.

3. The minimum value of B and an upper bound to the mass of moderately rotating strange stars

To determine the highest possible value of the maximum mass of static strange stars in the MIT bag model, it is enough to consider the e.o.s. of an ultrarelativistic Fermi gas in a volume with vacuum energy density $B > 0$. If the quarks are massive, the actual maximum mass of the star will be somewhat lower – as is evident from Fig. 1, at fixed values of the other parameters, the maximum mass of a strange star decreases with increasing quark mass.[1]

Currently, the actual value of B cannot be reliably derived from fits to hadronic masses of the quark-model of nucleons. Instead, we must rely on a different argument to find B_{min}. We require that neutrons do not combine to form plasma of deconfined up and down quarks, or equivalently, that quark matter composed of up and down quarks in 1:2 ratio is unstable to emission of neutrons through the reaction $u + 2d \to n$ (e.g., Haensel, 1987; Farhi, 1991), i.e., that the baryonic chemical potential at zero pressure of such quark matter satisfies

$$\mu_0^{u,d} > 939.57\,\mathrm{MeV}. \tag{2}$$

This condition provides a lower bound on the bag constant B, and consequently, by Eq. (1), an upper bound on the mass of static strange stars. We take the up and down quarks to be massless. Therefore, Eq. (2) implies $B \geq B_1(1-2\alpha_c/\pi)$. For noninteracting and massless quarks ($\alpha = 0, m_s = 0$), this value of the bag constant corresponds to a minimum density of strange matter (attained at zero pressure) of $\rho_0(0) = 4.20 \times 10^{14}$ g/cm^3 $\equiv 4B_1/c^2$, with a corresponding maximum mass of non-rotating strange stars of $M_{max}(0) = 1.98 M_\odot$.

For interacting (but massless) quarks, through lowest order in gluon exchange, the equation of state is identical to that of non-interacting quarks (Chapline & Nauenberg 1976), $p = (\rho - \rho_0)c^2/3$, the only difference being in that the lower bound on the density at zero pressure, following from conditions of neutron stability (Eq. (2)), is decreased with respect to the value for an ideal Fermi gas in a bag, by the same factor as the bag constant: $\rho_0(\alpha_c) = (1 - 2\alpha_c/\pi)\,\rho_0(0)$. Since the stellar mass scales as $\rho_0^{-1/2}$, this implies that the least upper bound on the mass of the star as a function of the QCD coupling constant is given for non-rotating strange stars by

$$M_{max}(\alpha_c) = \left(1 - \frac{2\alpha_c}{\pi}\right)^{-1/2} M_{max}(0)\,, \tag{3}$$

[1] The dependence of M on m_s is easily understood in the limit $\alpha_c = 0$. The mass of a static star modeled with ideal Fermi gas e.o.s. scales with fermion mass as $M \propto m^{-2}$ and attains a finite limit at $m \to 0$ (Oppenheimer & Volkoff 1939).

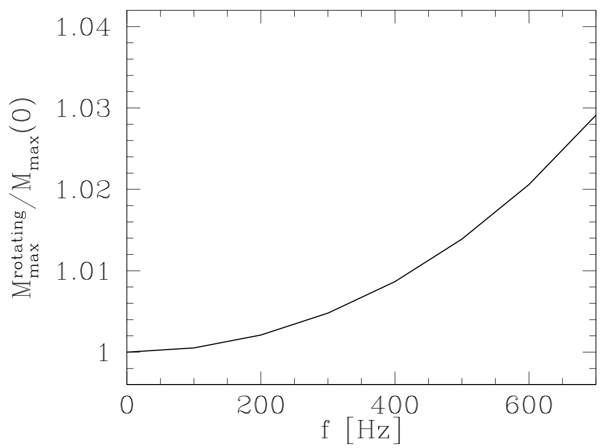

Fig. 2. The dependence on rotational frequency of the maximum mass of a strange star, as derived from sequences of fully relativistic numerical models for the e.o.s. of massless, noninteracting quarks (in a bag). In the figure, the stellar mass is scaled with the maximum mass of static models for this e.o.s., $1.98 M_\odot$. Note, that for all periods of stellar rotation in the currently observed range ($P \geq 1.6$ ms) the maximum mass of rotating strange stars differs from the static one by less than 4% (the same result holds also for the e.o.s. of strange matter with massive, interacting quarks [Gondek-Rosińska, et. al 2000]).

through first order in α_c (Prakash et al. 1990). For $\alpha_c = 0.4$, Eqs. (2), (3) give a maximum strange star mass of $2.29 M_\odot$, higher by 16% than the maximum mass which is obtained for $\alpha = 0$.

Stellar rotation at a frequency up to 700 Hz would increase the maximum mass by only a few percent. In Fig. 2, we present the maximum mass of a strange star as a function of the rotational frequency, $f = 1/P$. This is not the frequency dependence of the mass of a single star – what is plotted is the termination point, at each frequency, of a sequence of stellar models varying in mass. Thus, the baryon number of the maximum-mass model varies with frequency. In fact, for all $f > 0$, the maximum-mass stars are supramassive – if spun down at constant baryon number to $f = 0$, they would become unstable to collapse (compare the discussion of neutron-star models in Cook et al., 1994). We obtained the results presented in Fig. 2 with an accurate code based on spectral methods, developed by, and described in, Gourgoulhon et al. (1999).

4. The maximum mass of non-rotating strange stars as a function of m_s and α_c

To determine the maximum mass of such stars for specific values of model parameters (B, α_c, m_s), the e.o.s. of matter composed of interacting, massive quarks must be considered, and it can only be determined numerically. The maximum stellar mass then following from the lower bound on B implicit in Eq. (2) is exhibited in Fig. 3 (continuous lines) as a function of the strange-quark mass and of the QCD coupling constant.

From Fig. 3 we can conclude for example, that if a strange star of two solar masses ($2M_\odot$) were identified, and if α_c were

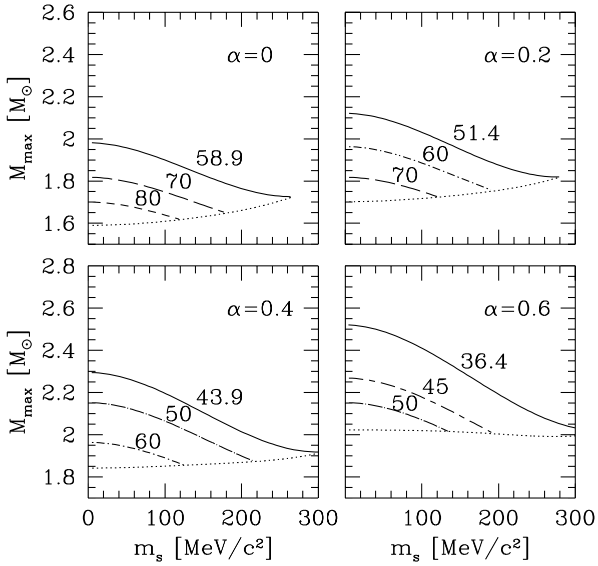

Fig. 3. The maximum mass of static strange stars in the MIT bag model, as a function of the strange quark mass, for various values of the QCD coupling constant. M_{max} is always above the dotted line, which corresponds to the upper bound on B (Sect. 5); and below the continuous line, which corresponds to the lower bound on the bag constant, $B_{min} = 58.9(1 - 2\alpha_c/\pi)$ MeV fm^{-3} (Eq. (2)). Also shown are lines corresponding to other fixed values of the bag constant: $B = 80,\ 70,\ 60,\ 50$, and 45 MeV fm^{-3}.

less than 0.4, then $m_s < 200$ MeV (with both quantities renormalized at 313 MeV).

If the density at zero pressure of quark matter is indeed close to its lowest bound given by the stability limit of Eq. (2), then for $\alpha_c > 0$ it can attain a value lower than 4×10^{14}g cm^{-3} and the mass of even a non-rotating strange star may be sufficiently high to allow an orbital frequency of 1.06 kHz in the marginally stable orbit. It would appear that in the model of interacting quarks considered here, and in contrast to models in which $\rho_0 > 4.2 \times 10^{14}$g cm^{-3}, the possible presence of moderately rotating strange stars in LMXBs could be compatible with the keplerian model of kHz QPOs – compare Bulik et al. (1999a).

5. The maximum value of B

There is another stability constraint limiting the parameters of quark matter, if such a substance is indeed the ground state of matter. The chemical potential at zero pressure of quark matter in three flavours and electrons in beta equilibrium, should be less than the rest energy per nucleon of the iron nucleus $\mu_0^{u,d,s} < \mu(^{56}\mathrm{Fe}) = 930.4\,\mathrm{MeV}$ (Farhi and Jaffe 1984). For massless quarks this corresponds to $B/(1 - 2\alpha_c/\pi) < 91.49\,\mathrm{MeV}$, but in general the constraint depends on the strange quark mass, and the maximum stellar mass corresponding to this upper bound on B is exhibited in each panel of Fig. 3 as a dotted line [for details see Prakash et al. (1990) and Zdunik et al. (2000)]. Note that the upper and lower constraints on B, when taken together, exclude high masses of the strange quark for low values of α_c, if strange quark gas is to be the stable form of matter.

6. Summary

We have investigated the question whether moderately rotating strange stars with masses somewhat higher than $2M_\odot$ are allowed by relativistic equations of stellar equilibrium, and found that the answer could, in principle, be positive within the MIT bag model of beta-stable quark matter. An extension of this discussion to other models of strange matter will be given elsewhere.

The physical constraints on the bag constant following from the basic hypothesis of stability of self-bound quark matter (Eq. (2)) allow B to be so small, that the corresponding mass of the star could be as high as $2.5M_\odot$ (Eq. (3), Fig. 3). However, the strange star mass cannot be higher than $2.6M_\odot$, even for stars of rotational period as short as 1.6 ms.

The perturbative approach used here is sensible only if the value of the QCD coupling constant α_c is small – we used $0 \leq \alpha_c \leq 0.6$. In this range, the direct dependence of M_{max} on the coupling is practically negligible (Fig. 1), but the window of allowed values of B does depend on α_c (Fig. 3). The actual value of B is subject to a very large uncertainty. Fits to the hadronic mass spectrum (DeGrand et al., 1975) gave $B = 59\,\mathrm{MeV\ fm^{-3}}$.

The stellar mass decreases with increasing mass, m_s, of the strange quark, and is lower by 10% to 20% than the one for massless-quark matter for the typical range considered in the literature, $150 \leq m_s c^2/\mathrm{MeV} \leq 300$ (Madsen 1999). Finally, the maximum stellar mass for strange stars rotating with a frequency up to 700 Hz, is larger than the one for non-rotating stars by less than 4% (Fig. 2).

The physical reason for which large values of mass are obtained for a low value of the bag constant, is that the latter is proportional to the density of quark matter at zero pressure, ρ_0, while the maximum mass of the star is proportional to $\rho_0^{-1/2}$. Thus, within the MIT bag model of quark matter, strange stars with mass $M > 2.0M_\odot$ must have surface densities $\rho_0 < \rho_1 \equiv 4.2 \times 10^{14}$ g/cm^3 (unless rotating with periods ≤ 1.5 ms).

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