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Strange stars – linear approximation of the EOS and maximum QPO frequency

J.L. Zdunik

N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, 00-716 Warszawa, Poland (jlz@camk.edu.pl)

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Abstract. In the present paper we study the equation of state of strange matter build of the u,d,s quarks in the framework of the MIT bag model. The scaling relations with bag constant are discussed and applied to the determination of maximum frequency of a particle in stable circular orbit around strange star. The highest QPO frequency of 1.33 kHz observed so far is consistent with the strange stars models for which the maximum QPO frequency is 1.7–2.4 kHz depending on the strange quark mass and the QCD coupling constant. The linear approximation of the equation of state is found and the parameters of this EOS are determined as a functions of strange quark mass, QCD coupling constant and bag constant. This approximation reproduces exact results within an error of the order of 1% and can be used for the complete study of the properties of strange stars including microscopic stability of strange matter and determination of the total baryon number of the star.

Key words: dense matter – equation of state – stars: neutron

1. Introduction

The idea of the compact stars build of strange matter was presented by Witten (1984) and the models of stars were calculated using various models of strange matter by Haensel et al. (1986) and Alcock et al. (1986). The main idea is that the u,d,s matter is ground state of matter at zero pressure (self-bound strange quark matter) i.e.:

$$\mu_0 \equiv \mu(P=0) < M(^{56}\mathrm{Fe}) = 930.4\ \mathrm{MeV} \quad (1)$$

There were two aspects of rather extensive studies of strange stars properties. The first one in the context of maximum rotational velocity of the star (Frieman & Olinto 1989, Glendenning 1989, Prakash et al. 1990, Zdunik & Haensel 1990) was related to the announcement reporting the detection of the half millisecond pulsar in 1989 (which after one year turned out to be erroneous). This observation stimulated detailed investigations of the limits on the rotational frequency of dense stars which excluded nearly all neutron stars models leaving strange stars as a possible explanation.

At present the increasing interest in strange stars is a result of QPOs observations in low-mass X-ray binaries LMBX and some difficulties with the explanations of its properties by neutron star models under the assumption that QPOs represent Keplerian frequencies of the particles in an accretion disk (Kaaret et al. 1997, Kluźniak 1998, Zhang et al. 1998, Miller et al. 1998, Thampan et al. 1999, Schaab & Weigel 1999). In this paper the maximum values of these frequencies are found for a broad set of the parameters describing a strange matter EOS. Strange star models are consistent with the maximum observed QPO frequency 1.33 kHz in 4U 0614+09 (van Straaten et al. 2000).

Frieman & Olinto (1989) mentioned the approximation of the EOS of strange matter by the linear function (see also Prakash et al. 1990). I present here the linear form of the EOS of strange matter with parameters expressed as a polynomial functions of the physical constants describing MIT bag model (strange quark mass and QCD coupling constant). These approximate formulae allow us to write down the pressure vs. density dependence in a simple form $P = a \cdot (\rho - \rho_0)$. The consequence of this form are scaling laws of all stellar quantities with appropriate powers of ρ_0. This linear EOS is complete in the sense that it contains not only pressure and energy density, which enter the relativistic stress-energy tensor and are sufficient for the determination of the main parameters of the star (mass, radius), but also the formula for baryon number density (or equivalently baryon chemical potential), necessary in the microscopic stability considerations and determination of the total baryon number of the star.

2. Strange stars and maximum QPO frequency

We describe the quark matter using the phenomenological MIT bag model (see, e.g., Baym, 1978). The quark matter is the mixture of the massless u and d quarks, electrons and massive s quarks. The model is described in detail in Farhi & Jaffe (1984), where the formulae for physical parameters of a strange matter are also presented. There are the following physical quantities entering this model: B – the bag constant, α_c – the QCD coupling constant and m_s – the mass of the strange quark. It is necessary to introduce also the parameter ρ_N – the renormalization point. Following Farhi & Jaffe (1984) we choose $\rho_N = 313$ MeV.

The consequence of this model of strange matter is scaling of all thermodynamic functions with some powers of B. Knowing the EOS for given α_c, m_s and B_0 we can obtain thermodynamic quantities for other value B from the following formulae:

$$P_{[B]} = P_{[B_0]} \cdot (B/B_0), \quad \rho_{[B]} = \rho_{[B_0]} \cdot (B/B_0),$$
$$n_{[B]} = n_{[B_0]} \cdot (B/B_0)^{3/4}, \quad \mu_{[B]} = \mu_{[B_0]} \cdot (B/B_0)^{1/4} \quad (2)$$

where the resulting EOS for B is determined for the same value of α_c but for the strange quark mass given by the relation:

$$m_s(B) = m_s(B_0) \cdot (B/B_0)^{1/4} \quad (3)$$

The advantage of this approximations is the elimination of one parameter (B) from the calculations of the EOS. The dependence of all parameters on B is very well defined and one can take this into account using simple scaling laws. It is therefore sufficient to study the EOS in two parameter space: α_c and m_s for chosen value of B_0 and then obtain results for other B from Eqs. (2).

Strictly speaking the scaling relations (2) hold exactly for the parameter ρ_N rescaled by the relation $\rho_N(B) = \rho_N(B_0) \cdot (B/B_0)^{1/4}$. However the renormalization point is fixed and this is the source of small discrepancy between the true results for different values of bag constant and the scaling relations presented in this paper. This difference is presented in Fig. 1 in the case of the baryon chemical potential at zero pressure calculated directly and using scaling relation (2). We choose μ_0 because this thermodynamic function is crucial in microscopic stability considerations. The models of strange quark matter correspond to two values of bag constant: $B_0 = 60\ \mathrm{MeV\,fm^{-3}}$ and $B = 90\ \mathrm{MeV\,fm^{-3}}$. The strange quark masses in this two cases fulfill the relation (3), namely $m_s(60) = 200$ MeV and $m_s(90) = 221.33$ MeV.

We see that scaling formulae give exact results for $\alpha_c = 0$ because then ρ_N does not enter the EOS. The maximum error of the scaling formulae (2) is less than 0.5%.

From the microscopic point of view the strange quark matter is unstable at zero pressure for some values of α_c and m_s discussed in this paper (large α_c and m_s), i.e. equation of state does not fulfill the relation (1). But from presented results we can obtain, using scaling formulae, the configurations which correspond to smaller value of B and are microscopically stable. The maximum value of B for which the strange quark matter is stable can be obtained from the equation:

$$\mu_0[B_0, \alpha_c, m_s(B_{\max}/B_0)^{-1/4}] \left(\frac{B_{\max}}{B_0}\right)^{1/4} = 930.4\ \mathrm{MeV} \quad (4)$$

The strange star configurations are calculated by solving Oppenheimer-Volkoff equations in the case of spherical symmetry. For given central pressure $P_{\rm centr}$ we obtain stellar parameters such as gravitational mass M, baryon number A, moment of inertia for slow rigid rotation I and gravitational surface redshift z. All these quantities are subject to the scaling formulae similar to those describing the EOS (Eq. 2). If we calculate the star for B_0 with central pressure equal to $P_{\rm centr}[B_0]$ we know all parameters of the corresponding star with $P_{\rm centr} = P_{\rm centr}[B_0]\, B/B_0$ in the model with bag constant B (cf. Witten 1984, Haensel et al. 1986). In general for the stellar parameter X the following equality holds:

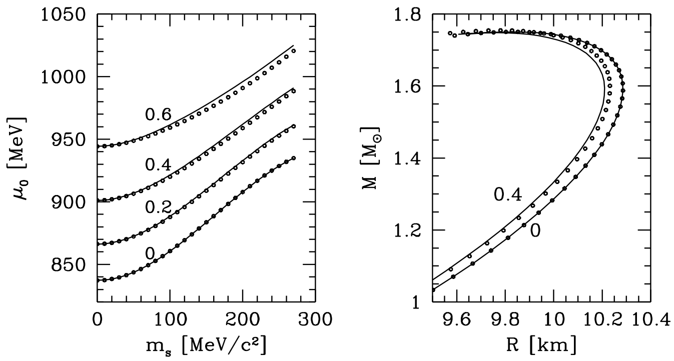

Fig. 1. The comparison between exact values of μ_0, M and R obtained by the calculation of the EOS for $B = 60\ \mathrm{MeV\,fm^{-3}}$ and $B = 90\ \mathrm{MeV\,fm^{-3}}$ and scaling relations. The values for $B = 90$ was rescaled to the value of $B_0 = 60$ using the relations $\mu \to \mu \cdot (B/B_0)^{-1/4}$, $m_s \to m_s \cdot (B/B_0)^{-1/4}$, $M \to M \cdot (B/B_0)^{1/2}$ and $R \to R \cdot (B/B_0)^{1/2}$. In the case of *exact* scaling relations the results for $B = 60$ and rescaled results for $B = 90$ would be the same.

$$X[B, \alpha_c, m_s] = X[B_0, \alpha_c, m_s(B/B_0)^{-1/4}]\,(B/B_0)^{-k} \quad (5)$$

where $k = 1/2$ in the case of mass and radius ($X = R, M$), $k = 3/4$ for $X = A$, $k = 3/2$ for I and $k = 0$ for z.

In Fig. 1 we see that scaling formulae (5) reproduce the stellar parameters M vs. R with an error less than 0.3%.

These scaling laws so far have been mainly exploited in the case of maximum mass of the star (or equivalently maximum rotational frequency). Of course these relations refer to all stellar configurations (e.g. the curve $M(R)$) and as an interesting example we can consider the point defined by the equality:

$$R = R_{\rm ms} = 3R_g = 6\frac{GM}{c^2} \quad (6)$$

which corresponds to the maximum possible frequency of a particle in stable circular orbit $\nu_{\max}$ around nonrotating star. Because the scaling properties of R and M are the same ($\sim B^{-1/2}$) one can solve the Eq. 6 independently of B. For stellar masses higher than the solution of the Eq. 6 the maximum Keplerian frequency of an orbiting particle is defined by the marginally stable orbit at $R_{\rm ms}$ and for lower by the stellar radius R (Fig. 2a). In both cases this frequency scales as $B^{1/2}$ and for other values of B the patterns of Fig. 2a do not change, provided one rescales the axes and m_s (Eqs. 3, 5, 7).

The maximum values of ν as a function of strange matter parameters are presented in Fig. 2b for $B_0 = 60\ \mathrm{MeV\,fm^{-3}}$. For other values of B these results scale as $B^{1/2}$ i.e.:

$$\nu_{\max}[B, \alpha_c, m_s] = \nu_{\max}[B_0, \alpha_c, m_s(B/B_0)^{-1/4}] \times \left(\frac{B}{B_0}\right)^{1/2} \quad (7)$$

which allows us to determine the absolute maximum value of $\nu_{\max}$ consistent with the requirement of the stability of strange matter at zero pressure (Eq. 4) by putting the maximum allowed value of B into Eq. (7). The result of this procedure

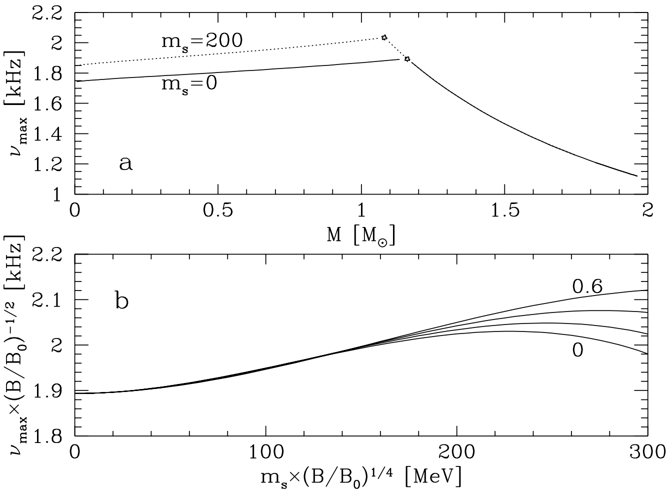

Fig. 2a and b. The value of the frequency of a particle in the innermost stable circular orbit around nonrotating strange star for strange matter models with $m_s = 0$ and $m_s = 200$ MeV (panel **a**) and the maxima of those functions as a function of m_s for $\alpha_c = 0,\ 0.2,\ 0.4,\ 0.6$ (panel **b**). The value of bag constant is fixed and equal to $B_0 = 60\,\mathrm{MeV\,fm^{-3}}$

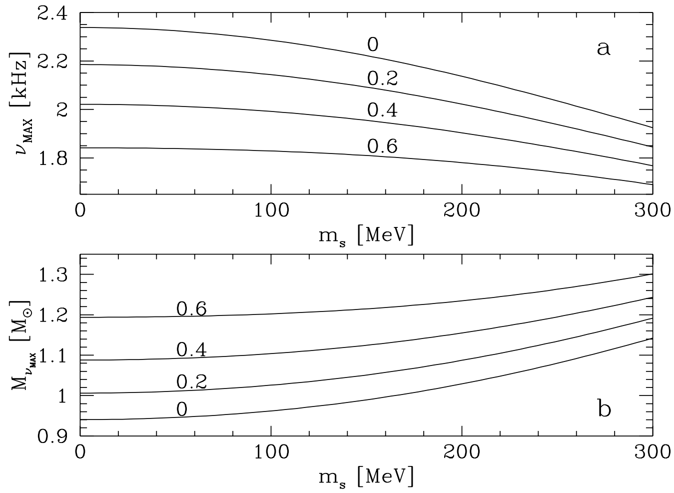

Fig. 3a and b. The absolute maximum value of the frequency of a particle in stable circular orbit around nonrotating strange star $\nu_{\rm MAX}$ obtained for $B = B_{\max}(m_s, \alpha_c)$ (Eq. 4) (panel **a**) and the corresponding values of stellar mass M – the minimum mass for which the existence of the marginally stable orbit is possible (panel **b**).

$\nu_{\rm MAX} \equiv \nu_{\max}(B_{\max})$ is presented in Fig. 3a. We see that this limit is well above the highest observed QPO frequency $\nu_{\rm obs} = 1.33\,\mathrm{kHz}$ recently reported by van Straaten et al. (2000). Thus at present the observations of the highest QPO frequency do not constrain strange matter models unless we do not make the assumptions about the mass of the star in LMBX. To set some limits on strange matter parameters the QPOs must be observed at frequencies larger than 1.8 kHz. Our conclusion is true also for moderate rotation rates of strange star due to the initial increase of the frequency at marginally stable orbit which in the case of stars with mass slightly above the solution of Eq. 6 results in $\nu_{\max}$ very close to the value corresponding to the nonrotating stars (Zdunik et al. 2000, Datta et al. 2000).

It should be mentioned that in principle analysis of the observational data can strongly support the existence of the marginally stable orbit. Such a conclusion have been recently presented by Kaaret et al. (1999) in the case of 4U 1820-30. This interpretation is equivalent to the condition $R < R_{ms}$ setting the lower bound for the mass of the star. This minimum mass limits are presented in Fig. 3b as a function of m_s and α_c. Below this mass the innermost stable orbit is located at the surface of the star.

3. Linear interpolation of EOS

In the present section we discuss the interpolation of equation of state of strange quark matter by the linear function of ρ. For a broad set of parameters α_c and m_s the dependence $P(\rho)$ can be very well approximated by the linear function, being exactly linear at the $m_s = 0$ limit. The EOS for strange quark matter is presented in Fig. 4.

The main parameters of the matter are described by the following formulae:

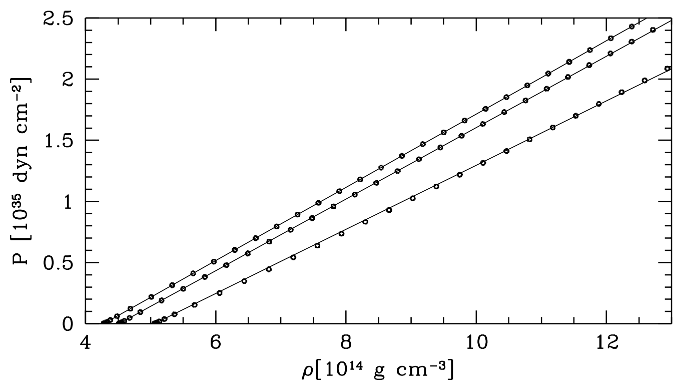

Fig. 4. The linear approximation of the EOS for strange quark matter for three choices of (m_s, α_c): $(0,0)$, $(100, 0.4)$, $(250, 0.6)$ (from left to right). The points represent exact results obtained by the numerical calculation of the EOS for $B = 60\ \mathrm{MeV\,fm^{-3}}$ and lines correspond to the approximate formula 8.

$$\begin{aligned} P(\rho) &= \tfrac{1}{3}(1+\varepsilon_{\rm fit})(\rho-\rho_0)c^2 \\ n(P) &= n_0 \cdot \left[1 + \left(4 - \frac{3\varepsilon_{\rm fit}}{1+\varepsilon_{\rm fit}}\right)\frac{P}{\rho_0 c^2}\right]^{3/(4+\varepsilon_{\rm fit})} \end{aligned} \qquad (8)$$

where ρ_0 and n_0 are energy and number density of the strange quark matter at zero pressure. The second equation is implied by the first law of thermodynamics.

The Eqs. (8) allow us to determine all thermodynamic quantities characterizing strange quark matter in linear interpolation of the EOS, e.g. the baryon chemical potential:

$$\mu(P) = \mu_0 \left(1 + \frac{4+\varepsilon_{\rm fit}}{1+\varepsilon_{\rm fit}}\frac{P}{\rho_0 c^2}\right)^{(1+\varepsilon_{\rm fit})/(4+\varepsilon_{\rm fit})} \qquad (9)$$

where $\mu_0 = \rho_0 c^2/n_0$.

Because for all thermodynamic quantities the scaling relations with B hold we can calculate all parameters needed for the linear EOS for chosen value of B and then rescale them using equations (2). Thus the main point is to determine three parameters entering equations (8), as a functions of α_c and m_s: $\rho_0(\alpha_c, m_s)$, $n_0(\alpha_c, m_s)$ and $\varepsilon_{\rm fit}(\alpha_c, m_s)$. These functions can be very well approximated by the polynomial of m_s with coefficients depending on the value of α_c. The formulae obtained by the least-squares fit to the exact results are following:

$$\begin{aligned} n_0 &= (a_0^n + a_2^n m_{s100}^2 + a_3^n m_{s100}^3)\overline{B}^{3/4} \qquad (10)\\ a_0^n &= 0.28660 C_\alpha^{1/4}\\ a_2^n &= (0.010788 + 0.0032046\alpha_c)/\overline{B}^{1/2}\\ a_3^n &= -0.0044248\sqrt{C_\alpha}/\overline{B}^{3/4}\\ \mu_0 &= (a_0^\mu + a_2^\mu m_{s100}^2 + a_3^\mu m_{s100}^3)\overline{B}^{1/4} \qquad (11)\\ a_0^\mu &= 837.260/C_\alpha^{1/4}\\ a_2^\mu &= (46.616 - 16.848/C_\alpha)/\overline{B}^{1/2}\\ a_3^\mu &= (-10.482 + 4.5211/C_\alpha)/\overline{B}^{3/4}\\ \varepsilon_{\rm fit} &= a_2^\epsilon m_{s100}^2 + a_3^\epsilon m_{s100}^3 \qquad (12)\\ a_2^\epsilon &= (-0.035745 + 0.013366\alpha_c + 0.023246\alpha_c^2)/\overline{B}^{1/2}\\ a_3^\epsilon &= (0.0055488 - 0.0054232\alpha_c - 0.0069193\alpha_c^2)/\overline{B}^{3/4} \end{aligned}$$

where m_{s100} is strange quark mass in units 100 MeV i.e. $m_{s100} = m_s c^2\,[\mathrm{MeV}]/100$, $\overline{B} = B\,[\mathrm{MeV\,fm^{-3}}]/60$ and $C_\alpha = 1 - 2\alpha_c/\pi$.

The energy-matter density at zero pressure is equal to $\rho_0 = n_0\mu_0/c^2$. The functions $\rho_0(m_s)$ and $\varepsilon_{\rm fit}(m_s)$ entering directly Eq. 8 and $\mu_0(m_s)$ for chosen values of α_c and accuracy of the above interpolations are presented in Fig. 5.

One can test the accuracy of the interpolation of the EOS by the linear function calculating the stellar configurations build of such a matter and comparing results with stars calculated using exact equation of state. Such a comparison is presented in Fig. 6. The relative error in this procedure is always less than 1% in the case of mass, radius and moment of inertia.

Except for the case of strange matter with massless quarks, studied in detail in the literature, the linear form of the equation of state of self-bound matter have been used so far mainly for the stellar mass and radius determination (e.g. Haensel & Zdunik 1989, Lattimer et al. 1990). The authors exploited the relation $P = a(\rho - \rho_0)$, because the quantities sufficient in these considerations are pressure and energy density, which enter the stress-energy tensor and TOV equations of hydrostatic equilibrium. However for the complete study of the properties of star the full microscopic description of the matter is needed, including baryon chemical potential and particle number density. In our EOS the formula for $n(P)$ (Eq. 8) enables us to determine the total baryon number of the stellar configuration and use this model of strange matter, for example, for the discussion of the conversion of neutron stars into strange stars (Olinto 1987, Cheng & Dai 1996, Bombaci & Datta 2000). During this process the total baryon number of the star is fixed. Using the relation (11) one can find the ranges of the values of m_s, α_c and B consistent with the requirement that strange matter is ground state of matter at zero pressure (Eq. 1). As a result the allowable parameters of strange stars (e.g. M, $\nu_{\rm MAX}$) can be found without complicated and time-consuming calculations of the microscopic properties of strange matter.

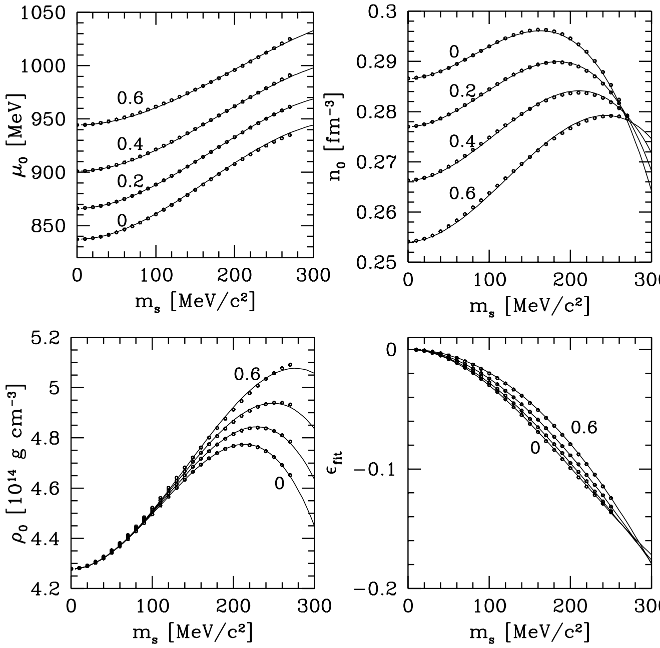

Fig. 5. The comparison between exact values of the main parameters of the strange matter (baryon chemical potential, particle number density, mass-energy density) at zero pressure and the parameter $\varepsilon_{\rm fit}$ entering the linear approximate EOS of strange quark matter $P = 1/3(1 + \varepsilon_{\rm fit})(\rho - \rho_0)$ obtained by the interpolation of the EOS for $B = 60\ \mathrm{MeV\,fm^{-3}}$ (points) and approximate formulae (10-12) (lines).

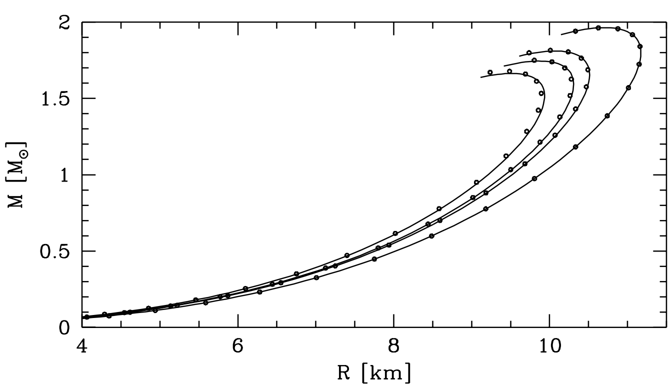

Fig. 6. The mass-radius relations for strange stars calculated using exact EOS (points) and the linear approximation of the EOS $P \sim (\rho - \rho_0)$ (lines) for $B = 60\,\mathrm{MeV\,fm^{-3}}$. The curves correspond to four different parameters of strange matter (m_s, α_c): $(0, 0)$, $(150, 0.3)$, $(200, 0)$, $(250, 0.6)$ (from right to left).

4. Discussion and conclusions

In the present paper I analyzed the accuracy of the scaling properties of strange matter parameters with the value of the bag constant B in the framework of MIT bag model of quark matter. The scaling formulae reproduce the main stellar parameters with an error less than 1%. This allow us to use them to determine the maximum possible frequency of a particle in stable circular orbit around strange star. The absolute maximum QPO frequency that can be accommodated by strange stars ranges from 1.7 to 2.4 kHz depending on the values of a strange matter parameters: strange quark mass and QCD coupling constant. Thus the present status of observational data ($\nu_{\max}^{\rm obs} = 1.33$ kHz) cannot exclude strange stars as source of QPOs in LMBX. The frequencies of QPO in the very high range (larger than 1.8 kHz), if observed, may set some bounds on parameters of strange matter, excluding large m_s and large α_c. We should mention that such a high values of $\nu_{\rm QPO}$ cannot be understood in terms of neutron stars (Thampan et al. 1999).

The high accuracy of the simple scaling laws with B is strictly connected with the possibility of the approximation of the equation of state by the linear function $P = a(\rho - \rho_0)$ for which such a scaling properties are well known and exact. For this linear EOS the parameters of the strange stars scale with the powers of ρ_0 for fixed value of a (analogous to the scaling laws with B, Eqs. 5, 7). It should be stressed that these scaling properties are valid not only for static stellar configurations but also in some dynamical problems (e.g. parameters of the rotating star, Gourgoulhon et al. 1999). For fixed ρ_0 one can apply also the approximate scalings of the stellar parameters at the maximum mass point with appropriate powers of a (Lattimer et al., 1990) which have been recently confirmed by Stergioulas et al. (1999) for stars rotating at Keplerian frequency.

The presented linear approximation of the equation of state allow us to determine the dependence of many properties of strange stars on the physical parameters of a matter (m_s, α_c) using very simple form of the EOS. For a broad set of m_s and α_c the values of $\varepsilon_{\rm fit}$ and ρ_0 entering linear EOS can be very accurately obtained by a polynomial formulae. In particular the formula for the baryon chemical potential enables us to study the microscopic stability of strange matter and make a complete discussion of the resulting constraints on strange stars parameters. The expression for baryon number density is necessary in the consideration of the conversions of neutron stars into strange stars with total baryon number conserved.

It is worth noticing that the linear approximation of $P(\rho)$ dependence (Eq. 8) can be used also for other models of strange matter, self-bound at high density ρ_0. The expressions for $n(P)$ and $\mu(P)$ allow us to determine all microscopic properties of the matter given the values of ρ_0 and n_0.

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