Detectability of the primordial origin of the gravitational wave background in the Universe

J. Lesgourgues[1], **D. Polarski**[2,3], **S. Prunet**[4,5], **and A.A. Starobinsky**[6]

[1] SISSA-ISAS, Via Beirut 4, 34014 Trieste, Italy
[2] LMPT, UPRES-A 6083 CNRS, Université de Tours, Parc de Grandmont, 37200 Tours, France
[3] DARC, Obs. de Paris-Meudon, 92195 Meudon Cedex, France
[4] IAP, CNRS, 98bis Boulevard Arago, 75014 Paris, France
[5] CITA, 60 St George Street, Toronto, Ontario M5S 3H8, Canada
[6] Landau Inst. for Theoretical Physics, Kosygina St. 2, Moscow 117334, Russia

Abstract. The appearance of peaks in the Fourier power spectra of various primordial fluctuations is a generic prediction of the inflationary scenario. We investigate whether future experiments, in particular the satellite experiment PLANCK, will be able to detect the possible appearance of these peaks in the B-mode polarization multipole power spectrum. This would yield a conclusive proof of the presence of a primordial background of gravitational waves.

Key words: cosmology: cosmic microwave background – gravitational waves

1. Introduction

Early Universe cosmology is reaching a stage where theories put forward for the generation of primordial fluctuations can be severely constrained by observations. It is already the case with present day observations and this will be even more so in the near future due in particular to the Cosmic Microwave Background (CMB) anisotropy measurements with unprecedented resolution by the satellites MAP (NASA) and PLANCK (ESA). At present, only inflationary scenarios seem capable to explain the existing bulk of data, in particular the acoustic (Doppler) peak in the CMB, and one hopes that the increasing amount of observations will finally lead us to the "right" inflationary model or at least restrict the remaining viable models to only a small number.

We would like here to deal with a generic aspect, one that is common to all inflationary models, namely the time coherence of the cosmological perturbations. All inflationary scenarios have in common an accelerated stage of expansion during which fluctuations are generated on super-horizon scales, i.e. with wavelength larger than the Hubble radius. The fluctuations responsible for the CMB fluctuations, whether temperature fluctuations or polarization, though they originate from vacuum quantum fluctuations, were for a long time on "super-horizon"

scales and this is why they appear to us as classical fluctuations with random amplitude and fixed temporal phase. In other words, soon after the end of inflation, cosmological perturbations appear to consist of only the growing, or quasi-isotropic, modes with an excellent accuracy. Remarkably enough, this coherence has a very distinct observational signature resulting in periodic acoustic peaks in the CMB temperature anisotropy multipoles C_l^{S} and also in the corresponding multipoles of the CMB polarization. Hence, the detection of these periodic peaks would be a dramatic confirmation of their primordial origin.

As well known, the generation of a gravitational wave (GW) background on a vast range of frequencies is also an important prediction of inflationary models (first quantitatively calculated in Starobinsky 1979), one that could constitute, if observed, a crucial experimental confirmation of these scenarios. In addition, what was said above concerning the time coherence of the fluctuations is equally valid for the primordial scalar fluctuations as well as for the primordial tensorial fluctuations, or primordial GW background. For them too, their primordial origin will uncover itself in the presence of a periodic structure in the multipole power spectrum which we call primordial peaks. Clearly, they are much more difficult to track than acoustic peaks produced by scalar (energy density) fluctuations. Note that these primordial peaks are periodic, with a periodicity (Polarski & Starobinsky 1996)

$$\Delta l = \pi \left(\frac{\eta_0}{\eta_{\mathrm{rec}}} - 1 \right), \tag{1}$$

which is approximately half the spacing between primordial acoustic peaks produced by scalar fluctuations (due to the difference between the light velocity which is relevant for (1) and the sound velocity in the baryon-photon plasma at recombination which enters into the corresponding expression for the spacing between acoustic peaks). Note that, strictly speaking, Eq. (1) becomes exact for $l \to \infty$ only. However (see Fig. 1), it turns out that Eq. (1) is already a good approximation for the spacing between the first and second peaks. In Eq. (1), $\eta \equiv \int^t \frac{dt'}{a(t')}$ and η_0, resp. η_{rec} are evaluated today, resp. at recombination.

Of course, the detection of these peaks is much more complicated than the discovery of a long-wave GW background in the Universe through the B-mode polarization of the CMB, though such a discovery would represent a great achievement in itself (for its prospects see, e.g., Kamionkowsky & Kosowsky 1998). However, the significantly smaller effect which we consider in this paper - the existence of multiple primordial peaks in the angular spectrum of the B-mode CMB polarization - is fundamental and remarkable enough to justify hard efforts to detect it for two reasons. The first reason, explained above, concerns the primordial origin of the GW background; the second one is related to the use of Eq. (1) in order to determine fundamental cosmological parameters.

The discovery of the (asymptotic) periodicity of the $\Delta T/T$ peaks produced by a primordial GW background will immediately give us an unbiased value of one of the most important parameters: the ratio of the present conformal time to the recombination conformal time $\eta_0/\eta_{\rm rec}$. Furthermore, by combining this result with the periodicity scale of acoustic peaks in the CMB anisotropy and the E-mode of the CMB polarization at $l > 200$ (which is much easier to measure) we can directly find the value of the sound velocity $c_{\rm s}$ in the cosmic photon-baryon plasma at the moment of recombination. This, in turn, leads to a new way of determining the present baryon density $n_{\rm B}$ which is free of "cosmic confusion".

Actually the observation of the peaks in the multipoles C_1^T due to the primordial GW is a hopeless experimental challenge with the presently existing technology. On the other hand, the observation of this coherence in a direct detection experiment of the primordial GW background is even worse: it would require a resolution in frequency $\Delta\nu \approx 10^{-18}$Hz (as briefly mentioned in Polarski & Starobinsky 1996, p.389), something that is clearly impossible to achieve (see also Allen et al. 2000 for a recent careful investigation).

A better prospect for the detection of these peaks might perhaps be offered by the measurement of the CMB polarization as scheduled by PLANCK. We expect the CMB to be also polarized and important physical information could be extracted from it. In particular, the scalar fluctuations will not contribute to the so-called B-mode polarization (Kamionkowski et al. 1997a; Seljak & Zaldarriaga 1997), therefore the latter bears the imprint of the primordial GW only. Hence, CMB polarization measurements might enable us to show the presence of a GW background of primordial origin. It is the aim of this letter to investigate whether the sensitivity of PLANCK is sufficient for this purpose. We will do this using a concrete, viable model (Lesgourgues et al. 1999a, 1999b) in which the generated GW background can be fairly high, with $C_{10}^{(T)} \leq C_{10}^{(S)}$ (note that here, $C_1^{(T)}$, resp. $C_1^{(S)}$, stands for the temperature anisotropy multipoles produced by tensorial, resp. scalar, perturbations).

2. The model and the induced polarization

The primordial GW produced during the inflationary stage originate from vacuum fluctuations of the quantized tensorial metric perturbations. Each polarization state λ – where $\lambda = \times, +$, and

the polarization tensor is normalized to $e_{\rm ij}(\mathbf{k})\, e^{\rm ij}(\mathbf{k}) = 1$ – has an amplitude h_λ (in Fourier space) given by

$$h_\lambda = \sqrt{32\pi G}\,\phi_\lambda \tag{2}$$

where ϕ_λ corresponds to a real massless scalar field. The production of a GW background is a generic feature of all inflationary models.

Let us briefly describe the BSI (Broken Scale Invariant) inflationary model used here. The power spectrum of this model has a characteristic scale which is due to a rapid change in slope of the inflaton potential $V(\varphi)$ from $A_+ > 0$ to $A_- > 0$ (when φ decreases) in some neighbourhood $\Delta\varphi$ of φ_0 (Starobinsky 1992). As a consequence, one of the two slow-roll conditions is violated and this is why the scalar perturbation spectrum $k^3\Phi^2(k)$ is non-flat around the scale $k_0 = a(t_{k_0})H_{k_0}$, which becomes larger than the Hubble radius when $\varphi(t_{k_0}) = \varphi_0$ ($H \equiv \dot{a}/a$ is the Hubble parameter). The spectrum can be basically represented as "step-like" while its shape is determined solely by the parameter $p \equiv \frac{A_-}{A_+}$ and is independent of the characteristic scale k_0. In particular, an inverted step is obtained for $p < 1$. This model could nicely account for the possible appearance of a spike in the matter power spectrum (Einasto et al. 1997). We will assume that the inflaton potential satisfies the slow-roll conditions far from the point k_0 and consider a particular behaviour of the spectral indices $n_{\rm T}(k)$ and $n_{\rm s}(k)$. This model was thoroughly investigated previously (Lesgourgues et al. 1999b; Lesgourgues et al. 1999; Polarski 1999) and it was found to be in agreement with observations in the presence of a large cosmological constant ($\Omega_\Lambda \approx 0.7$, as favoured by recent observations). We refer the interested reader to the literature for further technical details about our model and the possible observational hints in support of its BSI spectrum.

Also our model allows a high fraction of the temperature anisotropy to originate from tensorial fluctuations with $C_{10}^{(T)} \leq C_{10}^{(S)}$. This last property is significantly different from scale free single-field slow roll inflation for which the height of the Doppler peak precludes a high contribution of the GW to $\frac{\Delta T}{T}$ on large angular scales where the power spectrum gets normalised. It is this fact which is of interest to us here as we may hope that the B-polarization is large enough for our purposes.

We introduce now the polarization tensor and the multipole power spectra needed besides C_1^T, where

$$\langle a_{\rm lm}^{T*} a_{\rm l'm'}^T \rangle \equiv C_1^T\, \delta_{\rm ll'}\, \delta_{\rm mm'} \tag{3}$$

and the coefficients $a_{\rm lm}^T$ are defined through

$$\frac{\Delta T}{T} = \sum_{\rm l=0}^{\infty} \sum_{\rm m=-l}^{\rm m=l} a_{\rm lm}^T\, Y_{\rm lm}\,. \tag{4}$$

The symmetric, trace-free polarization tensor $P_{\rm ab}$ can be expanded as follows

$$\frac{P_{\rm ab}}{T} = \sum_{\rm l=0}^{\infty} \sum_{\rm m=-l}^{\rm m=l} \left(a_{\rm lm}^E\, Y_{\rm lm,ab}^E + a_{\rm lm}^B\, Y_{\rm lm,ab}^B \right), \tag{5}$$

where $Y_{\mathrm{lm}}^{E,B}$ are electric and magnetic type tensor spherical harmonics, with parity $(-1)^l$ and $(-1)^{l+1}$ respectively. A description of the CMB requires the three power spectra

$$C_1^T \equiv \langle |a_{\mathrm{lm}}^T|^2 \rangle, \quad C_1^E \equiv \langle |a_{\mathrm{lm}}^E|^2 \rangle, \quad C_1^B \equiv \langle |a_{\mathrm{lm}}^B|^2 \rangle , \tag{6}$$

together with the only non vanishing cross correlation function

$$C_1^{TE} \equiv \langle a_{\mathrm{lm}}^{T*} a_{\mathrm{lm}}^E \rangle . \tag{7}$$

Indeed, because of parity, the cross-correlation functions C_1^{TB}, C_1^{EB} vanish. Among the different types of primordial perturbations, only the primordial GW can produce B-mode polarization. Hence the latter offers a unique opportunity to probe the possible presence of a GW background and in particular its primordial origin.

3. Statistical analysis

We want first to investigate whether Planck has the required sensitivity in order to see possible small peaks in the power spectrum C_1^B. Our method will make use of the Fisher information matrix F_{ij}.

Using the CMB Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996), we compute the derivative of the C_l's with respect to each parameter θ_{i} on which the spectra may depend in a given model. The Fisher matrix (Jungman et al. 1996a, 1996b; Tegmark et al. 1997; see also Bond et al. 1997; Copeland et al. 1998; Eisenstein et al. 1998; Wang et al. 1999; Stompor & Efstathiou 1999) is then obtained by adding the derivatives, weighted by the inverse of the covariance matrix of the estimators of the polarized and unpolarized CMB power spectra for the PLANCK satellite mission, $\mathrm{Cov}(C_1^X, C_1^Y)$:

$$F_{\mathrm{ij}} = \sum_{l=2}^{+\infty} \sum_{X,Y} \frac{\partial C_1^X}{\partial \theta_{\mathrm{i}}} \mathrm{Cov}^{-1}\left(C_1^X, C_1^Y\right) \frac{\partial C_1^Y}{\partial \theta_{\mathrm{j}}} , \tag{8}$$

where $\{X, Y\} \in \{T, E, B, TE\}$ (Kamionkowski et al. 1997b; Zaldarriaga et al. 1997; Prunet et al. 1998a, 2000). The Fisher matrix F_{ij} measures basically the width and the shape of the likelihood function around the maximum likelihood point. Assuming that a fit to the PLANCK data yields a maximum likelihood for the model under consideration (for which the derivatives were computed), the $1 - \sigma$ error on the parameter θ_{i}, for any unbiased estimator of θ_{i} and however precise the observations may be, satisfies

$$\Delta \theta_{\mathrm{i}} \geq \sqrt{(F^{-1})_{\mathrm{ii}}} , \tag{9}$$

if all the parameters are estimated from the data, and

$$\Delta \theta_{\mathrm{i}} \geq F_{\mathrm{ii}}^{-\frac{1}{2}} , \tag{10}$$

when all other parameters are known.

Each multipole will be measured by Planck with unprecedented precision of the order of 1%, thereby allowing for an accurate extraction of the cosmological parameters. Still, one should remember that a given model with its spectra implies a set of parameters, each having a particular value, which define the model. Even though the power spectra C_1^X for some given parameter combination might be measured with very high precision, each parameter separately is usually constrained only at the percent level due to the possible degeneracy of the spectra with respect to a change in the parameter combination. In computing the covariance matrix of the CMB power spectra, we accounted for the presence of foregrounds (both polarized and unpolarized) in the measurement of the CMB power spectra, using the method described in Bouchet et al. 1999 (see also Prunet et al. 1998a, 2000).

In order to use this approach we need to quantify the appearance of peaks with the help of some additional parameter $\theta_{\mathrm{i}} \equiv s$. For this purpose, we adopt the following strategy: we compare the C_1^B curve of our inflationary model where peaks are present with a smoothed version $C_{1,\mathrm{sm}}^B$ which contains no peaks anymore. Obviously, we can write

$$C_1^B = C_{1,\mathrm{sm}}^B + s(C_1^B - C_{1,\mathrm{sm}}^B) . \tag{11}$$

Hence, the parameter s enters the Fisher matrix through the quantity

$$\frac{\partial C_1^B}{\partial s} = C_1^B - C_{1,\mathrm{sm}}^B. \tag{12}$$

Note that $s = 1$ corresponds to the original model which is assumed to be the correct one. We stress that it is perfectly self-consistent to smooth only the C_1^B spectrum since the possible appearance of peaks in the other spectra is due to the scalar perturbations only. This is well known for the temperature anisotropy, and it is also true for the E-mode polarization multipoles C_1^E. In summary, what we really measure with the help of the parameter s is the presence of a time-coherent GW background, in other words, a GW background which is of primordial origin.

For completeness, we take also into account the additional information provided by the T, E and TE modes: we smooth the tensor contibutions $C_1^{X,(T)}$, $X \in \{T, E, TE\}$, and calculate

$$\frac{\partial C_1^X}{\partial s} = C_1^{X,(T)} - C_{1,\mathrm{sm}}^{X,(T)}. \tag{13}$$

We stress that in general, statistical separation of the tensor contribution from the scalar contribution requires prior knowledge about the underlying theory (which is available here by assumption). Even so, this will change F_{ss} only by a small amount, due to observational uncertainties in tensor-scalar separation, a drawback which does not affect the B mode.

We fix the parameters of our model to $\Omega_{\mathrm{tot}} = 1$, $\Omega_\Lambda = 0.65$, $\Omega_{\mathrm{b}} = 0.04$, $h = 0.6$, $p = 0.58$, $k_0 = 0.016\ h\mathrm{Mpc}^{-1}$, $n_{\mathrm{S}}(k < k_0) = 1$, $n_{\mathrm{T}}(k_0) = -0.125$. For these parameters, Eq. (1) gives $\Delta l \approx 160$ (assuming 3 kinds of massless or very light neutrinos). As shown in (Lesgourgues et al. 1999b), this choice is consistent with current constraints, despite a fairly high GW contribution to the CMB temperature anisotropy with $C_{10}^{(T)}/C_{10}^{(S)} = 0.85$. We find that the $1 - \sigma$ error Δs on the parameter s satisfies

$$\Delta s \geq \sqrt{(F^{-1})_{\mathrm{ss}}} = 2.68 \tag{14}$$
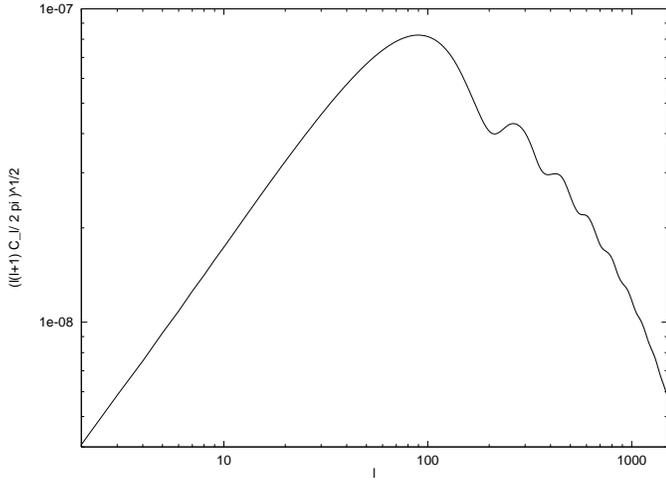
Fig. 1. C_l^B curve for the model under consideration. The $C_{l,\mathrm{sm}}^B$ curve is obtain by smoothing this curve for $l \geq 150$.

if all other parameters are extracted from the same data as well, while essentially the same result is obtained

$$\Delta s \geq F_{\mathrm{ss}}^{-\frac{1}{2}} = 2.63 \tag{15}$$

when all other parameters are known. This is not surprising since the error in the measurement of this parameter is dominated by the noise and the foregrounds and not by a possible degeneracy with the other parameters. Since in both cases $\Delta s \geq 1$, Planck clearly does not seem to have the level of sensitivity required in order to see the primordial peaks in the B-mode polarization, at least for our model. We recall however that our model admits a large GW background, in any case substantially larger than in usual single-field slow-roll inflationary models. Therefore, a negative result for this model is almost certain to imply, for the particular problem under consideration, rather gloomy prospects for most, if not all, viable inflationary models[1].

It is interesting to evaluate what is the sensitivity required for other future experiments. If we imagine an idealized experiment, with only one channel, and no foregrounds contamination at all, we find that only a sensitivity ten times higher than that achieved by Planck's best channel will allow a clear detection with $\Delta s \simeq 0.1$. The assumption of no foregrounds contamination is clearly an idealization if we compare the expected level of the dust polarized B-mode power spectrum (see for instance Prunet et al. 1998b) to the CMB spectrum shown in Fig. 1.

However, the level of contamination is very inhomogeneous on the sky, and one expects to find some locations where the contamination level by dust would be at least ten times smaller

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[1] Also, in our model, it is possible to neglect the gravitational lensing contamination of the B mode (Zaldarriaga & Seljak 1998), in contrast with models with a low tensor contribution. Indeed, in our model, gravitational lensing generates a B-polarized signal that dominates the primordial gravitational wave signal for $l > 140$. However, we checked with a specific Fisher matrix analysis that from the measurement of T, E, TE modes alone, the C_l^B contamination can be substracted with 4% accuracy, and therefore neglected up to $l = 350$, while our result for F_{ss} depends mainly on multipoles C_l^B with $150 < l < 350$.

than the mean level computed for a galactic latitude $\|b\| > 20°$. Of course, the draw-back of observing a smaller part of the sky is that it increases the sample variance. Indeed, in the no-foregrounds case, the sample variance part of the covariance of the estimator of a given B-mode multipole C_ℓ^B is approximately given by

$$\Delta C_\ell^B / C_\ell^B \simeq \sqrt{\frac{2 f_{\mathrm{sky}}}{2\ell + 1}} \tag{16}$$

where f_{sky} is the fraction of the sky covered by the experiment. However, since we are interested in multipoles $\ell \gtrsim 250$, a rather small region (typically $400 \deg^2$) should be sufficient for this sample variance to be smaller than the noise. Thus a dedicated, long-time observation of a particularly clean region of the sky, like the *Polatron* experiment[2], with possibly a poorer angular resolution than *Polatron* but with a significant gain in sensitivity, should be able to constrain the coherence parameter s to a reasonable accuracy, especially if we take into account the expected progress in bolometer technology.

In conclusion, it is not unreasonable to expect that in the upcoming decades, CMB polarization experiments, in addition to addressing the very existence of a cosmic gravitational wave background (which we think will already be settled by Planck), will also answer the fundamental question concerning the primordial origin of this background.

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[2] see http://astro.caltech.edu/ lgg/polatron/polatron.html

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