

Theoretical models for classical Cepheids

VII. Metallicity effects on the Cepheid distance scale

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Abstract. We use theoretical Period-Luminosity and Period-Luminosity-Color relations in the VI passbands, as based on nonlinear, nonlocal and time-dependent convective pulsating models, to predict the reddening and true distance modulus of distant Cepheids observed with the Hubble Space Telescope. By relying on the pulsating models with metal content $Z=0.008$, we find that the theoretical predictions agree to the values obtained by the Extragalactic Distance Scale Key Project on the basis of empirical Period-Luminosity relations referenced to LMC variables. In the meantime, from the theoretical relations with $Z=0.004$ and 0.02 we find that the predicted $E(B - V)$ and μ_0 decrease as the adopted metal content increases. This suggests a metallicity correction to LMC-based distances as given by $\Delta\mu_0/\Delta\log Z \sim -0.27 \text{ mag dex}^{-1}$, where $\Delta\log Z$ is the difference between the metallicity of the Cepheids whose distance we are estimating and the LMC value $Z=0.008$. Such a theoretical correction appears supported by an existing, although weak, correlation between the Cepheid distance and the $[O/H]$ metallicity of galaxies within a given group or cluster, as well as by a similar correlation between the H_0 estimate and the $[O/H]$ metallicity of the galaxies which calibrate the SNIa luminosity. On the contrary, the metallicity corrections earlier suggested on empirical grounds seem to be excluded. Eventually we suggest that the average value $\langle H_0 \rangle \sim 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$ provided by the Key Project team should increase *at least* up to $\sim 69 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Further observational evidences in support of the predicted scenario are finally presented.

Key words: stars: variables: Cepheids – stars: distances – galaxies: distances and redshifts

1. Introduction

The Cepheid Period-Luminosity (PL) relation is a classical tool widely used to estimate the distance to Local Group galaxies and to external galaxies with Hubble Space Telescope (HST) observations, as well as, through the calibration of secondary distance indicators, to even more distant stellar systems. This

explains the dominant role of these variables on the route to solve cosmological problems such as the evaluation of the Hubble constant H_0 and, in turn, of the age of the Universe.

The basic physics underlying the Cepheid variability suggests that the pulsation period P depends on the mass, luminosity and effective temperature of the pulsator. From stellar evolution theory one expects a close relation between mass and luminosity, and the natural result of these theoretical prescriptions is a Period-Luminosity-Color (PLC) relation, where the pulsator absolute magnitude M_j in each given photometric pass-band j is a linear function of the period and color index $[CI]$, as given by

$$M_j = \alpha + \beta \log P + \gamma [CI] \quad (1)$$

However, since the pulsation occurs within a finite zone of the HR diagram, the color term in the PLC relation is often neglected and Cepheid distances are usually estimated from the most favourite PL relation

$$\overline{M}_j = a + b \log P, \quad (2)$$

where \overline{M}_j is now the average of the Cepheid magnitudes for each given period. It should be noticed that whereas the PLC relation holds for any individual pulsator, PL is a “statistical” solution which depends both on the pulsation boundaries and on the distribution of pulsators within the instability strip. This explains why distance determinations based on PL relations require statistically significant numbers of Cepheids in order to reduce the effects of deviations from the ridge line.

The PL relation in bolometric magnitude is traditionally assumed to be metal-insensitive (see, e.g., Iben & Renzini 1984, Freedman & Madore 1990) and Cepheid distances are generally derived by adopting *universal* PL relations at different wavelengths, having the slope provided by the Cepheids in the LMC and the zero-point referenced to the LMC distance, as obtained with independent methods (see Freedman 1988; Kennicutt et al. 1998; Walker 1999, and reference therein), or to calibrating Galactic Cepheids (see Feast & Catchpole 1997; Lanoix et al. 1999).

In the last years, we have deeply investigated the Cepheid pulsational behavior through the computations of nonlinear, nonlocal and time-dependent convective pulsating models

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which take into account the coupling between pulsation and convection. With respect to linear-nonadiabatic models computed by different authors (e.g., Chiosi et al. 1993; Saio & Gautschi 1998; Alibert et al. 1999), our theoretical approach allows reliable predictions on the temperature of both the blue and red edges of the instability strip, as well as about the amplitude and morphology of the light-curve (see Bono et al. 1999a [Paper I], 2000a [Paper III], 2000b [Paper VI]). Concerning the assumptions on the input physics, computing procedures and the adopted mass-luminosity relation, the reader is referred to Paper I and Paper III, which contain also the detailed comparison of our models with the linear results in the literature. Here we only remark that from pulsating models computed with three different chemical abundances ($Z=0.004$, 0.008 and 0.02) we derived that the *bolometric* magnitude of metal-rich variables is, on average, fainter than that of metal-poor stars with the same period (Bono et al. 1999b [Paper II]). Moreover, we found that both the slope and zero-point of synthetic PL relations at different wavelengths depend on the pulsator metallicity, with the amplitude of the metallicity effect decreasing from visual to near-infrared magnitudes (see Paper II; Caputo et al. 2000 [Paper V]). Also the predicted PLC relations at the different wavelengths turned out to be, in various degrees, metallicity dependent: as an example, for a given period and $B - V$ color, metal-rich pulsators are brighter than metal-poor ones, whereas they are fainter if the $V - K$ color is adopted (see Paper V). On these grounds, it has been shown that Cepheid observations in three filters (BVI or BVK) allow to simultaneously constrain distance, reddening and metallicity of variables in the Magellanic Clouds (Caputo et al. 1999 [Paper IV]) and in the Milky Way (Caputo et al., in preparation).

In this paper we will apply our theoretical scenario to the Cepheids observed within two HST surveys: the ‘‘Extragalactic Distance Scale Key Project’’ (hereafter KP, see Freedman et al. 1994a) and the ‘‘Type Ia Supernova Calibration’’ (hereafter SNP, see Saha et al. 1994). The adopted procedure is briefly presented in Sect. 2 and the predicted reddenings and distances are given in Sect. 3. The metallicity effects on the Cepheid distance scale and on the value of the Hubble constant H_0 are discussed in Sect. 4. The concluding remarks close the paper.

2. The procedure

The results presented in Paper V predict that the intrinsic dispersion of the synthetic PL relations with fixed metallicity decreases with increasing the filter wavelength, in close agreement with the observed trend (see Madore & Freedman 1991; Tanvir 1999). Moreover, we found that the predicted PLC relations are almost independent of the pulsator distribution within the instability strip and reproduce the tight correlation between pulsation period and photometric parameters of individual pulsators, as earlier suggested by Sandage (1958), Sandage & Gratton (1963) and Sandage & Tammann (1968).

Since HST observations are in the two passbands V and I , we adopt the predicted $PL(I)$ and $PLC(VI)$ relations given in Paper V and reported here in Table 1 and Table 2, respec-

Table 1. Coefficients of theoretical PL relations in the I band for the labeled metal abundances.

Z	a	b	σ
$\overline{M_I} = a + b \log P$			
0.004	-2.00 ± 0.01	-3.00 ± 0.01	0.14
0.008	-2.03 ± 0.01	-2.88 ± 0.01	0.14
0.02	-2.18 ± 0.01	-2.53 ± 0.01	0.12

Table 2. Coefficients of theoretical PLC relation in the VI bands for the labeled metal abundances.

Z	α	β	γ	σ
$\langle M_V \rangle = \alpha + \beta \log P + \gamma [\langle V \rangle - \langle I \rangle]$				
0.004	-3.55 ± 0.03	-3.58 ± 0.03	3.75 ± 0.07	0.03
0.008	-3.54 ± 0.03	-3.59 ± 0.02	3.74 ± 0.06	0.03
0.02	-3.61 ± 0.03	-3.59 ± 0.04	3.85 ± 0.09	0.03

tively, to get the average reddening and true distance modulus of the Cepheid samples from the observed intensity-weighted mean magnitudes $\langle V \rangle$ and $\langle I \rangle$, for each assumption about the Cepheid metallicity. With respect to the classical method based on $PL(V)$ and $PL(I)$, our procedure aims at reducing the spurious effects caused by both the large dispersion of the observed $PL(V)$ relation and the uncertainty on the predicted $PL(V)$ relation as due to different pulsator distributions (see Paper II and Paper V).

The offset of each Cepheid from the predicted $PL(I)$ and $PLC(VI)$ relations gives the apparent distance moduli $\mu_1 = \mu_0 + A_I$ and $\mu_2 = \mu_0 + A_V - \gamma E(V - I)$, respectively. Adopting $A_V/E(B - V) = 3.1$ and $A_I/E(B - V) = 1.88$ from Cardelli et al. (1989), the reddening $E(B - V)$ and true distance modulus μ_0 of each Cepheid are derived. Eventually, by averaging over the sample of variables, we provide the predicted mean values, for the three adopted metallicities.

3. Predicted Cepheid distances to HST galaxies

Table 3 lists the KP galaxies together with the color excess $E(B - V)$ and true distance modulus μ_0 , as given by the authors (see references in the last column), and the oxygen-to-hydrogen abundance ratios (O/H) of HII regions, as listed in Ferrarese et al. (1999). For some of these galaxies the Cepheid data have been re-analyzed by the KP team and the new reddening and distance values (Ferrarese et al. 1999) are given in italics in the same table. The galaxies marked with an asterisk, originally studied by the SNP group, were extracted from the HST archive and re-processed by the KP team (Gibson et al. 2000). For them we list in Table 3 the KP values, while the SNP original distances are reported in Table 4.

By applying the theoretical relations in Table 1 and Table 2 to the full samples of Cepheids provided by KP and SNP studies (references in Table 3 and Table 4), we derive the predicted reddenings and true distance moduli listed in Table 5 and Table 6, respectively, for each of the three selected metal abundances. To

Table 3. Empirical reddening and distance from KP studies. The values in italic deal with the revision of KP galaxies, as given in Ferrarese et al. (1999). The asterisk marks the SNP galaxies re-processed by the KP team.

Galaxy	$E(B - V)$	μ_0	$12+\log(\text{O}/\text{H})$	Reference
NGC 1023 Group				
NGC 925	0.13 ± 0.02	29.84 ± 0.16	8.55 ± 0.15	Silbermann et al. 1996
Fornax Cluster				
NGC 1326A	0.05 ± 0.09	31.27 ± 0.21	8.50 ± 0.15	Prosser et al. 1999
	<i>0.00 ± 0.01</i>	<i>31.43 ± 0.17</i>		Ferrarese et al. 1999
NGC 1365	0.12 ± 0.06	31.31 ± 0.20	8.96 ± 0.20	Silbermann et al. 1999
	<i>0.15 ± 0.03</i>	<i>31.39 ± 0.19</i>		Ferrarese et al. 1999
NGC 1425	0.07 ± 0.03	31.73 ± 0.23	9.00 ± 0.15	Mould et al. 2000
	<i>0.07 ± 0.03</i>	<i>31.81 ± 0.17</i>		Ferrarese et al. 1999
M81 Group				
NGC 3031	0.03 ± 0.05	27.80 ± 0.20	8.75 ± 0.15	Freedman et al. 1994a
NGC 3184 Group				
NGC 3198	0.05 ± 0.03	30.80 ± 0.23	8.60 ± 0.15	Kelson et al. 1999
NGC 3319	0.06 ± 0.08	30.78 ± 0.17	8.38 ± 0.15	Sakai et al. 1999
Leo I Group				
NGC 3351	$0.12 \pm$	30.01 ± 0.19	9.24 ± 0.20	Graham et al. 1997
NGC 3368	$0.11 \pm$	30.20 ± 0.19	9.20 ± 0.20	Gibson et al. 2000
NGC 3627*	0.11 ± 0.04	30.06 ± 0.23	9.25 ± 0.15	Gibson et al. 2000
Coma I Cloud				
NGC 4414	0.01 ± 0.05	31.41 ± 0.19	9.20 ± 0.15	Turner et al. 1998
Coma II Cloud				
NGC 4725	0.19 ± 0.03	30.50 ± 0.16	8.92 ± 0.15	Gibson et al. 1999
	<i>0.19 ± 0.03</i>	<i>30.57 ± 0.18</i>		Ferrarese et al. 1999
M87 sub-cluster in the Virgo cluster				
NGC 4321	0.10 ± 0.06	31.04 ± 0.17	9.13 ± 0.20	Freedman et al. 1994b
NGC 4548	0.09 ± 0.03	31.01 ± 0.28	9.34 ± 0.15	Graham et al. 1999
NGC 4472 sub-cluster in the Virgo cluster				
NGC 4496A*	0.03 ± 0.05	31.02 ± 0.17	8.77 ± 0.15	Gibson et al. 2000
NGC 4535	0.08 ± 0.02	31.02 ± 0.26	9.20 ± 0.15	Macri et al. 1999
	<i>0.10 ± 0.03</i>	<i>31.10 ± 0.17</i>		Ferrarese et al. 1999
NGC 4536*	0.08 ± 0.03	30.95 ± 0.17	8.85 ± 0.15	Gibson et al. 2000
NGC 4649 sub-cluster in the Virgo cluster				
NGC 4639*	0.04 ± 0.03	31.80 ± 0.18	9.00 ± 0.15	Gibson et al. 2000
NGC 5128 Group				
NGC 5253*	$0.10 \pm$	27.61 ± 0.19	8.15 ± 0.15	Gibson et al. 2000
M101 Group				
NGC 5457	0.00 ± 0.04	29.34 ± 0.17	8.50 ± 0.15	Kelson et al. 1996
NGC 7331 Group				
NGC 7331	0.15 ± 0.05	30.89 ± 0.14	8.67 ± 0.15	Hughes et al. 1998
Other				
IC 4182*	-0.02 ± 0.03	28.36 ± 0.18	8.40 ± 0.20	Gibson et al. 2000
NGC 2090	0.07 ± 0.02	30.45 ± 0.16	8.80 ± 0.15	Phelps et al. 1998
NGC 2541	0.08 ± 0.05	30.47 ± 0.11	8.50 ± 0.15	Ferrarese et al. 1998
NGC 3621	0.23 ± 0.03	29.13 ± 0.18	8.75 ± 0.15	Rawson et al. 1997

avoid any bias, the predicted true distance modulus includes the correction for negative reddening, if present. The total errors to the predicted values are due to the observed scatter in the PL and PLC fits added in quadrature to the intrinsic dispersion of the theoretical relations.

Considering that both KP and SNP distances adopt $PL(V)$ and $PL(I)$ relations based on the Cepheids in the LMC, and since the average metallicity of LMC Cepheids is $\langle Z \rangle \sim 0.008$ (Luck et al. 1998), let us first compare the predicted reddening and distance with $Z=0.008$ to the LMC-based empirical values. It seems worth mentioning that we have already found a fair

Table 4. Empirical Cepheid distance to SNP galaxies, as published in the original papers by the SNP team.

Galaxy	μ_0	Reference
NGC 3627	30.22 ± 0.12	Saha et al. 1999
NGC 4496A	31.03 ± 0.14	Saha et al. 1996b
NGC 4536	31.10 ± 0.13	Saha et al. 1996a
NGC 4639	32.03 ± 0.22	Saha et al. 1997
NGC 5253	28.08 ± 0.10	Saha et al. 1995
IC 4182	28.36 ± 0.09	Saha et al. 1994

agreement between the empirical PL(V) and PL(I) relations and our predictions with $Z=0.008$ (see Paper V).

As for the galaxies in Table 3, we obtain that empirical and predicted results with $Z=0.008$ (Table 5) are in mutual agreement, with a difference (empirical minus predicted) of $\Delta\mu_0 \sim -0.05$ mag and $\Delta E(B - V) \sim +0.05$ mag, in the mean. We show in Fig. 1 that the discrepancy is for each galaxy well within a total uncertainty on the distance of the order of 10% (dashed lines). This is a reassuring result as to the reliability of the adopted method of analysis and our pulsating models with $Z=0.008$.

Moreover, one has to consider that both KP and SNP studies assume $\mu_{0,LMC}=18.50$ mag and $E(B - V)_{LMC}=0.10$ mag, and that the above differences of $\Delta\mu_0 \sim -0.05$ mag and $\Delta E(B - V) \sim +0.05$ mag with respect to the predictions with $Z=0.008$ could be easily removed by slightly increasing (decreasing) the adopted LMC distance (reddening). On the other hand, the theoretical results depend on assumptions such as the adopted mass-luminosity ratio, bolometric corrections and temperature-color relations. Thus, revisions of input physics and/or atmosphere models might modify the results in Table 5 and Table 6, but with marginal effects on the relative distances and reddenings. As a fact, by applying our method to the LMC Cepheids (Madore & Freedman 1991) we get $\mu_{0,LMC} \sim 18.6$ mag and $E(B - V)_{LMC} \sim 0.03$ mag. Accounting for these results, the actual discrepancy between empirical and predicted results is $\Delta\mu_0 \sim +0.05$ mag and $\Delta E(B - V) \sim -0.02$ mag, in the mean.

On the contrary, it seems important to notice that different distributions of Cepheids within the instability strip may produce non-systematic effects on both distance and reddening. As a matter of example, let us consider a Cepheid with absolute magnitudes M_V^* and M_I^* , true distance modulus μ_0^* and reddening $E^*(B - V)$, and let us assume that $M_j^* = \overline{M}_j - \epsilon_j$, where \overline{M}_j is given by the PL(j) relation. From the PL(V)+PL(I) procedure one derives $E(V - I) = E^*(V - I) - (\epsilon_V - \epsilon_I)$, while following our PL(I)+PLC(VI) method one has $E(V - I) = E^*(V - I) - 0.36\epsilon_I$. Since Cepheids close to the blue edge of the instability strip have positive values of ϵ_j , with $\epsilon_V > \epsilon_I$ as a consequence of the fact that the width of the instability strip decreases towards longer wavelengths (see Paper V), the result is that the actual reddening of Cepheids near the pulsation blue edge is underestimated, whereas the opposite occurs for variables close the red edge. Concerning the distance, from the

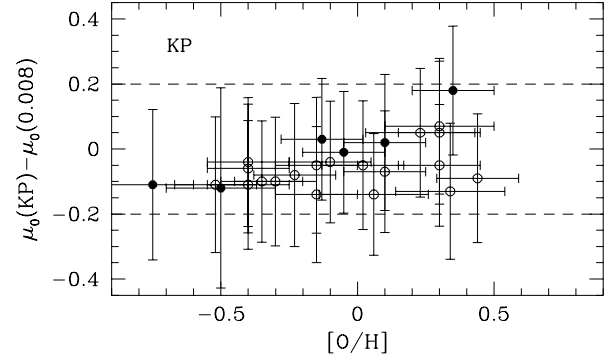


Fig. 1. The difference between the empirical Cepheid distance by KP studies and the predicted value with $Z = 0.008$, plotted against the O/H metallicity of the host galaxy. The Cepheid samples are original KP data (open circles) or refer to SNP galaxies extracted from the HST archive and re-processed by the KP team (filled circles). The two dashed lines depict a discrepancy on the distance of the order of $\pm 10\%$.

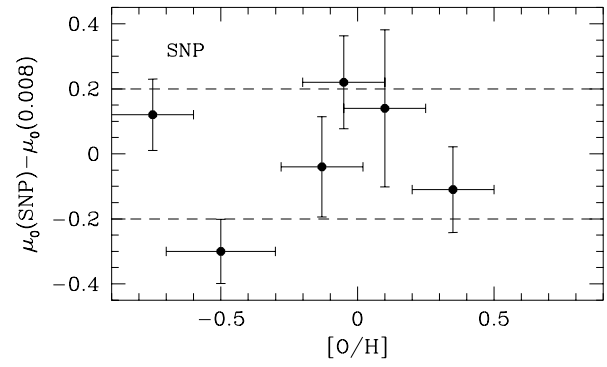


Fig. 2. As in Fig. 1, but with the Cepheid distances by the SNP group.

PL(V)+PL(I) procedure one has $\mu_0 = \mu_0^* + 1.54\epsilon_V - 2.54\epsilon_I$, while the PL(I)+PLC(VI) method yields $\mu_0 = \mu_0^* - 0.45\epsilon_I$. With $\epsilon_V \sim 0.35$ mag and $\epsilon_I \sim 0.25$ mag for Cepheids close to the pulsation blue edge¹, the derived reddening and true distance modulus are ~ 0.10 mag lower than the actual values, whatever is the adopted method of analysis. On these grounds, one understands that statistically significant samples of variables are required in order to get accurate distance and reddening determinations. Otherwise, selection effects on distant Cepheids (the variables close to the blue edge are brighter than those near the red edge, for fixed period) lead to underestimate both the distance and reddening.

As already mentioned, for some of the KP galaxies in Table 3 the Cepheid data have been reviewed by the same team, leading to a slight systematic increase on the distance. From the data listed in italics in Table 3 one finds that the average upward revision is of the order of ~ 0.09 mag. As discussed in Ferrarese et al. (1999), this is a consequence of the adopted lower cut-off in period, as the empirical distance modulus increases when the lower period cut-off is moved from 10 to 20–25 days. On the theoretical side, we show in Table 5 (see values in italics) that using the same selection as in Ferrarese et al. (1999) causes a

¹ Average values with $\log P \sim 1.0$ – 1.5 .

Table 5. Predicted Cepheid reddening and distance of KP galaxies for the labelled metal contents. The distance moduli based on the selected data used by Ferrarese et al. (1999) in their revision studies are labeled in italics. For the galaxies marked with the asterisk, originally observed by the SNP team, the photometric data provided by the KP group are adopted.

Galaxy	Z=0.004		Z=0.008		Z=0.02	
	$E(B - V)$	μ_0	$E(B - V)$	μ_0	$E(B - V)$	μ_0
IC 4182*	-0.06 ± 0.05	28.51 ± 0.18	-0.09 ± 0.05	28.48 ± 0.18	-0.15 ± 0.04	28.37 ± 0.15
NGC 925	0.10 ± 0.05	29.98 ± 0.17	0.06 ± 0.05	29.94 ± 0.17	-0.02 ± 0.04	29.80 ± 0.15
NGC 1326A	0.02 ± 0.05	31.42 ± 0.18	-0.03 ± 0.05	31.38 ± 0.18	-0.11 ± 0.04	31.22 ± 0.16
		<i>31.58 ± 0.18</i>		<i>31.53 ± 0.18</i>		<i>31.35 ± 0.16</i>
NGC 1365	0.07 ± 0.05	31.49 ± 0.17	0.02 ± 0.05	31.45 ± 0.17	-0.07 ± 0.04	31.27 ± 0.15
		<i>31.50 ± 0.18</i>		<i>31.45 ± 0.18</i>		<i>31.27 ± 0.16</i>
NGC 1425	0.04 ± 0.05	31.85 ± 0.18	0.00 ± 0.05	31.80 ± 0.17	-0.10 ± 0.04	31.63 ± 0.15
		<i>31.88 ± 0.18</i>		<i>31.83 ± 0.18</i>		<i>31.64 ± 0.15</i>
NGC 2090	0.06 ± 0.05	30.53 ± 0.17	0.01 ± 0.05	30.49 ± 0.17	-0.07 ± 0.04	30.34 ± 0.15
NGC 2541	0.07 ± 0.05	30.55 ± 0.18	0.02 ± 0.05	30.51 ± 0.18	-0.07 ± 0.04	30.35 ± 0.16
NGC 3198	0.04 ± 0.05	30.95 ± 0.18	-0.01 ± 0.05	30.90 ± 0.18	-0.10 ± 0.05	30.75 ± 0.16
NGC 3031	0.07 ± 0.05	27.89 ± 0.19	0.03 ± 0.05	27.85 ± 0.19	-0.05 ± 0.05	27.70 ± 0.16
NGC 3319	0.02 ± 0.05	30.94 ± 0.19	-0.02 ± 0.05	30.89 ± 0.19	-0.11 ± 0.05	30.74 ± 0.17
NGC 3351	0.09 ± 0.06	30.18 ± 0.19	0.05 ± 0.06	30.14 ± 0.19	-0.02 ± 0.05	30.00 ± 0.17
NGC 3368	0.09 ± 0.05	30.17 ± 0.20	0.04 ± 0.05	30.13 ± 0.19	-0.04 ± 0.05	29.97 ± 0.17
NGC 3621	0.20 ± 0.05	29.31 ± 0.18	0.15 ± 0.05	29.27 ± 0.18	0.07 ± 0.04	29.12 ± 0.16
NGC 3627*	0.12 ± 0.05	29.93 ± 0.18	0.08 ± 0.05	29.88 ± 0.18	-0.01 ± 0.04	29.72 ± 0.16
NGC 4321	0.07 ± 0.05	31.04 ± 0.18	0.02 ± 0.05	30.99 ± 0.18	-0.07 ± 0.04	30.82 ± 0.16
NGC 4414	0.04 ± 0.07	31.42 ± 0.21	-0.01 ± 0.07	31.36 ± 0.21	-0.11 ± 0.06	31.18 ± 0.19
NGC 4496A*	0.04 ± 0.04	31.04 ± 0.17	-0.01 ± 0.04	30.99 ± 0.17	-0.10 ± 0.04	30.83 ± 0.15
NGC 4535	0.06 ± 0.05	31.12 ± 0.17	0.02 ± 0.05	31.07 ± 0.17	-0.08 ± 0.04	30.90 ± 0.15
		<i>31.16 ± 0.18</i>		<i>31.11 ± 0.18</i>		<i>30.93 ± 0.16</i>
NGC 4536*	0.06 ± 0.05	31.01 ± 0.17	0.02 ± 0.05	30.96 ± 0.17	-0.07 ± 0.04	30.79 ± 0.15
NGC 4548	0.07 ± 0.05	31.15 ± 0.18	0.02 ± 0.05	31.10 ± 0.18	-0.06 ± 0.05	30.95 ± 0.16
NGC 4639*	0.05 ± 0.05	31.83 ± 0.19	-0.01 ± 0.05	31.78 ± 0.19	-0.11 ± 0.05	31.59 ± 0.16
NGC 4725	0.15 ± 0.05	30.60 ± 0.18	0.10 ± 0.05	30.55 ± 0.18	0.01 ± 0.04	30.39 ± 0.16
		<i>30.65 ± 0.18</i>		<i>30.60 ± 0.18</i>		<i>30.42 ± 0.16</i>
NGC 5253*	0.08 ± 0.06	27.75 ± 0.21	0.05 ± 0.06	27.72 ± 0.21	0.00 ± 0.06	27.62 ± 0.21
NGC 5457	0.01 ± 0.05	29.45 ± 0.18	-0.04 ± 0.05	29.40 ± 0.18	-0.13 ± 0.05	29.23 ± 0.16
NGC 7331	0.13 ± 0.06	31.01 ± 0.20	0.09 ± 0.06	30.97 ± 0.20	-0.00 ± 0.06	30.81 ± 0.18

systematic increase of the predicted true distance modulus by only ~ 0.05 mag. This suggests that the cut-off in period has minor influence upon our $PL(I)+PLC(VI)$ procedure, supporting the belief that robust distance determinations should adopt only PLC relations, for which Cepheid observations in at least three filters are needed.

Concerning the SNP galaxies, we give in Table 6 the predicted distances as obtained from the full set of Cepheid data provided in the original papers, while the values in italics refer to the Cepheid samples as selected by the SNP team. One notices that the adopted selection criteria yield rather discordant effects on the predicted distance. With $Z=0.008$, the true distance modulus of four galaxies increases (from 0.05 mag for NGC 4496A to 0.44 mag for NGC 3627), whereas it decreases by -0.11 mag for IC 4182. Moreover, limiting the comparison to the SNP empirical distances obtained from the selected samples (Table 4), the difference with respect to the predicted distance modulus with $Z=0.008$ (in italics in Table 7) ranges from -0.30 mag (IC 4182) to $+0.22$ mag (NGC 4536), which is somehow larger than that (from -0.12 mag for IC 4182 to

$+0.18$ mag for NGC 3627) found by adopting KP results for the same galaxies (see Table 3 and Table 5).

It is known that the disagreement between SNP and KP distances to the same galaxy (up to 0.47 mag for NGC 5253) deals with different pipeline processing procedures, selection criteria and data reduction strategies, and the interested reader can find a detailed discussion in Gibson et al. (2000) and Tammann et al. (2000). Here we have to face up to the evidence that by using the Cepheid data selected by SNP studies the overall discrepancy between empirical and predicted distances with $Z=0.008$ is somehow larger (see Fig. 2) than that found with KP data (filled circles in Fig. 1). For this reason, the rest of our analysis will deal exclusively with the results of KP studies.

Throughout the above discussion we used the Cepheid predicted distance with $Z=0.008$, neglecting the evidence that the oxygen abundance of the galaxies in Table 3 varies from $[O/H] \sim -0.50$ to $\sim +0.40^2$. Since for the LMC $[O/H] \sim -0.40$ (Pagel et al. 1978), one has that the HII metallicity of the Cepheid

² $[O/H] = \log(O/H) - \log(O/H)_{\odot}$, with $\log(O/H)_{\odot} = -3.10$.

Table 6. Predicted Cepheid distance to SNP galaxies for the labelled metal contents. The full sets of SNP photometric data are adopted. The results obtained from the selected data used by the SNP team are labeled in italics.

Galaxy	$Z = 0.004$	$Z = 0.008$	$Z = 0.02$
	μ_0	μ_0	μ_0
NGC 3627	29.94 ± 0.19	29.89 ± 0.19	29.73 ± 0.17
	<i>30.37 ± 0.18</i>	<i>30.33 ± 0.18</i>	<i>30.16 ± 0.16</i>
NGC 4496A	31.07 ± 0.17	31.02 ± 0.17	30.86 ± 0.15
	<i>31.12 ± 0.17</i>	<i>31.07 ± 0.17</i>	<i>30.89 ± 0.15</i>
NGC 4536	30.75 ± 0.18	30.71 ± 0.18	30.55 ± 0.16
	<i>30.93 ± 0.20</i>	<i>30.88 ± 0.20</i>	<i>30.70 ± 0.18</i>
NGC 4639	31.85 ± 0.23	31.80 ± 0.23	31.63 ± 0.21
	<i>31.94 ± 0.21</i>	<i>31.89 ± 0.21</i>	<i>31.71 ± 0.19</i>
NGC 5253	27.81 ± 0.24	27.78 ± 0.25	27.70 ± 0.24
	<i>27.99 ± 0.22</i>	<i>27.96 ± 0.22</i>	<i>27.89 ± 0.21</i>
IC 4182	28.81 ± 0.19	28.77 ± 0.19	28.66 ± 0.17
	<i>28.70 ± 0.19</i>	<i>28.66 ± 0.19</i>	<i>28.53 ± 0.17</i>

fields observed with HST may be up to 0.8 dex higher than the “reference” sample of LMC variables.

On the other hand, the data in Table 5 show that the predicted Cepheid reddening and true distance modulus decrease as the adopted metallicity Z increases. This is a consequence of the fact that our metal-rich pulsating models are, on average, fainter than metal-poor ones. Resuming the discussion in Sect. 3, let us assume that for our Cepheid holds $\epsilon_V = \epsilon_I = 0$ with respect to the PL relations with $Z=0.008$. This yields $E(V-I)_{0.008} = E^*(V-I)$ and $\mu_{0,0.008} = \mu_0^*$. However, the variable is brighter with respect to the PL relations with $Z=0.02$, by $\epsilon_V \sim 0.25$ mag and $\epsilon_I \sim 0.20$ at $\log P \sim 1.0$ (see Paper V). As a consequence, $E(V-I)_{0.02} \sim E^*(V-I) - 0.06$ mag and $\mu_{0,0.02} \sim \mu_0^* - 0.11$ mag. The decrease of the reddening with increasing the adopted metallicity yields that almost all the galaxies show unpleasant negative values of $E(B-V)$ with $Z=0.02$. However, as already mentioned, our theoretical scenario suggests $E(B-V) \sim 0.03$ mag for the Cepheids in the LMC, while both the KP and SNP studies assume $E(B-V)_{LMC} = 0.10$ mag. Thus, were our predictions calibrated on this galaxy, then all the reddening values in Table 5 and Table 6 should be increased by 0.07 mag.

Since our purpose is the evaluation of the metallicity effects on the distance scale, in the mean we derive $\delta\mu_0/\delta\log Z = -0.27 \pm 0.04$ mag dex $^{-1}$, at least within the explored range $0.004 \leq Z \leq 0.02$. The sense of this result is that when universal relations are used, the Cepheids more metal-rich than the calibrators will appear spuriously *brighter*. On this basis, the predicted correction (in magnitude) to empirical LMC-calibrated distance moduli is given by

$$c = -0.27\Delta\log Z, \quad (3)$$

where $\Delta\log Z$ is the difference between the Cepheid metallicity and the LMC value of $Z=0.008$.

In the following section we investigate into the effects of this metallicity correction to KP distances by assuming that the

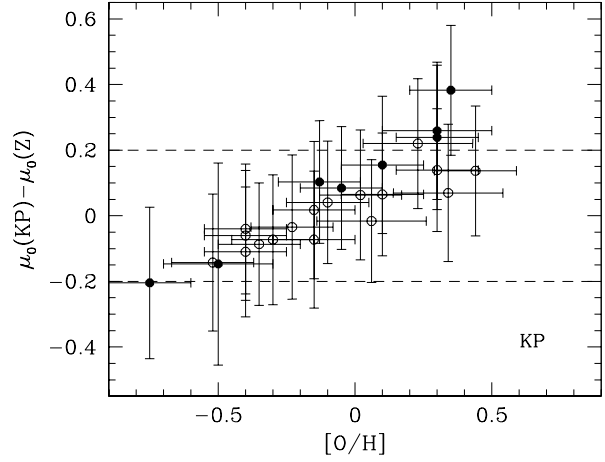


Fig. 3. The difference between KP empirical distances and predicted metallicity-corrected values, plotted against the HII metallicity of the host galaxy. The filled circles depict the eight galaxies which give Cepheid-calibrated SNIa luminosities.

Cepheid metal content scales with the HII region metallicity, i.e. adopting $\Delta\log Z = \Delta[O/H]$, the difference between the galaxy $[O/H]$ abundance and that of the LMC ($[O/H] = -0.40$). Observational pros and cons for this assumption will be discussed.

4. Metallicity effects on Cepheid distances and H_0

Fig. 3 shows the difference between the empirical KP data³ and the predicted true distance modulus when the oxygen abundance of the Cepheid fields is taken into account. As expected, the almost flat distribution in Fig. 1 now shows a clear correlation to the HII region metallicity, with the discrepancy between LMC-calibrated and theoretical distances increasing when moving from metal-poor to metal-rich galaxies. However, one notices that for very few galaxies the discrepancy overcomes the threshold of 10% (dashed line). Of particular interest is the behaviour of the eight galaxies (the six SNP galaxies plus NGC 3368 and NGC 4414; filled circles) which give Cepheid-calibrated SNIa luminosities (see later).

As a first straightforward test to the actual occurrence of a metallicity effect, we consider the KP distance to galaxies members of groups or clusters. The lower panel of Fig. 4 shows the residuals $\Delta_i\mu_0$ and $\Delta_i[O/H]$ of each galaxy from the distance modulus and O/H metallicity as averaged over the galaxies of the same group or cluster. A correlation between the metallicity and distance deviations from the mean values can be detected, best-fitted by the relation (dotted line)

$$\Delta_i\mu_0 = 0.28\Delta_i[O/H], \quad (4)$$

in the sense that galaxies whose O/H metallicity is larger than the average appear to have a *larger* distance. We believe that depth-effects within a given group or cluster cannot be invoked since there is no reason for which the metal-richest galaxies are also the most distant ones.

³ For sake of homogeneity, we adopt the Cepheid distances listed in Ferrarese et al. (1999).

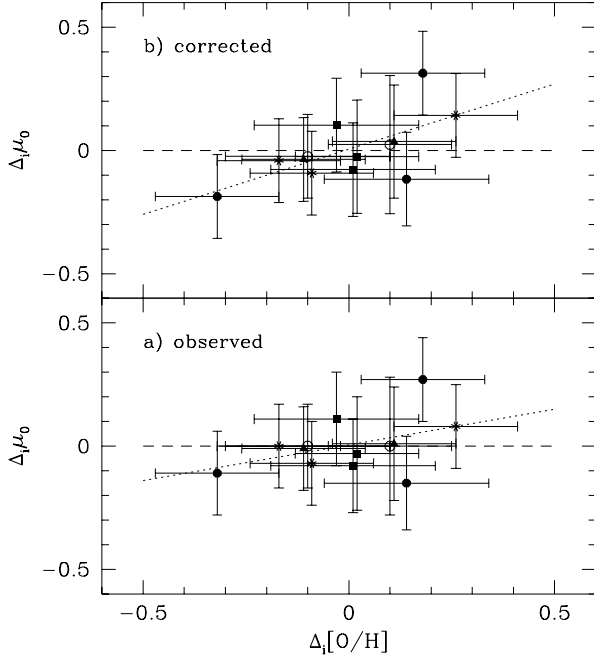


Fig. 4a and b. **a** Residuals of each galaxy from the average true distance modulus and O/H metallicity of the host cluster or group. Symbols as follows: Fornax cluster: filled circles, NGC 3184 group: triangles, Leo I group: squares, M87 sub-cluster: open circles, NGC 4472 sub-cluster: asterisks. The dotted line is the best fit to the points [Eq. (4)]. **b** As below, but with the Kennicutt et al. (1998) metallicity correction to the absolute distance moduli [Eq. (5)].

This unexpected result, which appears surprisingly in agreement with our predicted correction [Eq. (3)], disagrees with earlier observational clues. It is known that studies of different fields in M31 (Freedman & Madore 1990) and M101 (Kennicutt et al. 1998) suggested an opposite metallicity correction on distance (see also Kochanek 1997; Sasselov et al. 1997). Following Kennicutt et al. (1998), the correction (in magnitude) to the true distance modulus is given by

$$c = +0.24\Delta[O/H] \quad (5)$$

We show in the upper panel of the same Fig. 4 that adopting such an empirical correction would imply an even stronger correlation between distance and metallicity, with a slope of +0.53 (dotted line) hard to accept.

Applying the predicted metallicity correction to the absolute distance moduli [Eq. (3)], obviously results in a correction to the H_0 -values based on LMC-calibrated distances. Assuming $\Delta h_0 = \Delta H_0/H_0$, one has $\Delta h_0/\Delta[O/H] \sim +0.124 \text{ dex}^{-1}$, i.e. an increase of $\sim 6\%$ in the LMC-based H_0 value of any SNIa calibrator whose metallicity is 0.5 dex larger than that of the LMC. This is a not dramatic variation, nevertheless it seems interesting to settle whether it works to decrease or increase the present dispersion of H_0 values. Within this context, it is worth noticing that the well known disagreement between the “high” and “low” H_0 values claimed by KP and SNP studies, respectively, are not due entirely to the already mentioned different distances to SNIa calibrating galaxies. As discussed by Gibson et al. (2000),

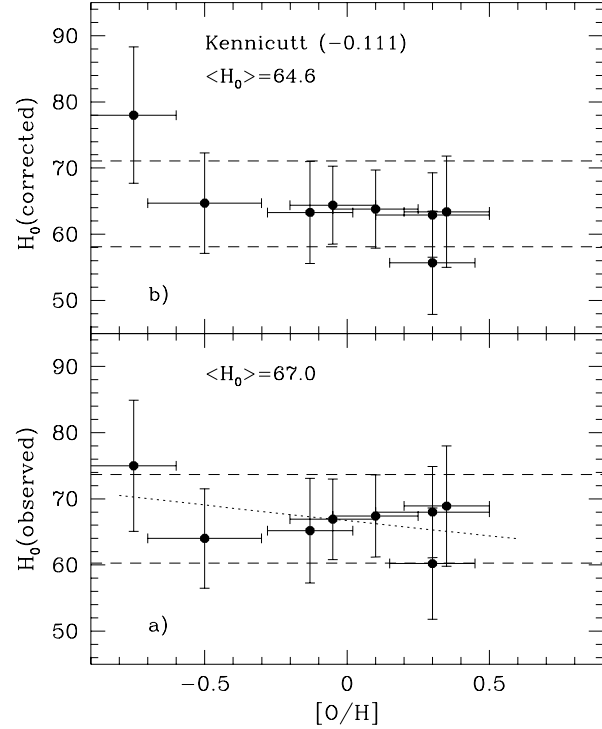


Fig. 5. a The KP $H_0(V)$ values from LMC-based Cepheid distance to the eight SNIa calibrating galaxies. The average value is labelled. The dashed lines depict the uncertainty of 10%. The dotted line depicts Eq. (6) in the text; **b** As below, but with the Kennicutt et al. (1998) metallicity correction to the absolute distance moduli. The resulting correction to H_0 is $\Delta h_0/\Delta[O/H] \sim -0.111 \text{ dex}^{-1}$ (see text).

the methodology adopted by the SNP group with KP distances leads to $H_0 \sim 63 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ dex}^{-1}$, which is 9% higher than the SNP average value ($H_0 \sim 58 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ dex}^{-1}$). Moreover, the calibration of SNIa luminosities presents a variety of different approaches and the same KP distances, coupled with the Suntzeff et al. (1999) procedure, would give $H_0 \sim 67 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ dex}^{-1}$. However, the absolute value of the Hubble constant is out the purpose of this paper. We aim only at determining the metallicity-correction to LMC-based H_0 values, with the hope of providing new elements for reducing the present uncertainty of $\sim 10\%$ to a 5% level.

Taking at the face value the $H_0(V)$ estimates given by Gibson et al. (2000, see their Table 6) we show in the lower panel of Fig. 5 that they agree to each other to within 10% (dashed lines), but with a mild tendency to increase as the oxygen abundance of the host galaxy decreases. The linear regression to the points (dotted line) is

$$\log H_0 = 1.82 - 0.03[O/H], \quad (6)$$

suggesting a metallicity correction as $\Delta h_0/\Delta[O/H] \sim +0.069 \text{ dex}^{-1}$ which is roughly half the amount predicted on the basis of Eq. (3), but it runs towards the same direction. On the contrary, applying the Kennicutt et al. (1998) correction to the true distance moduli [Eq. (5)], results in a correction as $\Delta h_0/\Delta[O/H] \sim -0.111 \text{ dex}^{-1}$ to the LMC-based H_0 values. The upper panel of Fig. 5 shows that this would pro-

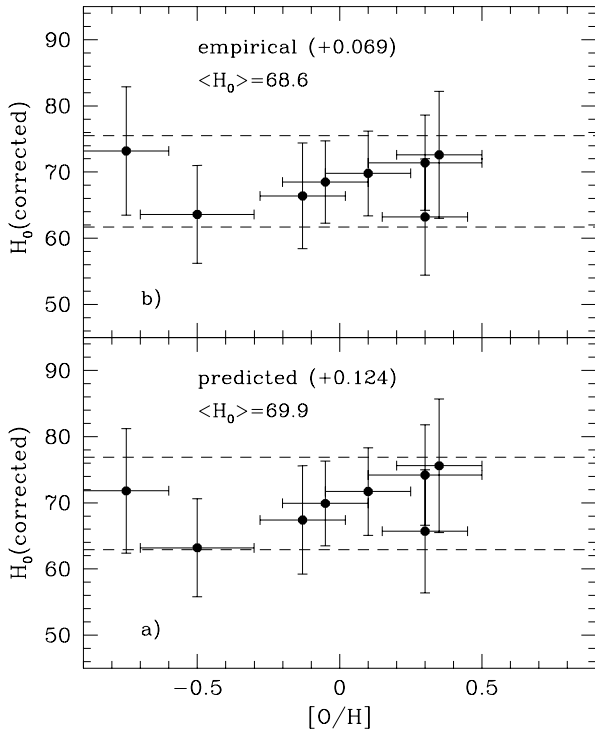


Fig. 6. **a** As in Fig. 5b, but with the predicted metallicity correction $\Delta h_0/\Delta[\text{O}/\text{H}] \sim +0.124 \text{ dex}^{-1}$; **b** As below, but with the metallicity correction $\Delta h_0/\Delta[\text{O}/\text{H}] \sim +0.069 \text{ dex}^{-1}$, as suggested by the dotted line in Fig. 5a.

duce a more evident correlation between H_0 and $[\text{O}/\text{H}]$, with two estimates overcoming the discrepancy of 10% from the average value (dashed lines). On the other hand, the lower panel of Fig. 6 shows that our predicted metallicity correction $\Delta h_0/\Delta[\text{O}/\text{H}] \sim 0.124 \text{ dex}^{-1}$ would yield H_0 values which agree to within 10%, but slightly increasing as the $[\text{O}/\text{H}]$ abundance increases. Eventually, in order to remove any dependence of H_0 on the galaxy metallicity (see upper panel in Fig. 6), we suggest that *at least* the empirical evidence in Fig. 5a should be taken into consideration, i.e., $\Delta h_0/\Delta[\text{O}/\text{H}] \sim 0.069 \text{ dex}^{-1}$, leading to a slight upward revision of the KP unweighted mean of $\langle H_0 \rangle = 67.0 \pm 4.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to $\langle H_0 \rangle = 68.6 \pm 3.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

As said before, the determination of H_0 deals with several factors apart from the distance to the SNIa calibrating galaxies. Thus, we are *not* giving the predicted value of the Hubble constant, but only the result of the predicted metallicity-correction to the KP H_0 estimates, still holding their adopted approach in the treatment of SNIa data and distance to LMC.

5. Concluding remarks

HST galaxies with published Cepheid distances have been studied to the light of a theoretical pulsational scenario based on nonlinear, nonlocal and time-dependent convective models with three selected metallicities ($Z=0.004, 0.008, 0.02$).

Theoretical PL and PLC relations in the VI passbands have been used to predict Cepheid distance and reddening for each

adopted metal content. The comparison of the predictions with $Z=0.008$ (the average metallicity of the LMC) to the LMC-calibrated empirical results provided by the KP group shows a mutual agreement, better than that found with SNP results.

The predicted PL and PLC relations suggest that the Cepheid distance should decrease with increasing the adopted metallicity and that LMC-based distance moduli should be corrected according to $c = -0.27 \Delta \log Z \text{ mag dex}^{-1}$, where $\Delta \log Z$ is the difference between the Cepheid metallicity and the LMC mean value of $Z=0.008$. Theoretical studies which adopt a linear pulsational approach, being unable to get firm predictions on the red edge of the instability strip, cannot disprove this result.

On the observational front, earlier evidences suggest an opposite metallicity correction as given by $c=+0.24 \Delta[\text{O}/\text{H}] \text{ mag dex}^{-1}$ (Kennicutt et al. 1998), where $\Delta[\text{O}/\text{H}]$ is the O/H metallicity difference between the Cepheid field and the LMC mean value ($[\text{O}/\text{H}] \sim -0.40$). However, we show that the KP true distance modulus to galaxies members of clusters or groups seems to be correlated with the galaxy O/H metallicity in good agreement with our predictions.

Moreover, we wish to mention that some recent observational clues give a further support to our theoretical scenario. They are:

- The comparison between Baade Wesselink radii for Galactic and Magellanic Clouds Cepheids (Laney 1998, 1999, 2000; Feast 1999; Storm 2000) suggests that metal-rich variables are fainter than the metal-poor ones.
- The purely “geometric” distance to the galaxy NGC 4258, as obtained from maser sources rotating around the central black hole, is $7.2 \pm 0.5 \text{ Mpc}$ (Herrnstein et al. 1999), whereas the value derived from LMC-calibrated PL relations is $8.1 \pm 0.4 \text{ Mpc}$ (Maoz et al. 1999). Since for the observed field one has $[\text{O}/\text{H}] \sim 0.05$ (Zaritsky et al. 1994), the Kennicutt et al. (1998) correction would increase the distance to $\sim 8.4 \text{ Mpc}$, whereas our predicted correction gives $\sim 7.8 \text{ Mpc}$.
- A carefully selected sample of 236 Cepheids from the HIPPARCOS catalogue suggests that the slope of the Galactic PL relations could be shallower than that observed for LMC variables (Groenewegen & Oudmaijer 2000), thus suggesting some caution in the adoption of universal PL relations.

As a whole, both the predictions and the empirical evidences seem to confirm that the metallicity effect on the Cepheid distance scale is not dramatic, with a correction factor of $\sim -0.14 \text{ mag}$ to the LMC-based distance modulus of any galaxy whose metallicity is 0.5 dex larger than that of the LMC.

The consequent predicted metallicity correction to the Hubble constant is $\Delta h_0/\Delta[\text{O}/\text{H}] \sim +0.124 \text{ dex}^{-1}$, which means an increase of $\sim 6.2\%$ in the LMC-based H_0 values of any SNIa calibrator whose metallicity is 0.5 dex larger than that of the LMC.

Taking the KP H_0 estimates at their face value, we show that the observed correlation with the $[\text{O}/\text{H}]$ metallicity of the Cepheid fields suggests $\Delta h_0/\Delta[\text{O}/\text{H}] \sim +0.069 \text{ dex}^{-1}$, which is more consistent with our prediction than the metallicity

correction based on the Kennicutt et al. (1998) empirical results ($\Delta h_0/\Delta[\text{O}/\text{H}] \sim -0.111 \text{ dex}^{-1}$.) On the basis of this, we suggest that the average value provided by the KP team ($\langle H_0 \rangle \sim 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$) should be increased *at least* up to ($\langle H_0 \rangle \sim 69 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

The adoption of a metallicity correction to LMC-based empirical estimates of H_0 might hopefully reduce the present uncertainty of 10% on the average value to a 5% level. Unfortunately, the variety of the current methods of calibrating SNIa luminosities give very different results for the absolute value of the Hubble constant. As an example (see Gibson et al. 2000), the Suntzeff et al. (1999) methodology yields a mean value which is from 3.7% to 8.3% larger than that found with the Saha et al. (1997) procedure, even adopting the same distance to the calibrators. In our belief this is a key problem on the route to settle the absolute value of the Hubble constant.

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