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Letter to the Editor

The distance modulus of the Small Magellanic Cloud based on double-mode Cepheids

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Abstract. We employ the very recent photometric data of the OGLE project together with stellar atmosphere and linear pulsation models to determine the distance modulus of the Small Magellanic Cloud from its double-mode Cepheids. Based on the requirement of obtaining the same distance modulus DM from the two types of variables (fundamental & first overtone and first & second overtone), we get $DM = 19.05$ mag, with a very small statistical error (standard deviation) of 0.017 mag. Various systematic and zero point ambiguities (primarily those of the color–temperature transformation) lead to an error of ± 0.13 mag (estimated 3σ deviation). This result is in very good agreement with the distance modulus of the Large Magellanic Cloud of 18.5 mag, derived earlier from cluster double-mode RR Lyrae stars.

Key words: stars: fundamental parameters – stars: distances – stars: variables: Cepheids – stars: oscillations – galaxies: Magellanic Clouds

1. Introduction

In a former paper (Kovács & Walker 1999, hereafter KW99) we have shown that the distances derived from double-mode RR Lyrae (RRd) stars are systematically larger than the ones obtained from the standard Baade-Wesselink (BW) analyses of RR Lyrae stars. The latter distances are in similar conflict also with the BW results of Cepheids (e.g., Gieren et al. 1998). The reason of this discrepancy is still unknown (Cacciari et al. 2000). This contributes further to the ambiguity at the 0.2–0.3 mag level over the luminosity scale of the RR Lyrae stars, and consequently, over the distances to the nearest globular clusters and galaxies.

The purpose of this Letter is to study the applicability of double-mode variables in the distance calibration in more detail. The discovery of a large sample of fundamental & first overtone (FU/FO) and first & second overtone (FO/SO) Cepheids in the Small Magellanic Cloud (SMC) by the OGLE team (Udalski et al. 1999a, hereafter U99) enables us to derive the distance to the Cloud and thereby adding another piece of information to the dispute over the distance of the Magellanic Clouds.

2. Data, method, models and temperature scales

Double-mode (or beat) Cepheids in the SMC have been discovered previously by the major microlensing projects (MACHO, Alcock et al. 1997; EROS, Beaulieu et al. 1997). However, it is only the OGLE team who publishes the data in standard colors (i.e., in Johnson V and in Kron-Cousins I_c). Therefore, for the time being, we decided to employ their data only.

We use the periods, average $V - I_c$ colors and V magnitudes of the 23 FU/FO and 70 FO/SO Cepheids published by U99.[1] Although they presented also $B - V$ colors, we decided not to use them, because of the few data points in B. For reddening we accepted their values derived for the various fields from a method based on red clump stars (see U99 and the *ftp* site mentioned above).

In computing the distance modulus, we follow almost entirely the method of KW99. Here we repeat only the basic steps and assumptions.

For any double-mode variable, from the pulsation models we obtain relations between the physical parameters and the periods

$$P_i = f_i(M, L, T_{\text{eff}}, X, Z), \tag{1}$$

where $i = 0, 1$ or $1, 2$ for the FU/FO and FO/SO variables, respectively. The other parameters have their usual meaning. In principle, from the observed color we can determine T_{eff} and from other information we also have approximate values for the hydrogen and metal abundances. Therefore, we can invert the above relations to compute the luminosity (and mass). Next, the distance modulus DM is computed simply from the comparison of the observed V magnitude and the calculated absolute magnitude from L by using a bolometric correction ($B.C.$) formula and proper interstellar reddening. The existence of the two types of beat Cepheids in SMC enables us to estimate DM in two different ways. Because of the different dependence of the periods on the physical parameters in the two types of variables, this will allow us to calculate DM in an optimum way and put constraints on Z, one of the most important parameters entering

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[1] Actually, we use the slightly revised data set, see the OGLE *ftp* site at *sirius.astrouw.edu.pl*

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in this method. The other crucial parameter is the *zero point* of $T_{\rm eff}$, which cannot be constrained from the present data, but, in principle, it is also possible to do.

The main assumptions entering in our approach are the following:

– FU/FO and FO/SO variables have the same chemical composition.
– Linear nonadiabatic and purely radiative pulsation model periods are applicable to the observed periods.
– The SMC has small spatial extent relative to its distance.
– Current color–temperature calibrations are reliable enough.

As regards the technical details, it should be mentioned that we used the same standard, purely radiative linear pulsation code as in KW99. A large number of models were computed with $X = 0.76$ and $Z = 0.001, 0.002, 0.003, 0.004, 0.008$ and using OPAL'96 opacities (Iglesias & Rogers 1996). The distribution of the various elements in Z corresponds to that of the Sun. Unlike in the case of RRd models, we could not find a simple way to invert Eq. (1). Therefore, in finding the values of (M, L) which fit the observed periods the best for the given $T_{\rm eff}$, X and Z, we used a straightforward search in the interpolated fine grid of the original model sequences.

As far as the $(V - I_c) \rightarrow T_{\rm eff}$ and $B.C.$ calibrations are concerned, we used the stellar atmosphere models of Castelli et al. (1997) and obtained the following formulae through least squares fits in a parameter space relevant for beat Cepheids (i.e., $T_{\rm eff} = 5000 - 7000$ K, $\log g = 2.0 - 3.5$, [M/H]$= -1.5 - 0.0$)

$$\log T_{\rm eff} = 3.9224 - 0.2470(V - I_c) + 0.0046 \log g$$
$$+ 0.0012[{\rm M/H}], \tag{2}$$
$$B.C. = 0.0411 + 2.0727\Delta T - 0.0274 \log g$$
$$+ 0.0482[{\rm M/H}] - 8.0634\Delta T^2 , \tag{3}$$

where $\Delta T = \log T_{\rm eff} - 3.7720$, and we used the standard notation for the relative heavy element abundance [M/H]$= \log Z/Z_\odot$, with $Z_\odot = 0.02$.

The above $B.C.$ is adjusted to $M_{\rm bol}(\odot) = 4.75$ and yields $B.C.(\odot) = -0.09$. The zero point of Eq. (2) is only marginally higher than that of Blackwell & Lynas-Gray (1994, hereafter BLG94) (see also Clementini et al. 1995), which is based on the InfraRed Flux Method (IRFM). Other, more recent IRFM-based $T_{\rm eff}$ calibrations (Blackwell & Lynas-Gray 1998, hereafter BLG98; Alonso et al. 1999, hereafter A99) yield lower zero points ($\log T_{\rm eff}(BLG98) - \log T_{\rm eff}(BLG94) \approx -0.004$, $\log T_{\rm eff}(A99) - \log T_{\rm eff}(BLG94) \approx -0.010$). We caution however, that these differences are based on $(V - I_c) \rightarrow T_{\rm eff}$ calibrations, and the corresponding formula of A99 can be employed only after applying a transformation from their Johnson I to I_c (Fernie 1983). This could perhaps be one of the reasons why we get (somewhat curiously) better agreement between BLG94 and A99 with the temperatures calibrated by $B - V$. In the next section we will check the sensitivity of the derived distance modulus against the various $T_{\rm eff}$ scales.

Table 1. Systematic errors in the distance moduli at $Z = 0.003$

Quantity		ΔDM_{01}	ΔDM_{12}
$\Delta \log T_{\rm eff}$	$= +0.01$	$+0.22$	$+0.11$
ΔV	$= +0.02$	-0.06	-0.03
ΔI_c	$= +0.02$	$+0.10$	$+0.05$
ΔE_{B-V}	$= +0.05$	$+0.15$	$+0.02$
ΔP_0	$= +0.01$	-0.00	....
$\Delta P_1/P_0$	$= +0.002$	-0.15	....
ΔP_1	$= +0.01$	....	-0.04
$\Delta P_2/P_1$	$= +0.002$	....	-0.01

Note: The period errors refer to the model values.

Finally we note that for the calculation of the gravity we used the following formula

$$\log g = 2.62 - 1.21 \log P_{FU} , \tag{4}$$

which has been derived from a simple pulsation equation and black-body relation. This formula has a maximum error of ± 0.05, assuming that $M/M_\odot = 3.0^{+2.0}_{-1.0}$.

3. Determination of the distance modulus

To characterize the quality of the fit, for each variable, we calculated the following quantity

$$\sigma = \sqrt{\Delta P_i^2 + \Delta P_{i+1}^2} , \tag{5}$$

where $i = 0$ or 1 for the FU/FO and FO/SO variables, respectively. The difference between the observed and calculated periods are denoted by ΔP_i. We found basic difference between the two types of variables in respect of the behavior of $\sigma(M, L)$. In Figs. 1 and 2 we show representative gray maps and one dimensional slices along the minimum values of σ. For better visibility, in the gray maps we used a relatively low resolution, and therefore we considered only the minimum value of σ in each pixel (the scans were performed on a much finer grid). We see that the minimum (indicated by the black pixels) is much more pronounced for the FU/FO than for the FO/SO variables. This is nicely seen also in the slices along the minimum values. In addition, there is also a slight offset of < 0.002 for P_2/P_1 between the observed and theoretical values. Although this problem might bear some theoretical significance, it is completely unimportant in the present context (see Table 1). Furthermore, the shallow minima of the FO/SO variables yielded more stable DMs, depending (much) less on the various parameter changes than those of the FU/FO variables. This, together with their large number, make them very valuable for the purpose of distance estimation.

The calculation of the distance modulus was performed for each Z listed in the previous section. By applying the temperature scale given by Eq. (2), the individual DMs are plotted in Figs. 3 and 4. We employed the 3σ criterion for filtering out a few outliers (the number of these variables never exceeded two). The most striking feature of these plots is the large sensitivity of DM_{01} (DM of the FU/FO variables) against the variation of Z.
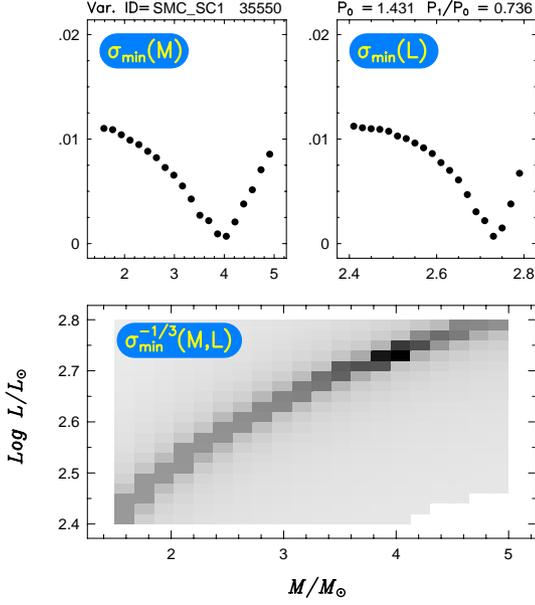
Fig. 1. *Lower panel:* Gray map of the period deviation (Eq. (5)) in the (M, L) parameter space. Darker areas indicate better fits to the observed periods. *Upper panels:* minimum period deviations as functions of M and $\log L$. An FU/FO variable is tested.
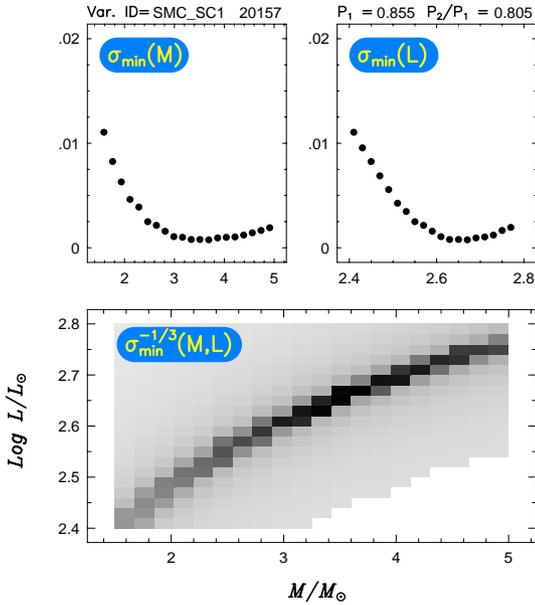

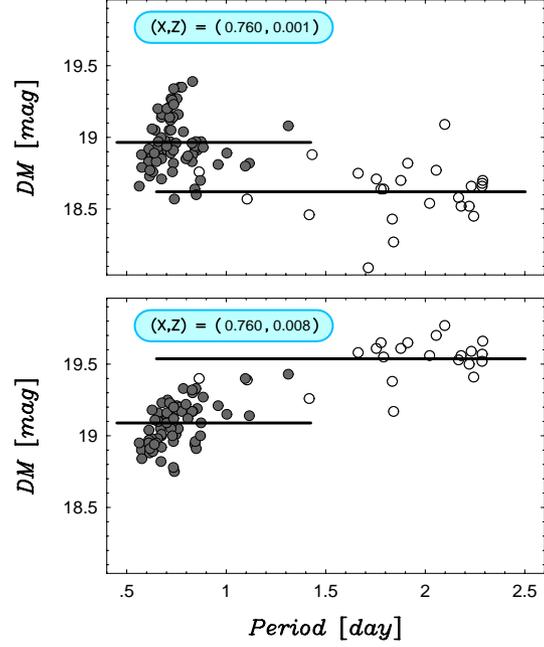**Fig. 2.** As in Fig. 1, but for an FO/SO variable.

Fig. 3. Individual and average distance moduli calculated for the FU/FO and FO/SO variables (shown by open and gray filled circles, respectively). Average DMs are shown by thick lines. Chemical compositions of the models used are given in the top left corners.
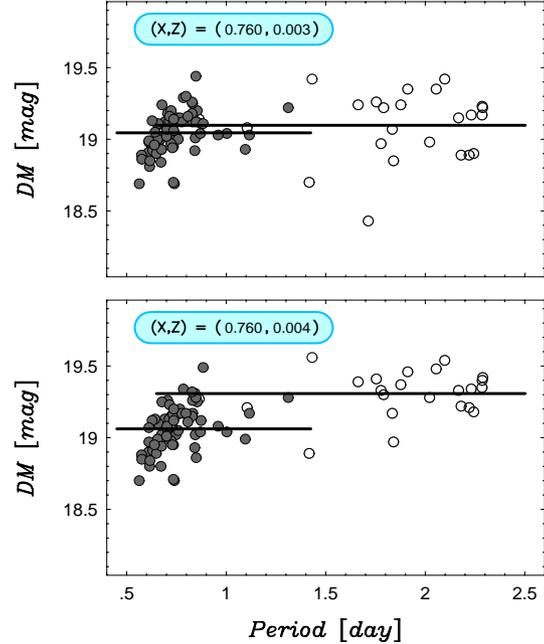

Fig. 4. As in Fig. 3, but for different heavy element abundances.

This is in contrast with the very *weak sensitivity* of DM_{12}. These properties and the $DM_{01} = DM_{12}$ condition allow us an optimum estimation of Z. (We note in passing that the other natural criterion, minimum dispersion of all individual distance moduli, leads to the same conclusion.) As it is seen in Fig. 4, the present results prefer a somewhat lower abundance than the one most often used in the context of the SMC Cepheids. With $Z_\odot = 0.02$, our close to optimum Z corresponds to [Fe/H]$= -0.8$, whereas the usually quoted value is -0.7 (corresponding to $Z = 0.004$, see Luck et al. 1998). We think that this is a fair agreement and

a good sign of the consistency between the present, completely independent estimation of Z and those obtained by direct spectroscopic observations (of other Cepheid variables).

Although Fig. 4 shows that the exact $DM_{01} = DM_{12}$ condition is still not satisfied for $Z = 0.003$, and a somewhat lower Z would be more appropriate, the difference is within the reasonable error limit, and therefore, in the estimation of DM, we

use the result obtained at $Z = 0.003$. When weighted by the number of stars, we get 19.05 mag for the average rounded distance modulus. By using the lower $T_{\rm eff}$ scale of A99, a similar match is found at $Z = 0.003$ between DM_{01} and DM_{12}. In this case we get $DM = 18.90$ mag. Due to the large number of variables, the formal statistical errors are very small in both cases: $\sigma_{\rm DM} = 0.017$ mag. Much more significant sources of errors are the various systematic effects and zero point ambiguities. Table 1 summarizes the changes in the distance moduli caused by the most significant potential sources of these kinds of errors. It is difficult to assess the size of the systematic errors in the various quantities. The numbers entering in the table are our best guesses on the 3σ errors (for notation simplicity we used positive changes everywhere). The assumed ambiguity in $T_{\rm eff}$ is based on the difference between A99 and BLG94. We think that this is a generous overestimation of the true error, because of the ambiguities mentioned in Sect. 2 and because of the smaller difference obtained in a comparison with the other current scale of BLG98 (see Sect. 2).

The following conclusions can be drawn from the table:

(a) FU/FO variables are (much) more sensitive to the systematic changes than FO/SO variables.
(b) The most serious source of error is the zero point ambiguity in the $T_{\rm eff}$ scale.
(c) Considering only the FO/SO variables, which dominate the value of the average distance modulus, and assuming that the various errors are independent, we get ± 0.13 mag for the total estimated systematic error.

4. Conclusions

By using the relative distance of 0.51 mag determined by Udalski et al. (1999b), the present determination of the SMC distance leads to an LMC distance modulus of 18.54 mag. This value is magically close to the value of 18.53 mag, derived from the application of the same method to Galactic double-mode RR Lyrae stars and using the relative distance of a few LMC globular clusters (Kovács 2000). Considering that this result was derived on a completely different data set, we think that the agreement is remarkable. This result suggest that the present models and input physics are more compatible with the observations than implied by Buchler et al. (1996) from their beat and bump Cepheid studies.

As given in Table 1, the most important source of ambiguity in this method is the potential error in the zero point of the temperature scale. Even if we consider this ambiguity, it is not possible to lower the distance modulus by more than ≈ 0.13 mag. This emphasizes further the contradiction between this 'long' and other 'short' distances, obtained e.g., by statistical parallax and red clump methods (Udalski et al. 1999b, see however Romaniello et al. 2000).

References

Alcock C. & the MACHO collaboration, 1997, astro-ph/9709025
Alonso A., Arribas S., Martínez-Roger C., 1999, A&AS 140, 261 (A99)
Beaulieu J. P. & the EROS collaboration, 1997, A&A 321, L5
Blackwell D. E., Lynas-Gray A. E., 1994, A&A 282, 899 (BLG94)
Blackwell D. E., Lynas-Gray A. E., 1998, A&AS 129, 505 (BLG98)
Buchler J. R., Kolláth Z., Beaulieu J. P., Goupil M-J., 1996, ApJ 462, L83
Cacciari C., Clementini G., Castelli F., Melandri F., 2000, ASP Conf. Ser. 203, 176
Castelli F., Gratton R. G., Kurucz R. L., 1997, A&A 318, 841
Clementini G., Carretta E., Gratton R., Merighi R., Mould J. R., McCarthy J. K. 1995, AJ 110, 2319
Fernie J. D., 1983, PASP 95, 782
Gieren W. P., Fouqué P., Gómez M., 1998, ApJ 496, 17
Iglesias C. A., Rogers F. J., 1996, ApJ 464, 943
Kovács G., Walker A. R., 1999, ApJ 512, 271 (KW99)
Kovács G., 2000, to appear in ASSL Series, Eds. M. Takeuti, D. D. Sasselov
Luck R. E., Moffett T. J., Barnes T. G., Gieren W. P. 1998, AJ 115, 605
Romaniello M., Salaris M., Cassisi S., Panagia N., 2000, ApJ 530, 738
Udalski A., Soszyński I., Szymański M., Kubiak M., Pietrzyński G., Woźniak P., Żebruń K., 1999a, Acta Astron. 49, 1
Udalski A., Szymański M., Kubiak M., Pietrzyński G., Soszyński I., Woźniak P., Żebruń K., 1999b, Acta Astron. 49, 201