

*Letter to the Editor***Confidence levels of evolutionary synthesis models****M. Cerviño<sup>1,2\*</sup>, V. Luridiana<sup>3</sup>, and F.J. Castander<sup>1</sup>**<sup>1</sup> Observatoire Midi-Pyrénées, 14, avenue Edouard Belin, 31400 Toulouse, France<sup>2</sup> Centre d'Etude Spatiale des Rayonnements, CNRS/UPS, B.P. 4346, 31028 Toulouse Cedex 4, France<sup>3</sup> Instituto de Astronomía, UNAM, Apdo. Postal 70-264, 04510 México D.F., México

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**Abstract.** The stochastic nature of the IMF in young stellar clusters implies that clusters of the same mass and age do not present the same unique values of their observed parameters. Instead they follow a distribution. We address the study of such distributions, parameterised in terms of their confidence limits, in evolutionary synthesis models. These confidence limits can be understood as the inherent uncertainties of the synthesis models. Here we concentrate on some parameters such as  $EW(H\beta)$  in emission. For instance, we show that for a cluster where  $10^5 M_{\odot}$  have been transformed into stars, the dispersion of  $EW(H\beta)$  is about 18% within the 90% confidence levels at ages between 3.5 and 5 Myrs.

**Key words:** Galaxies:Evolution**1. Introduction**

Since the development of evolutionary synthesis models by Tinsley (1980) several groups have developed models trying to reproduce observable quantities in systems in which the stellar content is not resolved. These models provide a powerful tool to study galactic and extragalactic HII regions. Several types of models can be found in the literature. For instance, Leitherer et al. (1996) give an extensive and comprehensive review on them.

The variety of external inputs used in the models (evolutionary tracks, stellar atmosphere libraries, etc) and the treatment of these input parameters, (for example, the interpolation methods), lead to slightly different model outputs from different authors, although “grosso modo” all of them produce similar results. One way to check the applicability of these models is to compare the results using different inputs. In this direction we highlight the work of Bruzual (2000) in which the author presents an extensive study on how the external inputs to the models influence the output results.

These problems can be exacerbated when modelling systems with small number of stars given that, in general, synthesis

models use continuous functions that do not reproduce exactly the discontinuous nature of star formation, especially in systems with low number of stars. In this letter we address this issue by studying the intrinsic variations of the evolutionary synthesis models outputs due to the deviations of the analytical Initial Mass Function (IMF) when modelling HII regions.

Here, we study the importance of such fluctuations in some typical observables of extragalactic HII regions. We characterise our results showing confidence levels of the distribution of a given parameter as a function of age and system mass. We focus on the study of the  $H\beta$  equivalent width in emission ( $EW(H\beta)$ ), that is extensively used as an age indicator of star forming regions. The results for the number of ionizing photons,  $Q(H^0)$ , and the ratio of the blue Wolf-Rayet (WR) bump luminosity to the  $H\beta$  luminosity are also presented. We differ the study of other quantities using different metallicities, IMF slopes, star formation regimes and evolutionary tracks to a future paper.

**2. Synthesis code and the Initial Mass Function**

The study of the IMF has been broadly covered in the astronomical literature. See Scalo (1986) for a comprehensive review. We define the IMF in the following way:

$$\Phi(m) = \frac{dN}{dm} = Am^{-\alpha} \quad (1)$$

where  $\alpha$  is the IMF slope, and  $A$  is a normalisation factor. This function gives us the number of stars in a given mass range. The widely used Salpeter IMF corresponds to  $\alpha = 2.35$  with this definition. The total mass of the system will be

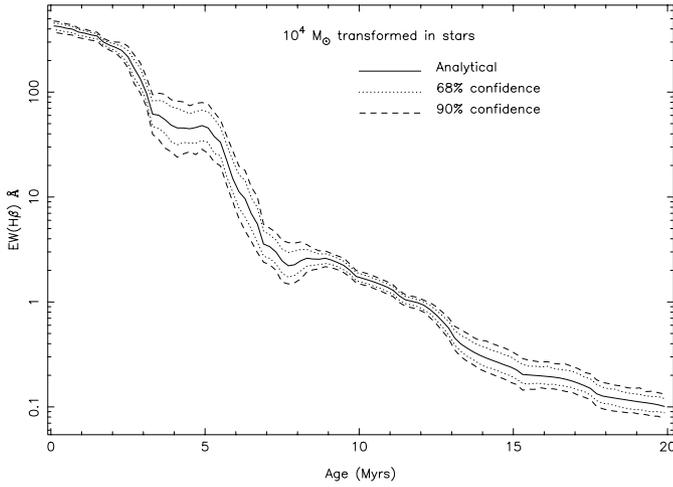
$$M = \int_{m_{\text{low}}}^{m_{\text{up}}} m\Phi(m)dm = \int_{m_{\text{low}}}^{m_{\text{up}}} mdN \quad (2)$$

In an evolutionary synthesis code, the amount of stars produced is usually generated binning the IMF (or using Monte-carlo simulations). The evolution of each bin (or star) is followed along the corresponding evolutionary track and the final result is computed from the overall evolution of all the stars (or masses) considered. In such a procedure, variations in the IMF affect directly the output results. The IMF and the synthesised

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**Fig. 1.** 90% and 68% Confidence levels of the EW(H $\beta$ ) distribution for the simulations with  $10^4 M_{\odot}$  transformed into stars.

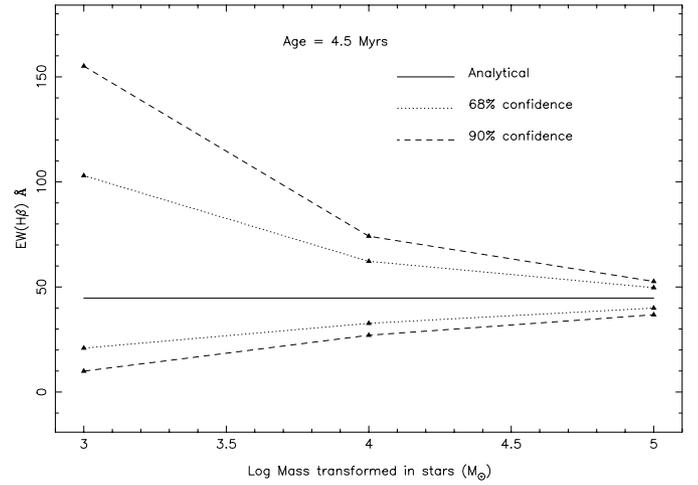
parameters are usually normalised to the total mass transformed into stars in the synthetic cluster. The comparison with real data is always carried out comparing an observed quantity with the corresponding one obtained with the code. It is assumed that the synthesised quantity is a good approximation to the theoretical analytical value and usually no further considerations are taken into account when comparing data and model outputs.

We have used an updated version of the evolutionary synthesis code of Cerviño & Mas-Hesse (1994), that uses Montecarlo realisations of the IMF to produce star forming regions of a given mass. We have also used the code to generate analytical simulations of clusters of the same mass, where analytical means a mass binning approximation of the IMF. We have tested that the analytical and the Montecarlo outputs show consistent results. The dispersion between them is lower than 1.5% when a large number of stars is used ( $5 \times 10^5$  stars, i.e.  $3 \times 10^6 M_{\odot}$  transformed into stars in the mass range 2 – 120  $M_{\odot}$ ). In this version of the code we use the solar metallicity evolutionary tracks of Meynet et al. (1994). The analytical results have also been compared with the predictions of the code of Leitherer et al. (1999), that uses the same evolutionary tracks. The results obtained with both codes are in general agreement.

We have performed 600 Montecarlo realisations of a cluster in which  $10^3 M_{\odot}$  are transformed into stars in the mass range 2 – 120  $M_{\odot}$ ; 400 realisations of a cluster of  $10^4 M_{\odot}$  and 200 of a cluster with  $10^5 M_{\odot}$ . The synthesis code is the same in all Montecarlo realisations, the only difference being the stochasticity of the Montecarlo process which reflects the fluctuations of the actual mass distribution with respect to the analytical continuous IMF.

### 3. Distribution of observed parameters: confidence levels

In this section we discuss the resulting distributions of the observable parameters obtained from the different Montecarlo runs for the masses mentioned above. We note that these distribution are in general non-Gaussian. The confidence levels presented



**Fig. 2.** 90% and 68% Confidence levels of the EW(H $\beta$ ) distribution for simulations of 4.5 Myrs as a function of mass transformed into stars.

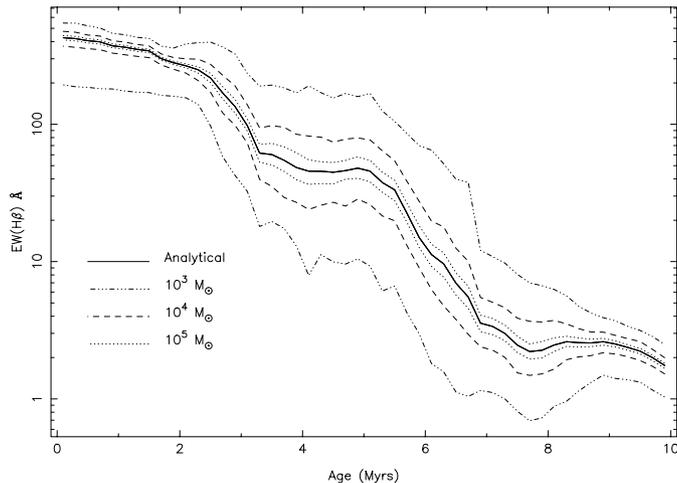
hereafter are computed integrating the resulting distributions to contain the 90% (68%) central values of the given parameter. We refer to these values of the parameters as the 90% (68%) confidence levels of the resulting parameter distribution obtained from the Montecarlo realisations.

Fig. 1. shows the 90% and 68% (equivalent to  $1\sigma$ ) confidence levels for EW(H $\beta$ ) resulting from the simulations in which  $10^4 M_{\odot}$  are transformed into stars. This is the mass range in which HII regions with  $L(H\alpha) \approx 10^{39}$  erg s $^{-1}$  lie. This value of luminosity of H $\alpha$  is usually used as the boundary between giant and normal HII regions.

It can be seen from the figure that the confidence levels change with time. The largest spread in EW(H $\beta$ ) is obtained between 3 and 9 Myrs (see also Fig. 3) when the fluctuations in the ionizing flux and the continuum can lead to variations of the order of 20% at the  $1\sigma$  level. The dispersion is reduced in the age range between 9 and 13 Myrs and increases again to an almost constant level for older ages.

The mass dependence of such fluctuations can be seen in Fig. 2, in which the 90% and 68% confidence levels are displayed for a fixed age of 4.5 Myr since the onset of the burst as a function of the mass transformed into stars. The figure shows clearly that the dispersion is not symmetric around the analytical value. The lower confidence level shows an almost linear correlation with the logarithm of the mass transformed into stars. The upper level does not show this linear behavior. An important implication regards the different interpretations of observational data when compared to models assuming that they provide an exact value. As can be seen from the figures, the 90% or 68% confidence levels of the parameter distributions are uncomfortably large, especially for low masses. Due to this behaviour of the EW(H $\beta$ ) evolution, the age determination between 3 and 5 Myrs are not well defined at the 90% confidence level. This situation may be improved using more diagnostic observables to estimate the age since the onset of the burst.

Fig. 3 presents the evolution with age of the 90% confidence level of the EW(H $\beta$ ) distribution for different amount of mass



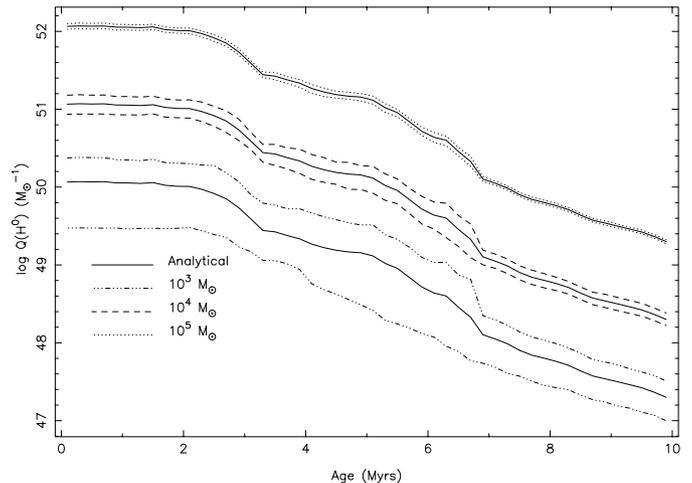
**Fig. 3.** Evolution of the 90% confidence level of the  $EW(H\beta)$  distribution for simulations with different amount of gas transformed into stars.

transformed into stars. This figure demonstrates the importance of taking into account the confidence levels of an observable parameter for a given mass range. For instance, for a  $10^5 M_{\odot}$  cluster, a value of the  $EW(H\beta)$  equal to  $50 \text{ \AA}$ , is compatible with any age between 3.5 and 5 Myrs within the 90% confidence levels.

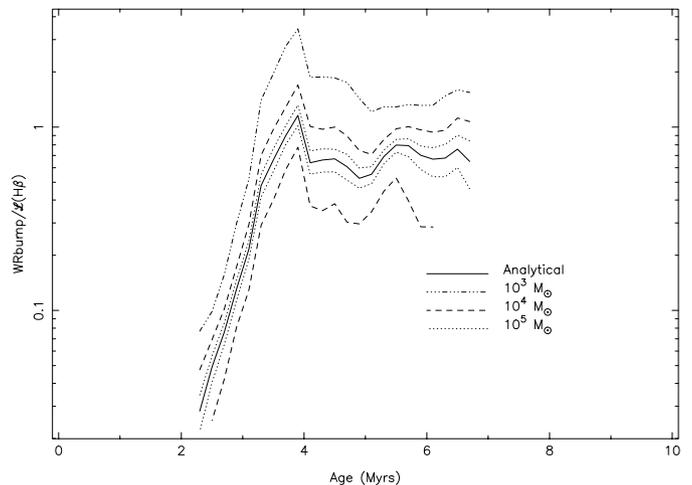
In Fig. 4 the evolution of the ionizing flux,  $Q(H^0)$ , is displayed against the mass transformed into stars. For a  $10^3 M_{\odot}$  system the fluctuations allow values that differ by an order of magnitude for a given age. The 90% confidence level values for the distribution of this observable show a relatively broad distribution up to 7 Myrs. Comparing this figure with the previous one, it is possible to figure out that the scatter in the continuum flux around  $4861 \text{ \AA}$ ,  $L_c(H\beta)$ , is the driving parameter producing the time evolution of the  $EW(H\beta)$  distribution. We have checked this point with the actual values and confirmed it to be the case. As a consistency check, we have also corroborated that the dispersion in  $L_c(H\beta)$  combined with the dispersion in  $Q(H^0)$  approximately reproduce the dispersion in  $EW(H\beta)$ .

Fig. 5 shows the evolution of the ratio of the Wolf-Rayet bump and  $H\beta$  luminosities,  $L(WRbump)/L(H\beta)$ , presenting the 90% confidence levels of its distribution. We would like to point out that the analytical value, close to 1, is 2.3 times higher than the one obtained in Cerviño & Mas-Hesse (1994). We have checked this value and confirm that it is the result of using different evolutionary tracks with enhanced mass loss rates. It is important to note that at the 90% confidence level, for a  $10^3 M_{\odot}$  cluster, values consistent with no detection of WR stars are allowed. The distribution at  $1\sigma$  level is also compatible with such non detection.

This study opens an additional question. If stochasticity of the stellar mass distribution of low mass clusters imply such dispersions in the distribution of observable parameters, what would its effect be in a system composed of several smaller groups? In order to address this question, we have simulated multiply-composed  $10^5 M_{\odot}$  clusters by adding randomly 6 sets



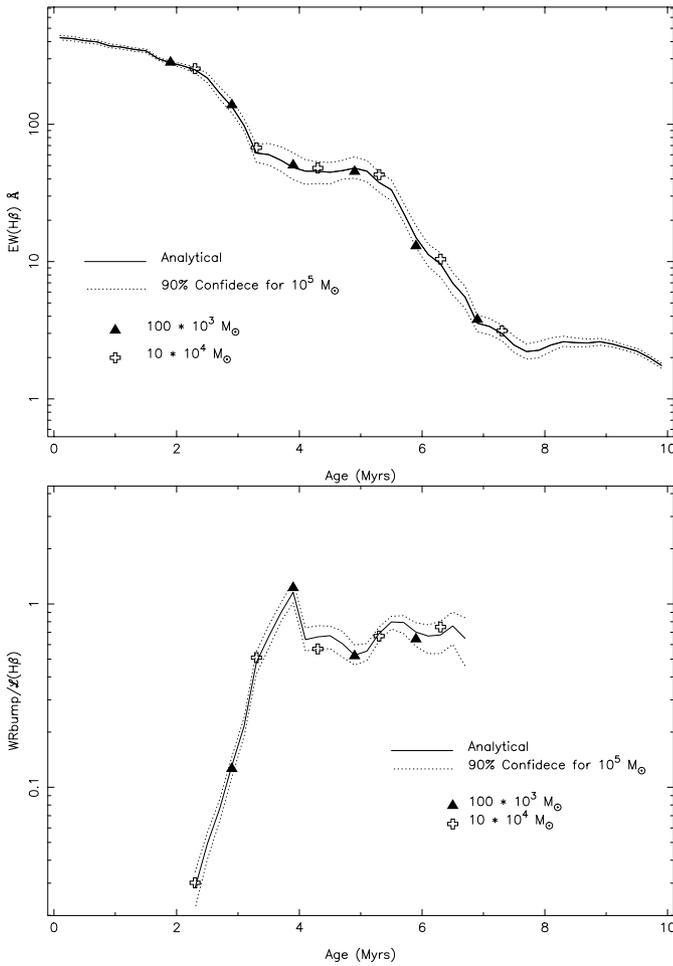
**Fig. 4.** Evolution of the 90% confidence level of the  $Q(H^0)$  distribution for simulations with different amount of gas transformed into stars.



**Fig. 5.** Evolution of the 90% confidence level of the  $L(WRbump)/L(H\beta)$  distribution for simulations with different amount of gas transformed into stars.

of 100 clusters with  $10^3 M_{\odot}$  and 6 sets of 10 clusters  $10^4 M_{\odot}$  to obtain the observables for a variety of ages. We have plotted the results for the  $EW(H\beta)$  and the  $L(WRbump)/L(H\beta)$  together with the analytical values and the 90% confidence level of the distributions for a  $10^5 M_{\odot}$  cluster in Fig. 6.

The figure shows that all the simulated points fall within the 90% confidence level for the  $10^5 M_{\odot}$  simulation. However, this is not the case for the 68% confidence level. In conclusion, the distribution of the observed parameters due to the stochasticity of the star formation process shrinks towards the analytical values of evolutionary synthesis models as larger masses are considered more or less independently of whether these systems are monolithic unique systems or composed by several coeval subgroups. We would like to stress that our previous simulations assumed the same age for the subgroups that form a system. We have not addressed the consequences of different ages for the different subgroups.



**Fig. 6.** Comparison of the values of  $EW(H\beta)$  and  $L(WRbump)/L(H\beta)$  obtained for multiply-composed  $10^5 M_{\odot}$  systems (triangles and crosses) with the analytical values and 90% confidence level for “single”  $10^5 M_{\odot}$  systems. (see text)

As a final note we would like to point out that the total masses used here have been computed using stellar masses in the range from 2 to  $120 M_{\odot}$ , which are the ones relevant for the observable parameters discussed. The corresponding total masses should be corrected by multiplying by a factor 1.36 for stellar masses in the range from 1 to  $120 M_{\odot}$ ; multiplying by 1.5 for the mass range 0.8 to  $120 M_{\odot}$  and multiplying by 3.75 for the mass range 0.08 to  $120 M_{\odot}$ . In all the cases a power law IMF with the Salpeter exponent has been assumed.

#### 4. Conclusions

The analysis presented in this letter allow us to draw the following conclusions:

- The intrinsic deviations between analytical and stochastic IMF must be taken into account when comparing synthesis models with observational data. The inherent uncertainties depend on the amount of gas transformed into stars.
- These deviations are independent of the synthesis code and represent a lower limit to the total uncertainties. The total deviations will depend on other parameters like the star formation history, the input ingredients, the numerical approximations, etc.
- The deviations depend on the evolutionary tracks used.
- The widths of the parameter distributions compared to the analytical values are proportional to the mass transformed into stars in the stellar cluster or groups of clusters. They also depend on the IMF slope and the evolutionary status of the cluster.
- The specific distribution of the deviation from the analytical value varies from observable to observable. As an extreme example, the 68% confidence level of the  $L(WRbump)/L(H\beta)$  ratio is compatible with no detection of WR stars.

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