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Mass-loss rates of Wolf–Rayet stars as a function of stellar parameters

T. Nugis[1] **and H.J.G.L.M. Lamers**[2,3]

[1] Tartu Observatory, 61602 Tõravere, Estonia (nugis@aai.ee)
[2] Astronomical Institute, University of Utrecht, Princetonplein 5, 3584CC Utrecht, The Netherlands (lamers@astro.uu.nl)
[3] SRON Laboratory for Space Research, Sorbonnelaan 2, 3584CA Utrecht, The Netherlands

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Abstract. Clumping-corrected mass-loss rates of 64 Galactic Wolf–Rayet (WR) stars are used to study the dependence of mass-loss rates, momentum transfer efficiencies and terminal velocities on the basic stellar parameters and chemical composition. The luminosities of the WR stars have been determined either directly from the masses, using the dependence of L on mass predicted by stellar evolution theory, or they were determined from the absolute visual magnitudes and the bolometric corrections. For this purpose we improved the relation between the bolometric correction and the spectral subclass.

(1) The momentum transfer efficiencies η (i.e. the ratio between the wind momentum loss and radiative momentum loss) of WR stars are found to lie in the range of 1.4 to 17.6, with the mean value of 6.2 for the 64 program stars. Such values can probably be explained by radiative driving due to multiple scattering of photons in a WR wind with an ionization stratification. However, there may be a problem in explaining the driving at low velocities.

(2) We derived the linear regression relations for the dependence of the terminal velocity, the momentum transfer efficiency and the mass-loss rates on luminosity and chemical composition. We found a tight relation between the terminal velocity of the wind and the parameters of the hydrostatic core. This relation enables the determination of the mass of the WR stars from their observed terminal velocities and chemical composition with an accuracy of about 0.1 dex for WN and WC stars. Using evolutionary models of WR stars, the luminosity can then be determined with an accuracy of 0.25 dex or better.

(3) We found that the mass-loss rates ($\dot{M}$) of WR stars depend strongly on luminosity and also quite strongly on chemical composition. For the combined sample of WN and WC stars we found that $\dot{M}$ in $M_\odot\ \mathrm{yr}^{-1}$ can be expressed as

$$\dot{M} \simeq 1.0 \times 10^{-11}\ (L/L_\odot)^{1.29}\ Y^{1.7}\ Z^{0.5} \qquad (1)$$

with an uncertainty of $\sigma = 0.19$ dex

(4) The new mass-loss rates are significantly smaller than adopted in evolutionary calculations, by about 0.2 to 0.6 dex, depending on the composition and on the evolutionary calculations. For H-rich WN stars the new mass-loss rates are 0.3 dex smaller than adopted in the evolutionary calculations of Meynet et al. (1994).

(5) The lower mass-loss rates, derived in this paper compared to previously adopted values, facilitate the formation of black holes as end points of the evolution of massive stars. However they might create a problem in explaining the observed WN/WC ratios, unless rotational mixing or mass-loss due to eruptions is important.

Key words: stars: atmospheres – stars: mass-loss – stars: emission-line, Be – stars: evolution – stars: Wolf-Rayet

1. Introduction

In this paper we study the dependence of mass-loss rates from Wolf–Rayet stars (WR stars) on the stellar parameters and on the chemical composition, using improved mass-loss rates and improved stellar parameters.

Wolf–Rayet stars are commonly believed to be evolved hot massive stars which have almost reached the end of their nuclear burning phase (Conti 1976, Maeder 1983, Lamers et al. 1991). Their formation depends strongly on the mass lost by the star in the previous evolutionary phases.

It is known that the stellar mass-loss rates of hot stars depend strongly on luminosity L. Several authors have tried to determine this dependence, e.g. de Jager et al. (1988), Nugis (1989), Lamers & Leitherer (1993) and Lamers et al. (2000a). Most authors describe the mass-loss rates as a function of the stellar parameters. Nugis (1989) also included the abundances and found a general mass-loss rate formula where $\dot{M}$ scales as $Y^{2.5}Z^{1.0}$ where Y and Z are the mass fractions of helium and the heavier elements respectively. Recently Vink et al. (2000a) has calculated theoretical mass-loss rates of O and B stars and showed that they agree very well with the observations. They describe a recipe for the calculations of $\dot{M}$ as a function of L, M and T_{eff}.

The mass-loss rate from WR stars is higher than the rates for O-stars of the same luminosities. The winds are thought to be driven by radiation pressure. In that case the efficiency of the momentum transfer from the radiation to the gas is expressed

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Send offprint requests to: H.J.G.L.M. Lamers in the momentum transfer efficiency (also called "wind performance number") η defined as

$$\eta \equiv \frac{\dot{M} v_\infty}{L/c}. \quad (2)$$

For WR stars the value of η is significantly larger than unity, viz. $1 < \eta < 70$ for WN stars (Hamann & Koesterke 1998a) and $10 < \eta < 170$ for WC stars (Koesterke & Hamann 1995). Values of $\eta >> 1$ require a very efficient momentum transfer with multiple scatterings. At present it is not clear which elements or ions could be responsible for such an efficient momentum transfer (Owocki & Gayley 1999). This has been a serious problem over many years.

In a recent paper Nugis et al. (1998) showed that the slope of the infrared and radio spectrum indicates that the wind is clumped with a distance dependent clumping factor (enhancement factor) being about 10–30 in the effective IR emission zone and of the order of unity in the radio emission zone. If this clumping is taken into account, the mass-loss rates of WR stars derived from the IR fluxes are reduced by about a factor 3 to 5. Similar or somewhat lower reductions of the mass-loss rates are expected if they are determined from the line luminosity. Schmutz (1997), Hamann & Koesterke (1998b), Koesterke et al. (1999), Crowther et al. (1999), Hillier & Miller (1999), Dessart et al. (2000) and De Marco et al. (2000) have reanalyzed the mass loss determinations from the strength of emission lines, taking clumping into account, and confirmed the reduction in mass loss by a factor of about two for WN stars and by somewhat higher factor for WC stars. The resulting wind momentum loss is still significantly higher than the radiative momentum loss.

Because of this momentum problem, Cassinelli (1991) proposed that the winds of WR stars are driven by magnetic effects, such as the fast magnetic rotator model. This model requires magnetic fields of order kiloGauss, which seem to be excessive if their presence is needed for the majority of WR stars. The model also predicts non-spherical winds, which in at least a few cases have indeed been confirmed by polarization measurements (Schulte-Ladbeck 1995).

The determination of realistic momentum transfer efficiency η requires reliable estimates of the luminosity of WR stars and this is one of the main tasks of the present paper. To this purpose we derive the stellar parameters of 34 WN and 30 WC stars. The other purpose of the present investigation is to derive the empirical dependence of mass-loss rates of WR stars on their stellar parameters. The results can be used to understand the mass-loss mechanism of WR stars.

Knowledge of the true mass-loss rates of WR stars is not only important for understanding the mass-loss mechanism, but also for the prediction of the final stages of evolution (final fate). With the high mass-loss rates that have been used in previous studies (i.e. without correction for clumping) it was found that the WR stars loose too much mass during the WR phase to form black holes as their end products. Wellstein & Langer (1999) found that with a reduction of the previously used mass-loss rates of WR stars by a factor 2 to 3 times it is possible to form black holes. In this context we would like to point out that although corrections due to clumping indeed reduce the WR mass-loss rates on the average by a factor 3, the final fate of the star still remains in many cases unclear. This is because observations show that some WR stars have passed through short-lived shell-ejection (superwind) phases during which a significant amount of mass has been lost (Marston 1999).

Our sample of Galactic WR stars contains all stars with well determined stellar and mass-loss parameters. The total number of Galactic WR stars studied by us is 64 which makes up about 29 % from the total number of 218 WR stars discovered so far (van der Hucht 1999). Our sample consists of the stars studied by Nugis et al. (1998) (except of WR 98, WR 110 and WR 145 with atypical spectral type) and of all the remaining WR stars with known distances and/or masses.

In Sect. 2 we describe the determination of the stellar parameters for stars with known distances and masses. This results in a new calibration of bolometric correction (BC) versus spectral type. This is based on the mass-luminosity relation of WR stars predicted by stellar evolution theory. In Sect. 3 we apply this BC-calibration to WR stars with known distances to derive the luminosity from the apparent visual magnitude, and the mass from the theoretical mass-luminosity relation. Sect. 4 gives a description of the chemical abundances of the program stars and in Sect. 5 we describe their temperatures and radii. In Sect. 6 we discuss the mass-loss rates and the terminal velocities of the winds. We then describe the dependence of v_∞ and η on the stellar parameters and composition in Sect. 7 and Sect. 8. In Sect. 9 we derive the empirical relations between the mass-loss rates and the stellar parameters and composition for WN stars and WC stars. Our mass-loss rates are compared with those used in evolutionary calculations in Sect. 10. The discussion and conclusions are in Sect. 11.

2. The program WR stars

Our sample of program stars contains 64 well observed WR stars for which distances are reasonably well known (from membership of associations or open clusters) or can be found by other methods. For 44 of program stars the mass has also been determined. These are the primary stars for this study. In this section we discuss the distances and masses of these 44 WR stars. We use this sample to derive an empirical relation between Bolometric Correction (BC) and spectral type, which we will use later for the secondary sample of stars with unknown masses to derive their luminosity.

The WN and WC stars with known mass and distance are listed in Table 2. Column 1 gives the number in the Sixth catalogue of WR stars (van der Hucht et al. 1981, 1988). The spectral types are from Smith et al. (1996) for WN stars and Smith et al. (1990) for WC stars.

2.1. *The distances of the primary program WR stars*

The distances of the primary program stars are derived by two methods: from membership of associations or clusters, or from the photometric M_V versus spectral type calibration. The method for the determination of the distance d is indicated in the Table 2.

2.1.1. Distances from association or cluster membership

In total 31 out of 44 primary program stars are members of associations or open clusters. The distances of the stars in open clusters or associations have been adopted mainly from the compilation by van der Hucht et al. (1988). Only for WR 78, WR 79, WR 95 and WR 133 the improved distances from the study of Smith et al. (1994) have been used. For three program stars (WR 11, WR 47 and WR 139), the membership of cluster/association is not confirmed, or the distance of cluster/association is strongly different from the commonly accepted value. For these three stars new distances have been derived by other methods as described in Nugis et al. (1998).

2.1.2. Distances derived from the M_V calibration

For stars which are not known to be the members of an open cluster or association, we used the mean absolute visual magnitudes of the spectral subtype of the WR star, or the companion O-star, with the exception of WR 6 whose distance has been obtained from the strength of interstellar lines (Howarth & Schmutz 1995).

Throughout the present paper we are using monochromatic (line-free) narrow band photometric magnitudes v_m ($\lambda \approx 5160$ Å) and b_m ($\lambda \approx 4270$ Å) which are taken from the papers of Torres-Dodgen & Massey (1988) and Massey (1984). If for some stars the monochromatic magnitude was not measured in these papers, then v_m was derived through the synthetic magnitudes v_S of Smith (1968) by using the mean differences between v_m and v_s for the subtype as derived by Torres-Dodgen & Massey (1988). Here v_s is the synthetic visual magnitude measured by Torres-Dodgen & Massey. To transform the observations by L. F. Smith into the Torres-Dodgen & Massey system we used the corrections: v_s=$v_S - 0.04$ for WN stars (Schmutz & Vacca 1991) and v_s=$v_S - 0.06$ for WC stars (our estimate).

The distance for some stars was found through the absolute visual magnitude from the relationship

$$M_v = \mathrm{v}_m - 10 - 5\log d - A_v - 2.5\log l_v, \tag{3}$$

where d is distance in kpc, A_v is interstellar extinction in the v-band in magnitudes and l_v is the fraction of the total light emitted in the v-band by the studied star. For single stars $l_v = 1$. In the case of binaries l_v^W is the fraction of the total light emitted by the WR component and $l_v^O = 1 - l_v^W$ is the fraction emitted by the O component in the v-band.

A_v can be expressed through the broad band Johnson system colour excess E_{B-V}. We adopted $A_v/E_{B-V} = 3.42$. This value is obtained as the mean derived from the extinction curves A_λ/E_{B-V} of Sapar & Kuusik (1978) and Seaton (1979) and from the mean relative interstellar absorption laws in the optical range derived by Ardeberg & Virdefors (1982) and Krelowski et al. (1986). In the latter cases $A(1/\lambda(\mu m) = 1.82)/E_{B-V} = 3.1$ was adopted. We remind that Turner (1982) and Lundström & Stenholm (1984) derived nearly the same value for this ratio ($A_v/E_{B-V} \approx 3.4$).

Table 1. Mean intrinsic colours and absolute visual magnitudes. Colons indicate that the result is based on only one star.

Subclass	$(b-v)_0$	$\langle M_v \rangle$
WN 2	−0.35:	−2.8:
WN 3	−0.35	−3.6
WN 4	−0.25	−3.7
WN 4b	−0.15	−4.5
WN 5	−0.25	−4.2
WN 6	−0.20	−5.4
WN 6b	−0.10	−5.4
WN 7–8	−0.20	−6.4
WN 9	−0.15	−6.4
WC 4	−0.32:	−3.0:
WC 5–6	−0.32	−3.7
WC 7–8	−0.30	−4.8
WC 9	−0.25	−4.8

The colour excesses E_{B-V} of WR stars have determined in the paper of Nugis et al. (1998). For those WR stars which were not studied by Nugis et al., we used the same method as in that paper. This means that E_{B-V} was taken from the mean of (i) the literature values obtained by nulling the $\lambda 2175$ interstellar feature, and (ii) intrinsic colours. The adopted relationship between E_{B-V} and E_{b-v} as well as the intrinsic colours $(b-v)_0$ as function of subtype are the same as used by Nugis et al. (1998).

The intrinsic colours $(b-v)_0$ for different subclasses were derived in the paper of Nugis & Niedzielski (1995) from the stars with well determined E_{B-V} from the nulling the $\lambda 2175$ interstellar feature. The intrinsic colours for WN stars have been redetermined using new spectral subtypes (Smith et al. 1996) and they are presented in Table 1. In that table we also present the adopted mean absolute visual magnitudes of different WR subtypes. The mean absolute visual magnitudes for WC stars were adopted from van der Hucht et al. (1988). For WN3 and WN4 stars we used the mean absolute visual magnitudes of LMC stars derived by Torres-Dodgen & Massey (1988). For other WN stars we derived the mean absolute visual magnitudes by using the following Galactic WN stars:

WN2 – WR 2;
WN4b – WR 1, WR 6;
WN5 – WR 133, WR 138, WR 139, WR 141, WR 157;
WN6/6b – WR 24, WR 25, WR 67, WR 115, WR 134, WR 136, WR 153, WR 155;
WN7–9 – WR 22, WR 78, WR 105.

To find the absolute visual magnitudes of WR components in the binary systems, we need to know the fractions of the total light emitted by the WR component in the v-band. The method for finding the fractions l_v^W is described in the paper of Nugis et al. (1998). The details of the determination of these fractions for the stars not studied in that paper are presented in the Appendix B. The absolute visual magnitudes of different O subtypes have been adopted from Vacca et al. (1996) with the correction $M_v - M_V = 0.1$ according to Turner (1982), where M_V is the absolute visual magnitude in the Johnson system.

2.2. The masses of the WR stars

2.2.1. The masses of WR binaries

The masses of WR stars in spectroscopic binary systems can be derived from their orbital parameters. Our list of program stars contains 21 WR stars with a mass determination from a spectroscopic orbit. These stars are listed in Table 2 with the reference indicated and the data in Appendix A. For WR stars in binaries with more than one reliable determination of the orbit and the mass, we have adopted the mean value from the different authors and indicate the uncertainty range.

2.2.2. The masses of single WR stars from the cluster age

The mass of single WR stars cannot be determined directly. However, if the WR star is a member of a cluster or association with a well determined age, the mass can be derived from evolutionary calculations. The mass, or rather the mass limits, can be derived from evolutionary tracks, using the age and the WR type (WN, WC or WO) to indicate the mass limits. This method has been applied by Smith et al. (1994) who used the evolutionary tracks of Meynet et al. (1994). The uncertainty in the mass of the WR stars derived from the cluster age is typically 0.4 dex, whereas the masses derived from the ages of associations is typically 0.4 to 0.7 dex (Smith et al. 1994). We adopted the mean value between the upper and lower limits derived by Smith et al. (1994). The masses and the uncertainties are indicated in Table 2.

2.3. The luminosities

For this study of the dependence of mass-loss on the stellar parameters we need the luminosities of WR stars. For association or cluster members the luminosity is derived from the known distance. For stars which are not cluster members but with a known mass, listed in Table 2, we use the mass luminosity relation predicted by stellar evolution. (This is not a circular method, because the evolutionary masses depend only on the age of the cluster or association and the spectral type and not on the luminosity of the WR star).

Schaerer & Maeder (1992) found for WNE/WC stars the relationship:

$$\log \frac{L}{L_\odot} = 3.032 + 2.695 \log \frac{M}{M_\odot} - 0.461 \left(\log \frac{M}{M_\odot} \right)^2 . \quad (4)$$

We adopted this formula, rather than the ones by Langer (1989a), because it based on improved evolutionary calculations. Using the Eq. (4) we can determine the luminosity, $\log L/L_\odot$, from the adopted mass. For massive WNL stars this formula may lead to wrong results. Smith et al. (1994) estimated that for massive WNL stars the luminosities may be overestimated up to 0.5 in $\log L$. Following these suggestions, we estimated from the evolutionary models of Meynet et al. (1994) that for WN stars with $M \geq 40 M_\odot$ and with hydrogen still present in their atmospheres the luminosities are about 0.25 dex smaller as compared to the Eq. (4). Thus we used for WR stars the following relationship between luminosity and mass:

$$\log \frac{L}{L_\odot} = C_M + 2.695 \log \frac{M}{M_\odot} - 0.461 \left(\log \frac{M}{M_\odot} \right)^2 , \quad (5)$$

where $C_M = 3.032$ if $M < 40 M_\odot$ or $N_{\rm H}/N_{\rm He} \leq 1$, and $C_M = 2.782$ if $M \geq 40 M_\odot$ and $N_{\rm H}/N_{\rm He} > 1$.

The resulting luminosites for stars with known mass are listed in Table 2, Column 8.

2.4. The bolometric corrections

The stars with known luminosity, either from the cluster or association distance or from their mass, listed in Table 2, can be used to derive a bolometric correction (BC) scale that will be adopted below to derive the luminosities of stars with known distance but with unknown mass.

The BC of the stars are derived from L and M_V in the usual way: $\log L/L_\odot = -0.4(M_V + BC - M^\odot_{BOL})$, where $M^\odot_{BOL} = +4.74$ (Bessell et al. 1998) is the bolometric magitude of the Sun. In the case of WR stars we use monochromatic apparent magnitudes (v_m) and respective narrow-band visual absolute visual magnitudes M_v (Eq. (3)) as explained in previous subsections. We will use in the present paper bolometric corrections BC_v which correspond to the use of absolute visual magnitudes M_v. The narrow band v_m magnitudes differ from those in the Johnson system by

$$M_v - M_V = v_m - V = BC_v - BC = 0.1 \quad (6)$$

(Turner 1982; van der Hucht et al. 1988), where BC is the bolometric correction in the Johnson system.

The values of M_v derived from the use of the Eq. (3) are listed in Table 2 Column 7. The resulting bolometric corrections are listed in Table 2, Column 9.

2.5. The bolometric correction scale

We derived the mean values of BC_v for different subclasses as the weighted means of the values in Table 2. The weights adopted for the stars are

– weight 5: the mass is determined from the binary solution with an uncertainty of M being less than 15% and with a well known distance

– weight 4: the mass is determined from the binary solution with an uncertainty of M being in the limits 15–30%

– weight 3: the mass is determined either from the mass ratio of the binary with well determined spectral type of the OB component and the mass of that component is adopted from the spectral type M relation by Vacca et al. (1996) or it is estimated from the cluster age study by Smith et al. (1994) with the maximum and minimum values of M given by Smith et al. differing by less than a factor 2

Table 2. Primary program stars: WR stars with well determined mass and distance

WR	Spectral Type Type	M_{WR} ($M_\odot$)	Ref	d (kpc)	E_{B-V}	M_v	log L ($L_\odot$)	BC_v	N_H/N_{He}	N_Z/N_{He}
2	WN 2b	10:	1	2.51 ^{a}	0.62	-2.79	5.27:	-5.64:	0.0	0.0043
127	WN 3b+O 9.5V	10.8:	2	4.37 ^{a}	0.50	-3.20	5.33:	-5.37:	0.0:	0.0043
1	WN 4b	10:	1	2.63 ^{a}	0.85	-4.47	5.27:	-4.00:	0.1:	0.0044
31	WN 4+O 8V	13.6:	2	4.65 ^{b}	0.71	-3.73	5.50:	-5.27:	0.0	0.0043
51	WN 4	10:	1	3.60 ^{a}	1.66	-3.74	5.27:	-4.69:	0.0	0.0043
151	WN 4+O 5V	25:	2	6.75 ^{b}	1.21	-3.81	5.90:	-5.20:	0.0	0.0043
10	WN 5+(A)	18:	1	4.57 ^{a}	0.59	-3.98	5.69:	-5.50:	0.0:	0.0043
21	WN 5+O4-6	12±2	2	4.26 ^{b}	0.69	-4.18	5.40	-4.59	0.0:	0.0043
97	WN 5b+O 7	9:	2	3.14 ^{b}	1.18	-4.06	5.18:	-4.16	0.0:	0.0043
133	WN 5+O 9I	10::	2	1.66 ^{a}	0.38	-3.40	5.27::	-5.03::	0.0:	0.0043
138	WN 5+B?	15::	1	1.82 ^{a}	0.65	-4.45	5.56::	-4.72::	0.8:	0.0051
139	WN 5+O 6V	9.3±0.5	2	1.13 ^{c}	0.88	-4.04	5.21	-4.24	0.2:	0.0045
141	WN 5+OB	24::	2	1.82 ^{a}	1.14	-4.67	5.87::	-5.27:	0.0:	0.0043
157	WN 5+(B1II)	10:	1	3.39 ^{a}	0.78	-4.39	5.26:	-4.04:	0.0:	0.0043
47	WN 6+O 5V	48±9	2	4.30 ^{d}	1.22	-5.75	6.01	-4.54	0.2:	0.0045
67	WN 6	11.5:	1	3.63 ^{a}	1.34	-5.22	5.37:	-3.47:	0.0:	0.0043
115	WN 6	12::	1	2.19 ^{a}	1.61	-4.86	5.40:	-3.91::	0.0:	0.0043
134	WN 6b	12::	1	2.09 ^{a}	0.46	-4.93	5.40:	-3.84::	0.2:	0.0045
136	WN 6b	20:	1	1.82 ^{a}	0.54	-5.51	5.76:	-4.15:	0.54	0.0048
153	WN 6/(CE?)+O6I:	14	2	3.47 ^{a}	0.65	-4.74	5.52	-4.31	0.1:	0.0044
155	WN 6+O 9:	16.7 ±3.0	2	3.47 ^{a}	0.66	-5.36	5.64	-4.00	0.24	0.0045
22	WN 7+OB	55.3±7.3	2	2.63 ^{a}	0.32	-6.63	6.08:	-3.83:	3.2	0.0077
78	WN 7	16.4:	1	1.58 ^{a}	0.51	-6.12	5.63:	-3.20:	0.4	0.0047
105	WN 9	23::	1	1.58 ^{a}	2.46	-6.49	5.60::	-2.76::	2.3	0.0070
142	WO 2	8:	1	0.95 ^{a}	1.87:	-2.33:	5.09:	-5.66:	0.0	0.60
30a	WC 4/WO 4+O 4	10.3:	2	7.46 ^{b}	1.37	-3.00	5.29:	-5.48:	0.0	0.44
9	WC 5+O 7	17.2:	2	2.06 ^{b}	1.46	-4.58	5.66:	-4.83:	0.0	0.44
111	WC 5	12::	1	1.58 ^{a}	0.30	-3.61	5.40:	-5.16::	0.0	0.50
114	WC 5	10::	1	2.19 ^{a}	1.44	-3.54	5.27::	-4.89::	0.0	0.26
23	WC 6	11::	1	2.63 ^{a}	0.44	-3.69	5.34::	-4.92::	0.0	0.33
30	WC 6+O 6-8	17.7:	2	9.48 ^{b}	0.69	-4.54	5.68:	-4.91:	0.0	0.32
48	WC 6+O 9.5I	$<$11.5	1	2.40 ^{a}	0.35	-4.28	$\langle$5.37	$\rangle$-4.41	0.0	0.32
154	WC 6	8:	1	3.47 ^{a}	0.76	-3.61	5.09:	-4.38:	0.0	0.27
42	WC 7+O 7V	12±3	2	3.02 ^{b}	0.37	-4.16	5.40	-4.61	0.0	0.32
50	WC 7+abs	$<$5.1	1	3.60 ^{a}	1.24	-4.31	$\langle$4.71	$\rangle$-2.72	0.0	0.40
79	WC 7+O 5-8	13.9:	2	1.58 ^{a}	0.48	-4.80	5.51:	-4.24:	0.0	0.32
93	WC 7+O 7-9	$<$8	1	1.74 ^{a}	1.82	-5.09	$\langle$5.09	$\rangle$-2.90	0.0	0.32
137	WC 7+OB	$<$17	1	1.82 ^{a}	0.66	-4.80	$\langle$5.65	$\rangle$-4.59	0.0	0.32
140	WC 7+O 4-5	23.1:	2	1.21 ^{b}	0.84	-5.32	5.85:	-4.56:	0.0	0.32
11	WC 8+O 8-9III	7±2	2	0.26 ^{e}	0.03	-3.65	4.98	-4.06	0.0	0.18
113	WC 8+O 8-9IV	13±2	2	2.00 ^{a}	1.02	-4.76	5.46	-4.16	0.0	0.22
135	WC 8	11::	1	2.09 ^{a}	0.38	-4.40	5.34::	-4.21:	0.0	0.16
70	WC 9+B 0I	9.8:	2	3.25 ^{f}	1.29	-4.80	5.25:	3.59:	0.0	0.22
95	WC 9	8:	1	2.09 ^{a}	2.15	-4.57	5.09:	3.42:	0.0	0.22

Notes: M_{WR}: a single colon gives uncertain value, uncertainty $<$ 30%
M_{WR}: a double colon gives uncertain value, uncertainty $>$ 30%
Sources
M_{WR}: ^{1} from association age (Smith et al. 1994); ^{2} from binary. For details see Appendix A
^{a} star is a member of association or cluster; ^{b} from M_v of the O-type companion
^{c} from L of the O6 V component (Nugis et al. 1998); ^{d} from M_v of both companions
d^{e} from parallax measurement; ^{f} from M_v of WR-companion

– weight 2: the mass is determined either from the mass ratio for the binary with uncertain spectral type of the OB component and the mass of that component is adopted from the spectral type M relation by Vacca et al. (1996) or the mass is from the cluster or

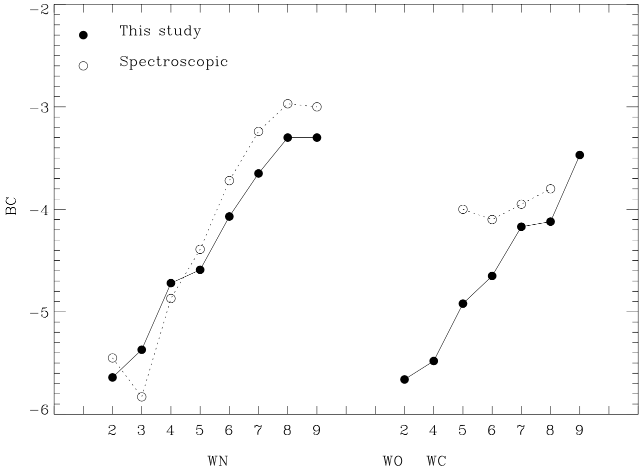

Fig. 1. The dependence of bolometric correction BC_v on spectral subtype for WN and WC stars. Filled dots are our data and open dots are the values obtained from the spectroscopic analyses of WN (Hamann & Koesterke 1998a) and WC stars (Koesterke & Hamann 1995).

Table 3. The mean BC_v for different sybtypes.

Subtype	BC_v this study	BC_v spectroscopic
WN 2	−5.64 (1)	−5.45 (1)
WN 3	−5.37 (1)	−5.83 (3)
WN 4	−4.72 (4)	−4.87 (10)
WN 5	−4.59 (8)	−4.39 (12)
WN 6	−4.07 (7)	−3.72 (10)
WN 7	−3.65 (2)	−3.24 (12)
WN 8–9	−3.3:	−2.98 (13)
WO 2	−5.66 (1)	
WC 4	−5.48 (1)	
WC 5	−4.92 (3)	−4.00:(9)
WC 6	−4.65 (4)	−4.10 (10)
WC 7	−4.17 (6)	−3.95 (5)
WC 8	−4.12 (3)	−3.80 (1)
WC 9	−3.47 (2)	

The number of stars used in deriving the mean values is given in parentheses. The uncertain data are followed by colons.

association age study by Smith et al. (1994) with the maximum and minimum values of M differing by a factor between 2 and 3

– weight 1: the mass is determined from the cluster or association age study by Smith et al. (1994) with the maximum and minimum values of M differing by a factor between 3 and 10

– weight 0: the mass is determined from the the cluster or association age study by Smith et al. (1994) but the estimated maximium and minimum limits of mass differ more than 10. If the distance is not reliably determined, i.e. it is estimated from the adopted value of M_v, then the weight of the individual BC_v estimate is reduced by one.

The resulting mean values of BC_v as a function of subtype are presented in Table 3. These BC_v differ somewhat from the results obtained by the Smith et al. (1994) who used practically the same method. These differences are mainly due to differences in spectroscopic masses and absolute visual magnitudes for some stars. We give for comparison the relation derived by Hamann & Koesterke (1998a) and Koesterke & Hamann (1995) from the spectroscopic analysis of the WR spectra, using standard non-LTE WR models. The comparison is plotted versus subtype in Fig. 1.

The mean BC_v values obtained from the spectroscopic analyses of WN stars do not differ much from the values obtained by us, but for some WC stars the two values of BC_v differ quite substantially. These differences are due to the neglect of clumping and metals (line blanketing and line blocking) in previous versions of standard models which lead to the underestimated luminosities in the standard models for some spectral subtypes (Hillier 1996, Schmutz 1997, Schmutz & De Marco 1999, Hillier & Miller 1999, Dessart et al. 2000).

3. Other program stars

Using the Bolometric Correction scale derived from the stars with known mass and luminosity in the previous section, we can now use this scale to determine the luminosities of the other well studied stars with unknown mass. These stars are listed in Table 4. The table lists the spectral types and the apparent monochromatic magnitude at 5160 Å (v_m). In case of a binary system, the listed value refers to the whole system. The distances are derived by different methods, as indicated. The values of E_{B-V} are from Nugis et al. (1998). For the stars which were not studied in that paper we found E_{B-V} by the method explained in Sect. 2. Column 6 gives the fraction of the visual light that is emitted by the WR component. In case of a binary, this fraction can be less than 1.0. The details of determination of these fractions for the studied stars are given in Appendix B. Column 7 gives the resulting absolute visual monochromatic magnitude of the WR stars. The values of BC_v are from Table 3. Column 9 gives the resulting luminosity. The masses, derived from L by means of Eq. (4), are listed in Column 10.

4. The chemical composition of the program stars

4.1. The He/H ratio

The ratios of N_{He}/N_H for WN stars were derived by taking into account the clumped structure of their winds (Nugis & Niedzielski 1995, Nugis et al. 1998). These estimates were obtained from comparisons of the observed line fluxes of He II, HeI and H I lines with theoretical line fluxes found by summing up the contributions from different layers of the clumped wind model. In these estimates only those lines have been used which are not blended with lines of other elements.

4.2. The chemical composition: Y and Z

The chemical composition, described by Y and Z, of the WN stars has been derived by the following scheme. The observed composition of the atmosphere (wind) is assumed to be a mixture of unprocessed matter and of CNO-cycle processed mate-

Table 4. Secondary program stars

WR	Sp. Type	v_m	d (kpc)	E_{B-V}	l_v^W	M_v	BC_v	log L $L_\odot$	M_{WR} $M_\odot$	N_H/N_{He}	N_Z/N_{He}
6	WN 4b	6.96	1.80 [g]	0.06	1.0	-4.52	-4.72	5.59	16	0.2:	0.0045
24	WN 6	6.50	2.63 [a]	0.18	1.0	-6.22	-4.07	6.01	48	2.4	0.0068
25	WN 6	8.18	2.63 [a]	0.73	1.0	-6.42	-4.07	6.09	57	3.8	0.0083
87	WN 7+abs	12.6	2.88 [a]	2.10	0.64	-6.40	-3.65	5.92	40	2.7	0.0071
16	WN 8	8.52	3.93 [f]	0.57	1.0	-6.40	-3.3	5.78	21	1.8	0.0062
40	WN 8	7.87	3.57 [f]	0.44	1.0	-6.40	-3.3	5.78	21	0.8	0.0051
89	WN 8+abs	11.55	2.88 [a]	1.72	0.81	-6.40	-3.3	5.78	21	1.0	0.0053
124	WN 8	11.60	6.4 [f]	1.16	1.0	-6.40	-3.3	5.78	21	1.9	0.0063
147	WN 8+OB	14.89	0.72 [f]	3.65	0.65	-6.40	-3.3	5.78	21	0.1	0.0044
148	WN 8+abs	10.46	8.90 [f]	0.75	0.66	-6.40	-3.3	5.78	21	0.6	0.0049
144	WC 4	15.45	1.12 [f]	2.40	1.0	-3.00	-5.48	5.29	10	0.0	0.44
15	WC 6	11.85	1.44 [f]	1.39	1.0	-3.70	-4.65	5.24	10	0.0	0.16
146	WC 6+O	13.91	0.70 [f]	2.80	0.33	-3.70	-4.65	5.24	10	0.0	0.10
14	WC 7	9.56	2.00 [a]	0.62	1.0	-4.07	-4.17	5.19	9	0.0	0.16
86	WC 7+B0I	9.72	2.16 [f]	0.99	0.61	-4.80	-4.17	5.48	13	0.0	0.60
65	WC 9	14.61	2.13 [f]	2.27	1.0	-4.80	-3.47	5.20	9	0.0	0.22
81	WC 9	12.99	2.23 [f]	1.77	1.0	-4.80	-3.47	5.20	9	0.0	0.22
103	WC 9	9.18	2.94 [f]	0.48	1.0	-4.80	-3.47	5.20	9	0.0	0.22
104	WC 9+abs	13.6	1.58 [a]	2.05	1.0	-4.40	-3.47	5.04	8	0.0	0.22
112	WC 9	19.1	1.30 [f]	3.90	1.0	-4.80	-3.47	5.20	9	0.0	0.22

[a] star is a member of association or cluster;
[f] from M_v of WR-companion;
[g] from IS absorption line strength.

rial. Unprocessed matter is assumed to have solar composition: $Y = 0.246$, $Z = 0.018$, $X = 0.736$ ($N_{He}/N_H = 0.0836$, $N_Z/N_{He} = 0.00142$, $A_Z \approx 17$). The mass fraction $Y_\odot$ follows from helioseismology (Basu & Antia 1995) and $Z_\odot$ is derived from the solar ratio $Z/X = 0.0244$ (Grevesse et al. 1996). Such a composition agrees well with modern evolutionary models for the Sun (Elliot 1998). The CNO-cycle processed gas of original solar composition has a composition: $Y = 0.983$, $Z = 0.0172$, $X = 0.0$. The mass fraction x of CNO processed hydrogen atoms of WN stars (the number-fraction of H-atoms transformed into helium) can be found from the observed ratio of N_{He}/N_H

$$x = \frac{N_{He}/N_H - [N_{He}/N_H]_\odot}{N_{He}/N_H + 0.25} \; . \tag{7}$$

For a given fraction x of the processed material the mass fractions Y and Z are obtained from the formulae:

$$Y = \frac{4(x/4 + [N_{He}/N_H]_\odot)}{1 - x + 4(x/4 + [N_{He}/N_H]_\odot) + [N_Z/N_H]_\odot A_Z} \; , \tag{8}$$

$$Z = \frac{[N_Z/N_H]_\odot A_Z}{1 - x + 4(x/4 + [N_{He}/N_H]_\odot) + [N_Z/N_H]_\odot A_Z} \; . \tag{9}$$

The chemical parameters Y and Z of WC stars have been derived by assuming that their observed compositions are due to partial He–burning of fully CNO-cycle processed matter with depleted ^{14}N due to reaction chain $^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^+)^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne}$. The mass fractions Y and Z are found from the observed number ratios N_C/N_{He} and N_O/N_{He} as follows:

$$Y = \frac{4}{4 + 12N_C/N_{He} + 16N_O/N_{He} + 0.1} \tag{10}$$

and

$$Z = 1 - Y. \tag{11}$$

For the WC stars with C and O abundances derived from detailed atmospheric modeling we adopted the individual determinations of the ratios N_C/N_{He} and N_O/N_{He}. This applies to the stars: WR 111 (Hillier & Miller 1999), WR 135, WR 146 (Dessart et al. 2000), WR 11 (De Marco et al. 2000), WR 14, WR 15, WR 23, WR 50, WR 86, WR 114 and WR 154 (Koesterke & Hamann 1995). For the other stars we used the mean values for the subtype which have been derived as the weighted means of different estimates. The weights were taken to be 1 for the stars with the estimates obtained by simple atmospheric models and recombination-type models (Torres 1988, Smith & Hummer 1988, Eenens & Williams 1992, Nugis 1991); the weights were taken to be 2 for the estimates of Koesterke & Hamann (1995) and the weights were taken to be 4 for the most correct estimates obtained by using the clumped wind models (Hillier & Miller 1999, Dessart et al. 2000 and De Marco et al. 2000).

The mean ratios N_C/N_{He} for the subtypes were found to be: WC 4-5: 0.36, WC 6-7: 0.26, WC 8-9: 0.18 and the mean ratios of N_C/N_O were estimated to be 5 for all WC subtypes. The individual values of N_C/N_{He} may differ from the mean value for the subtype by about 0.1. In the case of WR 142 (WO 2) we used the N_C/N_{He} and N_O/N_{He} determinations from Kingsburgh et al. (1995).

The resulting ratios N_H/N_{He} and N_Z/N_{He} of WR stars are listed in Tables 2 and 4. Note that N_Z means the sum over all elements heavier than helium. In the case of WN stars N_Z is predominantly the number of nitrogen atoms and in the case of WC stars it is the number of carbon atoms.

5. Temperatures and radii

The effective temperatures of WR stars are not well defined, because the radius of these stars depends on wavelength and hence the classical formula $L = 4\pi R_*^2 \sigma {T_{\rm eff}}^4$ loses its meaning. This is due to the high mass-loss rates of WR stars which results in an optically thick wind. Therefore we cannot simply use $T_{\rm eff}$ as one of the parameters for fitting the mass-loss rates, because $T_{\rm eff}$ itself will depend on the density distribution in the wind and hence on the mass-loss rate. The only "effective temperature" that is independent of $\dot{M}$ is the one that corresponds to the radius of the hydrostatic core, calculated in stellar evolution codes.

Schaerer & Maeder (1992) derived a formula for the hydrostatic core radius, $R_{\rm evol}$, of the WR stars from the evolutionary tracks

$$\log \frac{R_{\rm evol}}{R_\odot} = -1.845 + 0.338 \log \frac{L}{L_\odot} \tag{12}$$

and the corresponding effective core temperature, $T_{\rm evol}$, is

$$\log T_{\rm evol} = 4.684 + 0.0809 \ \log \frac{L}{L_\odot}. \tag{13}$$

These values are listed in Tables 5 and 6. The modesl of Schaerer & Maeder are not valid for the H-rich WNL stars. Therefore no values of these quantities are given for the stars WR 16, WR 22, WR 24, WR 25, WR 87, WR 89, WR 105 and WR 124.

6. The mass-loss rates and terminal velocities of the winds

The mass-loss rates of most of our WR program stars are from Nugis et al. (1998). They were derived from the radio emission power, with clumping taken into account. These values are indicated by "N" in Column 10 of Tables 5 and 6 whereas "N/d" means that the mass-loss rates have been corrected for the change of the distance. For the other program stars the mass-loss rates were found from the mass-loss rate versus emission line equivalent width relationships derived by Nugis et al. (1998), again taking into account the clumping of the WR winds. These mass-loss rate determinations are based on the equivalent widths of the emission lines at $\lambda\lambda$ 5411, 4945 and 7115 Å for WN stars and those at $\lambda\lambda$ 5411, 5471 and 5590 Å for WC stars. The equivalent widths are from:

– WN stars: Smith et al. (1996), Crowther (1993), Conti et al. (1990)

– WC stars: Koesterke & Hamann (1995), Torres-Dodgen (1985), Smith et al. (1990) and Kingsburgh et al. (1995) (WR 30a and WR 142).

In the case of WR 48 individual estimates of equivalent widths of these lines were absent and for the WC 6 component of this binary star we used the mean equivalent widths of its spectral subtype and took into account the depression due to the presence of the O component (through the derived value l_v^W). The stars with mass-loss rates derived from the equivalent widths are indicated in Tables 5 and 6 by the symbol "e".

The terminal velocities of the stellar winds of most of our program stars are from Nugis et al. (1998). For the stars not studied in that paper the sources of v_∞ are indicated in the Appendix C.

Tables 5 and 6 list the important parameters of the program stars. These are: spectral type, luminosity, mass, effective temperature of the hydrostatic core, the mass fractions of He and the metals Y, Z (the mass fraction of hydrogen X is equal to $1 - Y - Z$), the observed mass-loss rate, terminal velocity and the momentum transfer efficiency η.

7. The dependence of the terminal velocity on the stellar parameters

The line driven wind theory predicts that the terminal velocities are proportional to the effective escape velocity. This is indeed observed for the O and B-stars. The proportionality factor $v_\infty/v_{\rm esc}$ depends on the effective temperature. For OB stars the ratio is $v_\infty/v_{\rm esc} \simeq$ 2.6 for O-stars with $T_{\rm eff} > 22\,000$, $\simeq$1.3 for $10\,000 < T_{\rm eff} < 20\,000$ and $\simeq$0.7 for $8000 < T_{\rm eff} < 10\,000$ K (Lamers et al. 1995). Here we investigate if a similar relation holds for the WR stars. The problem with the very extended atmospheres of the WR stars is that the "effective radius" depends on the density structure of the wind and hence on the mass-loss rate. The effective radius, defined as the radius where some "effective temperature" is reached (e.g. where $\tau = 2/3$ or where $T({\rm electron}) = \{L/(4\pi\sigma R^2\}^{1/4}$) is larger than the hydrostatic radius by factors of the order of 10.

Since we want to determine the properties of the WR winds on their stellar parameters, we will compare v_∞ with the escape velocity of the hydrostatic core. The effective escape velocity from the core is defined as

$$v_{\rm esc}({\rm core}) = \sqrt{\frac{GM(1-\Gamma_e)}{R_{\rm evol}}} \tag{14}$$

where the factor $1 - \Gamma_e$ corrects the gravity for the effect of electron scattering with

$$\Gamma_e = 7.66 \times 10^{-5} \ \sigma_e \ L/M \tag{15}$$

if L and M are in solar units. The electron scattering coefficient is $\sigma_e \simeq 0.401(X + Y/2 + Z/4)$ cm^2. We have assumed that H and He are fully ionized near the surface of WR stars and that C, N and O are four times ionized (CV, NV, OV). The effective escape velocities are about 2000 km s^{-1} for all WR stars.

Table 5. Parameters and mass-loss rates of WN stars

Star	Sp. Type	$\log L$ ($L_\odot$)	M_{WR} ($M_\odot$)	$T_{\rm evol}$ (K)	Y	Z	$\dot{M}$(obs) (10^{-5}) ($M_\odot$ yr^{-1})	v_∞ (km s^{-1})	$\frac{\dot{M} v_\infty}{L/c}$ η
2	WN 2b	5.27	10.0	129 000	0.983	0.0172	0.40:^{e}	3100	3.29
127	WN 3b+O 9.5V	5.33	10.8	134 000	0.983	0.0172	0.47^{e}	1760	1.92
1	WN 4b	5.57	15.2	136 500	0.959	0.0172	2.40^{N}	2135	6.73
6	WN 4b	5.59	15.6	137 000	0.936	0.0172	1.90^{N}	1720	4.10
31	WN 4+O 8V	5.28	10.1	129 200	0.983	0.0172	0.86^{e}	1640	3.66
51	WN 4	5.28	10.2	129 300	0.983	0.0172	0.62^{e}	1460	2.33
151	WN 4+O 5V	5.71	18.5	140 000	0.983	0.0172	1.81^{e}	1500	2.61
10	WN 5+(A)	5.32	10.8	130 400	0.983	0.0172	0.51^{e}	1475	1.75
21	WN 5+O 4-6	5.40	12.0	132 300	0.983	0.0172	0.93^{e}	1660	2.98
97	WN 5b+O 7	5.36	11.3	131 200	0.983	0.0172	1.24^{e}	1900	5.09
133	WN 5+O 9I	5.09	8.0	124 900	0.983	0.0172	0.65^{N}	1625	4.19
138	WN 5+B?	5.51	13.9	135 000	0.819	0.0173	1.00^{N}	1345	2.03
139	WN 5+O 6V	5.21	9.3	127 600	0.936	0.0172	0.92^{N}	1785	4.96
141	WN 5+OB?	5.60	15.8	137 300	0.983	0.0172	1.20^{N}	1400	2.07
157	WN 5+(B1II)	5.49	13.5	134 400	0.983	0.0172	0.89^{e}	1230	1.76
24	WN 6	6.01	48.0		0.614	0.0176	2.95^{N}	2155	3.03
25	WN 6	6.09	57.0		0.503	0.0177	2.90^{N}	2455	2.82
47	WN 6+O 5V	5.92	40.0	145 700	0.936	0.0172	9.17^{e}	2460	13.30
67	WN 6	5.61	16.1	137 600	0.983	0.0172	4.56^{e}	1500	8.19
115	WN 6	5.47	13.1	133 900	0.983	0.0172	2.35^{N}	1150	4.51
134	WN 6b	5.50	13.6	134 600	0.936	0.0172	4.55^{N}	1905	13.60
136	WN 6b	5.73	19.1	140 600	0.866	0.0173	6.25^{N}	1605	9.20
153	WN 6/(CE?)+O6I	5.52	14.0	131 100	0.968	0.0172	3.51^{e}	1785	9.38
155	WN 6+O 9:	5.65	17.0	138 500	0.927	0.0172	3.04^{e}	1690	5.64
22	WN 7+OB	6.08	55.3		0.546	0.0176	4.20^{N}	1790	3.06
78	WN 7	5.80	21.5	142 600	0.893	0.0173	3.80^{N}	1365	3.99
87	WN 7+abs	5.92	40.0		0.586	0.0176	3.19^{e}	1500	2.85
16	WN 8	5.78	20.6		0.678	0.0175	2.83$^{N/d}$	740	1.72
40	WN 8	5.78	20.6	141 800	0.814	0.0173	4.18$^{N/d}$	910	3.12
89	WN 8+abs	5.78	20.6		0.786	0.0174	5.70^{N}	1500	7.02
124	WN 8	5.78	20.6		0.666	0.0175	2.45^{e}	710	1.43
147	WN 8+OB	5.78	20.6	141 800	0.959	0.0172	6.63$^{N/d}$	900	4.90
148	WN 8+abs	5.78	20.6	141 800	0.862	0.0173	7.48^{e}	1545	9.49
105	WN 9	5.81	21.8		0.624	0.0176	2.80:^{e}	1200	2.54

Notes: $\dot{M}^{N}$: mass loss rates are from Nugis et al. (1998), $\dot{M}^{N/d}$: with corrected distance.
$\dot{M}^{e}$: mass loss rates have been determined from the strength of emission lines using the formulae from Nugis et al. (1998).

Fig. 2 shows the dependence of the ratio $v_\infty/v_{\rm esc}({\rm core})$ on L for both the WN and the WC stars. We have used different symbols for different abundance ranges.

Fig. 2a shows that for WN stars $v_\infty/v_{\rm esc}({\rm core})$ has a mean value of about 0.8. The star that deviates most strongly is WR 2 (WN 2b), which has the highest terminal velocity of 3100 km s^{-1}. The H-rich WNL stars, for which the stellar radius $R_{\rm evol}$ is not known, are omitted. Fig. 2b shows a slow increase of log $v_\infty/v_{\rm esc}({\rm core})$ with increasing luminosity from about 0.6 at log(L)=5.0 to 1.3 at log(L)=5.7. The two highly discrepant stars are WR 142 (WO 2) and WR 30a (WC 4/ WO 4) which both have a very high terminal velocity in excess of 4000 km s^{-1}. The WC stars with $Z < 0.50$ have lower values of $v_\infty/v_{\rm esc}({\rm core})$ than the stars with higher Z-values. This, and the high values of v_∞ of the WO stars, shows that the terminal velocity of the winds of WR stars increases with increasing Z.

Linear regression relations in which we excluded the above mentioned stars WR 2, WR 142 and WR 30a give the following results:

$$\log\ v_\infty/v_{\rm esc}({\rm core}) = 0.61 - 0.13(\pm 0.09)\log L + 0.30(\pm 0.77)\log Y \qquad (16)$$

for WN stars (excluding the H-rich WNL stars) with a standard deviation of 0.084 dex and

$$\log\ v_\infty/v_{\rm esc}({\rm core}) = -2.37 + 0.43(\pm 0.13)\log L - 0.07(\pm 0.27)\log Z \qquad (17)$$

for WC stars with $\sigma = 0.13$ dex. If the degree of ionization near the hydrostatic core of WO stars is higher than for WC stars (WO stars have a higher degree of ionization *in the wind*), then the effect of continuum radiation pressure would be higher and

Table 6. Parameters and mass-loss rates of WC stars

Star	Sp. Type	$\log L$ ($L_\odot$)	M_{WR} ($M_\odot$)	$T_{\rm evol}$ (K)	Y	Z	$\dot{M}$(obs) (10^{-5}) $M_\odot$ yr^{-1}	v_∞ (km s^{-1})	$\frac{\dot{M} v_\infty c}{L}$ η
142	WO 2	5.09	8.0	124 800	0.342	0.658	$0.39{:}^{e}$	5500	8.55
30a	WC 4/WO4+O4	5.29	10.3	129 500	0.418	0.582	$1.20{:}^{e}$	4500	13.60
144	WC 4	5.29	10.3	129 500	0.418	0.582	1.10^{N}	2440	6.76
9	WC 5+O 7	5.70	18.2	139 700	0.418	0.582	2.30^{N}	3030	6.88
111	WC 5	5.31	10.6	130 000	0.381	0.619	1.00^{N}	2415	5.82
114	WC 5	5.28	10.2	129 300	0.535	0.465	0.68^{e}	2000	3.50
15	WC 6	5.24	9.6	128 300	0.658	0.342	1.20^{N}	2325	7.94
23	WC 6	5.23	9.6	128 200	0.488	0.512	1.29^{e}	2280	8.45
30	WC 6+O 6-8	5.57	15.2	136 500	0.497	0.503	2.40^{e}	2400	7.56
48	WC 6+O 9.5I	5.47	13.1	133 900	0.497	0.503	$1.81{:}^{e}$	2060	6.22
146	WC 6+O	5.24	9.6	128 300	0.744	0.256	1.70^{N}	2700	13.10
154	WC 6	5.20	9.2	127 400	0.537	0.463	1.10^{e}	2050	6.97
14	WC 7	5.19	9.1	127 200	0.658	0.342	1.88^{e}	1980	11.70
42	WC 7+O 7V	5.23	9.5	128 100	0.497	0.503	1.28^{e}	1645	6.11
50	WC 7+abs	5.29	10.3	129 500	0.439	0.561	2.15^{e}	2370	12.90
79	WC 7+O 5-8	5.48	13.4	134 300	0.497	0.503	2.40^{N}	2270	8.76
86	WC 7+B0I	5.48	13.4	134 300	0.342	0.658	1.70^{N}	1800	4.92
93	WC 7+O 7-9	5.60	15.8	137 300	0.497	0.503	2.50^{N}	2290	7.05
137	WC 7+OB	5.48	13.4	134 300	0.497	0.503	2.95^{N}	1885	8.94
140	WC 7+O 4-5	5.69	18.1	139 600	0.407	0.503	6.30^{N}	2800	17.60
11	WC 8+O 8-9III	5.00	7.2	122 700	0.627	0.373	1.08^{N}	1415	7.49
113	WC 8+O 8-9IV	5.46	13.0	133 800	0.585	0.415	2.44^{e}	1890	7.80
135	WC 8	5.30	10.5	129 900	0.651	0.349	1.50^{N}	1405	5.13
65	WC 9	5.20	9.2	127 500	0.585	0.415	1.50^{N}	1040	4.78
70	WC 9+B 0I	5.20	9.2	127 500	0.585	0.415	2.32^{e}	1250	8.88
81	WC 9	5.20	9.2	127 500	0.585	0.415	1.60^{N}	900	4.41
95	WC 9	5.11	8.1	125 300	0.585	0.415	2.37^{e}	1040	9.33
103	WC 9	5.20	9.2	127 500	0.585	0.415	2.40^{N}	1190	8.75
104	WC 9+abs	5.04	7.6	123 800	0.585	0.415	1.14^{e}	1180	5.96
112	WC 9	5.20	9.2	127 500	0.585	0.415	1.10^{N}	1100	3.71

Notes: $\dot{M}^{N}$: mass loss rates are from Nugis et al. (1998),
$\dot{M}^{e}$: mass loss rates have been determined from the strength of emission lines using the formulae from Nugis et al. (1998).

the effective escape velocity would be smaller for the WO stars compared to the WN and WC stars. This would result in an even higher value of $v_\infty/v_{\rm esc}$ for the WO stars.

The small values of σ of the fits of Eqs. (16) and (17) indicate that these two relations can be used to predict v_∞ quite accurately. Alternatively, these relations can be used to derive the effective escape velocity of the hydrostatic core from the observed terminal velocities.

8. The dependence of momentum transfer efficiency η on the stellar parameters

The momentum transfer efficiency η indicates the efficiency of the momentum transfer from the radiation to the wind.

Figs. 3a and 3b show the dependence of log η on L for the WN and the WC stars. Both figures show a large scatter, which indicates that η is not a simple function of luminosity only. Fig. 3a shows that for WN stars η depends on the helium abundance, because the points referring to stars with $Y < 0.85$ are concentrated on the lower right-hand corner. A least square linear regression of the data gives the following relation

$$\log\,\eta = -2.82 + 0.64(\pm 0.23)\log L + 2.84(\pm 1.02)\log Y \quad (18)$$

for WN stars, with a standard deviation of 0.25. This relation shows that η depends mainly on composition and only weakly on L: stars with a higher Helium abundance have a higher η.

Fig. 3b for WC stars also shows a large scatter. The linear regression relation, excluding the WO stars WR142 and WR30a, is

$$\begin{aligned}\log\,\eta = & -0.60 + 0.31(\pm 0.18)\log L \\ & + 0.65(\pm 0.42)\log Y \end{aligned} \quad (19)$$

for WC stars, with a standard deviation of 0.155.

We conclude that WN stars have values of η in the range of 0.2 to 1.0 dex, and WC stars have η in the range of 0.6 to 1.2 dex. The value of η increases with increasing L and Y for both the WN and the WC stars. Although the standard deviations of the

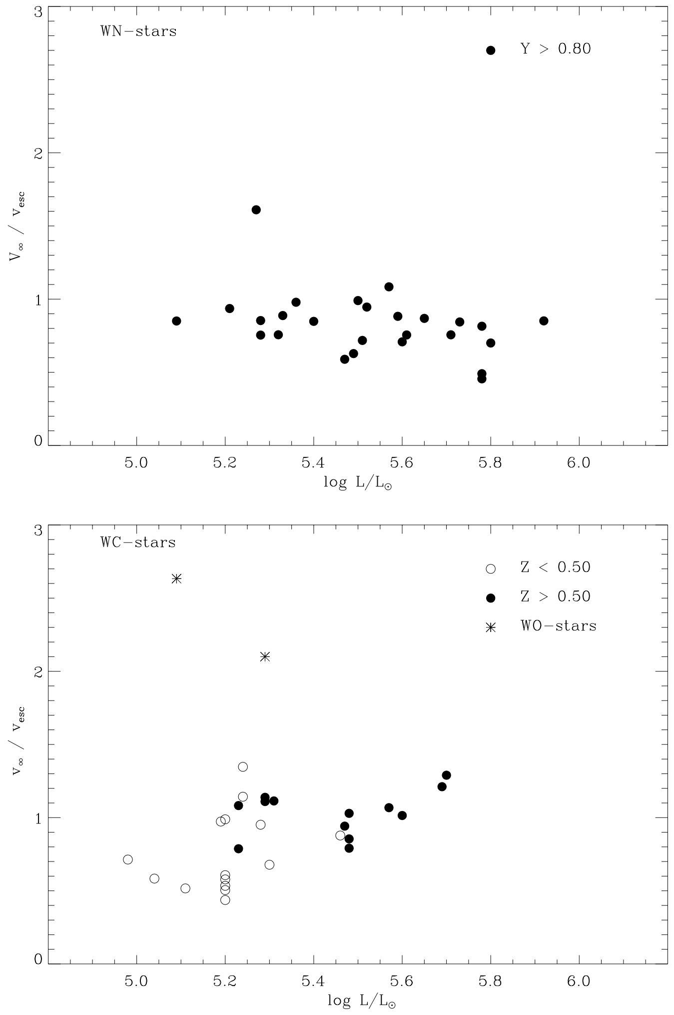

Fig. 2. The ratio $v_\infty/v_{\rm esc}({\rm core})$ of the H-poor WN stars (top panel) and the WC stars (bottom panel) as a function of L. The data show that this ratio is nearly constant with increasing L for WN stars and increases with the luminosity for WC stars. The strongly deviating points are from the hottest stars (earliest spectral types): WR 2 (WN 2b) in Fig. a. and the two WO stars WR 142 (WO2) and WR 30a (WC/O4+O4), indicated by an asterisk, in Fig. b. The scatter is large. The ratio depends on L and on the chemical composition, as indicated by different symbols for different ranges of Y or Z. The least square relations are given in the text.

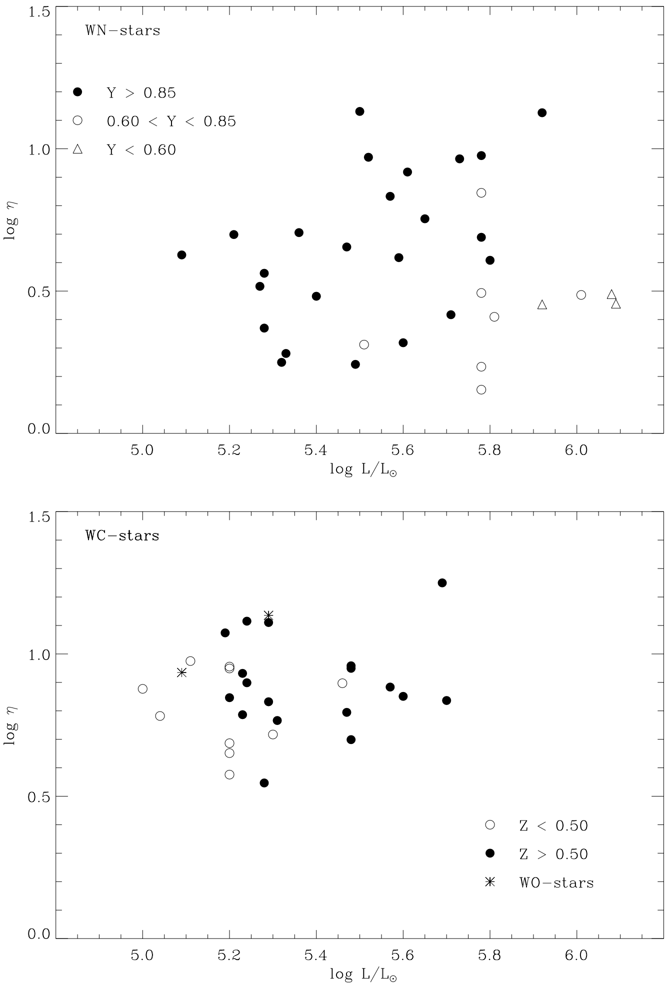

Fig. 3. The logarithmic momentum transfer efficiency η of the WN stars (top panel) and the WC stars (bottom panel) as a function of L. The two WO stars are indicated by an asterisk. The linear regression relations are given in the text.

regression relations are small, the coefficients that describe the dependence on composition are rather uncertain.

9. The dependence of the mass-loss on luminosity and composition

Figs. 4a and 4b show the dependence of $\dot{M}$ on luminosity for WN and WC stars respectively. Different symbols indicate different abundances. Fig. 4a shows that $\dot{M}$ increases with increasing L. The four stars with log(L)>5.9 and $Y < 0.85$ are the massive H-rich WN stars WR 22 (WN7 +OB), WR 24 (WN6), WR 25 (WN6) and WR 87 (WN7+abs). Including these H-rich stars, we derived the following regression relation for WN stars

$$\log\,\dot{M} = -13.60 + 1.63(\pm 0.21)\,\log L + 2.22(\pm 0.63)\log Y \tag{20}$$

with a standard deviation of 0.20 dex. The mass-loss rate increases with luminosity and with helium abundance.

Fig. 4b also shows a trend of $\dot{M}$ increasing with L for the WC stars, but the scatter is large. The star with the lowest mass-loss rate is WR 142 (WO2). The linear regression relation for the WC stars, excluding the two WO stars, is

$$\log\,\dot{M} = -8.30 + 0.84(\pm 0.17)\log L + 2.04(\pm 1.37)\log\,Y + 1.04(\pm 1.16)\log\,Z \tag{21}$$

with a standard deviation of 0.14 dex. The possible deviation in the values of Y and Z for individual stars from the adopted mean values of the WC subtypes (Sect. 4.2), is responsible for a significant fraction of the scatter. The two WO stars fit this relation within 0.08 dex, suggesting that the mass-loss rates of WO stars follow the same trend as that of the WC stars, although the terminal velocities of the WO stars are much higher than those of the WC stars.

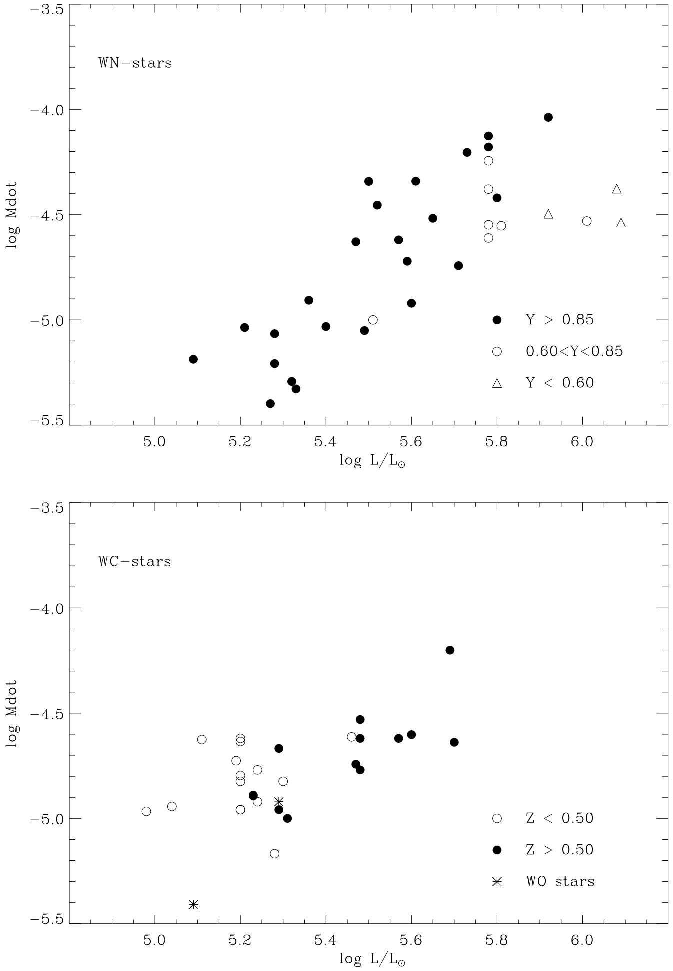

Fig. 4. The mass-loss rates in $M_\odot\ \mathrm{yr}^{-1}$ of WN stars (upper figure) and WC stars (lower figure) as a function of luminosity. Different symbols indicate different ranges in abundance. $\dot{M}$ increases with L, with Y and with Z (mainly C-abundance). The two WO stars are indicated by an asterisk. The linear regression relations are described in the text.

The relations (20) and (21) are only valid within the range of parameters of the program stars. For instance, if we would apply Eq. (21) of the WC stars, which have $0.26 < Z < 0.66$ in our sample, to the WN stars with $Z = 0.017$, the resulting predicted mass-loss rates of the WN stars would be an order of magnitude too small.

The full dependence of the mass-loss rates of WR stars on stellar parameters and composition can be found by combining both samples of WN and WC stars. This yields

$$\log\ \dot{M}\ =\ -11.00\ + 1.29(\pm 0.14)\ \log L \\ +1.73(\pm 0.42) \log Y\ + 0.47(\pm 0.09) \log Z \qquad (22)$$

for all stars, WN plus WC plus two WO stars, with a standard deviation of 0.19 dex. This equation shows that the mass-loss rate increases with luminosity as $\sim L^{1.3}$ with helium abundance as $\sim Y^{1.7}$ and with metallicity (mainly C in WC stars) $\sim Z^{0.5}$.

Several authors have assumed that the mass-loss rates of WR stars depend on their mass (e.g. Langer 1989a), following this original suggestion by Abbott et al. (1986). The data in Fig. 4a indeed suggest such a trend, because $\dot{M}$ increases with L and the luminosity of WR stars depends on the mass, as given in Eq. (5). Therefore we have also derived the relation between $\dot{M}$ and mass. The linear regression for the WN stars, including the four massive H-rich stars, is

$$\log\ \dot{M}\ =\ -5.99\ + 1.06(\pm 0.22)\ \log M \qquad (23)$$

with a standard deviation of 0.28 dex. For the WC stars the linear regression relation is

$$\log\ \dot{M}\ =\ -5.93\ + 1.13(\pm 0.26)\ \log M \qquad (24)$$

with $\sigma = 0.15$. For the full sample of WN plus WC stars, we find

$$\log\ \dot{M}\ =\ -5.73\ + 0.88(\pm 0.14)\ \log M \qquad (25)$$

with a standard deviation of 0.23 dex. These relations ignore the fact that the mass-loss rates increase with Y and Z, as we have shown before, so they have a larger standard deviation.

9.1. Comparison between predicted and observed mass-loss rates

Figs. 5a, 5b and 5c show the comparison between the mass-loss rates predicted by Eqs. (20) for the WN stars, (21) for the WC stars and (22) for the WN plus WC stars, and the observed rates. The figures show that the correlation is quite good, especially for the WC stars. It is interesting to see in Fig. 5c that there is no systematic shift between the WN and the WC stars, except maybe that the predicted rates of the stars with the highest observed mass-loss rates, $\log \dot{M} > -4.4$, are slightly too small by about 0.2 dex.

10. Comparison with mass-loss rates used in evolutionary calculations

Maeder (1991) and Maeder & Meynet (1994) assumed in the evolutionary calculations of the WR stars that the mass-loss rates of these stars vary as follows:

- for H-rich WN stars (called WNL by Maeder) he adopted a mass-loss rate of $4\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$.
- for H-free WN stars (called WNE by Maeder) and for WC stars he adopted the rates used by Langer (1989b)

  $$\log\ \dot{M}\ =\ -7.10(\pm 0.11)\ +\ 2.5\ \log\ M/M_\odot. \qquad (26)$$

  In this context H-free means $X < 0.05$, i.e. $Y + Z > 0.95$. We compare these adopted mass-loss rates with those derived in this paper.

The mean mass-loss rates of our sample of H-rich WN stars with $X > 0.05$ is $\log \dot{M} = -4.48 \pm 0.26$. This is only slightly smaller than the value of $\log \dot{M} = -4.40$, adopted by Maeder (1991).

In Fig. 6 we plot the dependence of $\dot{M}$ on mass of our H-poor ($X < 0.05$) program stars and we compare it with the relation (26) adopted by Langer (1989b) and Maeder (1991). The figure

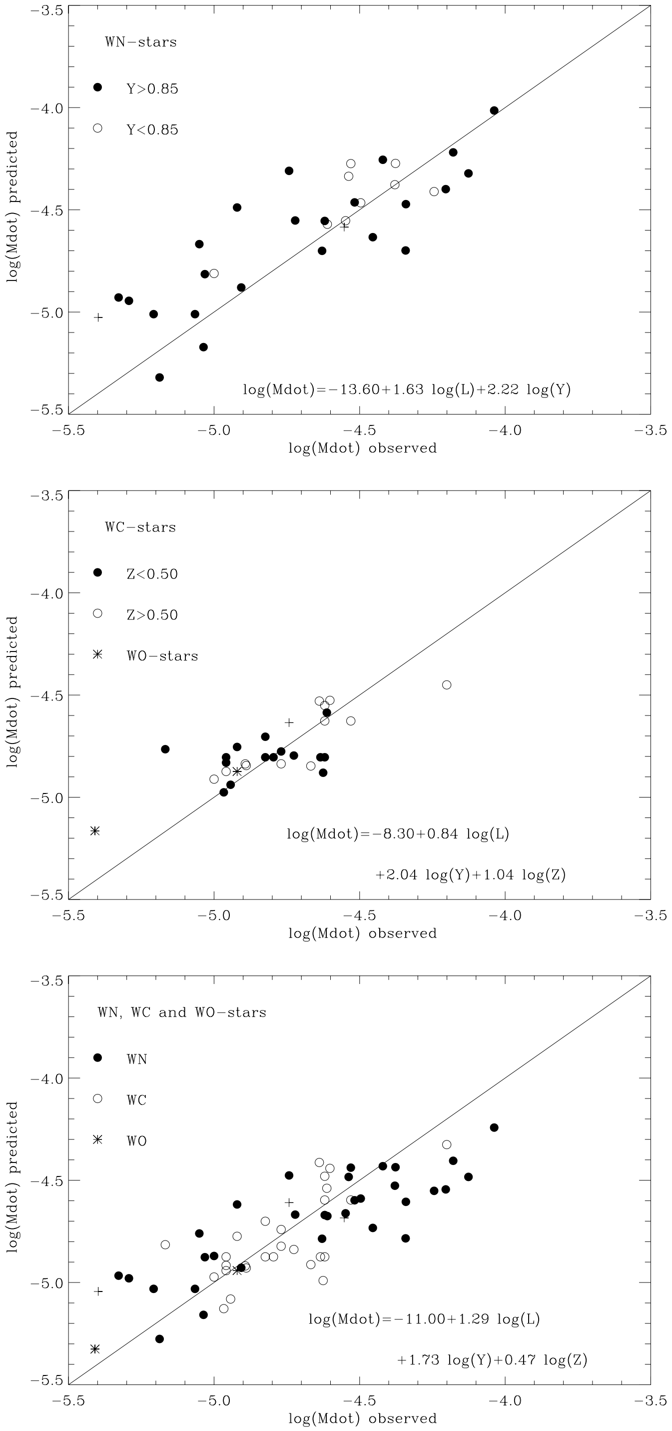

Fig. 5. The comparison between the observed and predicted mass-loss rates for WN stars (upper panel), WC-stars (middle panel) and WN+WC stars (lower panel). The adopted relations are indicated. In the upper two figures the symbols refer to different abundances, as indicated in the panels. In the lower figure open and filled symbols and stars indicate respectively WN, WC and WO stars. Plusses indicate uncertain values of the observed mass-loss rates.

shows that Eq. (26) overestimates the mass-loss rates on average by about 0.3 dex. The discrepancy is larger for the WN stars than for the WC stars.

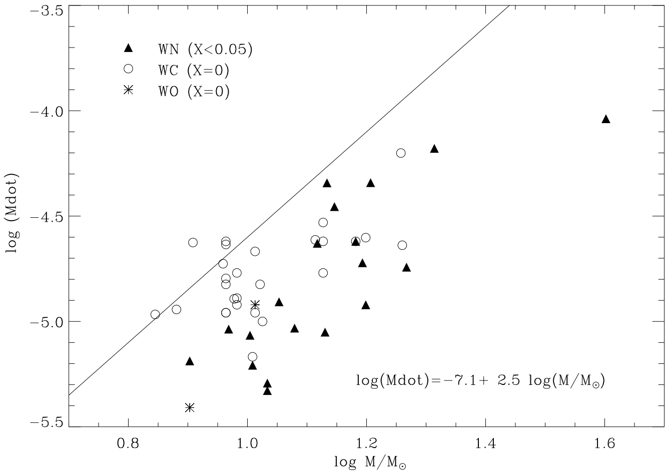

Fig. 6. The mass-loss rates in $M_\odot$ yr^{-1} of H-poor stars as a function of the stellar mass. Different symbols indicate different types of WR stars. The full line is the relation adopted in the evolutionary calculations by Maeder (1991).The star with the highest mass is WR 47 (WN6+O5) with $X = 0.047$.

In the frequently used evolutionary models of Meynet et al. (1994) the adopted mass-loss rate of WR stars is even higher than adopted by Langer (1989b) and Maeder and Meynet (1994) by a factor two. The mean discrepancy between our mass-loss rates and those adopted by Meynet et al. (1994) is 0.6 dex for WNE and WC stars and 0.3 dex for WNL stars.

11. Summary and discussion

In this paper we studied the dependence of the wind characteristics of WR stars on the stellar parameters and the composition. To this purpose we collected and rederived mass-loss rates and luminosities of 34 WN stars and 30 WC stars. The mass-loss rates are corrected for clumping, as described by Nugis et al. (1998).

Using these data we determined the dependence of the terminal velocity, v_∞, the momentum transfer efficiency, η, and the mass-loss rates on basic stellar parameters. The ratio $v_\infty/v_{\rm esc}({\rm core})$ of the WN stars is about constant at a mean value of 0.8, but the data for WC stars show a large scatter. We found that this ratio depends more strongly on composition than on luminosity (see Eq. (17)). The very high values of the WO stars confirm this trend. The high accuracy of the linear regression relations (16) and (17) shows that the effective escape velocities at the hydrostatic core can be derived quite accurately from the observed terminal velocity of the winds. Using the relations between mass, radius and luminosity, predicted by stellar evolution theory, it may be possible to derive the luminosity of the WR stars. The uncertainty of $\sigma = 0.084$ dex in the the ratio $v_\infty/v_{\rm esc}$ of the WN stars and 0.11 dex for WC stars suggests that the mass can be determined with an accuracy of about 0.1 dex, and from that the luminosity with an uncertainty 0.25 dex.

We found that the mass-loss rates of WR stars depend on luminosity L and are also quite strongly dependent on Y and Z. The linear regression relations for WN stars, WC stars and for the combined sample of WN and WC stars are given in Eqs. (20), (21) and (22). They show that $\dot{M}$ of the full sample of stars increases with luminosity as $\sim L^{1.3}$, with helium abundance as $\sim Y^{1.7}$ and with the metal abundance (mainly C in WC stars) $\sim Z^{0.5}$.

The momentum transfer efficiency $\eta = \dot{M}v_\infty/(L/c)$ of WR stars is found to lie in the range of $1.4 < \eta < 17.6$ with the average value of 64 stars being 6.2. This is substantially lower than the values obtained from wind models which neglect the clumping.

It is generally assumed that multiple scattering in a wind with an ionization stratification can provide the necessary driving force in the outer winds of WR stars if η is smaller or about 10. The reduced values of η derived in this paper might thus be in agreement with photon scattering as the main driver of the WR winds. However, hydrodynamic simulations carried out by Schmutz (1997) for the star WR 6 (HD 50896) showed that there is still insufficient driving in the inner wind regions where $v < v_{\rm esc}$. So there might be a need for some sort of "two-stage" driving process, in which some mechanism actually initiates the mass-loss, with radiative forces taking over to drive the winds to high terminal speeds (Glatzel et al. 1993, Owocki & Gayley 1999). Lamers & Casinelli (1999) and Vink et al. (2000a) have argued that for radiation driven winds the mass loss rate is set by the radiation pressure in the subsonic region and the terminal velocity depends on the radiation pressure in the supersonic region. (See also the discussion by Lamers et al. 2000a).

The dependence of mass-loss on the helium abundance and on the CNO abundances might point to two effects:

(1) The driving process deep in the wind might be dependent on abundance and possibly also on pulsations.
(2) The abundance of helium and CNO strongly affects the temperature and ionization structure in the wind, and hence the ionization and excitation structure of the elements that provide most of the radiative driving (CNO and the iron group elements).

The mass-loss rates derived in this paper are smaller than those adopted in evolutionary calculations. The difference is small, only about 0.1 dex, for the WN stars with $X > 0.05$, but it is significant for the H-poor WN stars and the WC stars. Following a suggestion by Abbott et al. (1986), Langer (1989b), Maeder (1991) and Maeder & Meynet (1994) assumed in the evolutionary calculations that the mass-loss varies only with mass, but not with composition, as described in Sect. 10.

Eq. (26) overestimates the mass-loss rates by about a factor two on average. The overestimate depends on the chemical composition and it is smaller for the WC stars, viz. about 0.2 dex, than for the H-poor WN stars, viz. 0.5 dex (see Fig. 6). The difference between our new mass loss rates and the higher rates adopted in the evolutionary calculations of Meynet et al. (1994) is even larger: about 0.6 dex for WNE and WC-stars, and 0.3 dex for WNL stars.

This has important consequences:

1. The lower mass-loss rates of the WR stars, compared to previous estimates, facilitate the formation of black holes as end stages of the evolution of massive stars (see the discussion in Wellstein & Langer 1999, and in de Koter 2000).
2. On the other hand, the lower mass-loss rates may also pose problems. Maeder (1991) showed that the evolutionary calculations predict the observed ratios of WN/WC/WO stars, if the high mass-loss rates of Langer's formula are adopted.
3. This then leaves us with the question: do WR stars suffer additional mass-loss, apart from the quiescence mass-loss that we derived from the observations, e.g. in the form of outbursts? The presence of nebulae around many WR stars may provide evidence for short phases of high mass-loss rates during the WR phase (Marston 1999).
4. The ratio of WN/WC stars may also be affected by rotation-induced mixing. Lamers et al. (2000b) have derived evidence for rotation-induced mixing in OB stars, from the study of the chemical composition of circumstellar nebulae.

It would be very interesting to calculate evolutionary models of WR stars with our new mass-loss rates, to see if additional mass-loss or mixing is needed to explain the observed evolutionary and nebular properties of WR stars.

(This is the first paper in a series on the mass-loss rates of hot stars. The next papers deals with the predicted mass-loss rates of O, B and A stars (Vink et al. 2000a), with the predicted dependence of mass loss on metallicity (Vink et al. 2000b) and with the observed mass-loss rates of O, B and A stars (Lamers et al. in preparation).

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Note added in press: After this paper was accepted, we received a preprint of the paper "WN stars in the LMC: parameters and atmospheric abundances" by W.R. Hamann & L. Koesterke (A&A, accepted) in which the authors claim that their new determinations of the mass-loss rates and stellar parameters of WN stars in the LMC do not agree with our relation (20) derived for Galactic WN stars.

(1) Hamann & Koesterke (hereafter called HK) use a "mean" clumping factor of $D = 4$ for all WN stars. This can result in errors of a factor two or three in $\dot{M}$, compared to the method for deriving clumping-corrected mass-loss rates (Nugis et al. 1998) that was adopted in our paper.

(2) Some stars of the HK sample may have an unrealible luminosity, as can be concluded from the large difference in bolometric correction of stars of the same subtype. This suspicion is supported by the large difference in luminosity of the star Brey 29 derived from the spectroscopic studies by HK and Crowther et al. (1995).

(3) If the luminosities of the LMC stars of the HK sample are derived from the absolute visual magnitudes, given by HK, and the mean bolometric corrections for their spectral types (our Table 3), then the LMC WN stars of the HK sample fit our relation

(20) on the average quite well. The mean difference in log $\dot{M}$ between the values of HK and our predictions by Eq. (20) is only -0.02 with a scatter of 0.18 (1 σ).

Appendix A: sources for masses M_{WR}

Below are given the sources of spectroscopic determinations of the masses of WR components in binary systems.

WR 9: q_M is taken to be the mean of the estimates of Niemela et al. (1984) and Niemela (1995) ($\overline{q} = 0.455$). With $M_{O7V} \approx 37.7 M_\odot$ (Vacca et al. 1996) we obtain $M_W = 17.2 M_\odot$.

WR11: with the angular size of the semimajor axis from the interferometric observations of Hanbury-Brown et al. (1970) and HIPPARCOS parallax we will obtain that $M_W \approx 7 M_\odot$ from the formula $M_W/M_\odot = (a''/p)^3/P^2/(1 + K_W/K_O)$, where a'' and p are in mas, orbital period P is in years and radial velocity semi-amplitudes (K) of Wolf–Rayet and O components are in km s^{-1} (P and K values are taken from the paper of Schmutz et al. 1997). The adopted mass is the mean of the estimates of Schaerer et al. (1997) and Schmutz et al. (1997).

WR 21: Niemela & Moffat (1982).

WR 22: Schweickhardt et al. (1999);.

WR 30: $q_M = 0.47 \pm 0.07$ comes from Niemela et al. (1983) and $M_{O7V} \approx 37.7 M_\odot$ from Vacca et al. (1996).

WR 30a: $q_M = 0.15$ comes from Niemela (1995) and $M_{O4V} = 68.9 M_\odot$ from Vacca et al. (1996).

WR 31: $q_M = 0.44 \pm 0.03$ (Niemela et al. 1985) and $M_O = 30.8 M_\odot$ (Vacca et al. 1996).

WR 42: $M_W \sin^3 i = 3.6 M_\odot$ (Davis et al. 1981) and i is adopted to be the mean of photometric and polarimetric estimates (Lamontagne et al. 1996) – $\overline{i} = 41.9°$.

WR 47: $M_W \sin^3 i = 40 \pm 4 M_\odot$ (Niemela et al. 1980) and $i = 70° \pm 4°$ (Moffat et al. 1990).

WR 70: $q_M = 0.45$ according to Niemela (1995) and $q_M = 0.2$ according to Golombek as cited by Smith & Maeder (1989). With the mean of q_M and $M_{B0I} \approx 30 M_\odot$ (Lang (1991) with the correction according to Lanz et al. 1996), we will get that $M_W \approx 9.8 M_\odot$.

WR 79: q_M is taken to be the mean of the estimates of Seggewiss (1974) and Lührs (1997) ($\overline{q} = 0.34$). With $M_{O6.5V} \approx 41 M_\odot$ (Vacca et al. 1996) we obtain $M_W = 13.9 M_\odot$.

WR 97: the mass is adopted to be the mean of the estimates obtained with the $q_M = M_W/M_O$ and $M_{O7V} = 37.7 M_\odot$ (Vacca et al. 1996) and with the $M_W \sin^3 i$ and $i = 85.°4$ (Lamontagne et al. 1996). The values of q_M and $M_W \sin^3 i$ are adopted to be the means of the spectroscopic studies of Niemela (1995) and Niemela (1982).

WR 113: with the spectroscopic orbit parameters from Niemela et al. (1996) and Massey & Niemela (1981) we will obtain that $M_W \sin^3 i \approx 11.9 M_\odot$ (P and e are from Niemela et al. (1996) and $K_{\rm abs}$ and $K_{\rm em}$ are the means of the estimates obtained in the cited papers (C III/C IV lines only are used in determining $K_{\rm em}$). The value of i is adopted to be the mean of photometric and polarimetric estimates (Lamontagne et al. 1996) – $\overline{i} = 75°$.

WR 127: $q_M = M_W/M_O = 0.465 \pm 0.1$ (Massey 1981) and $M_O = 23.3 M_\odot$ (Vacca et al. 1996).

WR 133: Bertrand (1995) estimated that $M_W \approx 15 M_\odot$ and Smith et al. (1994) found that $M_W \leq 10 M_\odot$. We adopted that $M_W = 10 M_\odot$.

WR 139: $M_W \sin^3 i = 8.8 \pm 0.4\ M_\odot$ (Marchenko et al. 1994) and $i = 78.7° \pm 0.5$ (Robert et al. 1990).

WR 140: with $q_M = 0.37$ according to Annuk (1995) and $M_{O4.5V} = 62.3 M_\odot$ (Vacca et al. 1996), we will get $M_W \approx 23.1 M_\odot$.

WR 141: The adopted mass is the mean of mass/age estimate (13 $M_\odot$ for binary evolution using the graphs of Smith et al. 1994) and of the spectroscopic orbit solutions of Grandchamps & Moffat (1991) (24 $M_\odot$) and of Marchenko et al. (1998) (45 $M_\odot$).

WR 151: $M_W \sin^3 i = 17.8 \pm 1.4\ M_\odot$ (Lewis et al. 1993) and $i \approx 64°$: (the mean of Lipunova & Cherepashchuk (1982a) and Schulte–Ladbeck & van der Hucht (1989) estimates).

WR 153: $M_W \sin^3 i \approx 13 M_\odot$ (Massey 1981) and $i = 78° \pm 2°$ (St-Louis et al. 1988).

WR 155: $M_W \sin^3 i = 14.4 \pm 1.1 M_\odot$ (Marchenko et al. 1995) and $i \approx 72° \pm 6°$ (the mean of Drissen et al. (1986) and Lipunova & Cherepashchuk (1982b) estimates).

Appendix B: determination l_v^W for WR stars in binaries

For binaries, we have determined the fraction of the total light emitted by the WR component in the v-band (l_v^W) using the strength (equivalent widths) of WR emission ($l_{\rm em}$) and O absorption ($l_{\rm abs}$) lines relative to single stars and absolute visual magnitudes of the components ($l_{\rm em}^W = {\rm EW(binary)/EW(expected)}$, $l_{\rm abs}^O = {\rm EW(binary)/EW(expected)}$, $M_v \propto -2.5 \log l_v$, $M_v^W - M_v^O = -2.5 \log{(l_v^W/l_v^O)}$). Smith et al. (1996) derived for WN subtypes the relationships EW(λ5411) = a FWHM(λ4686)+b, where EW is the expected equivalent width of the emission line, FWHM is the full width of the line at half measure (at half of the peak intensity) and a, b are the constants for a certain subclass. Smith et al. used single Galactic and LMC WN stars for deriving the constants a, b. The ratio EW(binary star)/EW(expected) gives the fractional luminosity (brightness) of the WN component in the continuum near the central line wavelength. We used $l_{\rm em}$ of WN stars as derived by Smith et al. (1996). For deriving $l_{\rm em}$ for WC binaries we used the mean EW of the lines $\lambda\lambda$ 5806, 5696, 5590, 5470, 5411 and 4860 of single Galactic WC stars of different subclasses. Equivalent widths of the emission lines of WC stars have been adopted from the sources: Koesterke & Hamann et al. 1995, Torres-Dodgen 1985, Niedzielski & Nugis 1991, Smith et al. 1990.

The expected mean absolute magnitudes of the O type stars are adopted from the paper of Vacca et al. (1996) and of the

Table 7. The mean equivalent widths of the emission lines λλ 5806, 5696, 5590, 5470, 5411 and 4860 for different WC subtypes used for determination of the fractions $l_{\rm em}^W$.

Sybtype	$\overline{W_\lambda}$(5806)	$\overline{W_\lambda}$(5696)	$\overline{W_\lambda}$(5590)	$\overline{W_\lambda}$(5470)	$\overline{W_\lambda}$(5411)	$\overline{W_\lambda}$(4860)
WC 4	943	50	89	32	24	
WC 5	1370	94.5	75.5	49.3	37.1	22
WC 6	1028	167	49.1	49.6	43.3	22.2
WC 7	440	278	35.4	36.6	32.9	23.7
WC 8	302	456	22	28.2	24.8	19.5
WC 9	92.7	358	8.75	10.1	10.0	10.9

Comments:
The following stars are used for finding the mean values for equivalent widths – WC 4: WR19, WR38, WR144; WC 5: WR17, WR33, WR41, WR52, WR111, WR114, WR150; WC 6: WR13, WR14, WR15, WR23, WR45, WR132, WR154; WC 7: WR56, WR57, WR68, WR90; WC 8: WR53, WR60, WR135; WC 9: WR59, WR65, WR66, WR73, WR80, WR81, WR88, WR92, WR95, WR96, WR103, WR106, WR119, WR121.

WR stars according to Table 1 of the present paper. For the stars studied in the paper of Nugis et al. (1998) we used l_v^W from that paper, only for WR11, WR87 and WR89 we redetermined l_W. The estimates of l_v^W for new stars and redetermined values of WR11, WR87 and WR89 were found as follows:

WR 10 (WN5+(A)) – $l_{\Delta M_v} \approx 0.60$ is found by Turner (1981) but l_v^W appears to be about unity if to use the mean absolute magnitude for the WN 5 component. We use the mean of these estimates (0.80) in our study.

WR 11 (WC8+O8-9II) – l_v^W is adopted to be the mean of the estimates of Hanbury Brown et al. (1970), Conti & Smith (1972) and Brownsberger & Conti (1993).

WR 21 (WN5+O4–6) – $l_{\rm em} \approx 0.24$ is used for l_v^W.

WR 30 (WC6+O6–8) – $l_{\rm em} \approx 0.40$ is used for l_v^W.

WR48 (WC6+O9.5I) – $l_{\rm em} \approx 0.045$, $l_{\Delta M_v} \approx 0.065$ if to use the expected absolute visual magnitudes for the components ($M_v^W = -3.7$, $M_v^O = -6.6$). The mean value 0.055 is used for l_v^W.

WR 30a (WC4/WO4+O4) – $l_{\Delta M_v} \approx 0.08$ using the expected absolute visual magnitudes for the components ($M_v^W = -3.0$, $M_v^O = -5.65$) and this value is used for l_v^W.

WR 31 (WN4+O8V) – $l_{\rm em} \approx 0.28$ and $l_{\Delta M_v} \approx 0.274$ if to use the expexted absolute visual magnitudes for the components ($M_v^W \approx -3.7$, $M_v^O = -4.76$). The mean value 0.28 is adopted for l_v^W.

WR 48 (WC6+O9.5I) – $l_{\rm em} \approx 0.045$, $l_{\Delta M_v} \approx 0.065$ if to use the expected absolute visual magnitudes for the components ($M_v^W = -3.7$, $M_v^O = -6.6$). The mean value 0.055 is used for l_v^W.

WR 50 (WC7+abs) – $l_{\rm em} \approx 0.63$, $l(M_v^W = -4.8) > 1.0$. The mean value 0.82 is used for l_v^W.

WR 70 (WC9+B0I) – $l_{\Delta M_v} \approx 0.16$ is obtained by adopting the expexted absolute visual magnitudes for the components ($M_v^W = -4.8$, $M_v^B = -6.6$) and this value is used for l_v^W.

WR 87 (WN7+abs) – $l_{M_v^W} \approx 0.64$ is used for l_v^W ($M_v^W = -6.4$).

WR 89 (WN7+abs) – $l_{M_v^W} \approx 0.81$ is used for l_v^W ($M_v^W = -6.4$).

WR 97 (WN5b+O7) – $l_{\Delta M_v} \approx 0.3$ if to use the expected absolute visual magnitudes for the components ($M_v^W = -4.2$, $M_v^O = -4.98$) and this value is used for l_v^W.

WR 104 (WC9+abs) – $l(M_v^W = -4.8) > 1.0$ and therefore l_v^W is adopted to be approximately 1.0.

WR 151 (WN4+O5V) – $l_{\rm em} \approx 0.55$ and $l_{\Delta M_v} \approx 0.17$ if to use the expexted absolute visual magnitudes for the components ($M_v^W = -3.7$, $M_v^O = -5.43$). The mean value 0.36 is adopted for l_v^W;

WR 157 (WN5+(B1II)) – $l_{\Delta M_v} \approx 0.45$ according to Turner et al. (1983), $l_{\rm em} \approx 0.35$. The mean of these estimates (0.40) is used for l_v^W.

Appendix C: sources for v_∞

The terminal velocities of WR stars are adopted from the paper of Nugis et al. (1998). The terminal velocities for the stars which were not studied in that paper are from the following sources:

WR 2, WR 87 – Abbott et al. (1986) with correction $v_\infty = 0.74 v_\infty$ (Abbott et al.) according to Willis (1991),

WR 10, WR 23 - Prinja et al. (1990),

WR 14 – v_∞ is adopted to be the mean of the estimates of Eenens & Williams (1994) and Prinja et al. (1990),

WR 21, WR 157 – v_∞ is found from the width of the line λ 4686 (Smith et al. 1996) using WN 5 stars with known v_∞ for scaling ($v_\infty \approx 61.6$ FWHM, where FWHM is in Å),

WR 30, WR 48 – v_∞ is found from the width of the line λ 5806 (Smith et al. 1990) using the mean scaling rule for WC 6 as derived by us: $v_\infty \approx 48.0$ FWHM, where FWHM is in Å,

WR 31, WR 51 – v_∞ is found from the width of the line λ 4686 (Smith et al. 1996) using WN 4 stars with known v_∞ for scaling ($v_\infty \approx 58.5$ FWHM, where FWHM is in Å and v in km s^{-1}),

WR 50, WR 95, WR 104 – Torres-Dodgen (1985) with correction $v_\infty = 0.74 v_\infty$ (Torres-Dodgen) according to Willis (1991).

WR 67– Hamann et al. (1995),

WR 70, WR 97, WR 105, WR 114 – Eenens & Williams (1994),

WR 124 – Crowther et al. (1995),

WR 142, WR 30a – Kingsburgh et al. (1995),

WR 151 – Lewis et al. (1993),

WR 154 – Koesterke & Hamann (1995)

References

Abbott D.C., Bieging J.H., Churchwell E., Torres-Dodgen A.V., 1986, ApJ 303, 239

Annuk K., 1995, In: van der Hucht K.A., Williams P.M. (eds.) Wolf-Rayet Stars: Binaries, Colliding Winds, Evolution. Proc. IAU Symp. 163, Kluwer, p. 231

Ardeberg A., Virdefors B., 1982, A&A115, 347

Basu S., Antia H.M., 1995, MNRAS 276, 1402

Bertrand J.-F., 1995, In: van der Hucht K.A., Williams P.M. (eds.) Wolf-Rayet Stars: Binaries, Colliding Winds, Evolution. Proc. IAU Symp. 163, Kluwer, p. 248

Bessell M.S., Castelli F., Plez B., 1998, A&A333, 231

Brownsberger K.R., Conti P.S., 1993, In: Cassinelli J.P., Churchwell E.B. (eds.) Massive Stars: Their Lives in the Interstellar Medium. ASP Conf. Proc. 35, ASP, San Fransisco, p. 275

Cassinelli J.P., 1991, In: van der Hucht K.A., Hidayat B. (eds.) Wolf-Rayet Stars and Interrelations with Other Massive Stars in Galaxies. Proc. IAU Symp. 143, Kluwer, p. 289

Conti P.S., 1976, Mèm. Soc. R. Sci. Liége, 6e Sèr., Tome IX, 193

Conti P.S., Smith L.F., 1972, ApJ 172, 623

Conti P.S., Massey P., Vreux J.-M., 1990, ApJ 354, 359

Crowther P.A., 1993, PhD Thesis, University of London

Crowther P.A., Smith L.J., Hillier D.J., Schmutz W., 1995, A&A293, 427

Crowther P.A., Pasquali A., De Marco O., et al., 1999, A&A293, 427

Davis A.B., Moffat A.F.J., Niemela V.S., 1981, ApJ 244, 528

de Jager C., Nieuwenhuizen H., van der Hucht K.A., 1988, A&A72, 259

de Koter A., 2000, In: Lamers H.J.G.L.M., Sapar A. (eds.) Thermal and Ionization Aspects of Flows From Hot Stars: Observations and Theory. ASP Conf. Series 204, p. 441

De Marco O., Schmutz W., Crowther P.A., et al., 2000, A&Asubmitted

Dessart L., Crowther P.A., Hillier D.J., et al., 2000, MNRAS submitted

Drissen L., Moffat A.F.J., Bastien P., Lamontagne R., Tapia S., 1986, ApJ 306, 215

Eenens P.R.J., Williams P.M., 1992, MNRAS 255, 227

Eenens P.R.J., Williams P.M., 1994, MNRAS 269, 1082

Elliott J.R., 1998, A&A334, 703

Glatzel W., Kiriakidis M., Fricke K.J., 1993, MNRAS 262, 7

Grandchamps A., Moffat A.F.J., 1991, In: van der Hucht K.A., Hidayat B. (eds.) Wolf-Rayet Stars and Interrelations with Other Massive Stars in Galaxies. Proc. IAU Symp. 143, Kluwer, p. 258

Grevesse N., Noels A., Sauval A.J., 1996, In: Holt S.S., Sonneborn G. (eds.) Cosmic Abundances. ASP Conf. Ser. 99, ASP, San Fransisco, p. 117

Hamann W.-R., Koesterke L., 1998a, A&A333, 251

Hamann W.-R., Koesterke L., 1998b, A&A335, 1003

Hamann W.-R., Koesterke L., Wessolowski U., 1995, A&A299, 151

Hanbury-Brown R., Davis J., Herbinson-Evans D., Allen L.R., 1970, MNRAS 148, 103

Hillier D.J., 1996, In: Vreux J.M., Detal A., Fraipont-Caro D., Gosset E., Rauw G. (eds.) Wolf-Rayet Stars in the Framework of Stellar Evolution. Proc. 33rd Liége International Astrophysical Colloquium, Universitè de Liége, p. 509

Hillier D.J., Miller D.L., 1999, ApJ 519, 354

Howarth I.D., Schmutz W., 1995, A&A294, 529

Kingsburgh R.L., Barlow M.J., Storey P.J., 1995, A&A295, 75

Koesterke L., Hamann W.-R., 1995, A&A299, 503

Koesterke L., Hamann W.-R., Gräefener G., 1999, In: van der Hucht K.A., Koenigsberger G., Eenens P.R.J. (eds.) Wolf-Rayet Phenomena in Massive Stars and Starburst Galaxies. Proc. IAU Symp. 193, ASP, p. 248

Krelowski J., Maszkowski R., Stroebel A., 1986, A&A166, 271

Lamers H.J.G.L.M., Cassinelli J.P., 1999, Introduction to Stellar Winds. Cambridge University Press

Lamers H.J.G.L.M., Leitherer C., 1993, ApJ 412, 771

Lamers H.J.G.L.M., Maeder A., Schmutz W., Cassinelli J.P., 1991, ApJ 368, 538

Lamers H.J.G.L.M., Snow T.P., Lindholm D.M., 1995, ApJ 455, 269

Lamers H.J.G.L.M., Nugis, T., Vink, J.S., De Koter A., 2000a, In: Lamers H.J.G.L.M., Sapar A. (eds.) Thermal and Ionization Aspects of Flows From Hot Stars: Observations and Theory. ASP Conf. Series 204, p. 395

Lamers H.J.G.L.M., Nota A., Panagia N., Smith L.F., Langer N., 2000b, ApJ submitted

Lamontagne R., Moffat A.F.J., Drissen L., Robert C., Matthews J.M., 1996, AJ 112, 2227

Lang K.R., 1991, Astrophysical Data: Planets and Stars. Springer

Langer N., 1989a, A&A210, 93

Langer N., 1989b, A&A220, 135

Lanz T., Koter A., Hubeny I., Heap S.R., 1996, ApJ 465, 359

Lewis D., Moffat A.F.J., Matthews J.M., Robert C., Marchenko S.V., 1993, ApJ 405, 312

Lipunova N.A., Cherepashchuk A.M., 1982a, SvA 26, 45

Lipunova N.A., Cherepashchuk A.M., 1982b, SvA 26, 569

Lundström I., Stenholm B., 1984, A&A58, 163

Lührs S., 1997, PASP 109, 504

Maeder A., 1983, A&A 120, 113

Maeder A., 1991, A&A 242, 93

Maeder A., Meynet G., 1994, A&A 287, 803

Marchenko S.V., Moffat A.F.J., Koenigsberger G., 1994, ApJ 422, 810

Marchenko S.V., Moffat A.F.J., Eenens P.R.J., Hill G.M., Grandchamps A., 1995, ApJ 450, 811

Marchenko S.V., Moffat A.F.J., Eenens P.R.J., 1998, PASP 110, 1416

Marston A.P., 1999, In: van der Hucht K.A., Koenigsberger G., Eenens P.R.J. (eds.) Wolf-Rayet Phenomena in Massive Stars and Starburst Galaxies. Proc. IAU Symp. 193, ASP, p. 306

Massey P., 1981, ApJ 244, 157

Massey P., 1984, ApJ 281, 789

Massey P., Niemela V.S., 1981, ApJ 245, 195

Meynet G., Maeder A., Schaller G., Schaerer D., Charbonnel C., 1994, A&AS103, 97

Moffat A.F.J., Drissen L., Robert C., et al., 1990, ApJ 350, 767

Niedzielski A., Nugis T., 1991, A&AS91, 273

Niemela V.S., 1982, In: de Loore C., Willis A.J. (eds.) Wolf-Rayet Stars: Observations, Physics, Evolution. Proc. IAU Symp. 99, Reidel, Dordrecht, p. 299

Niemela V.S., 1995, In: van der Hucht K.A., Williams P.M. (eds.) Wolf-Rayet Stars: Binaries, Colliding Winds, Evolution. Proc. IAU Symp. 163, Kluwer, p. 223

Niemela V.S., Moffat A.F.J., 1982, ApJ 259, 213

Niemela V.S., Conti P.S., Massey P., 1980, ApJ 241, 1050

Niemela V.S., Mendez R.H., Moffat A.F.J., 1983, ApJ 272, 190

Niemela V.S., Massey P., Conti P.S., 1984, PASP 96, 549

Niemela V.S., Mandrini C.H., Mendez R.H., 1985, Rev. Mex. Astron. Astrofis. 11, 143

Niemela V.S., Morrell N.I., Barbá R.H., Bosch G.L., 1996, In: Niemela V.S., Morrell N.I. (eds.) Colliding Winds in Binary Stars. Rev. Mex. Astron. Astrofis. Conf. Series 5, p. 100
Nugis T., 1989, SvA Lett. 15, 19
Nugis T., 1991, In: Michaud G., Tutukov A. (eds.) Evolution of Stars: the Photospheric Abundance Connection. Proc. IAU Symp. 145, Kluwer, p. 209
Nugis T., Niedzielski A., 1995, A&A300, 237
Nugis T., Willis A.J., Crowther P.A., 1998, A&A333, 956
Owocki S.P., Gayley K.G., 1999, In: van der Hucht K.A., Koenigsberger G., Eenens P.R.J. (eds.) Wolf-Rayet Phenomena in Massive Stars and Starburst Galaxies. Proc. IAU Symp. 193, ASP, p. 157
Prinja R.K., Barlow M.J., Howarth I.D., 1990, ApJ 361, 607
Robert C., Moffat A.F.J., Bastien P., St-Louis N., Drissen L., 1990, ApJ 359, 211
Sapar A., Kuusik I., 1978, Publ. Tartu Astrofüüs. Obs. 46, 71
Schaerer D., Maeder A., 1992, A&A263, 129
Schaerer D., Schmutz W., Grenon M., 1997, ApJ 484, L153
Schmutz W., 1997, A&A321, 268
Schmutz W., De Marco O., 1999, In: van der Hucht K.A., Koenigsberger G., Eenens P.R.J. (eds.) Wolf-Rayet Phenomena in Massive Stars and Starburst Galaxies. Proc. IAU Symp. 193, ASP, p. 147
Schmutz W., Vacca W.D., 1991, A&AS89, 259
Schmutz W., Schweickhart J., Stahl O., et al., 1997, A&A328, 219
Schulte-Ladbeck R.E., 1995, In: van der Hucht K.A., Williams P.M. (eds.) Wolf-Rayet Stars: Binaries, Colliding Winds, Evolution. Proc. IAU Symp. 163, Kluwer, p. 176
Schulte–Ladbeck R.E., van der Hucht K.A., 1989, ApJ 337, 872
Schweickhardt J., Schmutz W., Stahl O., Szeifert Th., Wolf B., 1999, A&A347, 127
Seaton M.J., 1979, MNRAS 187, 73P
Seggewiss W., 1974, A&A31, 211
Smith L.F., 1968, MNRAS 140, 409
Smith L.F., Hummer D.G., 1988, MNRAS 230, 511
Smith L.F., Maeder A., 1989, A&A211, 71
Smith L.F., Shara M.M., Moffat A.F.J., 1990, ApJ 358, 229
Smith L.F., Meynet G., Mermilliod J.-C., 1994, A&A287, 835
Smith L.F., Shara M.M., Moffat A.F.J., 1996, MNRAS 281, 163
St-Louis N., Moffat A.F.J., Drissen L., Bastien P., Robert C., 1988, ApJ 330, 289
Torres A.V., 1988, ApJ 325,759
Torres-Dodgen A.V., 1985, Ph.D. Thesis, JILA, Colorado
Torres-Dodgen A.V., Massey P., 1988, AJ 96, 1076
Turner D.G, 1981, AJ 86, 222
Turner D.G., 1982, In: de Loore C., Willis A.J. (eds.) Wolf-Rayet Stars: Observations, Physics, Evolution. Proc. IAU Symp. 99, Reidel, Dordrecht, p. 57
Turner D.G., Moffat A.F.J., Lamontagne R., Maitzen H.M., 1983, AJ 88, 1199
Vacca W.D., Garmany C.D., Shull J.M., 1996, ApJ 460, 914
van der Hucht K.A., 1999, In: van der Hucht K.A., Koenigsberger G., Eenens P.R.J. (eds.) Wolf-Rayet Phenomena in Massive Stars and Starburst Galaxies. Proc. IAU Symp. 193, ASP, p. 13
van der Hucht K.A., Conti P.S., Lundström I., Stenholm B., 1981, Space Sci. Rev. 28, 227
van der Hucht K.A., Hidayat B., Admiranto A.G., Supelli K.R., Doom C., 1988, A&A199, 217
Vink J.S., de Koter A., Lamers H.J.G.L.M., 2000a, A&Ain press
Vink J.S., de Koter A., Lamers H.J.G.L.M., 2000b, A&Asubmitted
Wellstein S., Langer N., 1999, A&A350, 148
Willis A.J., 1991, In: van der Hucht K.A., Hidayat B. (eds.) Wolf-Rayet Stars and Interrelations with Other Massive Stars in Galaxies. Proc. IAU Symp. 143, Kluwer, p. 265