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Polaris: astrometric orbit, position, and proper motion

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Abstract. We derive the astrometric orbit of the photo-center of the close pair α UMi AP (= α UMi Aa) of the Polaris multiple stellar system. The orbit is based on the spectroscopic orbit of the Cepheid α UMi A (orbital period of AP: 29.59 years), and on the difference $\Delta\mu$ between the quasi-instantaneously measured HIPPARCOS proper motion of Polaris and the long-term-averaged proper motion given by the FK5. There remains an ambiguity in the inclination i of the orbit, since $\Delta\mu$ cannot distinguish between a prograde orbit $(i = 50\overset{\circ}{.}1)$ and a retrograde one $(i = 130\overset{\circ}{.}2)$. Available photographic observations of Polaris favour strongly the retrograde orbit. For the semi-major axis of the photo-center of AP we find about 29 milliarcsec (mas). For the component P, we estimate a mass of 1.5 $\mathcal{M}_\odot$ and a magnitude difference with respect to the Cepheid of 6.5 mag. The present separation between A and P should be about 160 mas.

We obtain the proper motion of the center-of-mass of α UMi AP with a mean error of about 0.45 mas/year. Using the derived astrometric orbit, we find the position of the center-of-mass at the epoch 1991.31 with an accuracy of about 3.0 mas. Our ephemerides for the orbital correction, required for going from the position of the center-of-mass to the instantaneous position of the photo-center of AP at an arbitrary epoch, have a typical uncertainty of 5 mas. For epochs which differ from the HIPPARCOS epoch by more than a few years, a prediction for the actual position of Polaris based on our results should be significantly more accurate than using the HIPPARCOS data in a linear prediction, since the HIPPARCOS proper motion contains the instantaneous orbital motion of about 4.9 mas/year = 3.1 km/s. Finally we derive the galactic space motion of Polaris.

Key words: astrometry – stars: binaries: general – stars: variables: Cepheids

1. Introduction

Polaris (α Ursae Minoris, HR 424, HD 8890, ADS 1477, FK 907, HIP 11767) is a very interesting and important object, both from the astrophysical point of view and from the astrometric one. For astrophysics, the most remarkable feature of the multiple stellar system Polaris is the fact that its main component, namely α UMi A, is a Cepheid variable with a very unusual behaviour. In astrometry, Polaris is one of the most frequently and accurately observed objects, mainly because it is located so close to the North celestial pole and can be used for calibration purposes.

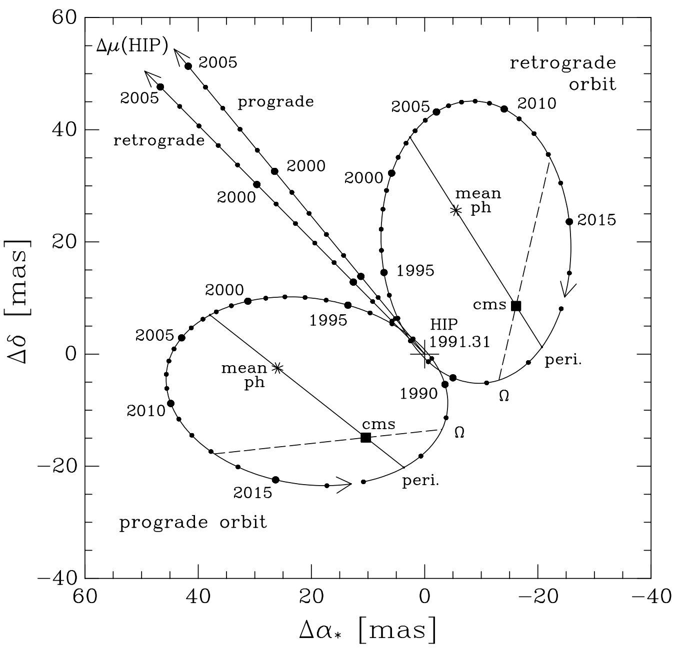

Fig. 1. Astrometric orbit (prograde or retrograde) of the photo-center of α UMi AP. The retrograde orbit is our preferred solution. For detailed explanations see Sect. 3.2.4

Up to now, the binary nature of Polaris was essentially neglected in ground-based fundamental astrometry, e.g in the FK5 (Fricke et al. 1988). This was justified by the limited accuracy reached by the meridian-circle observations. Now, the high-precision astrometric measurements carried out with the HIPPARCOS satellite (ESA 1997) require strongly to take into account the binary nature of Polaris in order to obtain an adequate astrometric description of α UMi. Similar procedures are required for many other binaries among the fundamental stars in order to be included properly into the Sixth Catalogue of

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Fundamental Stars (FK6; Part I: Wielen et al. 1999c; see also Wielen et al. 1998).

The main purpose of the present paper is to obtain a reliable astrometric orbit for Polaris, and to use this astrometric orbit for obtaining high-precision values for the position and proper motion of Polaris. This is done by combining the known spectroscopic orbit of Polaris with ground-based astrometric data given in the FK5 and with the HIPPARCOS results. Before doing so in the Sects. 3 and 4, we present in Sect. 2 an overview of the Polaris system.

2. Overview of the Polaris system

Polaris is a multiple stellar system, which consists of a close pair, α UMi A and α UMi P (= α UMi a), and a distant companion, α UMi B, and two distant components, α UMi C and α UMi D. We use here the designation 'P' for a close companion of A, which was used in the IDS and was adopted by the CCDM and by the HIPPARCOS Input Catalogue, rather than the traditional version 'a', which is used e.g. by the WDS and by CHARA).

2.1. The Cepheid α UMi A

The main component of Polaris is a low-amplitude Cepheid with a pulsational period of about 3.97 days. This period is increasing with time (e.g. Kamper & Fernie 1998). According to Feast & Catchpole (1997), α UMi A is a first-overtone pulsator (rather than a fundamental one), since α UMi A is too luminous for a fundamental pulsator, if they apply their period-luminosity relation (for fundamental pulsators) to Polaris. The fundamental period of α UMi A would follow as $P_0 = 5.64$ days, if the observed period is the first-overtone period P_1 (using the relation between P_1 and P_0 derived by Alcock et al. (1995) for Galactic Cepheids). An extraordinary property of α UMi A among the Cepheids is that the amplitude of its pulsation has been dramatically declined during the past 100 years, as seen both in the light curve and in the radial-velocity curve (Arellano Ferro 1983; Kamper & Fernie 1998, and other references given therein). The full amplitude was about $0.^{m}12$ in $m_{\rm V}$ and about 6 km/s in radial velocity before 1900, and seems now to be rather constant at a level of only $0.^{m}03$ in $m_{\rm V}$ and at 1.6 km/s in radial velocity. An earlier prediction (Fernie et al. 1993) that the pulsation should cease totally in the 1990s was invalid. A discussion of the HIPPARCOS parallax and of the absolute magnitude of α UMi A is given in the next Sect. 2.2.

2.2. The spectroscopic-astrometric binary α UMi AP

The Cepheid α UMi A is a member of the close binary system α UMi AP. This duplicity was first found from the corresponding variations in the radial velocity of α UMi A. However, the interpretation of the radial velocities of α UMi A in terms of a spectroscopic binary is obviously complicated by the fact that α UMi A itself is pulsating and that this pulsation varies with time. We use in this paper the spectroscopic orbit derived by Kamper (1996), which is based on radial velocity observations from 1896 to 1995. Kamper (1996) took into account changes in the amplitude of the pulsation and in the period of pulsation, but used otherwise a fixed sinusoid for fitting the pulsation curve. In an earlier paper, Roemer (1965) considered even 'annual' changes in the form of the pulsation curve. In Table 4, we list the elements of the spectroscopic orbit of A in the pair AP given by Kamper (1996, his Table III, DDO + Lick Data). The orbital period of α UMi AP is 29.59 $\pm$ 0.02 years, and the semi-amplitude is $K_{\rm A} = 3.72$ km/s. The value of $a_{\rm A} \sin i = 2.934$ AU corresponds to about 22 milliarcsec (mas), using the HIPPARCOS parallax.

Attempts to observe the secondary component α UMi P directly or in the integrated spectrum of α UMi AP have failed up to now. Burnham (1894) examined Polaris in 1889 with the 36-inch Lick refractor and found no close companion to α UMi A (nor to α UMi B). Wilson (1937) claimed to have observed a close companion by means of an interferometer attached to the 18-inch refractor of the Flower Observatory. Jeffers (according to Roemer (1965) and to the WDS Catalogue) was unable to confirm such a companion with an interferometer at the 36-inch refractor of the Lick Observatory. HIPPARCOS (ESA 1997) has not given any indication for the duplicity of Polaris. Speckle observations were also unsuccessful (McAlister 1978). All these failures to detect α UMi P directly are not astonishing in view of the probable magnitude difference of A and P of more than 6^{m} and a separation of A and P of less than $0.''2$ (see Sect. 3.2.5). Roemer and Herbig (Roemer 1965) and Evans (1988) searched without success for light from α UMi P in the combined spectrum of α UMi AP. From IUE spectra, Evans (1988) concluded that a main-sequence companion must be later than A8V. This is in agreement with our results for α UMi P, given in Table 5. A white-dwarf companion is ruled out by the upper limit on its effective temperature derived from IUE spectra and by considerations on its cooling age, which would be much higher than the age of the Cepheid α UMi A (Landsman et al. 1996).

After Polaris had become known as a long-period spectroscopic binary (Moore 1929), various attempts have been made to obtain an astrometric orbit for the pair α UMi AP. Meridian-circle observations were discussed by Gerasimovic (1936) and van Herk (1939). While van Herk did not find a regular variation with a period of 30 years, Gerasimovic claimed to have found such a modulation. However, the astrometric orbit of the visual photo-center of α UMi AP determined by Gerasimovich (1936) is most probably spurious, since he found for the semi-major axis of the orbit $a_{\rm ph(AP)} \sim 110$ mas, which is much too high in view of our present knowledge ($a_{\rm ph(AP)} = 29$ mas). More recent meridian-circle observations gave no indications of any significant perturbation. This is not astonishing in view of the small orbital displacements of the photo-center of AP of always less than $0.''04$. Long-focus photographic observations have been carried out at the Allegheny Observatory (during 1922–1964), the Greenwich Observatory, and the Sproul Observatory (during 1926–1956), mainly with the aim to determine the parallax of Polaris. The discussion of this material by Wyller (1957, Sproul data) and by Roemer (1965, Allegheny data) did not produce any significant results. The Allegheny plates were later remeasured and rediscussed by Kamper (1996), using his new spectroscopic orbital elements. Kamper also rediscussed the Sproul plates. While the Sproul data gave no relevant results for α UMi AP, the Allegheny data gave just barely significant results, such as $a_{\rm ph(AP)} = 19.5 \pm 6.5$ mas. For our purpose (see Sect. 3.2.3), the most important implication derived by Kamper (1996) from the Allegheny data is that the astrometric orbit of AP is most probably retrograde, not prograde.

In Sect. 3 we shall present a more reliable astrometric orbit of α UMi AP by combining ground-based FK5 data with HIPPARCOS results, using Kamper's (1996) spectroscopic orbit as a basis.

The HIPPARCOS astrometric satellite has obtained for α UMi AP a trigonometric parallax of $p_{\rm H} = 7.56 \pm 0.48$ mas, which corresponds to a distance from the Sun of $r_{\rm H} = 132 \pm 8$ pc. In the data reduction for HIPPARCOS, it was implicitely assumed that the photo-center of the pair AP moves linearly in space and time, i.e. a 'standard solution' was adopted. This is a fairly valid assumption, since the deviations from a linear fit over the period of observations by HIPPARCOS, about 3 years, are less than 1 mas (see Sect. 4.2). Hence the HIPPARCOS parallax obtained is most probably not significantly affected by the curvature of the orbit of AP. Nevertheless, it may be reassuring to repeat the data reduction of HIPPARCOS for α UMi, adopting the astrometric orbit derived here for implementing the curvature of the orbit of the photo-center of α UMi AP.

The mean apparent visual magnitude of the combined components A and P is $m_{\rm V,AP} = 1.982$ (Feast & Catchpole 1997). This agrees fairly well with the HIPPARCOS result (ESA 1997) $m_{\rm V,AP} = 1.97$. In accordance with most authors we assume that the reddening $E_{\rm B-V}$ and the extinction $A_{\rm V}$ of the Polaris system are essentially zero (e.g., Turner 1977, Gauthier & Fernie 1978), within a margin of ± 0.02 in $E_{\rm B-V}$ and ± 0.06 in $A_{\rm V}$. Using the HIPPARCOS parallax, we find for the mean absolute magnitude of AP $M_{\rm V,AP} = -3.63 \pm 0.14$. If we use our results of Table 5 for component P, i.e. $M_{\rm V,P} \sim +2.9$, and subtract the light of P from $M_{\rm V,AP}$, then the absolute magnitude of the Cepheid component A is $M_{\rm V,A} = -3.62 \pm 0.14$. Unfortunately, the pecularities in the pulsation of α UMi A are certainly not very favourable for using this nearest Cepheid as the main calibrator of the zero-point of the period-luminosity relation of classical Cepheids.

2.3. *The visual binary α UMi (AP)–B*

Already in 1779, W. Herschel (1782) discovered the visual-binary nature of Polaris. The present separation between AP and B is about $18.''2$. This separation corresponds to 2400 AU or 0.012 pc, if B has the same parallax as AP. Kamper (1996) has determined the tangential and radial velocity of B relative to AP. Both velocities of B agree with those of AP within about 1 km/s. Hence Kamper (1996) concludes that B is most probably a physical companion of AP, and not an optical component. The physical association between AP and B is also supported by the fair agreement between the HIPPARCOS parallax of AP ($r_{\rm H} = 132 \pm 8$ pc) and the spectroscopic parallax of B (114 pc, as mentioned below).

The spectral type of B is F3V. The magnitude difference between B and the combined light of AP is $\Delta m_{\rm V} = 6.61 \pm 0.04$ (Kamper 1996). Using $m_{\rm V,AP} = 1.98$, this implies for B an apparent magnitude of $m_{\rm V,B} = 8.59 \pm 0.04$. Adopting the HIPPARCOS parallax (and no extinction), we obtain for B an absolute magnitude of $M_{\rm V,B} = +2.98 \pm 0.15$. The standard value of $M_{\rm V}$ for an F3V star on the zero-age main sequence is $+3.3$. If we use this standard value for $M_{\rm V}$, we obtain for B a spectroscopic distance of $r = 114$ pc. Similar values of the spectroscopic distance were derived (or implied) by Fernie (1966), Turner (1977), and Gauthier & Fernie (1978). These authors were interested in the absolute magnitude (and hence in the distance) of B in order to calibrate the absolute magnitude of the Cepheid A. Now the use of the HIPPARCOS trigonometric parallax is, of course, better suited for this purpose.

The typical mass of an F3V star is $\mathcal{M}_{\rm B} = 1.5\,\mathcal{M}_\odot$. If we use for the masses of A and P the values adopted in Table 5 ($6.0 + 1.54\,\mathcal{M}_\odot$), we obtain for the triple system a total mass of $\mathcal{M}_{\rm tot} = 9.0\,\mathcal{M}_\odot$). We derive from $\rho_{\rm B-AP} = 18.''2$ and the statistical relation $a = 1.13\,\rho$ an estimate for the semi-major axis of the orbit of B relative to AP of $a_{\rm B-AP} \sim 21''$ or 2700 AU. From Kepler's Third Law, we get then an estimate of the orbital period of B, namely $P_{\rm B} \sim 50\,000$ years.

From the data given above, we can estimate the acceleration $g_{\rm AP}$ of the center-of-mass of the pair α UMi AP due to the gravitational attraction of α UMi B. If we project this estimate of $g_{\rm AP}$ on one arbitrarly chosen direction, we get for AP a typical 'one-dimensional' acceleration of about 0.003 (km/s)/century or 0.4 mas/century^2. Therefore, we should expect neither in the radial velocity nor in the tangential motion of AP a significant deviation from linear motion due to the gravitational force of B during the relevant periods of the observations used. For all present purposes, it is fully adequate to assume that the center-of-mass of the pair α UMi AP moves linearly in space and time. The same is true for the motion of B.

A modulation of the relative position of B with respect to the photo-center of AP with a period of about 30 years is not seen in the available observations of B. This is in accordance with our determination of the motion of the photo-center of AP with respect to the cms of AP, given in Table 7. The expected amplitude of the modulation is less than $0.''04$ and is obviously not large enough with respect to the typical measuring errors in the relative position of B.

The contribution of the orbital motion of the center-of-mass (cms) of AP, due to B, to the total space velocity of AP is of the order of a few tenth of a km/s. The expected value of the velocity of B relative to the cms of AP is of the order of 1 km/s.

2.4. *α UMi C and α UMi D*

In 1884 and 1890, Burnham (1894) measured two faint stars in the neighbourhood of α UMi AB. In 1890.79, the component C had a separation of $44.''68$ from A, and the component D $82.''83$.

Table 1. Mean proper motion of the photo-center of α UMi AP

		Prograde orbit				Retrograde orbit (Preferred solution)			
Quantity [mas/year]	System	$\mu_{\alpha*}$	m.e.	μ_δ	m.e.	$\mu_{\alpha*}$	m.e.	μ_δ	m.e.
$\mu_{\rm FK5}$	FK5	+ 38.30	0.23	– 15.20	0.35	+ 38.30	0.23	– 15.20	0.35
systematic correction		+ 3.20	0.94	– 1.53	0.66	+ 3.20	0.94	– 1.53	0.66
$\mu_{\rm FK5}$	HIP	+ 41.50	0.97	– 16.73	0.75	+ 41.50	0.97	– 16.73	0.75
μ_0	HIP	+ 41.05	0.58	– 15.05	0.45	+ 40.56	0.58	– 14.67	0.45
$\mu_{\rm FK5} - \mu_0$	HIP	+ 0.45	1.13	– 1.68	0.87	+ 0.94	1.13	– 2.06	0.87
$\mu_{\rm m}$	HIP	+ 41.17	0.50	– 15.49	0.39	+ 40.81	0.50	– 15.22	0.39
$\mu_{\rm H}$	HIP	+ 44.22	0.47	– 11.74	0.55	+ 44.22	0.47	– 11.74	0.55
$\Delta\mu = \mu_{\rm H} - \mu_{\rm m}$	HIP	+ 3.05	0.69	+ 3.75	0.67	+ 3.41	0.69	+ 3.48	0.67

According to the WDS Catalogue, the apparent magnitudes of C and D are $13\overset{m}{.}1$ and $12\overset{m}{.}1$.

The nature of the components C and D is unclear. The probability to find by chance a field star of the corresponding magnitude with the observed separation around α UMi A (galactic latitude $b = +26\overset{\circ}{.}46$ (Wielen 1974)) is of the order of 10 percent for each component. This favours on statistical grounds a physical relationship of the components C and D with A. If C and D are physical members of the Polaris system (instead of being optical components), their absolute magnitudes in V would be $+7\overset{m}{.}5$ and $+6\overset{m}{.}5$. Due to the low age of the Polaris system of about 70 million years (deduced from the Cepheid α UMi A), they would either just have reached the zero-age main sequence, or they may still be slightly above this sequence (i.e. pre-main-sequence objects, Fernie 1966).

3. Astrometric orbit of α UMi AP

In this section we determine the astrometric orbit of the photo-center of the pair α UMi AP (i.e. essentially of A) with respect to the center-of-mass of AP. We adopt all the elements of the spectroscopic orbit of A in the system AP, derived by Kamper (1996). The remaining elements, i.e. the orbital inclination i and the nodal length Ω, are basically obtained from the following considerations:

The observed difference $\Delta\mu$ between the instantaneous proper motion of α UMi A, provided by HIPPARCOS for an epoch $T_{\rm c,H} \sim 1991.31$, and the mean proper motion of α UMi A, provided by long-term, ground-based observations, summarized in the FK5, is equal to the tangential component of the orbital velocity of A with respect to the center-of-mass of the pair AP. Using the spectroscopic orbit of A and the HIPPARCOS parallax, we can predict $\Delta\mu$ for various adopted values of i and Ω. Comparing the predicted values of $\Delta\mu$ with the observed difference $\Delta\mu$, we find i and Ω. The length of the two-dimensional vector of $\Delta\mu$ gives us the inclination i; the direction of $\Delta\mu$ fixes then the ascending node Ω. Unfortunately, two values of i, namely i and $180° - i$, predict the same value for $\Delta\mu$ (see Fig. 1). This ambiguity corresponds to the fact that $\Delta\mu$ itself does not allow us to differentiate between a prograde orbit and a retrograde one. In the case of α UMi AP, it is fortunate that the ground-based observations of the Allegheny Observatory strongly favour the retrograde orbit over the prograde one.

3.1. The determination of $\Delta\mu$

3.1.1. The proper motion of the center-of-mass of AP

We determine first the proper motion $\mu_{\rm cms(AP)}$ of the center-of-mass of the pair AP. (μ is used here for $\mu_{\alpha*} = \mu_\alpha \cos\delta$ or for μ_δ). The proper motion $\mu_{\rm FK5}$ of α UMi given in the FK5 should be very close to $\mu_{\rm cms(AP)}$, since the ground-based data are averaged in the FK5 over about two centuries, which is much larger than the orbital period of AP of about 30 years. In Table 1, we list $\mu_{\rm FK5}$ in the FK5 system and, by applying appropriate systematic corrections, in the HIPPARCOS/ICRS system. The mean errors of $\mu_{\rm FK5}$ in the HIPPARCOS system include both the random error of $\mu_{\rm FK5}$ and the uncertainty of the systematic corrections.

Another determination of $\mu_{\rm cms(AP)}$ is based on the positions $x_{\rm H}(T_{\rm c,H})$ and $x_{\rm FK5}(T_{\rm c,FK5})$ at the central epochs $T_{\rm c,H}$ and $T_{\rm c,FK5}$ of the HIPPARCOS Catalogue and of the FK5. The designation x stands for $\alpha_* = \alpha\cos\delta$ or δ, where α is the right ascension and δ the declination of α UMi. The position $x_{\rm FK5}(T_{\rm c,FK5})$ represents a time-averaged, 'mean' position in the sense of Wielen (1997). Before being used, $x_{\rm FK5}(T_{\rm c,FK5})$ must be reduced to the HIPPARCOS/ICRS system.

The HIPPARCOS position is (approximately) an 'instantaneously' measured position of the photo-center of AP. Before combining the HIPPARCOS position with $x_{\rm FK5}$ to a mean proper motion μ_0, we have to reduce $x_{\rm H}$ to the mean position $x_{\rm mean\,ph(AP),H}(T_{\rm c,H})$ of the photo-center of AP at time $T_{\rm c,H}$. This is done by going first from $x_{\rm H}(T_{\rm c,H})$ to the center-of-mass $x_{\rm cms(AP),H}(T_{\rm c,H})$ by subtracting from $x_{\rm H}$ the orbital displacement $\Delta x_{\rm orb,ph(AP)}(T_{\rm c,H})$ predicted by the astrometric orbit of the photo-center of AP. Then we have to add to $x_{\rm cms(AP),H}(T_{\rm c,H})$ the (constant) off-set between the mean position of the photo-center $x_{\rm mean\,ph(AP)}$ and the center-of-mass (see Fig. 1). Using now α_* and δ, we obtain

$$\alpha_{*,\rm mean\,ph(AP),H}(T_{\rm c,H}) = \alpha_{*,\rm cms(AP),H}(T_{\rm c,H})$$

Table 2. Proper-motion difference $\Delta\mu$ between $\mu_{\rm H}$ and $\mu_{\rm m}$ of the photo-center of α UMi AP at the epoch $T_{\rm c,H} = 1991.31$

		Prograde orbit				Retrograde orbit (Preferred solution)			
Quantity	Unit	Observed		Predicted		Observed		Predicted	
$\Delta\mu_{\alpha*}$	mas/year	+ 3.05	± 0.69	+ 3.05		+ 3.41	± 0.69	+ 3.41	
$\Delta\mu_{\delta}$	mas/year	+ 3.75	± 0.67	+ 3.75		+ 3.48	± 0.67	+ 3.48	
$\Delta\mu_{\rm tot}$	mas/year	4.83	± 0.61	4.83	± 0.32	4.87	± 0.61	4.88	± 0.32
$\Theta_{\Delta\mu}$	°	39.1	± 8.8	39.1	± 3.5	44.4	± 8.8	44.4	± 3.4

$$- \frac{3}{2}\, e\, a_{\rm ph(AP)} (\cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i) \ , \qquad (1)$$

$$\delta_{\rm mean\,ph(AP),H}(T_{\rm c,H}) = \delta_{\rm cms(AP),H}(T_{\rm c,H})$$
$$- \frac{3}{2}\, e\, a_{\rm ph(AP)} (\cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i) \ , \qquad (2)$$

where $a_{\rm ph(AP)}$ is the semi-major axis of the orbit of the photo-center of AP around the center-of-mass of AP. The other elements of this orbit are: eccentricity e, inclination i, longitude of periastron ω, position angle of the ascending node Ω, orbital period P, epoch of periastron passage $T_{\rm peri}$. The quantities in the Eqs. (1) and (2) which follow after $-\frac{3}{2}\,e$ are just the Thiele-Innes elements B and A. The equations use the fact that, in the orbital plane, the time-averaged position is located on the major axis, towards the apastron, at a distance of $\frac{3}{2}\,e\,a$ from the center-of-mass.

If α_* and δ would change linearly with time, we could determine the mean proper motion μ_0 from

$$\mu_0 = \frac{x_{\rm mean\,ph(AP),H}(T_{\rm c,H}) - x_{\rm FK5}(T_{\rm c,FK5})}{T_{\rm c,H} - T_{\rm c,FK5}} \ . \qquad (3)$$

However, for Polaris we should use more accurate formulae because it is so close to the celestial pole. We determine μ_0 strictly by requiring that $\mu_0(T_{\rm c,H})$ is that proper motion which brings the object from $x_{\rm mean\,ph(AP),H}(T_{\rm c,H})$ to $x_{\rm FK5}(T_{\rm c,FK5})$. For calculating the (small) foreshortening effect, we have adopted the radial velocity of the center-of-mass of AP, $v_{\rm r} = \gamma = -16.42$ km/s (Kamper 1996).

The agreement between the two mean proper motions $\mu_{\rm FK5}$ and μ_0 is rather good (Table 1). For determining the best value $\mu_{\rm m}$ of the mean motion of the photo-center, which is equal to the proper motion of the center-of-mass, we take the weighted average of $\mu_{\rm FK5}$ and μ_0. Since the orbital corrections to $x_{\rm H}$ are different for the prograde and retrograde orbits, we have two values for μ_0 and hence for $\mu_{\rm m}$. In both cases, we had to iterate the determinations of the orbital elements (i and Ω) and of μ_0 (and hence $\mu_{\rm m}$), since μ_0 depends on the orbital corrections. The values for $\mu_{\rm m} = \mu_{\rm cms}$ finally adopted are listed in Table 1.

3.1.2. The HIPPARCOS proper motion $\mu_{\rm H}$

The HIPPARCOS proper motion $\mu_{\rm H}$ of Polaris (ESA 1997) refers to the photo-center of AP. Basically, $\mu_{\rm H}$ is the sum of the proper motion $\mu_{\rm cms(AP)}$ of the center-of-mass (cms) of AP and of the orbital motion $\Delta\mu_{\rm orb,ph(AP)}$ (abbreviated as $\Delta\mu$) of the photo-center of AP with respect to the cms of AP at time $T_{\rm c,H}$:

$$\mu_{\rm H}(T_{\rm c,H}) = \mu_{\rm cms(AP)} + \Delta\mu(T_{\rm c,H}) \ . \qquad (4)$$

During the reduction of the HIPPARCOS data, a linear 'standard' solution was applied to Polaris. The variation of $\Delta\mu(t)$ during the period of observations of about three years was neglected. This slightly complicates the comparison of the observed $\Delta\mu$ with the orbital ephemerides. In Sect. 3.1.5 we assume that $\mu_{\rm H}$ is obtained from a linear fit to quasi-continuously measured true positions over a time interval $D_{\rm H}$, centered at time $T_{\rm c,H}$. From the correlation coefficients given in the HIPPARCOS Catalogue, we derive for the central epochs $T_{\alpha,\rm H} = 1991.26$ and $T_{\delta,\rm H} = 1991.35$. We neglect the slight difference between $T_{\alpha,\rm H}$ and $T_{\delta,\rm H}$ and use the average of both, namely $T_{\rm c,H} = 1991.31$. From the epochs of the individual observations of Polaris by HIPPARCOS, we estimate $D_{\rm H} = 3.10$ years.

3.1.3. The observed value of $\Delta\mu$

The observed value of $\Delta\mu$ is derived from

$$\Delta\mu(T_{\rm c,H}) = \mu_{\rm H}(T_{\rm c,H}) - \mu_{\rm cms(AP)}(T_{\rm c,H}) \ . \qquad (5)$$

The values of $\Delta\mu$ in α_* and δ, derived from Eq. (5), are listed in Tables 1 and 2. Table 2 gives also the total length $\Delta\mu_{\rm tot}$ of the $\Delta\mu$ vector,

$$(\Delta\mu_{\rm tot})^2 = (\Delta\mu_{\alpha*})^2 + (\Delta\mu_{\delta})^2 \ , \qquad (6)$$

and the position angle $\Theta_{\Delta\mu}$ of the $\Delta\mu$ vector,

$$\Theta_{\Delta\mu} = \arctan(\Delta\mu_{\alpha*}/\Delta\mu_{\delta}) \ . \qquad (7)$$

All the values are valid for the equinox J2000 in the HIPPARCOS/ICRS system at the epoch $T_{\rm c,H} = 1991.31$.

Since $\mu_{\rm cms}$ depends on the direction of motion in the orbit (prograde or retrograde), this is also true for $\Delta\mu$, and we obtain therefore two values for $\Delta\mu$. Table 2 shows that $\Delta\mu_{\rm tot} \sim 5$ mas/year and $\Theta_{\Delta\mu}$ are statistically quite significant and rather well determined. A value of $\Delta\mu_{\rm tot} = 4.87$ mas/year corresponds to a tangential velocity of 3.05 km/s. Hence the 'instantaneous' HIPPARCOS proper motion of Polaris has a significant 'cosmic error' (Wielen 1995a, b 1997; Wielen et al. 1997, 1998, 1999a, b) with respect to the motion of the center-of-mass. If Polaris were not already known as a close binary, our $\Delta\mu$ method (Wielen et al. 1999a) would have detected Polaris to be a $\Delta\mu$ binary because of its large test parameter $F_{\rm FH} = 6.18$ for $\mu_{\rm FK5} - \mu_{\rm H}$.

3.1.4. The photo-center of α UMi AP

The HIPPARCOS observations refer to the photo-center of α UMi AP, since the pair is not resolved by HIPPARCOS. The 'phase' used in constructing the HIPPARCOS Catalogue is practically identical to the phase of the photo-center, because the magnitude difference $\Delta m_{\rm AP}$ of more than 6 mag between A and P is quite large and because the separation between A and P at $T_{\rm H}$ was rather moderate (about 93 mas). It can also be shown that the component B does not significantly affect the HIPPARCOS measurements of AP, because of $\Delta m_{\rm V,B-AP} = 6.61$, in spite of its separation $\rho = 18$". The HIPPARCOS observations have been carried out in a broad photometric band called Hp. The photo-center refers therefore to this photometric system.

The spectroscopic orbit, however, refers to component A. We have therefore to transform the value $a_{\rm A} \sin i$ of the spectroscopic orbit into $a_{\rm ph(AP)} \sin i$ for obtaining an astrometric orbit of the photo-center of AP. The relation between $a_{\rm A}$ and $a_{\rm ph(AP)}$ is given by

$$a_{\rm ph(AP)} = (1 - \frac{\beta}{B})\, a_{\rm A} \ , \tag{8}$$

where B and β are the fractions of the mass $\mathcal{M}$ and the luminosity L of the secondary component P:

$$B \ = \ \frac{\mathcal{M}_{\rm P}}{\mathcal{M}_{\rm A} + \mathcal{M}_{\rm P}} \ , \tag{9}$$

$$\beta \ = \ \frac{L_{\rm P}}{L_{\rm A} + L_{\rm P}} = \frac{1}{1 + 10^{0.4\, \Delta m_{\rm AP}}} \ . \tag{10}$$

$\Delta m_{\rm AP}$ is the magnitude difference between A and P:

$$\Delta m_{\rm AP} = m_{\rm P} - m_{\rm A} = M_{\rm P} - M_{\rm A} \ . \tag{11}$$

Using the results given in Table 5, we find for α UMi AP

$$1 - \frac{\beta}{B} = 0.988 \ , \tag{12}$$

with an estimated error of about ± 0.010. For calculating β, we have assumed that $\Delta m_{\rm AP}$ is the same in Hp as in V. This approximation is fully justified for our purpose. We derive (see the end of Sect. 3.1.6) from Kamper (1996) for component A:

$$a_{\rm A} \sin i = 2.934 \pm 0.028 \, {\rm AU} \ . \tag{13}$$

The HIPPARCOS parallax of Polaris (ESA 1997) is

$$p_{\rm H} = 7.56 \pm 0.48 \, {\rm mas} \ . \tag{14}$$

This leads to

$$a_{\rm A} \sin i = 22.18 \pm 1.42 \, {\rm mas} \ . \tag{15}$$

Using Eqs. (8), (12), and (15), we obtain for the photo-center

$$a_{\rm ph(AP)} \sin i = 21.91 \pm 1.42 \, {\rm mas} \ . \tag{16}$$

3.1.5. The predicted value of $\Delta\mu$

For predicting $\Delta\mu$, we use four elements $(P, e, T_{\rm peri}, \omega)$ of the spectroscopic orbit derived by Kamper (1996), and $a_{\rm ph(AP)} \sin i$ according to Eq. (16), all listed in Table 4. In addition we adopt various values of i and Ω in order to produce predicted values of $\Delta\mu$ at time $T_{\rm c,H}$ as a function of i and Ω.

Since the observed value of $\Delta\mu$ is not an instantaneously measured tangential velocity, we mimic the HIPPARCOS procedure of determining $\mu_{\rm H}$. We calculate the positions $\Delta x_{\rm orbit,ph(AP)}(t) \ \equiv \ \Delta x(t)$ of the photo-center of AP with respect to the cms of AP as a function of time, using standard programs for the ephemerides of double stars. We then carry out a linear least-square fit to these positions over a time interval of length $D_{\rm H} = 3.10$ years, centered at $T_{\rm c,H} = 1991.31$:

$$\Delta x_{\rm av}(T_{\rm c,H}) \ = \ \frac{1}{D_{\rm H}} \int\limits_{-D_{\rm H}/2}^{+D_{\rm H}/2} \Delta x(T_{\rm c,H} + \tau) d\tau \ , \tag{17}$$

$$\Delta \mu_{\rm av}(T_{\rm c,H}) \ = \ \frac{12}{D_{\rm H}^3} \int\limits_{-D_{\rm H}/2}^{+D_{\rm H}/2} \Delta x(T_{\rm c,H} + \tau) \tau \, d\tau \ . \tag{18}$$

Tests have shown that especially $\Delta\mu_{\rm av}$ is not very sensitive against small changes in the slightly uncertain quantity $D_{\rm H}$. Actual numbers for the predicted values $\Delta\mu_{\rm av}(T_{\rm c,H})$ are given in the Tables 2 and 3.

3.1.6. The problem of $T_{\rm peri}$ and of $a_{\rm A} \sin i$

In his paper, Kamper (1996, his Table III) gives for his best orbit (DDO+Lick Data) a value for $T_{\rm peri} \ = \ 1928.48 \pm 0.08$. This is exactly the value derived by Roemer (1965) from the Lick Data and also quoted in Kamper's Table III under 'Lick Data'. There are three possibilities for this coincidence: (1) Kamper has adopted this value of $T_{\rm peri}$ as a fixed input value from Roemer. Nothing is said about this in his paper. (2) Kamper found from a full least-square solution by chance the same values for $T_{\rm peri}$ and its mean error as quoted for Roemer. Such a mere accident is highly improbable. (3) The identical values of $T_{\rm peri}$ and its mean error in the two columns of Kamper's Table III occured due to a mistake or misprint. However, Kamper has not published any erratum in this direction.

Dr. Karl W. Kamper died in 1998 (Bolton 1998). We tried to get clarification on the problem of $T_{\rm peri}$ from colleagues of Dr. Kamper, but they were unfortunately unable to help us in this respect. Hence we are inclined to accept the possibility (1). However, even then there is an additional problem with the mean error of $T_{\rm peri}$. Kamper has obviously overlooked that Roemer (1965) gave *probable* errors instead of *mean* errors. Hence the mean error of $T_{\rm peri}$ according to Roemer should read $\pm\, 0.12$ in Kamper's Table III.

For our purpose, a value of $T_{\rm peri}$ closer to $T_{\rm c,H}$ should be chosen. Using $T_{\rm peri} = 1928.48 \pm 0.12$ and $P = 29.59 \pm 0.02$ years, we obtain an alternative value (two periods later) of

$$T_{\rm peri} = 1987.66 \pm 0.13 \ . \tag{19}$$

We have tested this value by carrying out an unweighted least-square fit to the mean radial velocities of α UMi A listed in

Table II of Kamper (1996). In this solution we solved for $T_{\rm peri}$ only, while we adopted all the other spectroscopic elements as given by Kamper (1996). We obtained $T_{\rm peri} = 1987.63 \pm 0.25$, in good agreement with Eq. (19). However, the formally most accurate radial velocity listed in the last line of Kamper's Table II does not fit perfectly (O–C = – 0.15 km/s) his final orbit with $T_{\rm peri}$ according to Eq. (19), but rather indicates the value of $T_{\rm peri} = 1987.27$. The independent radial-velocity data published by Dinshaw et al. (1989) lead us to $T_{\rm peri} = 1987.57$ with a very small formal error. This is in good agreement with Eq. (19). Hence we have finally adopted $T_{\rm peri}$ as given by Eq. (19). An error of $\pm\,0.13$ years introduces errors of $\pm 0\overset{\circ}{.}3$ in i and of $\pm 2\overset{\circ}{.}0$ in Ω, which are small compared to the errors in i and Ω due to the uncertainties in $a_{\rm ph(AP)} \sin i$ and in the observed value of $\Delta\mu$.

The values of $a_{\rm A} \sin i, K_{\rm A}, P$, and e given by Kamper (1996) in his Table III under DDO+Lick Data are unfortunately not consistent. If we accept $K_{\rm A}, P$, and e, we find $a_{\rm A} \sin i = 2.934$ AU, while Kamper gives in his Table III 2.90 AU. In the *text* of his paper, Kamper gives 2.9 AU for $a_{\rm A} \sin i$. Has he rounded 2.934 to 2.9 and later inserted this rounded value as 2.90 into his Table III? We prefer to trust $K_{\rm A}$ and e, and hence we use for $a_{\rm A} \sin i$ the value of 2.934 AU (see Sect. 3.1.4) in our investigation.

3.2. The astrometric orbit

3.2.1. Determination of the inclination i

In Table 3 we compare the *observed* values of $\Delta\mu_{\rm tot}$ (from Sect. 3.1.3 and Table 2) with the *predicted* values of $\Delta\mu_{\rm tot}$ (using the procedures described in Sect. 3.1.5 and the elements $P, e, T_{\rm peri}, \omega$ and $a_{\rm ph(AP)} \sin i$ given in Table 4) for different trial values of the inclination i. The length $\Delta\mu_{\rm tot}$ of the vector $\Delta\mu$ is obviously not a function of the nodal direction Ω. The mean error of the predicted value of $\Delta\mu_{\rm tot}$ includes the uncertainties in all the orbital elements except in i and Ω.

The best agreement between the observed and predicted values of $\Delta\mu_{\rm tot}$ occurs for $i = 50\overset{\circ}{.}1$ (prograde orbit) and for $i = 130\overset{\circ}{.}2$ (retrograde orbit). The uncertainties in the observed value of $\Delta\mu_{\rm tot}$ and in the orbital elements (mainly in $a_{\rm ph(AP)} \sin i$) lead to an uncertainty in i of $\pm 4\overset{\circ}{.}8$.

Our values for the inclination i of the two orbits do not fulfill strictly the expected relation $i_{\rm retrograde} = 180^\circ - i_{\rm prograde}$. The reason is the following: i is determined (Table 3) from two slightly different values $\Delta\mu$ for the prograde and retrograde orbits (Table 2). The difference in the $\Delta\mu$ values stems from a slight difference in μ_0 and hence in $\mu_{\rm m}$ (Table 1), and this difference in μ_0 is caused by a difference in $x_{\rm mean\,ph(AP),H}$ (Eq. (3)). The difference in the position $x_{\rm mean\,ph(AP),H}$ of the mean photo-center is due to the small, but totally different corrections which have to be added to the observed HIPPARCOS position $x_{\rm ph(AP),av,H}$ in order to obtain the mean photo-center (see Table 6 and Fig. 1). As mentioned already at the end of Sect. 3.1.1, we had to iterate our procedure of determining i and Ω, since the corrections depend on these orbital elements.

Table 3. Determination of the inclination i from $\Delta\mu_{\rm tot}$

Prograde orbit			Retrograde orbit (Preferred solution)		
	$\Delta\mu_{\rm tot}$ [mas/years]			$\Delta\mu_{\rm tot}$ [mas/years]	
Observed:	4.83	± 0.61	Observed:	4.87	± 0.61
Predicted for i [°]:			Predicted for i [°]:		
30	9.32		150	9.32	
40	6.58		140	6.58	
45	5.63		135	5.63	
50	4.85		130	4.85	
55	4.19		125	4.19	
60	3.64		120	3.64	
70	2.78		110	2.78	
50.1	4.83	± 0.32	130.2	4.88	± 0.32

The fit between the observed and predicted values of $\Delta\mu$ is rather pleasing. It is not granted that such a fit is always possible. In the case of Polaris, for example, the spectroscopic orbit and the HIPPARCOS parallax together require a minimum value of $\Delta\mu_{\rm tot}$ of 2.00 mas/year, which occurs for $i = 90^\circ$. There is no formal upper limit for $\Delta\mu_{\rm tot}$ for $i \to 0$. However, the requirement that the component P is not visible in the combined spectrum of AP gave for a main-sequence companion P a spectral type later than A8V (Sect. 2.2), or $\mathcal{M}_{\rm P} < 1.8\,\mathcal{M}_\odot$. Combined with the mass function of the spectroscopic orbit, $f(\mathcal{M}) = (\mathcal{M}_{\rm P} \sin i)^3/(\mathcal{M}_{\rm A} + \mathcal{M}_{\rm P})^2 = 0.02885\mathcal{M}_\odot$, and with a reasonable estimate of $\mathcal{M}_{\rm A} (\mathcal{M}_{\rm A} > 5\,\mathcal{M}_\odot)$, this gives a lower limit for i of about $i > 37^\circ$, which corresponds to $\Delta\mu_{\rm tot} < 7$ mas/year. Our observed value of $\Delta\mu_{\rm tot}$ of about 5 mas/year fulfills nicely the range condition of 2 mas/year $< \Delta\mu_{\rm tot} < 7$ mas/year.

3.2.2. Determination of the nodal length Ω

Having fixed the inclination i in Sect. 3.2.1, we now determine Ω from a comparison of the observed and predicted values of the direction $\Theta_{\Delta\mu}$ of the vector $\Delta\mu$. The difference (modulo 360°) between the observed value of $\Theta_{\Delta\mu}$ and the predicted value of $\Theta_{\Delta\mu}$ for $\Omega = 0$ gives just that desired value of Ω for which the observed and predicted values of $\Theta_{\Delta\mu}$ agree. We find $\Omega = 276\overset{\circ}{.}2$ for the prograde orbit and $\Omega = 167\overset{\circ}{.}1$ for the retrograde orbit. The uncertainties in the observed value of $\Theta_{\Delta\mu}$ and in the orbital elements (now mainly in i and $T_{\rm peri}$) lead to an uncertainty in Ω of $\pm 9\overset{\circ}{.}5$ or $\pm 9\overset{\circ}{.}4$.

The quality of the fit in the components of $\Delta\mu$ in α_* and δ can be judged from the data given in Table 2. The overall agreement is quite good.

3.2.3. The ambiguity problem of i

If we know only the vector $\Delta\mu$ at one epoch and the spectroscopic orbit of a binary, then there is an ambiguity (i or $180^\circ - i$)

in the inclination i, i.e. in the direction of motion in the astrometric orbit. In the prograde (or 'direct') orbit ($i < 90°$), the position angle of P relative to A increases with time, in the retrograde orbit ($i > 90°$) it decreases. The reason for the ambiguity is the fact that $\Delta\mu$ itself does not indicate whether the orbit will turn to the left-hand side or to the right-hand side (see Fig. 1).

In principle, the knowledge of the mean position of the photo-center predicted by the FK5 for $T_{c,H} = 1991.31$ would resolve the ambiguity. However, the mean errors of this predicted position of $\pm$ 72 mas in α_* and $\pm$ 66 mas in δ are so large with respect to the differences between $x_H(T_{c,H})$ and $x_{\mathrm{mean\,ph(AP)}}(T_{c,H})$, which are less than 26 mas (Table 6), that this method is not useful in our case.

At present, the best solution of the ambiguity problem is provided by the results of the photographic observations carried out at the Allegheny Observatory, which we discussed already in Sect. 2.2. While the full astrometric orbit based on the Allegheny data (Kamper 1996) is not very trustworthy, the Allegheny data give strong preference for a retrograde orbit (in contrast to a direct one). This can be seen best in Fig. 3 of Kamper (1996): The minimum of the residuals (dashed line) occurs for $i > 120°$ ($\cos i < -0.5$), and for this range of i the semi-major axis derived from the Allegheny data is quite reasonable. For our preferred value of i ($130\overset{\circ}{.}2$), we read off from Kamper's Fig. 3 a value of $a_{\mathrm{ph(AP)}} \sim 28$ mas with an estimated uncertainty of $\pm$ 9 mas. This is in very good agreement with our result, 28.7 $\pm$ 2.8 mas. Even the nodal length Ω derived by Kamper ($175°$) is compatible with our result ($167° \pm 9°$). Kamper's determination of i ($179°$) is very uncertain and therefore not in contradiction to our value (130°). He himself says in the text of the paper that 'all inclinations between 135° and 180° are equally satisfactory' in fitting the Allegheny data. (There is a small mistake in Kamper's discussion of this point: He claims in the text 'that the minimum scatter is for an inclination of almost 90°, which results in a face-on orbit'. The relative clause after 90°, his own Fig. 3 and his Table III all indicate that '90°' should be replaced by '180°'.)

A new astrometric space mission will immediately resolve the ambiguity, since it shall then be clear to which side of our $\Delta\mu$ vector (Fig. 1) the orbit will have turned over. Probably the much higher accuracy of a new space mission will allow to determine the direction (and amount) of the instantaneous acceleration (i.e. to obtain a 'G solution' in the HIPPARCOS terminology, if not even a full orbital 'O solution').

3.2.4. *Resulting orbits of α UMi AP*

The resulting orbits of the photo-center of α UMi AP are listed in Table 4. As explained in Sect. 3.2.3, the retrograde orbit should be preferred. The semi-major axes of the orbits of α UMi A and P itself, relative to the center-of-mass of AP, and that of P relative to A, are given in Table 5.

In Fig. 1, the two orbits (prograde and retrograde) of the photo-center of AP are illustrated. The zero-point of the coordinates $\Delta\alpha_*$ and δ is the HIPPARCOS position $x_H(T_{c,H})$ at epoch $T_{c,H} = 1991.31$. The zero-point is then comoving with the center-of-mass (cms) of either the prograde orbit or the retrograde one. Therefore the orbits stay fixed in these coordinates. Since the proper motion μ_{cms} of the cms of the two orbits differs slightly (Table 6), the linear motion of the position $x_H(t)$ predicted from the HIPPARCOS Catalogue, differs slightly for the two cases. The indicated motion of x_H corresponds to $\Delta\mu$ (Table 2). Hence by construction, the motion of x_H is a tangent to the corresponding orbit, except for the slight difference between the averaged position and the instantaneous position of the photo-center at $T_{c,H}$. The dots on the orbits mark the positions in intervals of one year, the years 1990, 1995, 2000, etc. being accentuated by a larger dot. We indicate also the true major axis, on which periastron, center-of-mass, mean photo-center, and apastron are located. In addition we plot the line of nodes. The position of the ascending node is indicated by Ω. Fig. 1 demonstrates clearly that the position predicted by HIPPARCOS is drifting away from the actual position of the photo-center of AP.

Table 4. Orbital elements of α UMi AP

Quantity	Prograde orbit	Retrograde orbit (Preferred solution)
$v_r = \gamma$ [km/s]	– 16.42 $\pm$ 0.03	
K_A [km/s]	3.72 $\pm$ 0.03	
$a_A \sin i$ [AU]	2.934 $\pm$ 0.028	
$a_A \sin i$ [mas]	22.18 $\pm$ 1.42	
$a_{\mathrm{ph(AP)}} \sin i$ [mas]	21.91 $\pm$ 1.42	
$a_{\mathrm{ph(AP)}}$ [mas]	28.56 $\pm$ 2.73	28.69 $\pm$ 2.75
a_A [mas]	28.91 $\pm$ 2.74	29.04 $\pm$ 2.77
P [years]	29.59 $\pm$ 0.02	
e	0.608 $\pm$ 0.005	
T_{peri}	1987.66 $\pm$ 0.13	
ω [°]	303.01 $\pm$ 0.75	
i [°]	50.1 $\pm$ 4.8	130.2 $\pm$ 4.8
Ω [°]	276.2 $\pm$ 9.5	167.1 $\pm$ 9.4

3.2.5. *Derived physical properties of α UMi P*

In Table 5, we summarize some physical properties of the components A and P of α UMi.

The mass of α UMi A is derived from the mass-luminosity relation for Cepheids given by Becker et al. (1977). Since we use the luminosity based on the HIPPARCOS distance (132 pc), our value of 6 $\mathcal{M}_\odot$ is higher than that of other authors who have used a smaller distance.

The age of α UMi A, and therefore of the whole system of Polaris, can be estimated from the period-age relation for Cepheids (Becker et al. 1977, Tammann 1969). Using $P_0 = 5.64$ days (see Sect. 2.1), we derive an age τ of about $7 \cdot 10^7$ years.

The spectroscopic orbit provides the mass function $f(\mathcal{M}) = 0.02885\,\mathcal{M}_\odot$. Adopting the inclination $i = 130\overset{\circ}{.}2$ of the retrograde orbit and $\mathcal{M}_A = 6.0\,\mathcal{M}_\odot$ for the Cepheid, we obtain $\mathcal{M}_P = 1.54\,\mathcal{M}_\odot$ for the component P. Using this value for $\mathcal{M}_P$,

Table 5. Physical properties of α UMi A and P

Quantity	Units	Combined A+P	Component A	Component P
$m_{\rm V}$	[mag]	+ 1.982	+ 1.985	+ 8.5 ± 0.4
$M_{\rm V}$	[mag]	– 3.63 ± 0.14	– 3.62 ± 0.14	+ 2.9 ± 0.4
$\mathcal{M}$	[$\mathcal{M}_\odot$]	7.54 ± 0.6	6.0 ± 0.5	1.54 ± 0.25
Spec. type			F7-F8 Ib-II	F0V
Age τ	[years]		$7 \cdot 10^7$	
a	[mas]	142 ± 21 *)	29.0 ± 2.8 c)	113 ± 21 c)
a	[AU]	18.8 ± 2.8 *)	3.84 ± 0.37 c)	15.0 ± 2.8 c)
Mean sep.	[mas]	135 *)		

*) orbit of P relative to A; c) orbit with respect to the cms of AP.

we estimate for a star on the zero-age main sequence an absolute magnitude of $M_{\rm V} = +2.9$ and a spectral type of F0V. The magnitude difference $\Delta m_{\rm V,AP}$ between A and P is then about $6\overset{m}{.}5$. As mentioned in Sect. 2.2, a White Dwarf is ruled out by the IUE spectra and the low age of Polaris. Our estimate for $\mathcal{M}_{\rm P}$ itself would not violate the Chandrasekhar limit for White Dwarfs, if we consider the uncertainty in $\mathcal{M}_{\rm P}$ of $\pm 0.25\,\mathcal{M}_\odot$. A neutron-star nature of P is possible, but not very likely. In any case, the adopted main-sequence nature of P is a rather probable solution which is in agreement with all observational constraints. Our derived astrometric orbit does not depend sensitively on the nature of P, since all the possible solutions indicate a very small value of β, so that the difference between the positions of A and of the photo-center of AP (see Sect. 3.1.4) is small in any case.

Using $\mathcal{M}_{\rm A}$ and $\mathcal{M}_{\rm P}$ as derived above, the predicted semi-major axis of the orbit of P relative to A is 142 ± 21 mas. Our Table 7 provides a prediction of the position of P relative to A, if the ephemerides for $\Delta x_{\rm orb,ph(AP)}(t)$ are multiplied by about -4.95. The separation between A and P should be presently about 160 mas, is slightly increasing to 186 mas until 2006, and is then decreasing to about 38 mas in 2017. Hence the next decade is especially favourable for resolving the pair α UMi AP. Of course, the large magnitude difference of more than 6^m makes a direct observation of α UMi P rather difficult. Since A and P seem to have nearly the same colour (as judged from the spectral types given in Table 5), the magnitude difference should be (unfortunately) rather the same in all the photometric bands. Nevertheless we hope that modern interferometric techniques or the use of other devices may be able to resolve the pair α UMi AP during the next decade. Our paper provides hopefully a fresh impetus for such investigations.

4. Proper motion and position of Polaris

4.1. Center-of-mass of α UMi AP

The proper motion $\mu_{\rm cms(AP)}$ of the center-of-mass (cms) of the closest components A and P of α UMi has already been derived in Sect. 3.1.1 for the epoch $T_{\rm c,H} = 1991.31$. This proper motion is then transformed to the other epochs by using strict formulae, assuming a linear motion of the cms of AP in space and time. The values of $\mu_{\rm cms(AP)}$ for the epochs 1991.25 and 2000.0 are given in Table 6.

In order to derive the position $x_{\rm cms(AP)}$ of the center-of-mass of α UMi AP (Table 6), we first transform the HIPPARCOS position $x_{\rm ph(AP),av,H}$ of the photo-center of AP from epoch 1991.25 to $T_{\rm c,H} = 1991.31$ using $\mu_{\rm ph(AP),av,H}$, since $T_{\rm c,H}$ corresponds best to the effective mean epoch of the HIPPARCOS observations. Then we subtract from $x_{\rm ph(AP),av,H}(1991.31)$ the orbital displacements $\Delta x_{\rm orb,ph(AP),av}(1991.31)$, where Δx is calculated from the derived astrometric orbits (prograde and retrograde), using the averaging method described by Eq. (17). This gives us the position $x_{\rm cms(AP)}(1991.31)$ at the epoch $T_{\rm c,H}$. Using the proper motion $\mu_{\rm cms(AP)}(T_{\rm c,H})$, we transform $x_{\rm cms(AP)}$ from the epoch $T_{\rm c,H} = 1991.31$ to the standard epoch 2000.0. For the convenience of those users who like to use the HIPPARCOS standard epoch, $T_{\rm H} = 1991.25$, we give also the position $x_{\rm cms(AP)}$ for this epoch $T_{\rm H}$. The values which should be used for predicting the position $x_{\rm cms(AP)}(t)$ and its mean error are given in Table 6 in bold face. The right ascension α is given alternatively in the classical notation (h, m, s) and, as done in the HIPPARCOS Catalogue, in degrees and decimals of degrees. As discussed in Sect. 3.2.3, we propose to use preferentially the retrograde orbit.

The position $x_{\rm cms(AP)}(t)$ at an arbitrary epoch t can be derived by using the strict formulae for epoch transformation, using the epochs 2000.0 or 1991.25 as a starting epoch. The mean error $\varepsilon_{x,\rm cms,(AP)}(t)$ of $x_{\rm cms(AP)}(t)$ should be derived from

$$\varepsilon^2_{x,\rm cms(AP)}(t) = \varepsilon^2_{x,\rm cms(AP)}(1991.31) + \varepsilon^2_{\mu,\rm cms(AP)}(t - 1991.31)^2 \ . \quad (20)$$

This equation assumes that $\mu_{\rm cms(AP)}$ and $x_{\rm cms(AP)}(1991.31)$ are not correlated. This assumption is not strictly true. However, for most applications it is not neccessary to allow for correlations, because for epoch differences $\Delta t = |t - 1991.31|$ larger than a few years, the second term in Eq. (20) is fully dominating. The correlation between $\mu_{\alpha *,\rm cms(AP)}$ and $\mu_{\delta,\rm cms(AP)}$ is negligably small (only caused by the tiny correlation between $\mu_{0,\alpha *}$ and $\mu_{0,\delta}$).

All the quantities given in Table 6 refer to the HIPPARCOS/ICRS system and to the equinox J2000 (but to various epochs).

4.2. Orbital corrections for the photo-center of α UMi AP

In order to obtain a prediction for the instantaneous position $x_{\rm ph(AP)}(t)$ of the photo-center of α UMi AP at an epoch t, one has to add the orbital correction $\Delta x_{\rm orb,ph(AP)}(t)$ to the position of the center-of-mass $x_{\rm cms(AP)}(t)$:

$$x_{\rm ph(AP)}(t) = x_{\rm cms(AP)}(t) + \Delta x_{\rm orb,ph(AP)}(t) \ . \quad (21)$$

The ephemerides for the orbit of the photo-center of AP are given in Table 7. The orbital elements used in calculating the ephemerides are those listed in Table 4. Usually it is allowed to neglect the effect that the $\alpha\delta$ system is slightly rotating ($\dot\Theta = +0\overset{\circ}{.}00088$/year), due to the motion of Polaris on a great

Table 6. Proper motion $\mu_{\rm cms(AP)}$ and position $x_{\rm cms(AP)}$ of the center-of-mass of α UMi AP

			Prograde orbit				Retrograde orbit (Preferred solution)			
Quantity	Unit	Epoch	in α_*	m.e.	in δ	m.e.	in α_*	m.e.	in δ	m.e.
$\mu_{\rm cms(AP)}$	[mas/year]	1991.31	+ 41.17	**0.50**	– 15.49	**0.39**	+ 40.81	**0.50**	– 15.22	**0.39**
$\mu_{\rm cms(AP)}$	[mas/year]	**1991.25**	**+ 41.17**		**– 15.49**		**+ 40.81**		**– 15.22**	
$\mu_{\rm cms(AP)}$	[mas/year]	**2000.00**	**+ 41.17**		**– 15.50**		**+ 40.81**		**– 15.23**	
$x_{\rm ph(AP),av,H}$ (+)	[mas]	1991.25	0.00	0.39	0.00	0.45	0.00	0.39	0.00	0.45
$x_{\rm ph(AP),av,H}$ (+)	[mas]	1991.31	0.00	0.39	0.00	0.45	0.00	0.39	0.00	0.45
$\Delta x_{\rm orb,ph(AP),av}$	[mas]	1991.31	– 10.37	2.67	+ 14.89	3.16	+ 16.15	3.08	– 8.58	2.78
$\Delta x_{\rm mean\,ph(AP)}$	[mas]	1991.31	+ 15.62	2.62	+ 12.40	3.54	+ 10.62	3.58	+ 17.05	2.58
$x_{\rm cms(AP)}$ (+)	[mas]	1991.31	+ 10.37	**2.70**	– 14.89	**3.19**	– 16.15	**3.11**	+ 8.58	**2.81**
$x_{\rm mean\,ph(AP)}$ (+)	[mas]	1991.31	+ 25.99	2.59	– 2.49	4.35	– 5.53	4.27	+ 25.63	2.72
			α		δ		α		δ	
$x_{\rm ph(AP),av,H}$		1991.25	$02^h\,31^m\,47\overset{s}{.}075254$		$+\,89^\circ\,15'\,50\overset{''}{.}89698$		$02^h\,31^m\,47\overset{s}{.}075254$		$+\,89^\circ\,15'\,50\overset{''}{.}89698$	
$x_{\rm ph(AP),av,H}$		1991.31	$47\overset{s}{.}089026$		$50\overset{''}{.}89628$		$47\overset{s}{.}089026$		$50\overset{''}{.}89628$	
$x_{\rm cms(AP)}$		1991.31	$47\overset{s}{.}142856$		$50\overset{''}{.}88139$		$47\overset{s}{.}005192$		$50\overset{''}{.}90486$	
$x_{\rm mean\,ph(AP)}$		1991.31	$47\overset{s}{.}223939$		$50\overset{''}{.}89379$		$47\overset{s}{.}060320$		$50\overset{''}{.}92191$	
$x_{\rm cms(AP)}$		**1991.25**	$\mathbf{02^h\,31^m\,47\overset{s}{.}130034}$		$\mathbf{+\,89^\circ\,15'\,50\overset{''}{.}88232}$		$\mathbf{02^h\,31^m\,46\overset{s}{.}992482}$		$\mathbf{+\,89^\circ\,15'\,50\overset{''}{.}90578}$	
$x_{\rm cms(AP)}$		**2000.00**	$\mathbf{02^h\,31^m\,48\overset{s}{.}999906}$		$\mathbf{+\,89^\circ\,15'\,50\overset{''}{.}74676}$		$\mathbf{02^h\,31^m\,48\overset{s}{.}846022}$		$\mathbf{+\,89^\circ\,15'\,50\overset{''}{.}77258}$	
$x_{\rm cms(AP)}$		**1991.25**	$\mathbf{37\overset{\circ}{.}94637514}$		$\mathbf{+\,89\overset{\circ}{.}26413398}$		$\mathbf{37\overset{\circ}{.}94580201}$		$\mathbf{+\,89\overset{\circ}{.}26414049}$	
$x_{\rm cms(AP)}$		**2000.00**	$\mathbf{37\overset{\circ}{.}95416628}$		$\mathbf{+\,89\overset{\circ}{.}26409632}$		$\mathbf{37\overset{\circ}{.}95352509}$		$\mathbf{+\,89\overset{\circ}{.}26410349}$	

Explanation for (+): To the quantities marked with (+) in the second part of Table 6, one has to add the HIPPARCOS positions $x_{\rm ph(AP),av,H}$ at the corresponding epochs which are given in the first two lines of the third part of Table 6.

circle. Table 7 lists also the position of the intantaneous photo-center at periastron, apastron, and at $T_{\rm c,H}$. The small difference between the instantaneous position and the averaged position (Sect. 3.1.5) of the photo-center at $T_{\rm c,H}$ shows that the deviations of the fitting straight line from the actual orbits remain mostly below 1 mas within the interval of $D_{\rm H} = 3.1$ years of the HIPPARCOS observations, since these deviations reach their maximum at the borders of $D_{\rm H}$, namely about twice the deviation at $T_{\rm c,H}$. The very small deviations from a straight line explain also why we were, during the HIPPARCOS data reduction, unable to obtain an orbital (O) solution or an acceleration (G) solution for Polaris, although we tried to do so.

At the end of Table 6, we give the (constant) off-set between the mean photo-center and the center-of-mass. All values are valid for the equinox J2000.0, and for the orientation of the $\alpha\delta$ system at epoch 1991.31 (which differs from that at epoch 2000.0 by $\Delta\Theta = -0\overset{\circ}{.}008$ only).

The typical mean error of $\Delta x_{\rm orb,ph(AP)}$, due to the uncertainties in the orbital elements (mainly in Ω), is about $\pm\,5$ mas. It varies, of course, with the orbital phase, approximately between $\pm\,2$ mas and $\pm\,7$ mas. However, a detailed calculation of this mean error is often unnecessary for deriving the mean error of the prediction for $x_{\rm ph(AP)}(t)$, since the mean error of $x_{\rm ph(AP)}(t)$ is governed by the mean error of $\mu_{\rm cms(AP)}$ for epoch differences $|t - T_{\rm c,H}|$ larger than about 20 years.

4.3. Comparison of positions

In Table 8 we compare positions predicted by our results with those predicted by HIPPARCOS and by the FK5.

At epoch $T_{\rm c,H} = 1991.31$, the positions of the photo-center of AP predicted by our results (for both types of orbits) agree with the HIPPARCOS position by construction (except for the slight difference between the instantaneous and averaged position).

From Fig. 1 we see that the HIPPARCOS predictions for small epoch differences $\Delta t = |t - T_{\rm c,H}|$, say for $\Delta t < 4$ years, are also in good agreement with our predictions, since the HIPPARCOS data are essentially a tangent to our astrometric orbits. In other words, the HIPPARCOS data are a good short-term prediction (relative to $T_{\rm c,H}$) in the terminology of Wielen (1997). For larger epoch differences (Table 8), the HIPPARCOS prediction for $x_{\rm ph(AP)}(t)$ starts to deviate significantly from our predictions. Going to the past, e.g. to $t = 1900$, the differences reach large values of about 300 mas = $0\overset{''}{.}3$ in each coordinate. Such differences are already larger than the measuring errors of some meridian circles at that time, especially for Polaris. (The formal mean errors in α_* and δ of the position predicted by the linear HIPPARCOS solution at the epoch 1900 are 43 mas and 50 mas only.) The reason for the failure of a linear prediction based directly on the HIPPARCOS Catalogue is the fact that the quasi-instantaneously measured HIPPARCOS proper motion of

Table 7. Orbital corrections $\Delta x_{\rm orb,ph(AP)}(t)$ for the photo-center of α UMi AP

		Prograde orbit		Retrograde orbit (Preferred solution)	
Quantity or epoch t		in α_*	in δ	in α_*	in δ
$\Delta x_{\rm orb,\,ph(AP)}(t)$					
1987.00		+ 0.44	– 7.89	– 7.97	– 0.50
1988.00		– 9.71	– 3.30	– 2.18	– 10.08
1989.00		– 14.16	+ 3.53	+ 5.19	– 13.71
1990.00		– 13.97	+ 9.49	+ 11.15	– 12.81
1991.00		– 11.63	+ 14.10	+ 15.51	– 9.93
1992.00		– 8.33	+ 17.62	+ 18.65	– 6.23
1993.00		– 4.61	+ 20.27	+ 20.88	– 2.20
1994.00		– 0.73	+ 22.23	+ 22.39	+ 1.91
1995.00		+ 3.18	+ 23.61	+ 23.33	+ 5.97
1996.00		+ 7.02	+ 24.51	+ 23.78	+ 9.91
1997.00		+ 10.75	+ 24.99	+ 23.83	+ 13.69
1998.00		+ 14.31	+ 25.09	+ 23.51	+ 17.26
1999.00		+ 17.69	+ 24.86	+ 22.89	+ 20.60
2000.00		+ 20.85	+ 24.32	+ 21.98	+ 23.69
2001.00		+ 23.77	+ 23.51	+ 20.82	+ 26.50
2002.00		+ 26.43	+ 22.43	+ 19.43	+ 29.02
2003.00		+ 28.79	+ 21.11	+ 17.83	+ 31.23
2004.00		+ 30.85	+ 19.56	+ 16.03	+ 33.10
2005.00		+ 32.56	+ 17.79	+ 14.06	+ 34.60
2006.00		+ 33.90	+ 15.81	+ 11.92	+ 35.69
2007.00		+ 34.81	+ 13.64	+ 9.64	+ 36.35
2008.00		+ 35.26	+ 11.29	+ 7.22	+ 36.52
2009.00		+ 35.17	+ 8.77	+ 4.70	+ 36.13
2010.00		+ 34.47	+ 6.09	+ 2.10	+ 35.12
2011.00		+ 33.05	+ 3.29	– 0.55	+ 33.36
2012.00		+ 30.75	+ 0.41	– 3.17	+ 30.73
2013.00		+ 27.37	– 2.48	– 5.68	+ 27.02
2014.00		+ 22.60	– 5.24	– 7.89	+ 21.92
2015.00		+ 15.96	– 7.53	– 9.42	+ 15.03
2016.00		+ 6.90	– 8.57	– 9.40	+ 5.86
2017.00		– 4.13	– 6.54	– 6.08	– 4.90
1987.66	Periastron	– 6.71	– 5.33	– 4.57	– 7.33
2002.46	Apastron	+ 27.54	+ 21.86	+ 18.73	+ 30.07
1991.31	instantan.	– 10.67	+ 15.30	+ 16.60	– 8.84
1991.31	averaged	– 10.37	+ 14.89	+ 16.15	– 8.58
$\Delta x_{\rm mean\,ph(AP)}$ (1991.31)		+ 15.62	+ 12.40	+ 10.62	+ 17.05

Polaris contains an orbital motion $\Delta\mu_{\rm tot}$ of about 5 mas/year as a 'cosmic error'.

Our data reproduce rather well the FK5 positions at the central FK5 epochs. This is to be expected, since we have made use of these positions in determining μ_0 (and hence $\mu_{\rm cms}$). For $T_{\rm c,H} = 1991.31$, the FK5 prediction deviates rather strongly from our values. This is in accordance with the mean error of $\mu_{\rm FK5}$ and the large epoch differences $T_{\rm c,H} - T_{\rm c,FK5}$. (The mean errors in α_* and δ of the FK5 position (reduced to the HIPPARCOS system) are at the central epochs (given in Table 8) 37 mas and 34 mas, and at epoch 1991.31 72 mas and 66 mas.)

Table 8. Comparison between predicted positions for the photo-center of α UMi AP

Difference	Epoch t	Prograde orbit in α_* [mas]	Prograde orbit in δ [mas]	Retrograde orbit (Preferred solution) in α_* [mas]	Retrograde orbit (Preferred solution) in δ [mas]
HIPPARCOS prediction minus this paper (instantaneous position):					
	1900.00	– 275	– 330	– 298	– 313
$T_{\rm c,H}$ =	1991.31	– 0	+ 0	+ 0	– 0
	2000.00	– 5	+ 23	+ 24	– 2
	2010.00	+ 12	+ 79	+ 78	+ 21
	2020.00	+ 90	+ 111	+ 101	+ 103
FK5 minus this paper (mean photo-center):					
	1900.00	– 1	– 14	– 2	– 18
$T_{\rm c,\delta,FK5}$ =	1916.08	+ 4	– 34	+ 9	– 42
$T_{\rm c,\alpha,FK5}$ =	1927.17	+ 8	– 47	+ 16	– 58
	1991.31	+ 29	– 126	+ 60	– 154
	2000.00	+ 31	– 137	+ 66	– 167
	2010.00	+ 35	– 149	+ 73	– 182
	2020.00	+ 38	– 162	+ 80	– 197
Prograde minus retrograde orbit (instantaneous positions):					
	1900.00	– 23	+ 17		
	1991.31	– 1	+ 1		
	2000.00	+ 29	– 25		
	2010.00	+ 66	– 56		
	2020.00	+ 11	– 8		

How large are the differences between the positions which we predict if we use either the retrograde orbit or the prograde one? At $T_{\rm c,H} = 1991.31$, the differences are nearly zero by construction. At other epochs, the orbital differences can be seen in Fig. 1. To these differences in the orbital corrections, we have to add the slight positional differences which are due to the differences in $\mu_{\rm cms}$ of both orbits. The total differences between the prograde and retrograde orbit are shown at the end of Table 8 for some epochs. An extremum in these differences occurs in α_* (+ 68 mas) and in δ (– 59 mas) at about the year 2012.

4.4. *Space velocity of Polaris*

From the derived proper motions $\mu_{\rm cms(AP)}$ of the center-of-mass of α UMi AP (Table 6, retrograde orbit), from the radial velocity $v_{\rm r} = \gamma$ (Table 4), and from the HIPPARCOS parallax $p_{\rm H}$ (Eq. 14), we derive the three components U, V, W of the space velocity **v** of Polaris (Table 9). We neglect a possible intrinsic K term in the pulsating atmosphere of the Cepheid α UMi A (Wielen 1974). This is probably justified, especially in view of the very small amplitude of the radial velocity due to pulsation.

The velocity component U points *towards* the galactic center, V in the direction of galactic rotation, and W towards the galactic north pole. The velocity $\mathbf{v}_{\rm S0}$ is measured relative to the Sun. The velocity $\mathbf{v}_{\rm L0}$ refers to the local standard of rest. For

Table 9. Space velocity of the center-of-mass of α UMi AP. For detailed explanations see Sect. 4.4.

	Prograde orbit				Retrograde orbit (Preferred solution)			
Velocity	U	V [km/s]	W	v	U	V [km/s]	W	v
$\mathbf{v}_{S0}$	– 14.4	– 28.2	– 5.5	32.1	– 14.2	– 28.0	– 5.4	31.9
m.e.	± 1.2	± 0.8	± 1.0		± 1.2	± 0.8	± 1.0	
$\mathbf{v}_{L0}$	– 5.4	– 16.2	+ 1.5	17.1	– 5.4	– 16.0	+ 1.6	17.0
$\mathbf{v}_{C0}$	– 8.0	– 16.0	+ 1.5	18.0	– 8.0	– 15.8	+ 1.6	17.8

the solar motion we use $\mathbf{v}_{\odot}$ = (+ 9, + 12, + 7) km/s, proposed by Delhaye (1965). The velocity $\mathbf{v}_{C0}$ is the peculiar velocity of Polaris with respect to the circular velocity at the position of Polaris (see Wielen 1974). For the required Oort constants of galactic rotation, we adopt $A = +14$ (km/s)/kpc and $B = -12$ (km/s)/kpc. As mentioned in Sect. 2.3, the velocity of the center-of-mass of α UMi AP may differ from that of α UMi AP+B by a few tenth of a km/s.

The peculiar velocity $\mathbf{v}_{C0}$ of Polaris is reasonable for a classical Cepheid. According to Wielen (1974), the velocity dispersions (σ_U, σ_V, σ_W) for nearby classical Cepheids are (8, 7, 5) km/s. Hence only the V component of $\mathbf{v}_{C0}$ of Polaris is slightly larger than expected on average.

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