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An analytical approximation for the MOID and its consequences

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Abstract. We give an analytical formulation for the approximation of the *Minimum Orbital Intersection Distance (MOID)* between two elliptical orbits, and we apply it to the case of the Earth and of an asteroid. With this formulation we are able to find an algorithm to compute the variance of the MOID for a given asteroid and we make the computation in the case of the asteroids whose orbit has been determined, even if with a great uncertainty, to find cases of asteroids that have a good probability of having a small MOID even if their nominal orbit gives a relatively large MOID.

Key words: celestial mechanics, stellar dynamics – minor planets, asteroids

1. Introduction

In the analysis of the problem of asteroids that can be dangerous for an impact with the Earth, one of the most important quantities computed is the so-called *Minimum Orbital Intersection Distance (MOID)*, that is a measure for the distance between the orbits of the asteroid and of the Earth, not considering the positions that the bodies occupy in them.

The computation of the MOID (see Sitarski 1968) of a given asteroid is of great importance for the study of the problem of possible impact of the asteroid with the Earth, but the problem is that the orbit of an asteroid is given by the fit of the observations, done using the method of the least squares, and this causes an indetermination. In fact, from the observations, we get the equinoctal elements of the asteroid along with their so-called *covariance matrix*. This matrix gives, in the space of the elements, the dimension of the ellipsoid of confidence of the asteroid, the set where the elements of the real orbit of the asteroid can be in. Our aim is the construction of an algorithm to propagate the uncertainty on the elements of the asteroid to the uncertainty on the calculation of the MOID. To make clearer the notations, we call *nominal MOID* the value of the MOID computed taking as orbital elements the elements at the centre of the ellipsoid of confidence.

Another problem that immediately we have to face to compute the variance of the MOID, that is its uncertainty, is that, in all the algorithms to compute the MOID, the calculation is done numerically. So there is no analytical formulation for the MOID, and in the equation for the propagation of the covariance matrix (see Eq. (32)) we need to explicitly compute the derivatives of the MOID with respect to its variables. This is a problem that cannot be solved, but we find an approximation to the MOID that is obtained analytically, so to compute the variance of this approximated MOID, and apply this variance to the nominal MOID. We will call this approximated MOID the *AMOID*, because we compute it only in the neighbourhood of the nodes of the asteroid.

2. Analytical theory

2.1. A first approximation

Given an asteroid with its observations, we introduce a reference system xyz with the origin at the Sun and such that the orbit of the Earth around the Sun lies on the plane xy. The x axis of the system is taken in the direction of the γ point of the Earth. This system is obtained by a small rotation from the usual system in which the orbital elements of an asteroid are considered, but notwithstanding this rotation has to be done to obtain the exact geometry of the orbit of an asteroid with respect to the orbit of the Earth. In our reference system the orbital elements of the asteroid are called *mutual elements*. Moreover we call *mutual nodes* the intersection points between the orbit of the asteroid and the xy plane. In the same way, we have the *mutual nodal line*.

Let a, e, i, ω, Ω be the mutual elements that identify geometrically the orbit of an asteroid in space. By a rotation around the z axis, it is possible to have $\Omega = 0$, taking the x axis in the direction of the ascending mutual node.

Let's assume in this paragraph (see Carusi et al. 1990) that the orbit of the Earth is circular and that the nodal distance, that is the distance between the two orbits along the mutual nodal line, at one of the mutual nodes is small. For example let's assume without loss of generality that the nodal distance at the ascending node is small. Moreover, let the radius of the orbit of the Earth be equal to 1, Earth's velocity along its orbit be equal to 1 and let's scale everything such to have all the constants equal to 1. We assume to have the asteroid at its ascending mutual node and the Earth at the intersection between its orbit and the x axis. Then, if

$$T = \frac{1}{a} + 2\sqrt{a(1-e^2)}\cos i \tag{1}$$

is the so-called *Tisserand constant*, we have

$$|\boldsymbol{U}| = \sqrt{3-T}, \tag{2}$$

where $\boldsymbol{U}$ is the geocentric velocity of an asteroid entering the influence sphere of the Earth. So, let's assume $\boldsymbol{U}$ to be the geocentric velocity of the asteroid at its ascending mutual node.

At this point, we introduce a new reference system XYZ by a translation along the x axis such that the Earth is at the origin. Let's call x_0 the nodal distance. As x_0 is small, we can suppose that $\boldsymbol{U}$ has its base point at the origin, and we define θ as the angle between $\boldsymbol{U}$ and the Y axis, that is in the direction of the velocity of the Earth, and φ as the angle between the projection of $\boldsymbol{U}$ on the XZ plane and the Z axis. $\boldsymbol{U}\theta\varphi$ defines a new reference system, called the *Öpik reference system* (see Öpik 1976). Then, the following holds

$$U_X = |\boldsymbol{U}|\sin\theta\sin\varphi = \pm\sqrt{2-\frac{1}{a}-a(1-e^2)} \tag{3}$$

$$U_Y = |\boldsymbol{U}|\cos\theta = \sqrt{a(1-e^2)}\cos i - 1 \tag{4}$$

$$U_Z = |\boldsymbol{U}|\sin\theta\cos\varphi = \sqrt{a(1-e^2)}\sin i. \tag{5}$$

Then, as a first approximation for the analytical computation of the AMOID (see Valsecchi et al., in prep.), we consider the minimum distance between the two lines tangent to the orbits of the Earth and the asteroid at the intersection points between the orbits and the mutual nodal line. With this assumption, we can define the two lines as

$$\begin{cases} X = \frac{U_X}{U_Y}\,Y + x_0 \\ Z = \frac{U_Z}{U_Y}\,Y \end{cases} \tag{6}$$

for the asteroid, and

$$\begin{cases} X' = 0 \\ Z' = 0 \end{cases} \tag{7}$$

for the Earth. The minimum distance between these lines is given by the minimum of the function

$$D^2(Y) = (X-X')^2 + (Y-Y')^2 + (Z-Z')^2, \tag{8}$$

that is the value of the function at the point $(Y_{\min}, Y'_{\min})$. It is obtained as the solution of

$$\begin{cases} \frac{\partial D^2}{\partial Y} = 0 \\ \frac{\partial D^2}{\partial Y'} = 0. \end{cases}$$

The equations are

$$\begin{cases} \frac{\partial D^2}{\partial Y} = \frac{2\,(U_X^2+U_Z^2)}{U_Y^2}\,Y + 2\,(Y-Y') + \frac{2\,U_X}{U_Y}\,x_0 \\ \frac{\partial D^2}{\partial Y'} = 2\,(Y'-Y), \end{cases} \tag{9}$$

that have their zero at

$$\begin{cases} Y_{\min} = -\frac{U_X U_Y}{U_X^2+U_Z^2}\,x_0 = -\frac{\cos\theta\sin\varphi}{\sin\theta}\,x_0 \\ Y'_{\min} = Y_{\min}, \end{cases} \tag{10}$$

so, we have

$$D^2_{\min} = \frac{x_0^2\,U_Z^2}{U_X^2+U_Z^2} = x_0^2\,\cos^2\varphi. \tag{11}$$

This becomes, in terms of the orbital mutual elements,

$$D^2_{\min} = \frac{x_0^2 a^2(1-e^2)\sin^2 i}{2a-1-a^2(1-e^2)\cos^2 i}. \tag{12}$$

2.2. Nonlinearity in nodal distance

The first step to generalize Eq. (12) is to eliminate the assumption that x_0 is small. In fact we want to deal also with asteroids with an AMOID not too small, for example about 0.5AU, and such asteroids must have a not small nodal distance, as can be seen from equations (11) and (12). Our effort in this direction leads to a different expression for the velocity vector $\boldsymbol{U}$.

Let's consider the same problem as before. In the reference system xyz with origin in the Sun, we suppose that the orbit of the Earth is circular with radius equal to 1 and that the velocity of the Earth is constant and equal to 1, while the asteroid has an elliptical orbit, given by the mutual orbital elements a, e, i, ω, with $\Omega = 0$. We do the same calculations as before, so the AMOID is obtained by the Eq. (11).

The only difference is the calculation of the components of $\boldsymbol{U}$. In fact, if x_0 is not small, we cannot assume $\boldsymbol{U}$ to have its base point where the Earth is. So, we use the equations given by classical celestial mechanics to evaluate the velocity of a body along its elliptical orbit and the equations for its angular momentum. If $\boldsymbol{V}$ is the heliocentric velocity of the asteroid at its ascending mutual node and $\boldsymbol{V'} = (0,1,0)$ is the heliocentric velocity of the Earth, we have $\boldsymbol{U} = \boldsymbol{V} - \boldsymbol{V'}$, that is, in components, $U_x = V_x$, $U_y = V_y - 1$ and $U_z = V_z$. Moreover we have

$$|\boldsymbol{V}|^2 = \frac{2}{1+x_0} - \frac{1}{a} \tag{13}$$

$$|\boldsymbol{h}|^2 = a(1-e^2) \tag{14}$$

where $\boldsymbol{h}$ is the angular momentum. The equations to compute U_y and U_z come from the definition of the angular momentum, while U_x is obtained by the Eq. (13) and by the relation between $\boldsymbol{U}$ and $\boldsymbol{V}$. We have

$$h_x = 0 \tag{15}$$

$$h_y = \sqrt{a(1-e^2)}\sin i = -(1+x_0)U_z \tag{16}$$

$$h_z = \sqrt{a(1-e^2)}\cos i = (1+x_0)(U_y+1) \tag{17}$$

from which we get

$$U_y = \frac{\sqrt{a(1-e^2)}\cos i}{1+x_0} - 1 \tag{18}$$

$$U_z = \frac{\sqrt{a(1-e^2)}\sin i}{1+x_0}, \tag{19}$$

and

$$\frac{2}{1+x_0} - \frac{1}{a} = U_x^2 + \frac{a(1-e^2)\cos^2 i}{(1+x_0)^2} + \frac{a(1-e^2)\sin^2 i}{(1+x_0)^2},$$

that leads to

$$U_x = \pm\sqrt{\frac{2}{1+x_0} - \frac{1}{a} - \frac{a(1-e^2)}{(1+x_0)^2}}. \qquad (20)$$

Now we can substitute the equations for the components of $\boldsymbol{U}$ to get $D^2_{\min}$. We use Eq. (11)

$$D^2_{\min} = \frac{x_0^2\, U_Z^2}{U_X^2 + U_Z^2}$$

to get

$$D^2_{\min} = \frac{x_0^2 a^2 (1-e^2)\sin^2 i}{2a(1+x_0) - (1+x_0)^2 - a^2(1-e^2)\cos^2 i}. \qquad (21)$$

Eq. (21) is obtained in the same way as Eq. (12). However the different way in the calculation of the components of the velocity $\boldsymbol{U}$ leads to the generalization of equation (11), that can be obtained from Eq. (21) expanding the denominator in power series of x_0 and keeping only the first term of the series. In this sense we say that Eq. (21) is a formulation for the AMOID nonlinear in the nodal distance x_0.

2.3. Earth on elliptical orbit

A generalization of Eq. (21) can be achieved by eliminating the assumption that the Earth is on a circular orbit. Then we have to introduce orbital elements also for the Earth, so in the reference system xyz we call a', e', i', ω' these elements, and we have $i' = 0$, while Ω' cannot be defined. Moreover let $\boldsymbol{V'} = (V'_x, V'_y, 0)$ be Earth's velocity along its orbit.

If we want to eliminate all the approximations relative to the nodal distance we made in the previous paragraphs, we have to use the heliocentric velocity $\boldsymbol{V}$ to parametrise the line tangent to the asteroid's orbit at the ascending mutual node. By the way, we will remark that with this improvement we get the same result for the AMOID, so this is just an improvement by a theoretical point of view.

Let r' be the distance of the Earth from the Sun at the point where its orbit intersects the x axis. We can write

$$r' = \frac{a'(1-e'^2)}{1+e'\cos\omega'}.$$

Then, to evaluate $\boldsymbol{V}$ we use the same method as before (see Eqs. (20),(18) and (19)), so we get

$$V_x = U_x + V'_x = \pm\sqrt{\frac{2}{r'+x_0} - \frac{1}{a} - \frac{a(1-e^2)}{(r'+x_0)^2}} \qquad (22)$$

$$V_y = U_y + V'_y = \frac{\sqrt{a(1-e^2)}\cos i}{r'+x_0} \qquad (23)$$

$$V_z = U_z = \frac{\sqrt{a(1-e^2)}\sin i}{r'+x_0}, \qquad (24)$$

while to evaluate $\boldsymbol{V'}$ we use the equations to obtain the components of the velocity of a body on an elliptical orbit along the radius and orthogonal to the radius. These equations give

$$V'_x = |\boldsymbol{V'}|\sin\alpha \qquad (25)$$

$$V'_y = |\boldsymbol{V'}|\cos\alpha, \qquad (26)$$

where

$$\cos\alpha = \sqrt{\frac{a'^2(1-e'^2)}{r'(2a'-r')}}$$

and

$$|\boldsymbol{V'}|^2 = \frac{2}{r'} - \frac{1}{a'}.$$

At this point, we can write the tangent lines as

$$\begin{cases} x = \frac{V_x}{V_y} y + x_0 \\ z = \frac{V_z}{V_y} y \end{cases} \qquad (27)$$

for the asteroid, and

$$\begin{cases} x' = \frac{V'_x}{V'_y} y' \\ z' = \frac{V'_z}{V'_y} y' \end{cases} \qquad (28)$$

for the Earth. Then we look for the minimum of the function

$$D^2(y, y') = (x-x')^2 + (y-y')^2 + (z-z')^2. \qquad (29)$$

The minimum is achieved at the point (y_0, y'_0), the solution of

$$\begin{cases} \frac{\partial D^2}{\partial y} = 0 \\ \frac{\partial D^2}{\partial y'} = 0 \end{cases}$$

and we get $D^2(y_0, y'_0) = D^2_{\min}$. The equation for $D^2_{\min}$ in terms of the orbital mutual elements a, e, i, ω and of x_0 is too long and too complicated to be written here, and it is also beyond our aim, since it's much easier to write the equation in terms of the velocities $\boldsymbol{V}$ and $\boldsymbol{V'}$ and it's much easier to evaluate the derivatives of this equation, and this will be important in the next paragraph. What we get is

$$D^2_{\min} = \frac{x_0^2\, T_x^2}{|\boldsymbol{T}|^2}, \qquad (30)$$

where $\boldsymbol{T}$ is the wedge product $\boldsymbol{T} = \boldsymbol{V} \wedge \boldsymbol{V'}$. By Eq. (30)we notice that the AMOID would be the same parametrising the lines with $\boldsymbol{U}$ and $\boldsymbol{V'}$, since $\boldsymbol{T} = \boldsymbol{V} \wedge \boldsymbol{V'} = \boldsymbol{U} \wedge \boldsymbol{V'}$. Eq. (30) reduces to Eq. (21) if $\boldsymbol{V'} = (0, 1, 0)$.

Eq. (30) is better understood as a particular case of the equation for the distance between two given lines

$$D^2_{\min} = \frac{(\boldsymbol{R}\cdot\boldsymbol{T})^2}{|\boldsymbol{T}|^2}, \qquad (31)$$

where $\boldsymbol{R}$ is the vector joining two given points on the lines and $\boldsymbol{T}$ is the wedge product between the velocity vectors of the lines.

2.4. Variance of the AMOID

Having obtained the analytical formulation for the AMOID, we have to evaluate the variance of the AMOID in terms of the covariance matrix of the equinoctal elements given by the observations of the asteroid (see Milani 1999). To do that, we have to construct an algorithm that in various steps leads from the observations to the AMOID. It is important to have a equation for the AMOID much simpler in its dependence on the variables because to obtain its variance we need to compute all the derivatives with respect to the variables. In fact, if X is a vector of data and Γ_X is its covariance matrix, then the covariance matrix Γ_Y of a vector $Y(X)$, function of X, is given by

$$\Gamma_Y = \left(\frac{\partial Y}{\partial X}\right)\ \Gamma_X\ \left(\frac{\partial Y}{\partial X}\right)^t . \tag{32}$$

In the first part of the algorithm we make a change of coordinates from the system $\xi\eta\zeta$ (where the equinoctal elements of an asteroid are obtained) to our system xyz.

Let's start changing the elements of the asteroid from equinoctal to keplerian, in order to study the geometry of the orbit of the asteroid and identify the rotations we have to do. Let A, h, k, p, q, l be the equinoctal elements. If we write $a_1, e_1, i_1, \omega_1, \Omega_1, l_1$ for the keplerian elements, we have

$$a_1 = A \tag{33}$$
$$e_1 = \sqrt{h^2 + k^2} \tag{34}$$
$$i_1 = 2\ \arctan\left(\sqrt{p^2 + q^2}\right) \tag{35}$$
$$\omega_1 = \arctan\left(\frac{h}{k}\right) - \arctan\left(\frac{p}{q}\right) \tag{36}$$
$$\Omega_1 = \arctan\left(\frac{p}{q}\right) \tag{37}$$
$$l_1 = l. \tag{38}$$

The second step we need is the change of coordinates from the system $\xi\eta\zeta$ to a new system $\xi'\eta'\zeta'$, where the plane $\xi'\eta'$ is the plane of the orbit of the Earth, and the ξ' axis is in direction of the γ point. Let $a_2, e_2, i_2, \omega_2, \Omega_2, l_2$ be the new keplerian elements, and let i_0 and Ω_0 be, respectively, the inclination and the longitude of the ascending node of the Earth in the system $\xi\eta\zeta$. Then the change of coordinates leads to the following transformations for the elements

$$a_2 = a_1 \tag{39}$$
$$e_2 = e_2 \tag{40}$$
$$i_2 = \arccos\left(C\ \sin i_0\ \sin i_1 + \cos i_0\ \cos i_1\right) \tag{41}$$
$$\Omega_2 = -\arctan\left(\frac{D\ \sin i_1}{C\ \cos i_0\ \sin i_1 - \sin i_0\ \cos i_1}\right) \tag{42}$$

where

$$C = \cos\Omega_0\ \cos\Omega_1 + \sin\Omega_0\ \sin\Omega_1$$
$$D = \sin\Omega_0\ \cos\Omega_1 - \cos\Omega_0\ \sin\Omega_1.$$

We still have to find the transformation equation for ω_2. Let

$$r = \frac{a_1\,(1 - e_1^2)}{1 + e_1\ \cos l_1}$$

and

$$\xi_{ast} = r\ \cos i_1\,(\cos\Omega_1\ \cos(\omega_1 + l_1) - \sin\Omega_1\ \sin(\omega_1 + l_1))$$
$$\eta_{ast} = r\ \cos i_1\,(\sin\Omega_1\ \cos(\omega_1 + l_1) + \cos\Omega_1\ \sin(\omega_1 + l_1))$$
$$\zeta_{ast} = r\ \sin(\omega_1 + l_1)\ \sin i_1,$$

being $\xi_{ast}, \eta_{ast}, \zeta_{ast}$ the cartesian coordinates of the asteroid in the system $\xi\eta\zeta$. Then the composition of a rotation of an angle equal to Ω_0 around the ζ axis and a rotation of an angle equal to i_0 around the new ξ' axis gives the new cartesian coordinates of the asteroid in the system $\xi'\eta'\zeta'$, that are

$$\xi'_{ast} = \xi_{ast}\ \cos\Omega_0 + \eta_{ast}\ \sin\Omega_0$$
$$\eta'_{ast} = \cos i_0\,(\eta_{ast}\ \cos\Omega_0 - \xi_{ast}\ \sin\Omega_0) + \zeta_{ast}\ \sin i_0$$
$$\zeta'_{ast} = -\sin i_0\,(\eta_{ast}\ \cos\Omega_0 - \xi_{ast}\ \sin\Omega_0) + \zeta_{ast}\ \cos i_0.$$

To get ω_2 at this point, we use the transformation equations from cartesian coordinates to keplerian elements. We get

$$\omega_2 = \arctan\left(\frac{\zeta'_{ast}}{\sin i_2\,(\xi'_{ast}\ \cos\Omega_2 + \eta'_{ast}\ \sin\Omega_2)}\right) - l_1. \tag{43}$$

The last step consists of a rotation around ζ' axis of an angle equal to Ω_2, the longitude of the ascending node of the asteroid in the system $\xi'\eta'\zeta'$, to get the mutual elements a, e, i, ω in the system xyz and the elements a', e', ω' of the Earth. For the mutual elements

$$a = a_2 \tag{44}$$
$$e = e_2 \tag{45}$$
$$i = i_2 \tag{46}$$
$$\omega = \omega_2, \tag{47}$$

while for the elements of the Earth, a' and e' are fixed given the time of the observations of the asteroid, while for ω', the argument of the pericentre of the Earth in the system xyz, we have

$$\omega' = \omega_0 - \Omega_2 \tag{48}$$

where ω_0 is the argument of the pericentre of the Earth in the system $\xi\eta\zeta$.

At this point by the Eqs. (44), (45), (46), (47) and (48), we can get the nodal distance x_0 given by

$$x_0 = \frac{a\,(1 - e^2)}{1 + e\ \cos\omega} - \frac{a'\,(1 - e'^2)}{1 + e'\ \cos\omega'}, \tag{49}$$

the velocities of the Earth and of the asteroid by the Eqs. (25), (26) and (22), (23), (24), and the minimum distance between the tangent lines (28) and (27) by Eq. (30).

Obviously at each step of the algorithm we use Eq. (32) to get the variance of the AMOID.

3. Applications

In the last paragraph we derived an analytical formulation for the AMOID near one of the mutual nodes for an asteroid crossing the plane of the orbit of the Earth around the Sun, and constructed an algorithm to obtain the AMOID and its variance. Now we apply the algorithm to two different kinds of real cases of asteroids, the Near Earth Asteroids and a large catalog of unnumbered asteroids.

Obviously the importance of the algorithm lies in the possibility of computation of the variance of the AMOID, and not

Table 1. Virtual PHAs among NEA.

Name	Nom. Min.	Minim. Value
1984QY1	0.1781838560	-0.7525736331
1994XG	0.0561041431	0.0481921878
1997XV11	0.5842455386	-0.1820207990
2000AQ219	0.2083623575	-0.0271626559

in the explicit computation of the AMOID. In fact the AMOID can be just an approximation for the nominal MOID, that is computed more accurately by other algorithms. It is important also to notice that the AMOID is a good approximation only in some cases, so first of all we have to discriminate these cases. It is obvious that the AMOID can be a good approximation of the nominal MOID only in cases with high mutual inclination, so we expect to find inaccurate results if we compare the nominal MOID and the AMOID in cases of small mutual inclination.

We take a list of asteroids along with the covariance matrix of their equinoctal elements, then we use our algorithm to obtain the orbital mutual elements of these asteroids in our reference system xyz. At this point we use an algorithm invented by G.F. Gronchi to obtain the nominal MOID, all the stationary points of the function that gives the distance between two points on elliptical orbits and the eccentric anomalies of the stationary points (see Gronchi, in prep.). In some cases we obtain two minima, so the nominal MOID is the smaller. Then we compute the AMOID near the ascending and the descending mutual nodes and the variance. What we want to do after this is to apply the variance of the AMOID to the nominal minima. To do that, we have to remember that we computed the variance of the AMOID near the mutual nodes, so we cannot simply apply the variance to the minima. So we have to choose, among all the asteroids in a list, only the ones who have at least one minimum of the distance function, what we'll call a *nominal minimum*, near a mutual node. The limit we imposed is that the angular distance between the point of minimum and the node should be less than or equal to 45^o along the orbit. Finally we compute the minimum value the nominal minima can achieve, that is obtained via the following equation

$$\text{Min. Value} = \text{Nom. Min.} - 3 * \text{Stand. Deviation}, \tag{50}$$

where the *standard deviation* is the square root of the variance of the corresponding AMOID.

This minimum value is of great relevance if it reveals that the asteroid could approach the Earth at a small distance, for example the distance Earth-Moon, even if the probability of this approach is small, in fact, in this case, the asteroid obviously needs to be studied better to compute its orbit more accurately. Anyway, it is also important to know when an asteroid can approach the Earth at a distance less than or equal to 0.05AU, so there is a list of objects with a nominal MOID less than 0.05AU, and we call these asteroids *Low MOID Asteroid (LMA)*. By our algorithm we can find some asteroids that don't have a nominal MOID less than 0.05AU, but such that one of their minimum values could be less than 0.05AU. We call these asteroids *Virtual PHA*. So, our algorithm could enlarge the number of asteroids classified as LMA, and so could turn the attention of astronomical observatories to a larger number of objects that need to be observed more to increase the accuracy in the specification of their orbits.

3.1. Near Earth Asteroids

The first list that we examined with the algorithm described above, is the list of the so-called *Near Earth Asteroid (NEA)*, that are asteroids with a perihelion less than 1.3AU, obtained from NEODyS (http://newton.dm.unipi.it/neodys/), updated to March 1st, 2000. At the moment there are 922 asteroids classified as NEA.

A large number of them, 324, are already classified as LMA, since their nominal MOID is less than 0.05AU, but our algorithm presents some cases of Virtual PHAs, and Table 1 below lists these asteroids.

In the table, we have in the second column the nominal minimum, that is the minimum of the distance function, while in the third column there is the minimum value that the distance can achieve at its minimum point obtained considering the variance of the AMOID near to the minimum point.

First of all we notice that all the asteroids listed above don't have a small inclination, so that we can suppose that the AMOID is not a bad approximation for the nominal minimum. Then in all the cases the nominal minimum actually corresponds to the nominal MOID.

Not really interesting is 1994XG since its minimum value is just less than 0.05AU, while its nominal MOID is just more than 0.05AU. In the other three cases, we can see that the uncertainty in the observations leads to a relatively big standard deviation, making the three asteroids Virtual PHA. Anyway, a big standard deviation means that, even if the minimum value is less than 0.05AU, the probability that the asteroids are actually LMA is quite small.

Running our algorithm on the NEA list, we also noticed that sometimes we have strange configurations for the position of the nominal minima on the orbit of an asteroid. For example we found a list of 13 asteroids with both the minima points further than 45^o from both the nodes, but with an inclination larger than 10^o, and a list of 98 asteroids with only one minimum further than 45^o from both the nodes, always with an inclination larger than 10^o. Sometimes this is due to a particular position of the perihelion with respect to the mutual nodes, but in some other cases it is not clear why this phenomenon occurs.

3.2. Unnumbered asteroids

The second list that we examined with our algorithm is a list of 56505 asteroids, whose orbits have been computed by A. Milani using the software OrbFit version 2.0.2 (http://newton.dm.unipi.it/~asteroid/orbfit/) on the base of the observations, updated to January 24th, 2000, published by the Minor Planet Center. This list of asteroids' orbits is much bigger than the previous we used, because of the adding of a program

Table 2. Distribution of Virtual PHAs by probability.

Range of Probability	Number of Asteroids
0 - 3	5020
3 - 6	1071
6 - 9	173
9 - 12	38
12 - 15	21
15 - 18	4
18 - 21	1
21 - 24	4
24 - 27	3
27 - 30	0
30 - 33	2
33 - 36	3
36 - 39	1
39 - 42	0
42 - 45	0
45 - 48	1
48 - 50	1

by Z. Knezevic and based on a Vaisala algorithm, that allowed the calculation of orbits also for asteroids with very few observations. In this list all the NEAs are included, so besides the 324 LMAs already found, using our algorithm, we found 6368 asteroids that become LMAs only if we consider one of the minima with the variance of the nearer AMOID. Actually this number becomes 6343 if we consider only asteroids with a nominal minimum less than 10AU. Moreover 6225 asteroids have a nominal minimum less than 2AU.

Some of the Virtual PHAs have very poorly determined orbits and the uncertainty in the calculation of the AMOID is very high, for example this happens for the 118 asteroids with a nominal minimum bigger than 2 AU. These cases are not particularly interesting. But for the other cases we expect to find very interesting results.

At this point it seemed to us that it could have been interesting to compute the probability that an asteroid in our Virtual PHAs list has to have its real MOID less than 0.05AU. Actually, since the minimum value we find is a signed quantity, we want to find the probability that the real MOID is between -0.05AU and 0.05AU. To do this, we simply considered the uniform distribution on the segment

$$[\text{Nom. Min.} - 3 * \text{St. Dev.}, \text{Nom. Min.} + 3 * \text{St. Dev.}],$$

and computed the probability of the intersection with the segment [-0.05,0.05]. We added the probability of all our Virtual PHAs, and found as value 127.803. This means that besides the 324 LMAs found in the list of NEAs, there are other 127.803 LMAs, distributed on the 6343 Virtual PHAs. This implies that the estimation on the number of LMAs could be wrong of a factor approximately equal to 39.4 per cent. In Table 2 we show how the probabilities of being LMA are distributed on the 6343 Virtual PHAs.

The range of probability is expressed in percentage, so there are, for example, 5020 asteroids with a probability of being LMAs less than 3 per cent. Most interesting are the cases of the two asteroids with a percentage more than 45 per cent. These two are 1999VS112 with a probability of 47.14 per cent and 1999AG3 with a probability of 48.07 per cent.

The asteroid designated 1999VS112 was observed in two nights separated by three days, and the observed proper motion was 0.21 degrees per day, mostly in right ascension. This seems to indicate that this asteroid is actually a *Main Belt Asteroid (MBA)*, but with our algorithm we found a high probability for this asteroid to be a LMA. Probably in this case the probability of being a LMA has to be lowered taking into account the much larger number density of MBAs.

Completely different is the case for 1999AG3. This asteroid was observed in two nights separated by four days, and the observed proper motion was 0.37 degrees per day, out of which 0.35 were in declination. So, it is much less likely for 1999AG3 to be a MBA, and it seems that the high probability of being a LMA is a realistic estimate. In fact it also has to be lowered due to two main reasons, being the first that our computation is an approximation of the real situation. If we define the *elongation* of an asteroid to be the angle between the lines Earth-Sun and Earth-asteroid, then an asteroid is said to be at its *quadrature* if its elongation is approximately 90 degrees. The second reason to lower our computation of the probability of being a LMA is then that when the asteroid is observed at its quadrature, as in this case, the computation of its orbit presents a problem. In fact in the step of calculating preliminar orbits, discrete multiple solutions are found, and so there are separate confidence regions obtained by the least squares method, and the problem is which region to choose. This problem is not encountered when the asteroid is observed at its *opposition*, that is when its elongation is 180 degrees. So we can say that 1999AG3 has a probability of 48.07 per cent of being a LMA in the region of the parameters space given by the ellipsoid of confidence found with respect to one of its possible orbits. But we expect that the real probability is not to be much smaller, so it still remains an interesting case.

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