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# The effect of diffusion on pulsations of stars on the upper main sequence

## $\delta$ Scuti and metallic A stars

**S. Turcotte**[1,2], **J. Richer**[3], **G. Michaud**[3], **and J. Christensen-Dalsgaard**[2,4]

[1] Service d'Astrophysique/DAPNIA, CEA-Saclay, 91191 Gif sur Yvette, France (turcotte@discovery.saclay.cea.fr)
[2] Teoretisk Astrofysik Center, Danmarks Grundforskningsfond
[3] Université de Montréal and Centre de Recherche en Calcul Appliqué, Montréal, Canada (richer,michaudg@cerca.umontreal.ca)
[4] Institut for Fysik og Astronomi, Aarhus Universitet, 8000 Århus C, Denmark (jcd@ifa.au.dk)

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**Abstract.** Recent dramatic improvements in the modeling of abundance evolution due to diffusion in A stars have been achieved with the help of monochromatic opacity tables from the OPAL group. An important result in the context of stellar pulsations is the substantial helium abundance shown to be left over in the driving region of $\delta$ Scuti-type pulsations in chemically peculiar Am stars. An accurate opacity profile in the entire stellar envelope including the full effect of heavy elements is also now available for the first time.

Pulsations are shown to be excluded for young Am stars but occur naturally when these stars evolve off the ZAMS. The predicted variable metallic A stars all lie towards the red edge of the instability strip, in qualitative agreement with the observed variable $\delta$ Delphini and mild Am stars.

Results show little direct excitation from iron-peak elements in A-type stars. The main abundance effect is due to the settling of helium, along with a marginal effect due to the enhancement of hydrogen.

**Key words:** diffusion – stars: abundances – stars: evolution – stars: variables: $\delta$ Sct

### 1. Introduction

Until very recently the theory for the formation of chemically peculiar (CP) stars and the observations of variable stars agreed that CP stars were not variable and variable stars were chemically normal. A few exceptions were known for many years but their rarity and their mild chemical anomalies did not challenge the standard picture.

In recent years this picture has evolved considerably and the dichotomy between CP stars and variable stars is no longer clear-cut. Indeed, many types of CP stars are now known to exhibit various types of pulsations. Significantly, most of the variable CP stars belong to the magnetic Ap and Bp families. But some non-magnetic CP stars are known to be variable and it has been claimed that in some cases the observed variability cannot be easily reconciled with the theory of diffusion for CP stars.

In this paper we examine how recent progress in modeling Am stars affects our understanding of the problem of variability in A-type CP stars. We first review the current observational and theoretical pictures for variability in CP stars. The improved models of diffusion in A stars are then presented. The linear nonadiabatic oscillation equations are solved in selected models of A-type stars, both with and without diffusion. These results highlight the differential effect of diffusion on the stability of the models, and allow to draw conclusions on how well the models with diffusion can be reconciled with observations.

### 2. Metallicism and pulsations: observations

CP stars are found almost everywhere in the HR diagram. Some of these stars are evolved and their observed abundance peculiarities reflect nuclear processes. Others are compact objects in which diffusion is well established (Michaud & Fontaine 1984). Many of these objects are variable and they are very favourable for asteroseismology.

We concentrate mainly on main-sequence stars in which diffusion is thought to be the principal cause of abundance anomalies. These are found from early F-type (Fm) stars to late B-type (HgMn) stars, including many varieties of A stars [Am, Ap, $\lambda$ Bootis, $\rho$ Puppis]. Within these spectral classes many types of variable stars are also found [$\gamma$ Doradus, several classes of $\delta$ Scuti, and Slowly Pulsating B (SPB) stars].

In A-type stars, almost 70% of non-CP stars are $\delta$ Scuti variables at current levels of sensitivity. Most non-variable A stars are Am stars.

Amongst the CP stars which happen to be pulsating, the most conspicuous are the "rapidly oscillating Ap stars" (roAp) first discovered by Kurtz (1978a). These are found amongst the coolest of the magnetic Ap stars which exhibit large abundance anomalies of many heavy elements. They have generated considerable interest because of the relatively large number of ob-

*Send offprint requests to*: S. Turcotte

served overstable modes which makes them promising objects for asteroseismology. See, for example, Matthews (1991) for a review.

In non-magnetic stars, however, variability and anomalous abundances are found in very few stars simultaneously. Over the years some mildly metallic stars have been found to exhibit some variability. Baglin (1972) suggested that if diffusion is the cause of the Am phenomenon, Am stars should not pulsate. Some mild Am stars and evolved CP stars were then already known to be variable (Kurtz 1976) but all classical Am stars which were thought to be variable in the early 70's were subsequently found to be stable. Since then Kurtz and his collaborators have been the principal investigators in the search for variable CP stars. They have assembled a short list of metallic stars, most if not all of them fairly evolved, which are also $\delta$ Scuti-type pulsators (Kurtz 1978b, 1984, 1989; Kurtz et al. 1995; Kurtz 2000).

Amongst these stars, some have been thought to be problematic. In particular, Kurtz (1989) claimed to have discovered a pulsating classical Am star. The problem lies in its apparently large abundance anomalies, not typical of the other known variable non-magnetic CP stars. Also, the variable evolved Am star HD40765 has been considered by Kurtz et al. (1995) to be problematic because of the possibly large surface velocities involved. A point that has been made repeatedly is that the large velocities caused by pulsations, estimated in this star to be of the order of $14\,\mathrm{km\,s^{-1}}$ at the surface, might generate turbulence in the interior which would in turn hinder the formation of the required surface-abundance anomalies.

In addition to variable CP stars there are some $\delta$ Scuti stars that are particularly interesting from our point of view. First there are some otherwise seemingly run-of-the-mill $\delta$ Scuti stars[1] in which peculiar abundances (Russell 1995; Rachkovskaya 1994) superficially consistent with what is expected as a result of diffusion seem to have been found.

Second, the high-amplitude $\delta$ Scuti stars are interesting because they are characterized by very small $v \sin i$ (Solano & Fernley 1997). As CP stars are mostly slow rotators ($v_{\mathrm{rot}} < 100\,\mathrm{km\,s^{-1}}$), one should find different velocity distributions in CP stars and in $\delta$ Scuti stars. Indeed, observations show that $\delta$ Scuti stars are on the average fairly fast-rotating stars, with the exception of the high-amplitude stars. In other respects the high-amplitude $\delta$ Scuti stars do not differ from their more common low-amplitude counterparts. They are, however, known to be evolved stars and as such might not feature significant abundance anomalies.

Although observations do not completely rule out variability in Am stars, they do pose rigorous constraints on them. Either it is an extremely rare occurrence or pulsations are of extremely low amplitude. An extensive search of the Hipparcos database by many authors revealed many new variable stars (Aerts et al. 1998; Waelkens et al. 1998; Paunzen & Maitzen 1998). Significantly, all the newly discovered variable CP stars were found to be magnetic (Ap or Bp). All the other variable stars found in this way are of the known families of variable stars, i.e., $\gamma$ Doradus, SPB and a few $\beta$ Cephei stars. The expected sensitivity of these surveys ranges from as low as 3 to 55 mmag depending on the brightness of the object (Eyer & Grenon 1998).

The CP stars found most recently to be variable are $\lambda$ Bootis stars in which nonradial pulsations have been detected (Paunzen et al. 1998).

## 3. Metallicism and pulsations: theory

Elements migrate with respect to each other because of differential forces mostly due to inward gravity and outward radiative pressure. This segregation of different atomic species is what is termed here diffusion. Diffusion is a rather fragile process because typical diffusion velocities are of the order of fractions of cm per second. Large-scale motion of matter or turbulence quickly overwhelm diffusion and homogenize the chemical composition outside of nuclear burning regions. Diffusion is efficient only in stars where the competing processes are weak. This explains why CP stars are mainly of spectral types ranging from early F to late B-type stars. In these spectral types the surface convection zone is thin enough and the mass loss rates small enough (a mass loss of around $10^{-14}\,\mathrm{M_\odot\,yr^{-1}}$ is large enough to remove all anomalies) to permit the development of significant abundance anomalies. As a result, CP stars are typically relatively unevolved, slowly rotating stars. There are some CP stars, Ap or $\lambda$ Bootis stars, for example, in which other factors come into play.

In the standard picture for FmAm, HgMn and Ap stars, the observed abundance anomalies of heavy elements develop as a consequence of the settling of helium. As it disappears from the superficial convection zone it can no longer provide the opacity to sustain convection in the He II zone and the result is a much thinner convection zone due to the ionization of H I. At this depth, the radiative forces on the various heavy elements are compatible with the pattern of surface-abundance anomalies. This model agrees qualitatively with observations but requires additional assumptions to obtain a quantitative agreement as the predicted anomalies are generally too large. No calculation based on this model has ever reproduced the abundances of individual Am stars.

The major consequence of this model for stellar stability is that helium is no longer present to excite pulsations typical of the classical instability strip. Many studies related to this problem have been carried out. The most thorough, by Cox et al. (1979), showed that variability is possible with a low helium content but that the width of the instability strip decreases as the helium abundance decreases. Their conclusion was that classical Am stars should not be variable but that metallicism and variability were not mutually exclusive in the red part of the classical instability strip if the surface helium abundance fell marginally below 0.1 but without dropping to a very low value.

When the diffusion of heavy elements is included in a consistent fashion this picture changes drastically. These models are dubbed here The New Montreal Models (NMM).

[1] Amongst these is 20CVn which was classified as $\delta$ Delphini by Kurtz (1976)

Magnetic Ap stars and $\lambda$ Bootis stars will be ignored at this time. We note in passing that for roAp stars a few models have been proposed based on the so-called $\kappa$ mechanism, where the dominant driving comes from the effects of the opacity $\kappa$. As only microscopic diffusion can explain the observed abundances at present, the proposed models include its effects. Amongst the possibilities are: the replenishment of superficial helium by advection from mass loss (Vauclair & Dolez 1990), the replacement of helium by silicon pushed in the driving region by radiative levitation (Matthews 1988), and hydrogen overabundances in the H I ionization zone as a result of helium settling (Dziembowski & Goode 1996). As for $\lambda$ Bootis stars, the currently preferred but so far unproven accretion models for these stars assume that the helium abundance would remain normal in the driving region (Turcotte & Charbonneau 1993), accounting for the necessary opacity for the $\kappa$ mechanism to work.

## 4. The new Montreal models

### *4.1. New results with the NMM*

The NMM include the diffusion of all major elements up to nickel consistently by using monochromatic opacity tables of the Livermore group (Iglesias & Roger 1996) to compute the opacity and the radiative forces accurately at all points in the star and for the local chemical composition; the basic procedures were outlined by Turcotte et al. (1998a) and Richer et al. (1998).

The evolution of the abundances and of the structure is completely consistent through the opacity. One very important property of these models is the presence of a convectively unstable zone around $200\,000$ K where iron-group elements dominate the opacity. This convection zone appears naturally as the consequence of abundance changes if they are large enough. The NMM then assume that this deeper convection zone is connected to the helium and hydrogen convection zones through convective overshoot.

The large depth of the convective mixing relative to standard models for chemically peculiar stars results in much smaller surface-abundance variations, more in line with observations. Richer et al. (2000) showed that the radiative forces at the base of the iron convection zone follow the correct pattern over a relatively narrow region for the Am signature to be recovered without additional assumptions. A quantitative agreement with observed abundances for Am stars does require additional turbulence below the superficial convective envelope. In the NMM, the mixing necessary to reproduce the abundances observed in Am stars prevents the formation of this convection zone.

A significant result in the context of stellar pulsations is that helium is still substantially present in the He II driving region in these new models. Also, as the opacity in the vicinity of the so-called "metal opacity bump"[2] is increased relative to standard models, one can expect that this region will contribute to the excitation of longer-period modes. On the other hand, in some extreme cases this region might be convective in contrast to standard models, which would reduce the radiative flux and by consequence the driving in that region.

We shall examine the consequence of these results on the possible instability of Am stars. As a complement, we shall also estimate the effect of mild abundance anomalies on predicted pulsations of $\delta$ Scuti stars.

### *4.2. The basic properties of the NMM*

The NMM include the detailed diffusion of 21 major elements from H through Ni plus several light elements and isotopes for a total of 28 species. The opacity data used in the evolution code and in the following analysis are the OPAL monochromatic opacity tables (Iglesias & Rogers 1996) which allow us to calculate accurate Rosseland mean opacities and radiative forces for any peculiar chemical composition necessary. At low temperatures, for which the OPAL data is lacking, we supplement them with the Kurucz (1991) opacity tables. Although it would be desirable to take into account changes in the composition of individual heavy elements in the atmosphere, the transition occurs at such low temperatures that it might be of little consequence for the stellar interior.

The models incorporate standard procedures for the equation of state and the nuclear reaction rates. The mixing-length formalism for convection is used and is calibrated using the Sun (Turcotte et al. 1998a). All models have an identical, homogeneous, initial chemical composition as specified by Turcotte et al. (1998b). All models are one-dimensional, non-rotating, and non-magnetic.

Individual models for a given stellar mass only differ in the parameters adopted for the coefficient of turbulent diffusion. In all calculations presented in this paper, the coefficients are chosen so that the zone mixed by turbulence goes from the surface to somewhat below the iron convection zone. The coefficient of turbulent diffusion is modeled with the following three-parameter expression

$$D_{\rm T} = D_0 \left(\frac{\rho_0}{\rho}\right)^n , \qquad (1)$$

where the free parameters are $D_0$, $\rho_0$ and $n$. The evolution of the abundances is very sensitive to the depth of the mixing but not so much to the profile of $D_{\rm T}$. The models are named with reference to the number $R$ which specifies the ratio of $D_{\rm T}$ to the coefficient of atomic diffusion of helium $D_{\rm He}$ at the point where the density $\rho$ is equal to the reference value $\rho_0$. For example, model 1.90R1000-2 is a 1.90 $M_\odot$ star with $R = 1000$ and $n = 2$; for simplicity, '10K' is used to refer to models with $R = 10\,000$. In all the models discussed in this paper $\rho_0$ is $8 \times 10^{-6}$ g cm$^{-3}$. The reader is referred to Richer et al. (2000) and Richard et al. (2000) for further details.

For every stellar mass examined, one model that does not include any effects of diffusion is also included. For computational efficiency this comparison model uses the mean Rosseland opacity tables of Iglesias & Rogers (1996). Assuming that there is no separation of elements in the star implies that some

[2] We shall refrain from using this expression and otherwise use the expression "iron-peak-element opacity bump", as those elements dominate the opacity there

unspecified mixing is necessarily assumed. This mixing is required to be large and deep enough to keep superficial regions at a constant chemical composition without mixing too deeply, to avoid dredging up nuclearly processed matter to the surface. They are named according to the mass and are labeled with the tag "ND" (e.g., 1.90-ND).

## 5. Diffusion and the $\kappa$ mechanism

To determine the frequencies of modes of oscillation for a star requires only that we solve the adiabatic equations. Solving the full nonadiabatic equations of stellar oscillation allows us to calculate the growth rates of the modes, and hence to determine which of the modes are overstable; also, by considering the work integral we can investigate the contributions of the different parts of the star to the excitation and damping of the mode.

The nonadiabatic oscillation package used was generously provided to us by W. Dziembowski and follows the procedure first described by Dziembowski (1977). We are mainly concerned here with excitation via the $\kappa$ mechanism on which abundance variations have a direct impact. We note, however, that the present calculations lack a good modeling of the effect of convection; this must be kept in mind in the analysis of the results.

The physics of the $\kappa$ mechanism has been reviewed extensively (e.g. Cox 1980; Unno et al. 1979; Gautschy & Saio 1995). As a quick reminder, generating pulsations in a star requires that the energy gained by an oscillation mode over a complete cycle be larger than the energy lost. We are then looking for a positive net work over the entire star over one cycle. In the case of the $\kappa$ mechanism, the energy is transferred from the outward radiation flux to the oscillation mode via the opacity. A mode becomes overstable by this mechanism if the opacity profile and its derivatives have the right features.

Following Unno et al. (1979), from the definition of the work ($W$) as the variation of the kinetic energy ($E$) over a cycle,

$$W = \oint \frac{\mathrm{d}E}{\mathrm{d}t}\mathrm{d}t\,, \tag{2}$$

one can write

$$W = \frac{\pi}{\omega}\int_0^{M_r} \frac{\delta T}{T}\delta\left[\epsilon_{\mathrm{N}} - \frac{1}{\rho}\nabla\dot{(}F_{\mathrm{R}} + F_{\mathrm{c}})\right]\mathrm{d}M_r\,, \tag{3}$$

where $\delta$ denotes Lagrangian perturbations. Also, $\omega$ is the (angular) oscillation frequency, $T$ is temperature, $M_r$ is the mass interior to the radius $r$, $\epsilon_{\mathrm{N}}$ is the nuclear energy generation rate, and $F_{\mathrm{R}}$ and $F_{\mathrm{c}}$ are the radiative and convective fluxes. If one neglects the contribution from the nuclear ($\epsilon_{\mathrm{N}}$) and convective terms ($F_{\mathrm{c}}$), and only keeps the perturbation of the radiative flux ($F_{\mathrm{R}}$), one can isolate the contribution of the $\kappa$ mechanism to the driving of a given mode of oscillation. To obtain a simple estimate of this contribution to the work integral we make the quasi-adiabatic approximation (i.e., evaluate the work integral by means of adiabatic eigenfunctions), and furthermore assume that the adiabatic thermodynamic derivatives $\Gamma_1$, $\Gamma_3$ and $\nabla_{\mathrm{ad}}$ are constant. Then the work done by the $\kappa$ mechanism is proportional to

$$\int\left(\frac{\delta T}{T}\right)^2\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(\kappa_T + \frac{\kappa_\rho}{\Gamma_3 - 1}\right)L_r\right]\mathrm{d}M_r\,, \tag{4}$$

where $L_r$ is the luminosity at $r$, and $\kappa_T = (\partial\ln\kappa/\partial\ln T)_\rho$, $\kappa_\rho = (\partial\ln\kappa/\partial\ln\rho)_T$; thus regions where

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\kappa_T + \frac{\kappa_\rho}{\Gamma_3 - 1}\right) > 0 \tag{5}$$

contribute to the excitation. Local increases in the logarithmic derivatives of $\kappa$ are necessary and a decrease in $\Gamma_3 - 1$ in partial ionization zones of a dominant species (H or He) is helpful. It also follows that regions where the gradients of $\kappa_T$ and $\kappa_\rho$ are negative contribute to damping of the pulsation. The $\kappa_T$ term usually dominates over the other term.

The numerical results reported in the text, including the growth parameters and work integrals, are computed using the full nonadiabatic procedure of the Dziembowski code.

In order that the excitation by the $\kappa$ mechanism should not be cancelled by damping elsewhere, it is necessary that the driving region lie in the so-called transition zone between the quasi-adiabatic and nonadiabatic regimes; in that case, the oscillations are strongly nonadiabatic outside the driving region, and this part of the star therefore does not contribute to the damping, giving rise to net driving. This leads to an approximate relation between the period $\Pi$ of a given mode of pulsation and the position of the transition region in a star (Cox 1980):

$$\frac{\langle c_v T\rangle_{\mathrm{tr}}\Delta M_{\mathrm{tr}}}{L\Pi} \simeq 1\,, \tag{6}$$

here $\Delta M_{\mathrm{tr}}$ is the mass outside the transition region, $\langle\cdots\rangle_{\mathrm{tr}}$ is the average over that part of the star, $c_v$ being the specific heat, and $L$ is the luminosity.

The normalized growth rate is defined as

$$\eta = \int\frac{\mathrm{d}W}{\mathrm{d}\log T}\mathrm{d}\log T \Big/ \int\left|\frac{\mathrm{d}W}{\mathrm{d}\log T}\right|\mathrm{d}\log T\ . \tag{7}$$

In this formulation, $\eta$ varies from $+1$, if there is driving in the entire star, to $-1$, if there is damping in the entire star. The value of zero defines neutral stability.

Diffusion affects the $\kappa$ mechanism by decreasing driving from helium in favour of driving from metals. As a consequence of Eq. (6), the pulsation period of the unstable modes depends on the depth of the driving region. During a star's evolution the helium ionization zone gradually shifts deeper in the star, thereby increasing the period of the observed pulsation modes. Additionally, as the driving in the deeper iron-peak driving region increases while the driving due to helium decreases as a result of diffusion, one might expect the observed pulsation periods to shift to even longer periods. The effect of abundance variations on the opacity profiles is discussed below for selected models (see Fig. 3).

**Table 1.** Physical parameters of the 1.9 $M_\odot$ models

| Age (Myr) | $L/L_\odot$ | $T_{\rm eff}$ | $R/R_\odot$ | $\log g$ |
|---|---|---|---|---|
| 1.90-ND | | | | |
| 101 | 13.50 | 8710 | 1.628 | 4.293 |
| 300 | 14.30 | 8501 | 1.759 | 4.226 |
| 502 | 15.17 | 8248 | 1.924 | 4.148 |
| 670 | 15.94 | 7952 | 2.123 | 4.063 |
| 1 003 | 17.04 | 7002 | 2.830 | 3.813 |
| 1.90R10K-2 | | | | |
| 101 | 13.09 | 8669 | 1.615 | 4.291 |
| 300 | 14.11 | 8431 | 1.633 | 4.217 |
| 503 | 15.01 | 8148 | 1.961 | 4.131 |
| 670 | 15.76 | 7848 | 2.167 | 4.045 |
| 1 002 | 16.84 | 6919 | 2.881 | 3.797 |
| 1 105 | 23.72 | 7063 | 3.281 | 3.684 |
| 1.90R1000-2 | | | | |
| 101 | 13.34 | 8652 | 1.640 | 4.287 |
| 300 | 14.11 | 8407 | 1.787 | 4.212 |
| 503 | 15.04 | 8125 | 1.975 | 4.125 |
| 670 | 15.82 | 7830 | 2.181 | 4.039 |
| 1 001 | 16.96 | 6918 | 2.893 | 3.794 |
| 1 094 | 19.29 | 7052 | 2.969 | 3.771 |
| 1.90R300-2 | | | | |
| 101 | 13.34 | 8640 | 1.645 | 4.284 |
| 300 | 14.11 | 8387 | 1.795 | 4.208 |
| 503 | 15.04 | 8103 | 1.985 | 4.121 |
| 670 | 15.82 | 7809 | 2.192 | 4.035 |
| 1 003 | 16.98 | 6898 | 2.912 | 3.788 |
| 1 098 | 19.00 | 6958 | 3.027 | 3.755 |

**Table 2.** Physical parameters of the 2.0 $M_\odot$ models

| Age (Myr) | $L/L_\odot$ | $T_{\rm eff}$ | $R/R_\odot$ | $\log g$ |
|---|---|---|---|---|
| 2.00-ND | | | | |
| 300 | 17.84 | 8767 | 1.847 | 4.206 |
| 670 | 20.27 | 7979 | 2.377 | 3.987 |
| 801 | 20.85 | 7501 | 2.728 | 3.867 |
| 852 | 21.06 | 7290 | 2.903 | 3.813 |
| 2.00R2500-0.75 | | | | |
| 300 | 17.62 | 8731 | 1.851 | 4.204 |
| 670 | 20.06 | 7958 | 2.378 | 3.986 |
| 801 | 20.69 | 7497 | 2.720 | 3.870 |
| 854 | 20.88 | 7274 | 2.903 | 3.813 |
| 2.00R1000-2 | | | | |
| 300 | 17.59 | 8663 | 1.879 | 4.191 |
| 670 | 19.97 | 7862 | 2.430 | 3.967 |
| 751 | 20.37 | 7594 | 2.631 | 3.898 |
| 854 | 20.75 | 7186 | 2.966 | 3.794 |

**Table 3.** Physical parameters of the 2.2 $M_\odot$ models

| Age (Myr) | $L/L_\odot$ | $T_{\rm eff}$ | $R/R_\odot$ | $\log g$ |
|---|---|---|---|---|
| 2.20-ND | | | | |
| 500 | 29.88 | 8595 | 2.487 | 3.989 |
| 603 | 30.97 | 8082 | 2.864 | 3.866 |
| 670 | 31.71 | 7700 | 3.193 | 3.772 |
| 2.20R1000-2 | | | | |
| 300 | 26.78 | 9116 | 2.093 | 4.138 |
| 501 | 29.48 | 8470 | 2.545 | 3.969 |
| 597 | 30.52 | 8001 | 2.901 | 3.855 |

## 6. Opacity and its derivatives

We recomputed many thermodynamic quantities and the opacities in the process of preparing the models for input to the nonadiabatic oscillation package. Every effort has been made to be consistent with the procedures followed in the evolution code. The most tricky operation at this point is the determination of accurate opacity derivatives. The opacities are interpolated linearly and smoothed locally in the evolution code. As first and second derivatives of the opacity are needed in the oscillation package a more refined interpolation procedure had to be adopted to guarantee smoother derivatives. We chose the Houdek (Houdek & Rogl 1996) routines which use two-dimensional rational splines. For each mesh point of the models we constructed a $7 \times 7$ opacity grid in the $(\rho, T)$ plane in which the splines were fitted and the opacity derivatives were determined.

## 7. The models of A stars

Amongst the numerous NMM models some were selected to provide a satisfactory sampling of the instability strip and of Am stars. The selected models of 1.9, 2.0 and 2.2 $M_\odot$ are summarized in Tables 1, 2, and 3 respectively. (The naming convention for the models was described in the discussion relating to Eq. 1.) Of these, the models with $R = 1000$ and $n = 2$ reproduce reasonably well the abundances of Am stars (Richer et al. 2000). They are not optimal, however, and models with $R = 1000$ and with shallower turbulence, $n = 3$ or 4, would be more accurate. We feel that using such models would not alter the essence of the interpretation of the results.

## 8. $\delta$ Scuti-type pulsations in Am stars

Most pulsating A stars lie in the He II instability strip. Consequently, most attempts to model the pulsations of chemically peculiar A stars have centered on how helium can remain or be replenished in the He II zone, or possibly which element can replace it as the motor of the excitation mechanism. As previously discussed, an important characteristic of the NMM is the relatively high superficial helium abundance remaining throughout the evolution in Am stars (see Sect. 4). This section aims at answering some questions raised by these new models: Is helium sufficiently abundant to generate variability in Am stars? Do the NMM clash with our current observational understanding

**Table 4.** Overstable radial modes in models 1.90R1000-2 and 1.90-ND for several ages. + identifies models for which there is at least one mode with $\eta > 0.0$.

| | $l = 0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $n$ | | | | | | | |
| age (Myr) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1.90R1000-2 | | | | | | | | |
| 101 | | | | | | | | |
| 300 | | | | | | | | |
| 503 | | | | | | | | |
| 670 | | | | | | | | |
| 1 003 | + | + | + | + | + | | | |
| 1 098 | + | + | + | + | | | | |
| 1.90-ND | | | | | | | | |
| 101 | | | | | | | | + |
| 300 | | | | | | | + | + |
| 502 | | | | | | + | + | |
| 670 | | | | + | + | + | + | |
| 1 003 | + | + | + | + | + | + | + | |
| 1 105 | + | + | + | + | + | + | | |

**Table 5.** Overstable nonradial ($l = 1$) p modes in models 1.90R1000-2 and 1.90-ND for several ages. + identifies models for which there is at least one mode with $\eta > 0.0$.

| | $l = 1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $n$ | | | | | | | |
| age (Myr) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1.90R1000-2 | | | | | | | | |
| 101 | | | | | | | | |
| 300 | | | | | | | | |
| 503 | | | | | | | | |
| 670 | | | | | | | | |
| 1 003 | + | + | + | + | | | | |
| 1 098 | + | + | + | | | | | |
| 1.90-ND | | | | | | | | |
| 101 | | | | | | + | + | + |
| 300 | | | | | | + | + | |
| 502 | | | | | + | + | + | |
| 670 | | + | + | + | + | + | | |
| 1 003 | + | + | + | + | + | | | |
| 1 105 | | | | | | | | |

of metallicism and pulsations? Does the larger driving in the iron-peak opacity bump play a role in exciting g modes in CP stars?

### *8.1. The variability of an Am star across the instability strip*

The evolution of a 1.9 $M_\odot$ star from the ZAMS to hydrogen exhaustion in the core spans almost the full width of the instability strip as shown in Fig. 1. Studying the evolution of instability in models for such a star as it evolves gives a first idea of the behaviour of Am stars in general. The luminosity dependence of the instability strip implies that the lower part of the instability strip will be populated by young stars and the upper part by generally more evolved stars. This should be taken into account if one attempts to extrapolate the following results to stars of different masses.

Tables 4 and 5 show the general pattern of overstability for radial ($l = 0$) and nonradial ($l = 1$) p modes, respectively, in an evolving 1.9 $M_\odot$ model with and without diffusion. The model with diffusion shown here is an adequate representation of a classical Am star (see Fig. 22 of Richer et al. 2000).

First, one can see that no overstable modes, radial or otherwise, are found in the young model with diffusion (Am star) whereas high-order modes are predicted to be overstable in the model without diffusion (a standard model for $\delta$ Scuti stars, i.e., a homogeneous envelope with solar composition). As the stars age, the lower-order modes are gradually more excited and eventually the fundamental mode becomes overstable in both models. In the model with diffusion, the superficial helium abundance reaches its lowest value at 750 Myr where it is 0.114 by mass. The model is stable at that time. The more evolved models tend to be more unstable. At an age of 1.098 Gyr, for example, the helium mass fraction rises to 0.126. The small difference in the helium abundance for these two models suggests that the pulsations in evolved stars occur without relying on the dredge-up of helium nor on hypothetic mass-loss but only as a result of their evolution. This result is in agreement with Cox et al. (1979) and earlier work which showed that stars would be

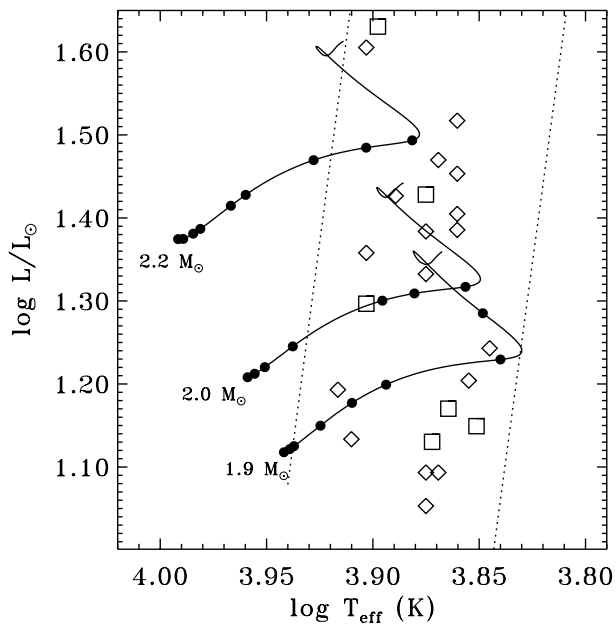


**Fig. 1.** Various classes of peculiar and variable A stars in the HR diagram and evolutionary paths for some of our models whose mass are indicated in the graph (these paths are for models R1000-2; paths for other models at the same masses would be indistinguishable at this scale). The squares identify the stars of Kurtz (1976) while the diamonds identify the $\delta$ Scuti stars observed by Russell (1995). The black dots mark the position of the models listed in Tables 1 through 3. The dotted lines identify the approximate boundaries of the instability strip.

expected to be variable in the red part of the instability strip if their helium abundance were of the order of 0.1 or higher.

As far as nonradial modes are concerned, the same overall behaviour as for radial modes is repeated, although the labeling of the modes presents some problems. Nonetheless, a presentation such as Table 5 illustrates the general evolution of overstability in nonradial modes; here the mathematical mode orders, defined such that the mode order of a given mode is constant during the evolution of the model, are shown. This is not directly related to the physical nature of the mode, however. For p modes, including radial modes, the dimensionless frequency[3], normalized with the dynamical timescale of the star, is approximately constant during the evolution. In contrast, the dimensionless frequency increases for g modes, which are strongly affected by the composition structure in the stellar interior. Where the frequency of a g modes meets the frequency of a p mode, an avoided crossing takes place, at which occurs a shift in mode order for a given frequency, or conversely a shift in frequency for a given mode order (e.g. Osaki 1975). Thus, in contrast to radial modes, there is no firm correspondence between mode order and dimensionless frequency through the life of a star. Even so, a table corresponding to Table 5 but based on a definition of the order more closely related to the physical nature of the modes, would have shown a similar overall pattern. The more evolved models feature overstable nonradial p and g modes. In the most evolved model, the last model of 1.90-ND, the overstable modes all have the physical nature of g modes, spanning periods of 1 to 3.5 hours. Some modes, at avoided crossings, are mixed modes which share p and g mode characters.

Two models with a turbulence depth bracketing that of the model discussed so far are also available at this mass. In the model with shallower mixing (1.90R300-2) only one mode is found to have a positive growth rate, the $l = 0, n = 3$ mode for the model at 1.098 Gyr. No overstable nonradial modes have been found. The minimum helium abundance at 750 Myr is 0.0734 and increases to 0.0871 at 1.098 Gyr for this star. In the model with deeper mixing (1.90R10K-2) the minimum helium abundance reached is higher (0.180) and the youngest age at which overstability is found is 850 Myr. This confirms that the blue edge of the instability strip is sensitive to the helium abundance, which has long been established (Cox et al. 1979 and references therein).

In an attempt to give a better picture of the evolution of the excitation of various modes in the star, Fig. 2 shows the change in the growth rate as a function of time for 3 different radial modes in all four models of the 1.9 $M_\odot$ star. This figure reflects the preceding discussion in the sense that the growth rates of low-order modes increase with passing time and that high-order modes become less excited. It also shows that the effect of diffusion is much stronger for higher-order modes. The different evolution of the growth rates of the $l = 0, n = 8$ mode after 700 Myr in the models with and without diffusion is noteworthy. This will be discussed again shortly.

[3] e.g. $\sigma$, defined by $\sigma^2 = R^3\omega^2/(GM)$ where $M$ and $R$ are total mass and surface radius of the star, and $G$ is the gravitational constant.

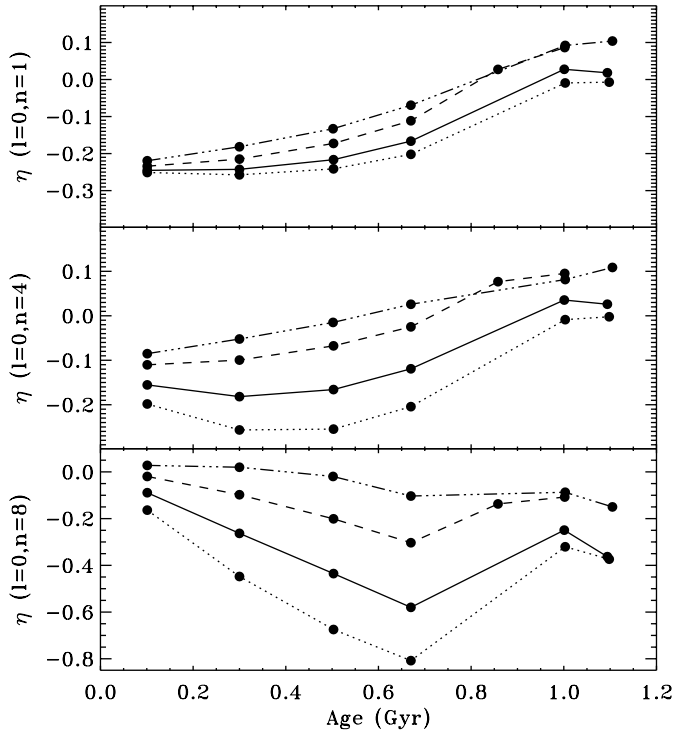


**Fig. 2.** Variation of the growth rate of selected radial modes as a function of age in models of 1.90 $M_\odot$. The dot-dashed line represents model 1.90-ND, the dashed line 1.90R10K-2, the solid line 1.90R1000-2, and the dotted line 1.90R300-2.

The detailed effect of diffusion on the opacities of A stars is illustrated in Fig. 3 where the opacity is shown for the same four models at 670 Myr. The other panels show the evolution of the argument of Eq. (5) for ages of roughly 100, 670 and 1000 Myr. The most striking effect is the increase around $\log T \simeq 5.3$ caused by the heightened iron-peak opacity bump. The effect of the settling of helium around $\log T \simeq 4.5$ appears minor in the figure, but this ends up as being the dominant effect.

Considering just Fig. 3 can give a false impression of the importance of diffusion in different regions. A truer impression is obtained from Fig. 4 where the growth rates for radial modes are shown together with the work integrals for two of those modes, the $l = 0, n = 1$ and the $l = 0, n = 8$, again at 100, 670 and 1000 Myr.

Evidently, the work integrals for different models are close to each other in most regions of the star. The differences are larger for the $n = 8$ modes which is reflected in the growth rates. The driving due to helium at $\log T \simeq 4.7$ is the feature most obviously affected by diffusion, decreasing as helium settles out of the convection zone. Diffusion has some marginal effect in the iron-peak opacity bump for the $n = 1$ mode where it partially compensates the effect of He settling but it has no effect whatsoever for the higher-frequency $n = 8$ mode. The differences from model to model are more easily seen in the growth rates which have already been discussed.

Comparing the evolution of driving in the models with and without diffusion in Fig. 4 explains the different evolution of

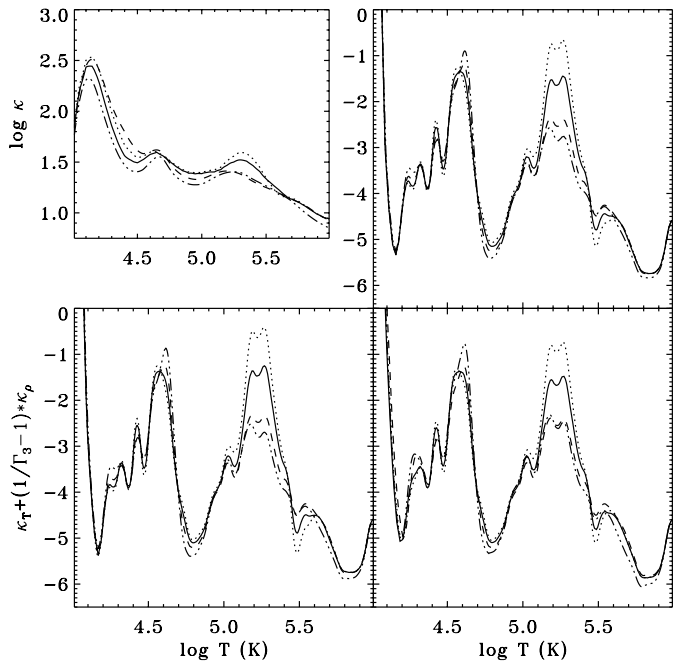


**Fig. 3.** Evolution of the opacity in the various 1.90 $M_\odot$ models discussed in the text. The panel on the upper left shows the logarithm of the Rosseland mean opacity at an age of 670 Myr for all models. The other three panels show the variation of the argument of Eq. (5) at three ages: upper right at 100 Myr, lower left at 670 Myr, and lower right at 1000 Myr. The curves are associated with the following models: dot-dashed line for 1.90-ND, dashed line for 1.90R10K-2, solid line for 1.90R1000-2, and dotted line for 1.90R300-2.

the growth rates of low and high-order modes in the presence of diffusion presented in Fig. 2. In Fig. 4, the height of the peaks due to H I, at $\log T \simeq 4.1$, increases with time at a rate quite similar in all models shown. However, the evolution of the peak due to helium, at $\log T \simeq 4.7$, differs significantly for models with and without diffusion. As discussed previously diffusion causes a decrease in helium driving, but it has little effect on driving from hydrogen in the H I ionization zone. The relative contribution of hydrogen to the net excitation of a mode is larger in higher-order modes, as illustrated by comparing the evolution of the $n = 8$ and the $n = 1$ modes in Fig. 4, and is also larger in models with diffusion because of the lower excitation from helium.

As a result, with the passage of time, the net excitation of a high-order mode is less affected by diffusion than a low-order mode. This can explain why, for higher-order modes shown in the bottom panel of Fig. 2, the growth rates are less adversely affected by diffusion than would have been expected from a simple extrapolation of the evolution in the young stars.

There are some apparent numerical effects in the work integrals that are caused by the manipulation of the opacity tables. First, there are obvious spurious oscillations of the work integrals for the $n = 1$ mode. Although these oscillations change the growth rates slightly, one still gets an accurate assessment of the differential effect of diffusion as the numerical features are the same for all models. Also, the apparent temperature shift towards the surface at 1.0 Gyr between the 1.90-ND and other models is probably an age effect. The model without diffusion is slightly more evolved than the models with diffusion at a given evolution time because the former was computed using the Livermore Rosseland mean opacity tables rather than the monochromatic opacity tables.

Fig. 5 illustrates the dependence of the growth rates on the surface helium abundance for different ages of the 1.9 $M_\odot$ models. Three modes were selected, the fundamental ($l = 0, n = 1$), a higher-order p mode ($l = 0, n = 8$) and a g mode ($l = 1, n = -14$) at 100, 670 and 1000 Myr. As one could have predicted, both p modes are very sensitive to the depletion of helium at the surface. The higher the frequency of the mode, the larger the slope of the correlation. For the g mode, there is a slight anti-correlation which shows that it is weakly correlated to the level of driving in the iron-peak opacity bump.

The periods of the modes predicted to be overstable are summarized in Table 6. First one can verify that the periods are within the observed period range for $\delta$ Scuti stars. Their periods are known to range from 30 minutes to 6 hours. The young models discussed here feature periods of 20 minutes while older models are characterized by longer periods, roughly between one and four hours. While we find periods well below the currently observed lower limit for $\delta$ Scuti stars, we know of no reason to exclude their existence outright.

Diffusion has little effect on the periods of oscillation. There is a systematic trend for diffusion to increase slightly the period of a given mode. The relative difference between the same modes in the models shown in Table 6 is of the order of a few per cent with no clear dependence on the order of the modes. Actually, most of the difference could be consistent with only evolutionary effects related to the use of Rosseland opacity tables for the ND model. If we compare the periods of the radial modes of models 1.90R1000-2 and 1.90R10K-2, which are both computed with the monochromatic opacity tables, we find period differences typically only a fifth of the differences between the models calculated with differing opacities.

### *8.2. The effect of diffusion on the instability strip*

The effect of the settling of helium on the width of the instability strip has already been studied by Cox et al. (1979). They found that the main effect was a reduction of its width, the blue edge shifting towards the red edge, eventually leading to the disappearance of instability when helium was sufficiently depleted. As has been touched upon in the preceding section, the present models seem to follow the same pattern.

In order to gain a somewhat more complete vision of the effect of diffusion on the width of the instability strip we need to examine a larger sample of models of A-type stars with and without diffusion. For this purpose, additional models of 2.0 and 2.2 $M_\odot$ with various assumed turbulence were selected.

The effect of diffusion on the width of the classical instability strip can be estimated from Fig. 6 where our models which exhibit variability are shown in the HR diagram. In each panel of the figure, all models have the same turbulence parameteri-

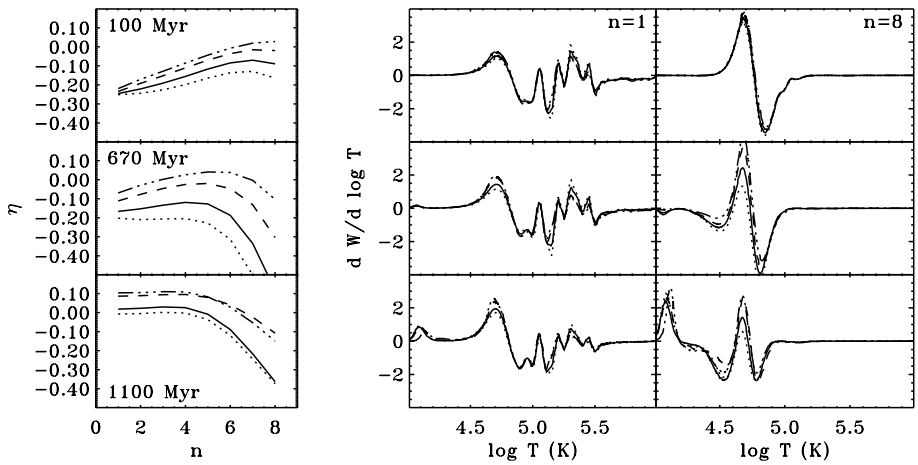


**Fig. 4.** Growth rates ($\eta$) for low-order radial modes $l = 0$ (left) and the work integrals for the fundamental $l = 0, n = 1$ (centre) and the shorter period $n = 8$ (right) modes for models of 1.9 $M_\odot$ at 100, 670, and 1100 Myr from top to bottom respectively. The dot-dashed line represents model 1.90-ND, the dashed line 1.90R10K-2, the solid line 1.90R1000-2, and the dotted line 1.90R300-2.

**Table 6.** Periods in minutes of overstable modes in models of a 1.9 $M_\odot$ star with and without diffusion. The upper table lists the periods of radial modes and the lower table that of nonradial $l = 1$ p modes. Only the models in which at least one overstable mode was found are shown.

Radial modes

| age | | | | $(l,n)$ | | | | |
|---|---|---|---|---|---|---|---|---|
| (Myr) | (0,1) | (0,2) | (0,3) | (0,4) | (0,5) | (0,6) | (0,7) | (0,8) |
| | | | | 1.90R1000-2 | | | | |
| 1 003 | 164.30 | 130.37 | 104.85 | 87.32 | 75.22 | | | |
| 1 098 | 170.70 | 135.32 | 108.52 | 90.39 | | | | |
| | | | | 1.90-ND | | | | |
| 101 | | | | | | | | 22.38 |
| 300 | | | | | | | 28.03 | 25.14 |
| 502 | | | | | | 36.11 | 32.00 | |
| 670 | | | | 55.39 | 47.65 | 41.66 | 37.01 | |
| 1 003 | 163.47 | 126.41 | 101.49 | 84.55 | 72.54 | 63.69 | 56.66 | |
| 1 105 | 209.26 | 161.28 | 128.88 | 106.93 | 91.75 | 80.46 | | |

Nonradial modes

| age | | | | $(l,n)$ | | | | |
|---|---|---|---|---|---|---|---|---|
| (Myr) | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) | (1,7) | (1,8) |
| | | | | 1.90R1000-2 | | | | |
| 1 003 | 126.46 | 101.74 | 85.00 | 74.22 | | | | |
| 1 098 | 105.73 | 97.90 | 87.68 | | | | | |
| | | | | 1.90-ND | | | | |
| 101 | | | | | | 26.52 | 23.60 | 21.29 |
| 300 | | | | | | 29.77 | 26.51 | |
| 502 | | | | | 38.91 | 34.06 | 30.37 | |
| 670 | | | 63.04 | 52.57 | 45.03 | 39.48 | | |
| 1 003 | 122.10 | 98.37 | 82.08 | 71.42 | 66.40 | 60.43 | | |
| 1 105 | | | | | | | | |

zation. Clearly, the blue edge of the instability strip is sensitive to the efficiency of the diffusion processes, which means that it is sensitive to the assumed turbulence. It provides additional and independent constraints on the turbulence model. The superficial metal abundances constrain the turbulence (see Richer et al. 2000). However, the superficial He abundance is poorly determined by observations. It is much better determined via the driving of $\delta$ Scuti-type pulsations.

In practice, each mode of oscillation has an individual blue edge and each mode is affected differently by diffusion. Tables 4 and 5 and Fig. 5 suggest that the blue edge for the fundamental mode is only slightly shifted to the red compared with normal stars except if turbulence is better represented by the R300-2

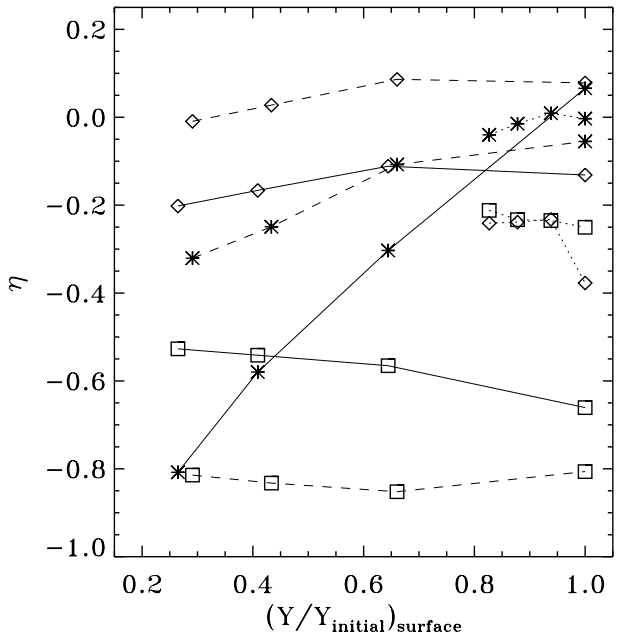


**Fig. 5.** The dependence of the growth rate $\eta$ on the superficial helium abundance is shown for four models of 1.9 $M_\odot$ with varying turbulence. The lines link the values of $\eta$ for the various modes for different models of the same age: the dotted lines are for models at 30 Myr, the solid line at 670 Myr and the dashed line at 1 Gyr. The higher is the turbulence in a model the more to the right it will be placed. Models without diffusion always have $Y/Y_{\rm initial} = 1.0$. The models are in order of increasing turbulence: 1.90R300-2, 1.90R1000-2, 1.900R10K-2 and 1.90-ND. Three modes are shown: $l = 0, n = 1$ as diamonds, $l = 0, n = 8$ as asterisks, and $l = 1, n = -14$ (g mode) as squares.

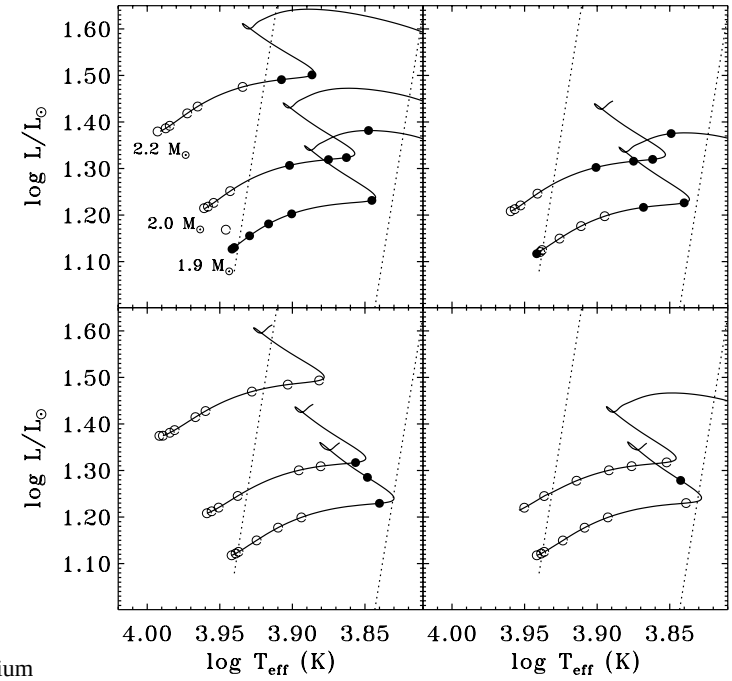


**Fig. 6.** The position of our models of 1.9, 2.0 and 2.2 $M_\odot$ in the HR diagram. Models for which at least one mode is predicted to be overstable are marked as filled circles, whereas those where we found no overstable modes are marked as open circles. Each of the four panels show models with similar turbulence: The upper left for models without diffusion, upper right for models with large turbulence (e.g. R10K-2), the lower left for models with turbulence approximately representative of Am stars (R1000-2), and lower right for models with less turbulence (e.g. R300-2).

model. The results of Richer et al. (2000) do not exclude this model, at least not for all Am stars. The blue edge for higher-frequency modes is shifted significantly as soon as the He abundance is slightly reduced.

As indicated in Fig. 6 we have considered only models hotter than the observational red edge of the instability strip. It is likely that the return to stability for cooler models would be dominated by convective effects (e.g. Houdek et al. 1999), which are ignored here. On the basis of time-dependent mixing-length calculations (Gough 1977; Balmforth 1992) one finds that the convective effects grow rapidly as the red edge is approached (e.g. Houdek 2000), although they have significant influence on the stability properties also in somewhat hotter stars. Even so, we expect that our qualitative conclusions, and the results on differential effects of diffusion, are robust.

### *8.3. Comparison with variable evolved CP stars*

The $\delta$ Delphini spectral sub-type has been shown to be a very inhomogeneous group of stars consisting of classical Am stars, evolved Am stars and other stars which are essentially normal A stars (Gray & Garrison 1989). Amongst those, the $\rho$ Puppis stars are those which are thought to be evolved Am stars. Some of these evolved Am stars have been shown to be variable. It was shown in Sect. 8.1 that models of evolved Am stars can be variable; thus, it remains to be seen whether our evolved Am-star models are comparable with the observed variable CP stars. To verify if this is the case, the most evolved models of 1.90R1000-2, 2.00R1000-2 and 2.20R1000-2 are compared here with the variable metallic stars observed by Kurtz (1976).

The positions of these stars are first shown in a temperature – gravity diagram (Fig. 7) to see if the models are in an evolutionary state comparable with the stars in our reference sample. The variable 1.90R1000-2 model at 1.094 Myr and the variable 2.00R1000-2 model at 850 Myr are both on the cool side (roughly 500 K cooler than the average) of the observed stars. Surprisingly, the 2.20R1000-2 model at 670 Myr, which is apparently a close match to two variable metallic A stars and which is much more compatible with the sequence of variable metallic stars than the cooler models, is predicted to be stable. This may or may not be a serious difficulty given the uncertainty of effective-temperature determinations for CP stars.

The surface-abundance signatures of the Kurtz (1976) $\delta$ Delphini stars and of the three selected models of the evolved Am stars are compared in Fig. 8. In a second panel we repeat the surface abundance of the evolved 2.20 $M_\odot$ but then compared with the two stars closest to it in Fig. 7, which are HR1706 and HR6561. These two stars have very nearly normal abundances that are closer to the surface abundances of a 2.20R10K-2 model

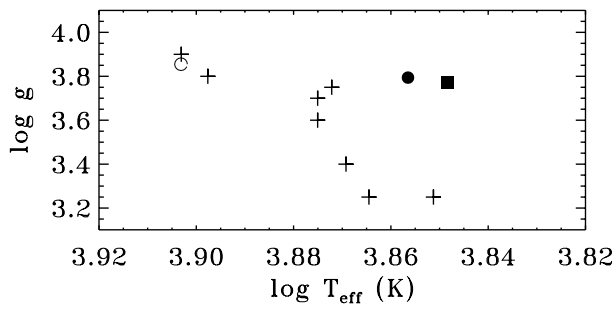


**Fig. 7.** Effective temperature vs gravity for evolved CP A stars. The $\delta$ Delphini stars of Kurtz (1976) discussed in the text are identified by the crosses. The models: filled square is for 1.90R1000-2 at 1094 Myr, the filled circle is for 2.00R1000-2 at 854 Myr and the open circle is for 2.20R1000-2 at 670 Myr. Filled symbols represent models in which overstable modes are found, the open symbol represents a model in which NO overstable mode was found.

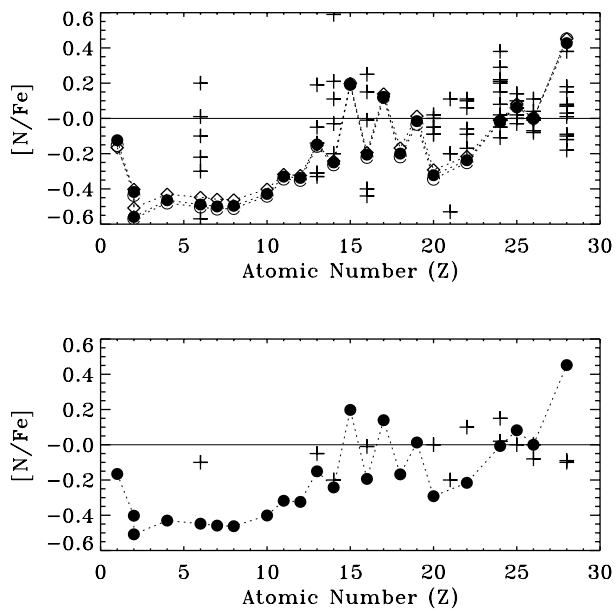


**Fig. 8.** The logarithm of the observed and predicted abundances in evolved Am stars and $\delta$ Delphini stars are shown as a function of the atomic number in both panels. The crosses are the observed values of Kurtz (1976) for $\delta$ Delphini stars. In the top panel, three models are shown: 1.90R1000-2 at 1094 Myr (filled circles), 2.00R1000-2 at 854 Myr (open circles) and 2.20R1000-2 at 670 Myr (open diamonds). In the bottom panel, the predicted abundances for model 2.20R1000-2 at 670 Myr are repeated along with the observed abundances (Kurtz 1976) of HR1706 and HR6561.

which would probably pulsate at that $T_{\rm eff}$ and $\log g$, if it behaves similarly to the 1.9R10K-2 model.

Fig. 8 shows that there is little variation in the surface-abundance profiles between the three evolved Am-star models. This is due to the fact that the depth of the mixed region is similar in these models since we adopted the same turbulence parameterization in all three. There is a significant scatter in the observed abundances of these stars. Nonetheless, the models lie in general within the envelope defined by the observations. As Richer et al. (2000) pointed out, care must be taken when comparing with observed abundances due to large variations in the abundances determined by different observers. Although one cannot claim that the global properties of the present models are a perfect match to those of individual variable CP stars, there is a qualitative agreement for the stars taken as a group. Better observational data for these stars would greatly help the interpretation of the results.

### *8.4. The excitation of g modes in A stars with diffusion*

As the driving regions shift deeper into the star as it evolves, the g modes become gradually more and more excited. The effect of diffusion on the excitation of p modes and g modes of a 1.9 $M_\odot$ star was discussed in Sect. 8.2. It was shown there that p modes are stabilized through diffusion whereas g modes tend to be excited as a result of that process. In the more evolved stars presented here, some g modes are predicted to become overstable. For example, we find all overstable modes in model 1.90-ND at 1.105 Gyr to be of the physical nature of g modes (the mathematical mode orders are very confusing for that model). However, the modes which are found to be overstable in our models are low-order modes for which the driving from helium is significant. Therefore, the effect of diffusion on their excitation is similar to that of low-order p modes, i.e., the settling of helium leads to a reduction of their excitation.

As an example for one of these g modes, we get the following growth parameters for the $l = 1, n = -1$ mode in the different models of 1.9 $M_\odot$ at roughly 1 Gyr, in order of decreasing helium content in the driving region: 0.0864 for 1.9-ND, 0.0891 for 1.9R10K-2, 0.0294 for 1.9R1000-2 and $-0.0098$ for 1.9R300-2.

The detection of g modes in $\delta$ Scuti stars has been problematic. Models for evolved A stars predict a very dense spectrum of overstable g modes while observations so far show little evidence of such variations (Guzik 2000). In addition, Breger & Beichbuchner (1996) show that there is no observational evidence for long-period g modes (of the nature of those observed in cooler $\gamma$ Doradus stars) in $\delta$ Scuti stars. The models suggest that they would not be easier to find in evolved Am stars, especially considering that convective effects should become increasingly important in models leaving the main sequence.

## 9. Is there a seismic signature of diffusion in $\delta$ Scuti stars?

In general, $\delta$ Scuti stars are fairly fast rotators in which diffusion is not expected to be important. In those stars for which diffusion could be important it is legitimate to ask in what measure it could influence their modeling.

The high-amplitude $\delta$ Scuti (HADS) stars are typically slow rotators and as such are candidates to become Am stars. They are characterized by high-amplitude pulsations compared with their normal counterparts. All evidence currently points to HADS being normal stars (Høg & Petersen 1997) following normal stellar evolution (Petersen & Christensen-Dalsgaard 1996). They are not distributed uniformly in the instability strip but occupy only

a strip some 300 K wide. If the high amplitudes were linked to a particular non-linear effect caused by the low rotational velocity this could have interesting repercussions on the stability of other slowly rotating A stars, such as the Am stars. Certainly, nothing in our results would lead us to believe that diffusion, by itself, would generate higher-amplitude modes in A stars. The lack of abundance anomalies in HADS might simply be due to their advanced evolutionary state and the depth of the convection zone at that time.

As has been briefly touched upon in the introduction (Sect. 2) some $\delta$ Scuti stars may exhibit abundance anomalies. These anomalies are not small contrary to what one might expect. In fact, one can see a scatter of [Ca/Fe] from $-0.6$ to $0.7$ and depletions of carbon of up to [C/Fe] $\simeq -1.3$ in Russell's (1995) results. Referring to Fig. 8 it is evident that the anomalies reported by Russell are higher than those observed in evolved Am stars. Similar results have been also published by Rachkovskaya (1994) and her other work referenced therein.

Although the anomalies have the overall appearance of diffusion, e.g. depleted C and Ca, normal Si and enhanced Fe and Ti, the scatter of the observed abundances is so large as to make any correlation between the abundances of the different elements very difficult. There is no systematic signature of diffusion as found in Am stars (cf. Richer et al. 2000) or even in the more evolved Am stars shown in Fig. 8. Moreover, some of those stars exhibit a $v \sin i$ as large as 100 to 150 km s$^{-1}$.

In other more standard $\delta$ Scuti stars the abundance anomalies are expected to be small. In such cases, the effect of diffusion on the seismology of those stars would be subtle. Previous experience in lower-mass stars (Turcotte & Christensen-Dalsgaard 1998) suggests that any seismic signature of diffusion would be overwhelmed by other effects such as convective-core overshoot, for example. In the event that the iron convection zone would develop, one could expect a clear seismic signature if a sufficient number of oscillation modes were identified. Guzik (1993) pointed out that helium settling may have some consequence on observed properties of $\delta$ Scuti stars, such as the light curve.

## 10. Pulsation and turbulence

In their discussion of metallicism and pulsations, Kurtz and collaborators have made the point that pulsations involve rapid motion to a substantial depth in the star. They argue that velocity of the displacement generated by the pulsation remains high throughout the region in which the abundance anomalies are thought to be formed (referring for example to the eigenfunctions displayed in Fig. 8.2 of Cox 1980). They speculate that the high velocities might generate turbulent mixing and therefore inhibit the occurrence of metallicism.

Although the velocity of the radial displacement might be large, turbulence is expected to be generated not by the speed of the displacement itself but rather by velocity gradients: a fast but uniform displacement will not become turbulent. The scale on which the displacement occurs is much greater than other relevant scales, e.g. the pressure scale height. The gradient of the displacement velocity is small over those scales and one would not expect turbulence to be necessarily generated as a result.

Our understanding of the extent to which pulsations are laminar or generate turbulence is too incomplete for us to speculate further on the link between the turbulence assumed in Am stars and the observed pulsations.

## 11. Conclusion

We have investigated the impact of diffusion on the stability of A-type main-sequence Population I stars. The models include the consistent abundance evolution of all important elements and its impact on stellar structure. In these models helium remains present in the He II ionization zone and the opacity in the iron opacity bump increases substantially, raising the possibility of a strong relationship between variability and diffusion.

We present evidence that young Am stars are stable against driving from the $\kappa$ mechanism and that, as the stars evolve, they become unstable, but only when near the red edge of the instability strip. Hot Am stars need to be more evolved than cool Am stars before variability can occur. The blue edge of the instability strip for metallic A stars is sensitive to the magnitude of the abundance variations and is thus indicative of the depth of mixing by turbulence.

The stability of A stars is more sensitive to the evolution of the abundance of helium than to the accumulation of iron-peak elements. In stars with very little turbulence the iron-peak driving region can become convectively unstable thereby reducing the radiative flux there and negating the driving effect of the enhanced opacity bump. However, in the models which are representative of Am stars, the turbulence is high enough to prevent the formation of that iron convection zone while allowing a significant increase of the opacity bump due to iron-peak elements. Still, only a marginal positive effect can be seen in the long-period g modes. The higher-frequency modes depend mostly on the helium abundance and somewhat on the hydrogen abundance.

There are a number of caveats relevant to the present work.

There is no direct link between the normalized growth rates $\eta$ and the actual amplitude of the pulsations as evidenced by the lower number of modes observed in $\delta$ Scuti stars relative to their predicted number. Additionally, comparisons to models at solar composition by J. Christensen-Dalsgaard and W. Dziembowski have shown that the growth rates are sensitive to the details of the modeling. In general, in the stars we compared at 1.8 and 1.9 $M_\odot$, the growth rates were lower in our models than in either of their models.

The treatment of convection in the present work is simplistic. Convection and turbulence are known to damp pulsation to a certain extent. The most striking and well known evidence of the effect of convective damping is the red edge of the instability strip [Gough (1977); Gonczi & Osaki (1980); Balmforth & Gough (1988); see also Buchler et al. (1999) for a recent review]. Turbulence affects pulsations in two ways: 1) through turbulent viscosity, which always dampens pulsations, and 2)

through the phase difference between entropy variations and the modulation of the convective flux, which can either excite or dampen pulsations. It is the latter effect which is responsible for the red edge of the classical instability strip. While the interaction between turbulent convection and pulsations has been studied, the effect of turbulence outside of the convection zones is unknown.

In the models including diffusion, the coupling between turbulence and diffusion might be very important. For the present models reproducing Am stars, the turbulence extends to a significant depth below the convection zone. While the energy flux related to the turbulence is expected to be very small, the effect of turbulent viscosity might not be negligible. For stars in which the iron convection zone is allowed to form, three separate convection zones exist with highly turbulent regions between them. The behavior of pulsations in such stars might be heavily affected by a proper treatment of convective effects.

While it is true that our analysis does not take this into account, one should not forget that in the standard models without diffusion an ad hoc mixing mechanism is implied to prevent the formation of abundance gradients. This presumably turbulent mixing (at least in the more slowly rotating $\delta$ Scuti stars) is also never taken into account. It is, by hypothesis, larger than that in the models where atomic diffusion is important. Under these circumstances, we may perhaps consider the simple treatment of convection and turbulence applied here as adequate for identifying the differential effect of diffusion on stellar stability.

The NMM used here still contain an element of arbitrariness in that they extend turbulent mixing a little beyond that expected from iron convection zones, without providing a physical mechanism for this extension. This extension is required to fit the observed surface abundances of Am stars. The extension is, however, sufficient to cause the iron-peak abundances to decrease sufficiently for the convection zone to disappear in models representative of these stars. The models do not include the potential effects of mass loss or rotation, although differential rotation is one mechanism that is now being investigated as a source of the instabilities that could provide this mixing zone extension.

When compared against each other the present models do, however, illustrate the effect of diffusion on the stability of main-sequence A stars as well as present modeling allows.

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