

*Letter to the Editor***Torsional oscillations in the solar convection zone****E. Covas<sup>1</sup>, R. Tavakol<sup>1</sup>, D. Moss<sup>2</sup>, and A. Tworkowski<sup>3</sup>**<sup>1</sup> Astronomy Unit, School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, UK (eoc,reza@maths.qmw.ac.uk)<sup>2</sup> Department of Mathematics, The University, Manchester M13 9PL, UK (moss@ma.man.ac.uk)<sup>3</sup> MRC, School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, UK (ast@maths.qmw.ac.uk)

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**Abstract.** Recent analysis of the helioseismic observations indicate that the previously observed surface torsional oscillations extend significantly downwards into the solar convection zone.

In an attempt to understand these oscillations, we study the nonlinear coupling between the magnetic field and the solar differential rotation in the context of a mean field dynamo model, in which the nonlinearity is due to the action of the azimuthal component of the Lorentz force of the dynamo generated magnetic field on the solar angular velocity. The underlying zero order angular velocity is chosen to be consistent with the most recent helioseismic data.

The model produces butterfly diagrams which are in qualitative agreement with the observations. It displays torsional oscillations that penetrate into the convection zone, and which with time migrate towards the equator. The period of these oscillations is found to be half that of the period of the global magnetic fields. This is compatible with the observed period of the surface torsional oscillations. Inside the convection zone, this is a testable prediction that is not ruled out by the observations so far available.

**Key words:** Sun: magnetic fields – chaos – magnetic fields

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**1. Introduction**

An important feature of the solar convection zone is the presence of differential rotation in the form of a decrease in angular velocity from equator to the pole. This has been observed both in the surface layers (e.g. Snodgrass 1984) and deeper in the convection zone, as inferred from helioseismic measurements (e.g. Thompson et al. 1996). Furthermore, the differential rotation on the surface has been observed to vary with time (e.g. Howard & LaBonte 1980; Snodgrass, Howard, & Webster 1985). These so called torsional oscillations, which have periods of about 11 years, manifest themselves in the form of four alternating latitudinal bands of slightly faster and slower than average zonal flows which migrate towards the equator in about 22 years.

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These oscillations have also been confirmed by the analysis of the helioseismic data from the Solar and Heliospheric Observatory (SOHO) spacecraft for the present solar cycle (Kosovichev & Schou 1997; Schou et al. 1998).

Recent analysis of the helioseismic data, from both the Michelson Doppler Imager (MDI) instrument on board the SOHO spacecraft and the Global Oscillation Network Group (GONG) project, has also produced evidence that this banding signature is not merely a surface feature, but extends into the convection zone, to a depth of at least 8 percent in radius (Howe et al. 2000). These authors present data on departures of the reconstructed rotation rate from its temporal averages - the residuals - as a function of latitude at several target depths, which behave in a manner similar to the migration of sunspots during the solar cycle (the ‘butterfly diagram’). This finding is also supported by the analysis of Antia & Basu (2000), who use different data sets from GONG and independent inversion techniques. The time-base of these observations is only a few years, less than a complete solar cycle.

These torsional oscillations are thought to be produced as a consequence of the nonlinear interactions between the magnetic fields and the solar differential rotation. A zero order ‘mean’ differential rotation is assumed to be maintained by the Reynolds stresses of the turbulence; this can be included as a constant ‘background’ angular velocity, or explicitly parametrized, e.g. by the so called  $\Lambda$ -effect (e.g. Rüdiger 1989). Attempts have been made to explain the surface oscillations in terms of the effects of the Lorentz force exerted by the large scale magnetic field on the azimuthal velocity field (e.g. Brandenburg & Tuominen 1988), or as a consequence of the ‘quenching’ of the turbulence-dependent quantities by the magnetic field (Kitchatinov et al. 1999).

Here we study this nonlinear coupling in the context of a two dimensional axisymmetric mean field dynamo model, in a spherical shell, in which the only nonlinearity is the action of the azimuthal component of the Lorentz force of the dynamo generated magnetic field on the solar angular velocity. Obtaining torsional oscillations in this way is also of interest in view of the fact that the Lorentz force is of second order in the magnetic

field, thus naturally leading to the excitation of hydrodynamical oscillations of about half the period of the magnetic oscillations.

In the next section we introduce our model. Sect. 3 contains our results and Sect. 4 gives a brief discussion.

## 2. The model

Here we shall assume that the gross features of the large scale solar magnetic field can be described by a mean field dynamo model, with the standard equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B} - \eta \nabla \times \mathbf{B}), \quad (1)$$

where  $\mathbf{u} = v\hat{\phi} - \frac{1}{2}\nabla\eta$ , the term proportional to  $\nabla\eta$  represents the effects of turbulent diamagnetism, and where the velocity field is taken to be of the form

$$v = v_0 + v', \quad (2)$$

where  $v_0 = \Omega_0 r \sin \theta$ ,  $\Omega_0$  is a prescribed underlying rotation law and the component  $v'$  satisfies

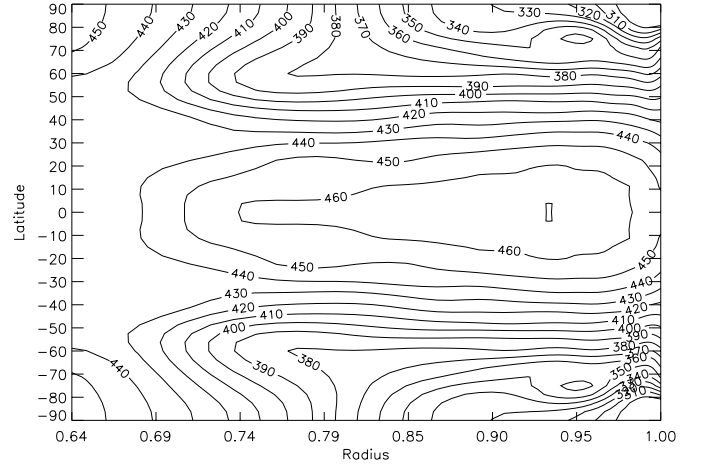
$$\frac{\partial v'}{\partial t} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\mu_0 \rho r \sin \theta} \cdot \hat{\phi} + \nu D^2 v', \quad (3)$$

where  $D^2$  is the operator  $\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2 \sin^2 \theta} (\frac{\partial}{\partial \theta} (\sin \theta \frac{\partial}{\partial \theta}) - \frac{1}{\sin^2 \theta})$  and  $\mu_0$  is the induction constant. The assumption of axisymmetry allows the field  $\mathbf{B}$  to be split into toroidal and poloidal parts,  $\mathbf{B} = \mathbf{B}_T + \mathbf{B}_P = B\hat{\phi} + \nabla \times A\hat{\phi}$ , and results in Eq. (1) yielding two scalar equations for  $A$  and  $B$ . Nondimensionalizing in terms of the solar radius  $R_\odot$  and time  $R_\odot^2/\eta_0$ , where  $\eta_0$  is the maximum value of  $\eta$  and letting  $\Omega = \Omega^* \tilde{\Omega}$ ,  $\alpha = \alpha_0 \tilde{\alpha}$ ,  $\eta = \eta_0 \tilde{\eta}$ ,  $\mathbf{B} = B_0 \tilde{\mathbf{B}}$  and  $v' = \Omega^* R_\odot \tilde{v}'$ , results in a system of equations for  $A, B$  and  $v'$ , with the dynamo parameters  $R_\alpha = \alpha_0 R_\odot / \eta_0$ ,  $R_\omega = \Omega^* R_\odot^2 / \eta_0$ ,  $P_r = \nu_0 / \eta_0$ , and  $\tilde{\eta} = \eta / \eta_0$ , where  $\Omega^*$  is the solar surface equatorial angular velocity (see Moss & Brooke 2000 for details). Here  $\nu_0$  and  $\eta_0$  are the turbulent magnetic diffusivity and viscosity respectively and  $P_r$  is the turbulent Prandtl number. The density  $\rho$  is assumed to be uniform, and stress free boundary conditions ensure angular momentum conservation.

These equations were solved using the code and boundary conditions described in Moss & Brooke (2000), over the range  $r_0 \leq r \leq 1$ ,  $0 \leq \theta \leq \pi$ , with uniform spacing in both  $r$  and  $\theta$ . The computational domain is the region  $r_0 = 0.64 \leq r \leq 1$ ; with the solar convection zone proper being thought to occupy the region  $r > 0.7$ , the region  $r_0 \leq r \leq 0.7$  can be thought of as an overshoot region/tachocline. In the following simulations we used a mesh resolution of 61 points uniformly distributed radially and 101 points uniformly distributed latitudinally (over  $0 \leq \theta \leq \pi$ ), but test runs were carried out at higher spatial resolutions.

In this investigation, we took  $\Omega_0$  to be given in  $0.64 \leq r \leq 1$  by an interpolation on the MDI data obtained from 1996 to 1999 (Howe et al. 2000), depicted in Fig. 1. For  $\alpha$  we took  $\tilde{\alpha} = \alpha_r(r)f(\theta)$ , where  $f(\theta) = \sin^2 \theta \cos \theta$  (cf. Rüdiger & Brandenburg 1995) and

$$\alpha_r = 1; \quad 0.7 \leq r \leq 0.8 \quad (4)$$



**Fig. 1.** Isolines of the time average of the angular velocity of the solar rotation, obtained by inverse techniques using the MDI data (Howe et al. 2000). Contours are labelled in units of nHz.

with cubic interpolation to zero at  $r = r_0$  and  $r = 1$ , with the convention that  $\alpha_r > 0$  and  $R_\alpha < 0$ . Also, in order to take into account the likely decrease in the turbulent diffusion coefficient  $\eta$  in the overshoot region, we allowed a simple linear decrease from  $\tilde{\eta} = 1$  at  $r = 0.8$  to  $\tilde{\eta} = 0.5$  in  $r < 0.7$ .

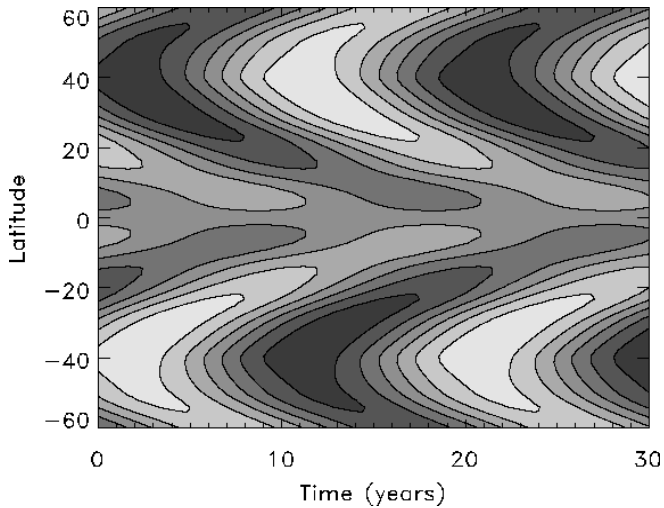
We monitor the time evolution of the total magnetic energy  $E$  and the global parity of the magnetic field, defined as  $P = \frac{E^S - E^A}{E^S + E^A}$ , where  $S$  and  $A$  refer respectively to the parts of the magnetic field that have symmetry or antisymmetry with respect to the equatorial plane (see also Brandenburg et al. 1989). Thus  $P = +1$  and  $-1$  correspond to symmetric and antisymmetric fields respectively.

## 3. Results

We calibrated our model so that near marginal excitation the cycle period was about 22 years. This determined  $R_\omega = 6 \times 10^4$ , corresponding to  $\eta_0 \approx 2.5 \times 10^{11} \text{ cm}^2 \text{ s}^{-1}$ , given the known values of  $\Omega^*$  and  $R_\odot$ . The first solutions to be excited in the linear theory are odd parity ( $P = -1$ ) limit cycles, which in this case have marginal dynamo number  $R_\alpha \sim -3.12$ . The even parity ( $P = +1$ ) solutions are also excited at similar marginal dynamo numbers of  $R_\alpha \sim -3.16$ . We considered two values of the Prandtl numbers,  $P_r = 0.1$  and  $P_r = 1.0$ .

With these parameter values, we found that this model, with underlying zero order angular velocity chosen to be consistent with the recent (MDI) helioseismic data (Fig. 1), is capable of producing butterfly diagrams which are in qualitative agreement with the observations. An example of this is depicted in Fig. 2. The polar feature is rather too strong – we have checked that this can be rectified by modifying slightly the spatial dependence of  $\alpha$ , by for example choosing  $f(\theta) = \sin^4 \theta \cos \theta$ .

We also found that this model successfully produced torsional oscillations in the convection zone, similar to those deduced from recent helioseismic data. To compare our model with these results, we have plotted in Figs. 3 and 4 the variations of the rotation rate with latitude and time (‘butterfly diagrams’) to



**Fig. 2.** Butterfly diagram of the toroidal component of the magnetic field  $B$  at  $R = 0.95R_{\odot}$ . Dark and light shadows correspond to positive and negative  $B_{\phi}$  respectively. Parameters values are  $R_{\alpha} = -3.2$ ,  $P_r = 1.0$  and  $R_{\omega} = 6 \times 10^4$ .

reveal the migrating banded zonal flows. In these models, the basic magnetic field parity is odd ( $P = -1$ ).

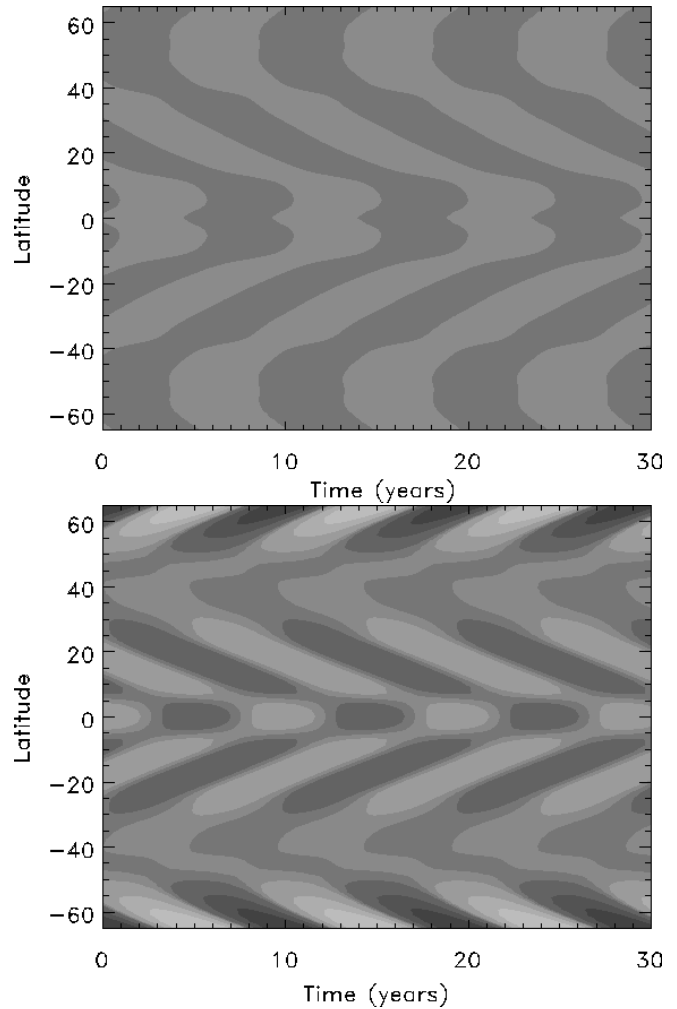
For the sake of comparison with observational results, we have also plotted in Fig. 5 the evolution of the residual rotation rate with time, at radius  $0.84$  and latitude  $60$  degrees.

As can be seen, consistent with the observations, in each hemisphere there are alternating latitudinal bands, with the width of approximately  $10$  degrees, of slightly faster and slower than average zonal flows. These migrate towards the equator in about  $22$  years, and extend deep into the convection zone. The amplitudes of these oscillations increase with depth below the surface and depend on the parameters of our model, in particular the Prandtl number. For  $P_r = 1.0$ , these amplitudes range from about  $0.07$  nHz at the surface to more than  $0.4$  nHz towards the bottom of the convection zone, somewhat lower than, but in principle compatible with, the results of Howe et al. 2000. The torsional oscillations present in our model have periods half that of the period of the global magnetic field which is compatible with the observed period of the oscillations at the surface and consistent with the observed behaviour inside the convection zone.

The strictly odd parity models presented have an equatorial feature in the ‘butterfly diagrams’ for the velocity perturbations which does not appear to be present in the current inversions of the observational data. This becomes weaker as Prandtl number increases. We also note in passing that these figures are almost identical for cases where even parity solutions are found, except that this feature is then absent. We further point out that the large scale solar magnetic field is probably of *mixed* global parity (predominantly odd) – see, e.g., Pulkinnen et al. (1999).

#### 4. Discussion

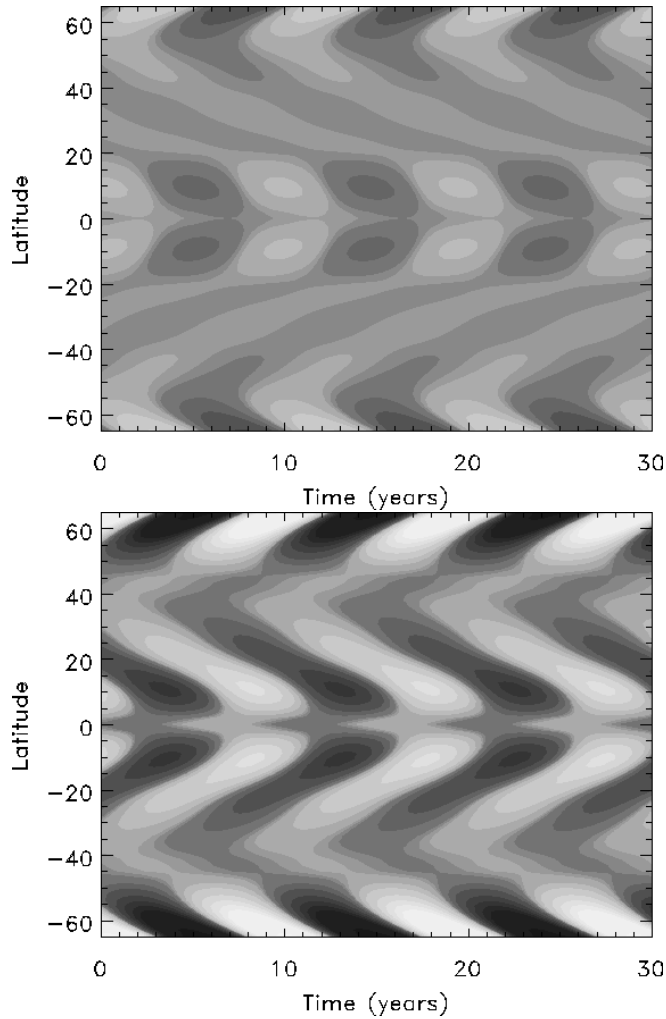
We have studied a solar dynamo model, calibrated to have the correct cycle period, with a mean rotation law given by recent



**Fig. 3.** Variation of rotation rate with latitude and time from which a temporal average has been subtracted to reveal the migrating banded zonal flows, at  $R = 0.95R_{\odot}$  (top) and  $R = 0.84R_{\odot}$  (bottom). Darker and lighter regions represent positive and negative deviations from the time averaged background rotation rate. Parameters values are  $R_{\alpha} = -3.2$ ,  $P_r = 0.1$  and  $R_{\omega} = 6 \times 10^4$ .

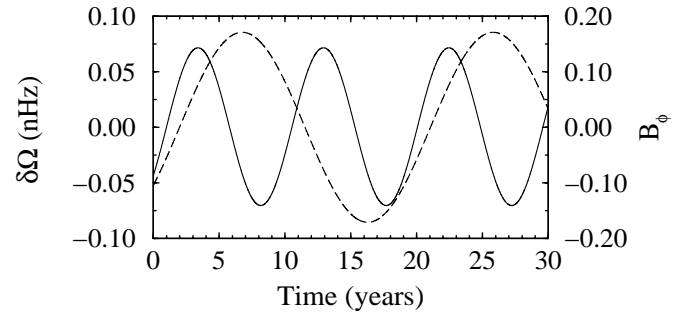
helioseismic observations. This model produces butterfly diagrams in qualitative agreements with those observed.

We have shown that a nonlinear coupling between the magnetic field and the solar differential rotation, where the nonlinearity is due to the action of the azimuthal component of the Lorentz force, is capable of producing torsional oscillations, with period of about  $11$  years, which penetrate into the convection zone and which migrate towards the equator in about  $22$  years. The period of these oscillations is about half that of the period of the global magnetic fields. This is in agreement with the observed period of the torsional oscillations at the surface. For the oscillations inside the convection zone, this is a testable prediction, not contradicted by the current helioseismic observations, which so far extend over an interval less than a solar cycle. We note also that solutions with even parity ( $P = +1$ ) show slightly larger amplitudes without the equatorial feature.



**Fig. 4.** Variation of rotation rate with latitude and time from which a temporal average has been subtracted to reveal the migrating banded zonal flows, at  $R = 0.95R_{\odot}$  (top) and  $R = 0.84R_{\odot}$  (bottom). Parameters values are  $R_{\alpha} = -3.2$ ,  $P_r = 1.0$  and  $R_{\omega} = 6 \times 10^4$ .

The current inversions of the helioseismological data seem to suggest that the torsional oscillations largely disappear below about  $R = 0.9R_{\odot}$ , in contrast to our model oscillations. However, there are uncertainties in these inversions, specially at the deeper levels. At the same time our dynamical model is oversimplified and substantial improvements can be made. In particular, the predicted amplitudes for torsional oscillations in our model are likely to be affected by our assumption of uniform density in Eq. (3). Nevertheless, we find it interesting that



**Fig. 5.** Variation of rotation rate at radius  $0.84R_{\odot}$  and latitude 60 degrees. Also depicted is the (nondimensionalized) toroidal field at the same radius and latitude (dashed line), which shows double the period. Parameters values are as in Fig. 4.

some of the major features in the torsional oscillations can be readily reproduced. A more detailed study of these oscillations, including more realistic density profiles, is in progress.

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