On the energy equation and efficiency parameter of the common envelope evolution

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Abstract. We have investigated the structure of evolved giant stars with masses $3 - 10 \, \mathrm{M_\odot}$ in order to evaluate the binding energy of the envelope to the core prior to mass transfer in close binary systems. This binding energy is expressed by a parameter λ which is crucial for determining the outcome of binaries evolving through a common envelope (CE) and spiral-in phase. We discuss the λ–parameter and the efficiency of envelope ejection in the CE-phase, and show that λ depends strongly on the evolutionary stage (i.e. stellar radius) of the donor star at the onset of the mass transfer. The existence of this relation enables us to introduce a new approach for solving the energy equation. For a given observed binary system we can derive a unique solution for the original mass and age of the donor star, as well as the pre-CE orbital period.

We find that the value of λ is typically between 0.2 and 0.8. But in some cases, particularly on the asymptotic giant branch of lower-mass stars, it is possible that $\lambda > 5$. A high value of λ (rather than *assuming* a high efficiency parameter, $\eta_{\mathrm{CE}} > 1$) is sufficient to explain the long final orbital periods observed among those binary millisecond pulsars which are believed to have evolved through a CE-phase.

We also present a tabulation of λ as a function of stellar radius and mass, which is useful for a quick estimation of the orbital decay during a common envelope and spiral-in phase.

Key words: stars: binaries: general – stars: evolution – stars: mass-loss – methods: numerical – stars: pulsars: individual: PSR J1454-5846

1. Introduction

A very important stage of the evolution of close binaries is the formation of a common envelope. Generally, if a star in a binary fills its Roche-lobe (either because of the swelling up of the envelope due to exhaustion of nuclear fuel at the centre – or reduction in the separation due to loss of orbital angular momentum) there will be an attempt to transfer mass to the companion.

However, if the thermal re-adjustment timescale of the accreting star is larger than the mass-transfer timescale, the accreted layer piles up above the companion, heats up and expands so that the accreting star will also fill its Roche-lobe. The result is that the transfered matter will form a common envelope embedding both stars (Paczynski 1976; Ostriker 1976). When the mass ratio is large the CE-phase is accompanied by the creation of a drag-force, arising from the motion of the companion star through the envelope of the evolved star, which leads to dissipation of orbital angular momentum (spiral-in process) and deposition of orbital energy in the envelope. Hence, the global outcome of a CE-phase is reduction of the binary separation and often ejection of the envelope. For a general review on common envelopes, see e.g. Iben & Livio (1993).

There is clear evidence of orbital shrinkage (as brought about by frictional torques in a CE-phase) in some observed close binary pulsars and white dwarf binaries (e.g. PSR 1913+16, L 870-2 – see Iben & Livio 1993 and references therein). In these systems it is clear that the precursor of the last-formed degenerate star must have achieved a radius much larger than the current orbital separation. Since the present short separation of degenerate stars in some binaries can not be explained by gravitational wave radiation or a magnetic wind as the main source of the loss of J_{orb}, frictional angular momentum loss was probably responsible for the very close orbits observed. An alternative process recently investigated (King & Begelman 1999; Tauris et al. 2000) is a highly Super-Eddington mass transfer on a subthermal timescale. However, as shown by the latter authors this process can not explain the observed properties of all systems. They conclude that CE and spiral-in evolution is still needed to account for the formation of double neutron star systems and mildly recycled pulsars with a heavy white dwarf companion and $P_{\mathrm{orb}} < 3$ days.

There are many uncertainties involved in calculations of the spiral-in phase during the CE evolution. The evolution is tidally unstable and the angular momentum transfer, dissipation of orbital energy and structural changes of the donor star take place on very short timescales (10^3–10^4 yr). A complete study of the problem would also require very detailed multi-dimensional hydrodynamical calculations which is beyond the scope of this

paper. Here we aim to improve the simple formalism developed by Webbink (1984) and de Kool (1990) for estimating the orbital evolution of binaries evolving through a CE and spiral-in phase. An important quantity in their formalism is the binding energy between the envelope and the core of the donor star. This quantity is expressed by a parameter λ which depends on the stellar density distribution, and consequently also on the evolutionary stage of the star. With a detailed evolutionary calculation, Bisscheroux (1998) found that $\lambda \sim 0.4$–0.6 on the red and asymptotic giant branches of a 5 $M_\odot$ star. In the literature this parameter is usually taken to be a constant (e.g. $\lambda = 0.5$) disregarding the evolutionary stage (structure) of the donor star at the onset of the mass transfer process. This in turn leads to a misleading (over)estimation of the so-called efficiency parameter, η_{CE}, often discussed by various authors. From observations one is able to put constraints on the product $\eta_{CE}\lambda$ (see Eq. 4 below). However, by underestimating λ the derived values for η_{CE} will be overestimated – e.g. van den Heuvel (1994) and Tauris (1996) concluded $\eta_{CE} > 1$ by assuming $\lambda = 0.5$. By calculating λ as a real function of stellar radius we are able to improve the estimates of post-CE orbital separations and hence put more realistic constraints on η_{CE}.

Detailed calculations on the binding energy of the envelope to determine the outcome of binaries evolving through a CE-phase have been done by Han et al. (1994, 1995). The method and the stellar evolution code used to calculate the binding energy in their work is similar to the one applied in our work. However, here we discuss the binding energy within the context of the λ-parameter in the Webbink-formalism. We present calculated values of λ for stars with initial mass in the range of 3–10 $M_\odot$ at different evolutionary stages. Such values are usefull for an estimation of the orbital decay during a common envelope and spiral-in phase. We also demonstrate a new approach to find a unique solution for the stellar and binary parameters of systems evolving through a CE and spiral-in phase.

We will in particular consider binaries in which the companion of the evolved star is a neutron star. Such binaries are the progenitors of binary millisecond pulsars (BMSPs) with a massive degenerate companion (e.g. van den Heuvel 1994) and they may appear in nature as short lived intermediate-mass X-ray binaries (IMXBs).

In Sect. 2 we describe the basic concepts of common envelope evolution concerning the energy equation and the problems of defining the binding energy. Sect. 3 introduces briefly the numerical stellar code used to evaluate the parameter λ. The results are presented in Sect. 4 and discussed in Sect. 5. We summarize our conclusions in Sect. 6.

2. Common envelope and spiral-in phase

2.1. The energy equation and orbital evolution

A simple estimation on the reduction of the orbit can be found by simply equating the difference in orbital energy (before and after the CE-phase) to the binding energy of the envelope of the (sub)giant donor. Following the formalism of Webbink (1984), the original binding energy of the envelope at the onset

of mass transfer will be of the order: $-GM_{\mathrm{donor}}M_{\mathrm{env}}/a_i r_L$, where M_{donor} is the mass of the donor star at the beginning of the CE-phase; M_{env} is the mass of the hydrogen-rich envelope of the donor; a_i is the orbital separation at the onset of the CE and $r_L = R_L/a_i$ is the dimensionless Roche-lobe radius of the donor star, so that $a_i r_L = R_L \approx R_{\mathrm{donor}}$. A parameter λ was introduced by de Kool (1990) as a numerical factor (of order unity) which depends on the stellar density distribution, such that the real binding energy of the envelope can be expressed as:

$$E_{\mathrm{env}} = -\frac{GM_{\mathrm{donor}}M_{\mathrm{env}}}{\lambda a_i r_L}. \tag{1}$$

The total change in orbital energy is given by:

$$\Delta E_{\mathrm{orb}} = -\frac{GM_{\mathrm{core}}M_2}{2a_f} + \frac{GM_{\mathrm{donor}}M_2}{2a_i} \tag{2}$$

where $M_{\mathrm{core}} = M_{\mathrm{donor}} - M_{\mathrm{env}}$ is the mass of the helium core of the evolved donor star; M_2 is the mass of the companion star and a_f is the final orbital separation after the CE-phase.

Let η_{CE} describe the efficiency of ejecting the envelope, i.e. of converting orbital energy into the kinetic energy that provides the outward motion of the envelope: $E_{\mathrm{env}} \equiv \eta_{CE} \Delta E_{\mathrm{orb}}$ or, by equating Eqs. (1) and (2):

$$\frac{GM_{\mathrm{donor}}M_{\mathrm{env}}}{\lambda a_i r_L} = \eta_{CE} \left[\frac{GM_{\mathrm{core}}M_2}{2a_f} - \frac{GM_{\mathrm{donor}}M_2}{2a_i} \right] \tag{3}$$

which yields:

$$\frac{a_f}{a_i} = \frac{M_{\mathrm{core}}M_2}{M_{\mathrm{donor}}} \frac{1}{M_2 + 2M_{\mathrm{env}}/(\eta_{CE}\lambda r_L)}. \tag{4}$$

2.2. The binding energy of the envelope

The total binding energy of the envelope to the core is given by:

$$E_{\mathrm{bind}} = \int_{M_{\mathrm{core}}}^{M_{\mathrm{donor}}} \left(-\frac{GM(r)}{r} + U \right) dm \tag{5}$$

where the first term is the gravitational binding energy and U is the internal thermodynamic energy. The latter involves the basic thermal energy for a simple perfect gas ($3\Re T/2\,\mu$), the energy of radiation ($aT^4/3\rho$), as well as terms due to ionization of atoms and dissociation of molecules and the Fermi energy of a degenerate electron gas (Han et al. 1994, 1995).

The binding energy of the envelope can be expressed by Eq. (5), under the assumption that the entire internal energy is used efficiently in the ejection process. However, it is not known how much of the internal energy is involved to unbind the envelope. Han et al. (1995) introduced a parameter α_{th} and expressed the envelope binding energy as:

$$E_{\mathrm{env}} = -\int_{M_{\mathrm{core}}}^{M_{\mathrm{donor}}} \frac{GM(r)}{r}dm + \alpha_{\mathrm{th}} \int_{M_{\mathrm{core}}}^{M_{\mathrm{donor}}} U\,dm. \tag{6}$$

The value of α_{th} depends on the details of the ejection process, which is very uncertain. A value of α_{th} equal to 0 or 1 corresponds to maximum and minimum envelope binding energy,

respectively. By simply equating Eqs. (1) and (6) we calculated the parameter λ for different evolutionary stages of a given star. The minimum and maximum derived values of λ are denoted by λ_{g} ($\alpha_{\mathrm{th}} = 0$) and λ_{b} ($\alpha_{\mathrm{th}} = 1$).

By taking the core mass M_{core} as the lower boundary of the integral, we assume that the structure of the core does not change during the envelope ejection process, and therefore we assume that there is no exchange of energy between the core and the envelope. Here, we define the core mass as the central mass which contains less than 10% hydrogen – see discussion in Sect. 5.2.

For the case of binary stars we use the radii of the stars to calculate the separations at the onset of the mass transfer assuming a companion star of mass M_2. An estimation on the Roche-lobe radius is given by Eggleton (1983):

$$R_{\mathrm{L}} = \frac{0.49\,a_{\mathrm{i}}}{0.6 + q^{-2/3}\ln(1 + q^{1/3})} \qquad (7)$$

where $q = M_{\mathrm{donor}}/M_2$ is the mass ratio.

3. A brief introduction to the numerical computer code

We used an updated version of the numerical stellar evolution code of Eggleton (1971, 1972, 1973). This code uses a self-adaptive, non-Lagrangian mesh-spacing which is a function of local pressure, temperature, Lagrangian mass and radius. It treats both convective and semi-convective mixing as a diffusion process and finds a simultaneous and implicit solution of both the stellar structure equations and the diffusion equations for the chemical composition. New improvements are the inclusion of pressure ionization and Coulomb interactions in the equation-of-state, and the incorporation of recent opacity tables, nuclear reaction rates and neutrino loss rates. The most important recent updates of this code are described in Pols et al. (1995, 1998) and some are explained in Han et al. (1994).

We performed such detailed numerical stellar evolution calculations in our work since they should yield more realistic results compared to models based on complete, composite or condensed polytropes.

4. Results

The above-mentioned detailed stellar evolution code has been used to probe the structure of donor stars at the onset of mass transfer. We evolved stars with an initial mass in the interval of 3–10 $M_{\odot}$ assuming a chemical composition of ($X = 0.70$, $Z = 0.02$) and using a mixing-length parameter of $\alpha = l/H_p = 2.0$. Convective overshooting is taken into account in the same way as in Pols et al. (1998) using an overshooting constant $\delta_{\mathrm{ov}} = 0.1$. We used de Jager's wind mass-loss rate (de Jager et al. 1988; Nieuwenhuijzen & de Jager 1990) to estimate the mass loss prior to the Roche-lobe overflow and CE evolution. The wind mass loss from the donor star (and subsequent widening of the orbit) prior to the Roche-lobe overflow is typically only $\sim$ 1–4% on the AGB and hence this effect is not very significant for the stars considered here.

With the Eggleton code, the total binding energy is computed for each mesh-point. Included here are the ionization of H^+, H, He^{++}, He^+, He, dissociation of H_2, rotational and vibrational modes of H_2, as well as ionization of seven heavier elements (C, N, O, Ne, Mg, Si, Fe) which are assumed to be fully ionized at all temperatures and densities (Pols et al. 1995). By integrating from the core to the photosphere, we found the total binding energy of the envelope. We used this energy to derive the parameter λ. In Fig. 1 we present the relation between λ and stellar radius for each of the different donor stars considered. Here λ was determined either from the total binding energy of the envelope (i.e. $\alpha_{\mathrm{th}} = 1$) or from the gravitational binding energy alone (i.e. $\alpha_{\mathrm{th}} = 0$).

For a donor star in a binary system the stellar radius at the onset of the CE-phase is roughly equivalent to its Roche-lobe radius, and given a mass of its companion star one finds the corresponding orbital period of the system. In Table 1, assuming that the companion is a 1.3 $M_{\odot}$ neutron star, we list the relation between the orbital period of the binary and λ at the onset of the mass transfer (again due to the wind mass loss, this period might be slightly larger than the initial ZAMS period). We distinguish λ_{g} (as derived from gravitational binding energy alone) from λ_{b} (calculated from the total binding energy). The gyration radius of the star, $k^2 = I/MR^2$, is also presented to provide information about the orbital tidal instability. Mass transfer becomes tidally unstable if the rotational angular momentum of the donor exceeds one third of the orbital angular momentum (Darwin 1908).

5. Discussion

5.1. On the binding energy of the envelope

In Fig. 2 we have plotted the binding energy of the envelope as well as the density profile of two stars with an initial mass of 4 and 10 $M_{\odot}$. When the star reaches the asymptotic giant branch, the core is surrounded by a convective envelope, and the density gradient around the core mass is very steep. As the star ascends this giant branch, the convective envelope becomes deeper and the density contrast higher.

Assuming a star to be a non-rotating globe of non-degenerate ionized hydrogen and helium in hydrostatic equilibrium (i.e. nearly an ideal monatomic gas) one would expect from the virial theorem: $\lambda_{\mathrm{b}} \simeq 2\lambda_{\mathrm{g}}$. However at late stellar evolutionary stages this picture no longer holds. After the second dredge-up stage of evolution, the gravitational binding energy becomes smaller as the stellar radius expands to a supergiant dimension. Meanwhile the ionization and dissociation energy becomes more significant when the temperature decreases in the envelope (Bisscheroux 1998) – raising the internal energy above 0.5 times the gravitational binding energy in absolute value. Hence, the total binding energy becomes smaller in absolute value relative to the gravitational binding energy. This in turn causes a larger discrepancy between λ_{g} and λ_{b}, see upper right panel in Fig. 1.

Especially for low-mass stars (3–6 $M_{\odot}$) we see that the contribution of the internal energy to the total binding energy becomes dominating. At some point on the asymptotic giant
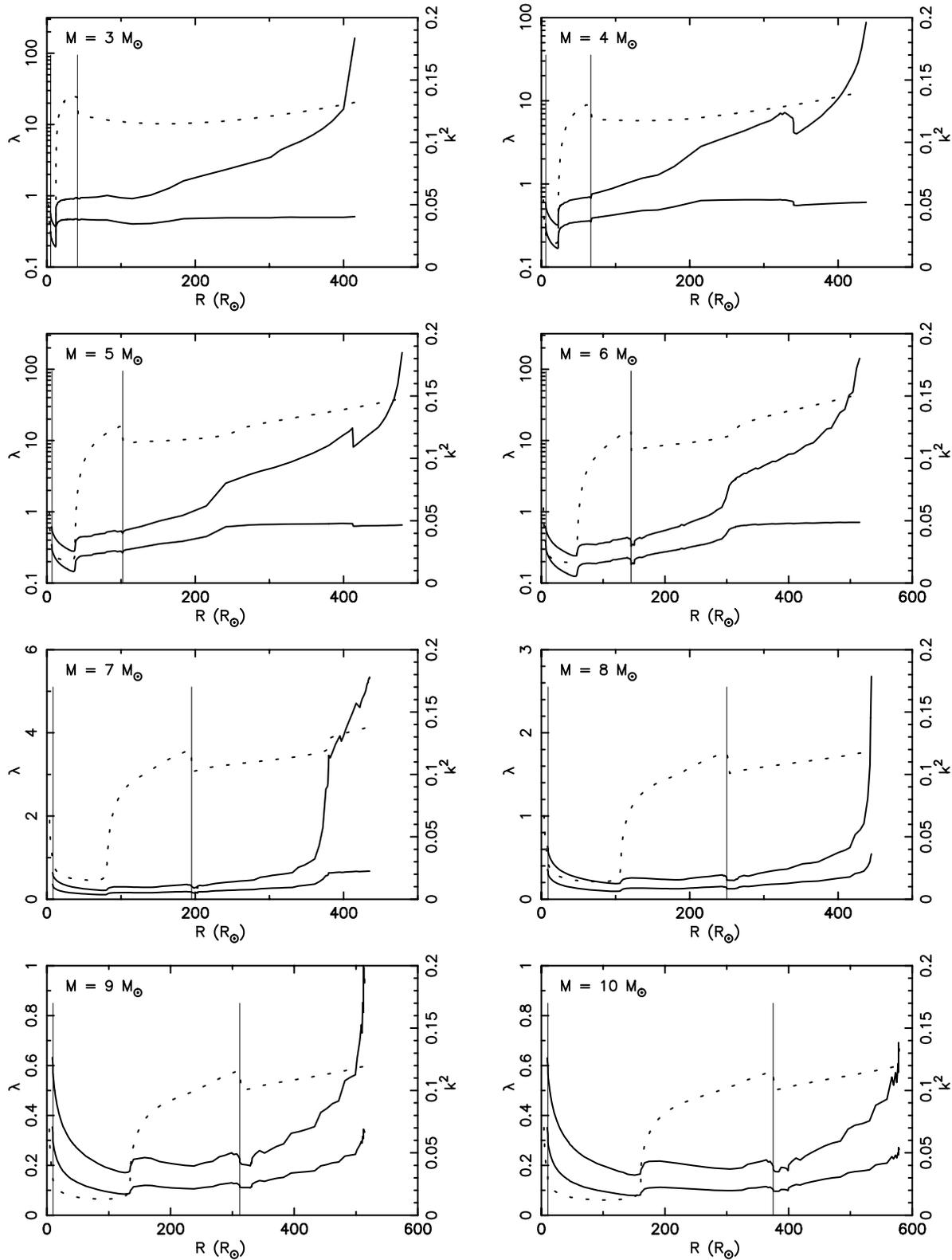
Fig. 1. The parameter λ as a function of stellar radius, for stars with initial mass of 3–10 $M_\odot$. Upper and lower solid lines represent the value of λ derived from the total (λ_b) and gravitational (λ_g) binding energy, respectively – see text. For stars with mass in the interval of 3–6 $M_\odot$, the λ_b-curves are plotted until the moment before the binding energy becomes positive (and the λ_b-value would become negative, see Sect. 5.1 for an explanation). The dotted line in each panel presents the gyration radius of the star ($k^2 = I/MR^2$) with a scale given on the right y-axis. The vertical lines separate the regions of case A (left-), case B (middle-) and case C (right-side) mass transfer from a binary donor.
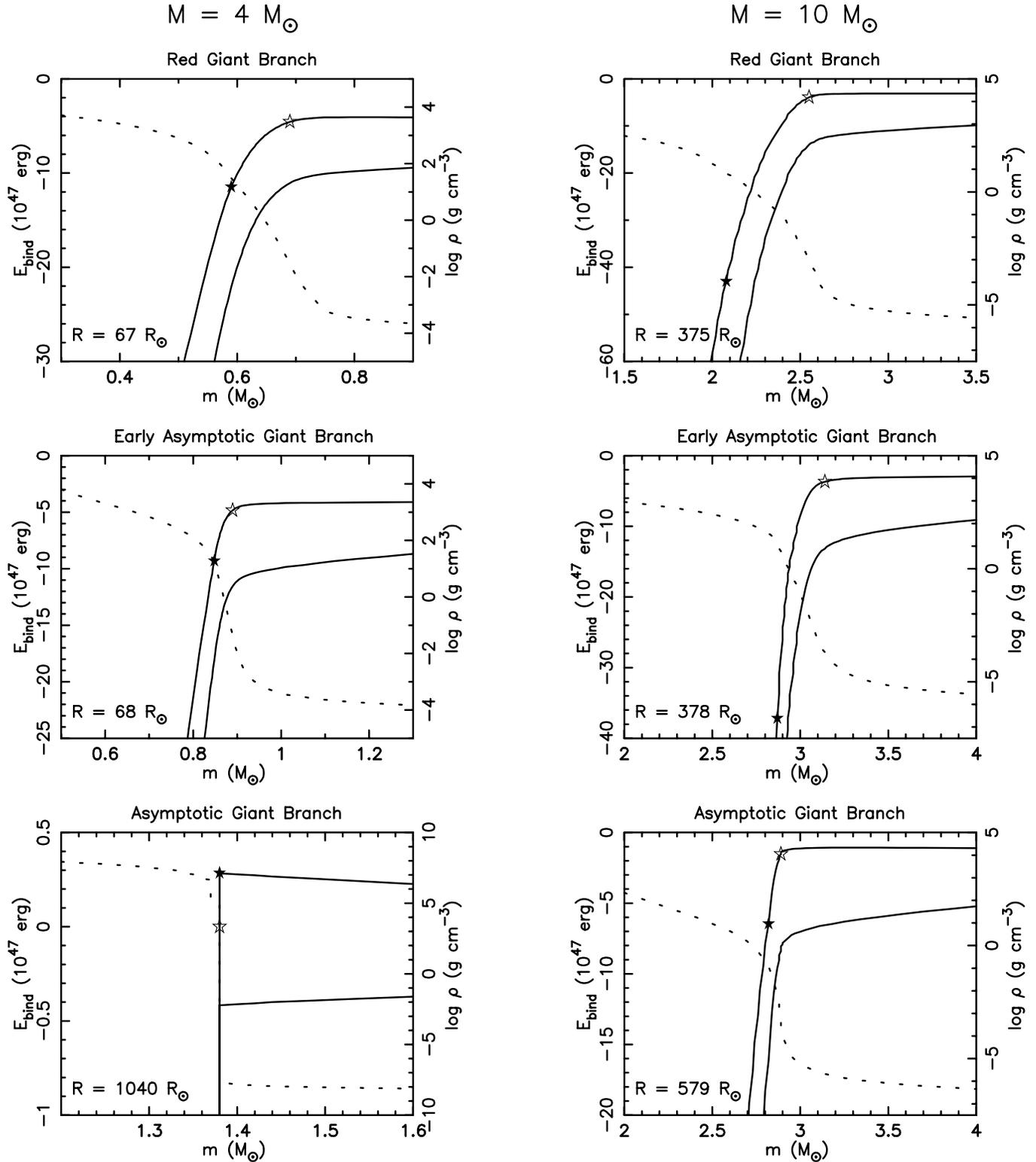
Fig. 2. Total and gravitational binding energy (upper and lower solid line, respectively) around the core of stars with an initial mass of 4 $M_\odot$ (left-side panels) and 10 $M_\odot$ (right-side panels) at three giant stages. The stellar radius corresponding to each stage is indicated at the bottom of the panels. The density profile is represented by a dotted line (with the scale given on the right y-axis). Solid and open stars indicate the core mass chosen to determine the binding energy using our choice and the definition of Han et al. (1994), respectively.

branch, the internal energy dominates over the gravitational binding energy, and hence the total binding energy of the envelope reaches a positive value. One might regard this alteration from a negative to a positive value of the total binding energy as the ejection of the giant's envelope, or as a point of the onset of a pulsationally driven (Mira) superwind (Han et al. 1994 and references therein). However, when the binding energy becomes positive, the derived parameter λ_b will be negative and our formalism in Sect. 2.1 breaks down – cf. resulting "negative" final separation in Eq. (4). On the contrary λ_g is relatively constant during this late stage of evolution.

The evolution of stars above 7 M$_\odot$ is cut short by carbon burning. Therefore they never move as far up the AGB as lower mass stars, and the internal (recombination) energy never becomes so important as to dominate the total binding energy. This is why the difference between λ_g and λ_b is less pronounced as we go to the more massive stars.

5.2. On the definition of the core mass boundary

Our results are sensitive to the definition of core mass. In our work we have chosen to define the core mass as the central mass which contains less than 10% hydrogen. Using other boundary conditions such as $\partial^2 \log \rho/\partial m^2 = 0$ (e.g. Bisscheroux 1998) or the alternative method by Han et al. (1994) will lead to different values for the binding energy of the envelope and hence different estimates of λ. In general the above mentioned methods will result in a core mass boundary further out in the star compared to our definition (see symbols on the dashed line in Fig. 2). Hence their methods lead to smaller envelope binding energies and thus larger values of λ (factor ~ 2) on the RGB and EAGB. For a 10 M$_\odot$ star, this difference is much higher than a factor of 2. Further up the AGB, where the density gradient is steeper, the differences become smaller and the result more reliable. Nevertheless, in accordance with our findings using any of the two other boundary conditions mentioned will also result in large values of λ on the AGB (for low-mass stars: $\lambda_b \gg 1$) and this is the important issue.

5.3. Dependency of the λ-parameter on the chemical composition and mixing-length parameter

In order to investigate the dependency of the λ-parameter on chemical composition and mixing-length parameter, we have plotted in Fig. 3 the computed $\lambda_b(R)$ for both Pop. I and Pop. II chemical abundances, as well as for two different values of the mixing-length parameter for a 6 M$_\odot$ donor star.

From the ZAMS to the base of the RGB ($R \sim 50$ M$_\odot$) there are no differences on the calculated values of λ_b. It is also seen that there is hardly any dependency on the chemical composition for the rest of the evolutionary track far up the AGB. However, from the horizontal branch ($R \sim 150$ M$_\odot$) and halfway up the AGB ($R \sim 300$ M$_\odot$) a higher value of λ_b is found using a larger value of the mixing-length parameter. This picture is reversed near the top of the AGB ($R > 400$ M$_\odot$). In general we can conclude that λ_b is almost independent of the

Table 1. The estimated λ (see text) as a function of stellar radius for different donor stars (graphs shown in Fig. 1). R is in units of R$_\odot$ and $k^2 = I/MR^2$ is the gyration radius of the star. The corresponding orbital period, P_{orb} (days) is also given at the onset of Roche-lobe overflow, assuming a 1.3 M$_\odot$ neutron star companion.

$M = 3.0$ M$_\odot$					$M = 4.0$ M$_\odot$				
R	λ_g	λ_b	k^2	P_{orb}	R	λ_g	λ_b	k^2	P_{orb}
20.0 [*]	0.46	0.87	0.13	16.4	20.0	0.17	0.33	0.02	13.6
40.0 [*]	0.47	0.94	0.14	46.4	40.0 [*]	0.34	0.64	0.13	38.6
70.0 [**]	0.46	0.97	0.12	107.6	70.0 [**]	0.39	0.76	0.12	89.2
100.0 [**]	0.42	0.94	0.12	183.9	100.0 [**]	0.43	0.91	0.12	152.6
200.0 [**]	0.48	1.81	0.12	519.5	200.0 [**]	0.59	2.32	0.12	432.1
300.0 [**]	0.49	3.44	0.12	953.5	300.0 [**]	0.64	5.43	0.13	791.0
400.0 [**]	0.50	16.50	0.13	1470.0	400.0 [**]	0.59	10.60	0.14	1220.0
500.0 [**]	0.51		0.14	2048.9	500.0 [**]	0.61		0.15	1703.4
600.0 [**]	0.54		0.15	2700.7	600.0 [**]	0.63		0.16	2242.4

$M = 5.0$ M$_\odot$					$M = 6.0$ M$_\odot$				
R	λ_g	λ_b	k^2	P_{orb}	R	λ_g	λ_b	k^2	P_{orb}
20.0	0.18	0.34	0.02	11.7	20.0	0.18	0.36	0.02	10.4
40.0	0.22	0.41	0.04	33.3	40.0	0.14	0.27	0.02	29.5
70.0 [*]	0.26	0.49	0.11	77.0	70.0 [*]	0.19	0.35	0.11	68.3
100.0 [*]	0.28	0.54	0.13	131.0	100.0 [*]	0.20	0.37	0.12	116.4
200.0 [**]	0.42	1.06	0.12	374.8	200.0 [**]	0.27	0.55	0.11	332.3
300.0 [**]	0.67	4.03	0.13	687.7	300.0 [**]	0.54	2.07	0.13	611.0
400.0 [**]	0.69	12.00	0.14	1058.0	400.0 [**]	0.69	6.44	0.14	941.0
500.0 [**]	0.66		0.15	1490.0	500.0 [**]	0.71	44.87	0.15	1316.7
600.0 [**]	0.69		0.17	1960.5	600.0 [**]	0.70		0.18	1733.2

$M = 7.0$ M$_\odot$					$M = 8.0$ M$_\odot$				
R	λ_g	λ_b	k^2	P_{orb}	R	λ_g	λ_b	k^2	P_{orb}
20.0	0.19	0.37	0.02	9.4	20.0	0.20	0.39	0.02	8.6
40.0	0.14	0.28	0.02	26.6	40.0	0.14	0.29	0.02	24.4
70.0	0.11	0.22	0.02	61.6	70.0	0.11	0.22	0.02	56.6
100.0 [*]	0.16	0.30	0.10	105.5	100.0	0.10	0.19	0.02	96.2
200.0 [**]	0.15	0.28	0.11	300.7	200.0 [*]	0.13	0.25	0.11	272.0
300.0 [**]	0.26	0.56	0.11	553.0	300.0 [**]	0.17	0.32	0.11	508.5
400.0 [**]	0.65	3.93	0.13	853.0	400.0 [**]	0.25	0.56	0.11	783.0

$M = 9.0$ M$_\odot$					$M = 10.0$ M$_\odot$				
R	λ_g	λ_b	k^2	P_{orb}	R	λ_g	λ_b	k^2	P_{orb}
20.0	0.20	0.40	0.02	8.0	20.0	0.21	0.42	0.02	7.4
40.0	0.15	0.29	0.02	22.5	40.0	0.15	0.31	0.02	21.1
70.0	0.11	0.23	0.02	52.5	70.0	0.11	0.23	0.02	48.7
100.0	0.09	0.19	0.02	89.4	100.0	0.10	0.20	0.02	83.8
200.0 [*]	0.11	0.21	0.10	252.3	200.0 [*]	0.11	0.22	0.08	235.2
300.0 [*]	0.13	0.25	0.12	465.7	300.0 [*]	0.10	0.19	0.10	432.2
400.0 [**]	0.17	0.33	0.11	727.9	400.0 [**]	0.11	0.20	0.10	681.0
500.0 [**]	0.25	0.59	0.12	1013.3	500.0 [**]	0.17	0.35	0.11	952.6

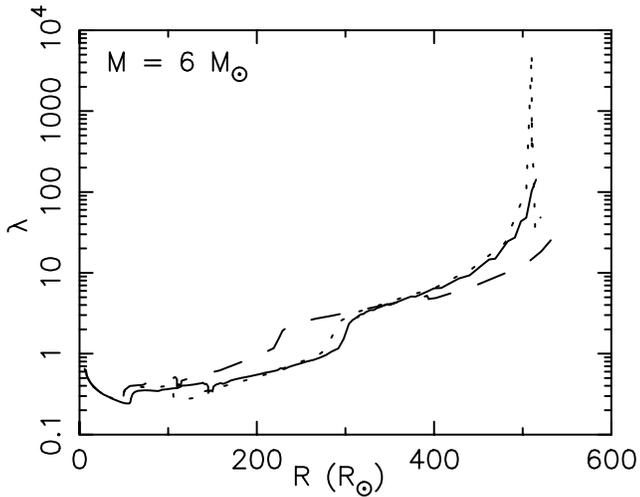[*] Red giant branch
[**] Asymptotic giant branch

Fig. 3. λ_b as a function of radius for a 6 $M_\odot$ star. The full line is for $X = 0.70, Z = 0.02$ and $\alpha = 2.0$. The dashed line is for the same chemical composition with $\alpha = 3.0$. The dotted line is for a star with $X = 0.75, Z = 0.001$ and $\alpha = 2.0$. See text for a discussion.

chemical composition of the star, but it depends on the choice of the mixing-length parameter.

5.4. The efficiency parameter of the ejection process

In order to explain the long orbital periods observed among those of the BMSPs which are likely to have evolved through a CE-phase, Tauris (1996) and van den Heuvel (1994) found it necessary to invoke efficiency parameters larger than unity (e.g. $1 < \eta_{CE} < 4$). However, their simple estimates were based on a constant value of $\lambda = 0.5$. Here we suggest that a high value of λ (rather than a high value of η_{CE}) is the natural explanation for the long orbital periods observed. This in turn indicates that if the internal energy is used efficiently ($\alpha_{th} = 1$) to unbind the envelope, there is no need to assume $\eta_{CE} > 1$ for these systems. This result was also concluded by Han et al. (1995).

Let us now consider a system with a 9 $M_\odot$ donor star and a 1.3 $M_\odot$ neutron star. Such a binary is the progenitor of a pulsar with a massive degenerate companion. We computed the final orbital separation of the binary after a CE-phase by using $E_{env} = \eta_{CE} \Delta E_{orb}$ combined with Eqs. (2) and (6). Here we applied $\eta_{CE} = 1$ and $\alpha_{th} = 1$ (i.e. including the internal thermodynamic energy, $\lambda = \lambda_b$), and present the result in the upper panel of Fig. 4. As a comparison, we also show the final separation computed by using Eq. (4) with a constant efficiency parameter $\lambda = 0.5$ (commonly used in the literature).

A binary will survive the CE-phase only if the final orbit is large enough so that the core of the donor will not instantly fill its Roche-lobe. In the case of a binary with a 9 $M_\odot$ donor and a 1.3 $M_\odot$ neutron star, this condition is reached if the binary separation at the onset of the CE-phase is larger than 435 $R_\odot$ ($P_{orb} > 327^d$). If we use the constant $\lambda = 0.5$, this separation is reduced to 175 $R_\odot$ ($P_{orb} > 84^d$). In the latter case the donor star still has a radiative envelope with only a very thin convec-
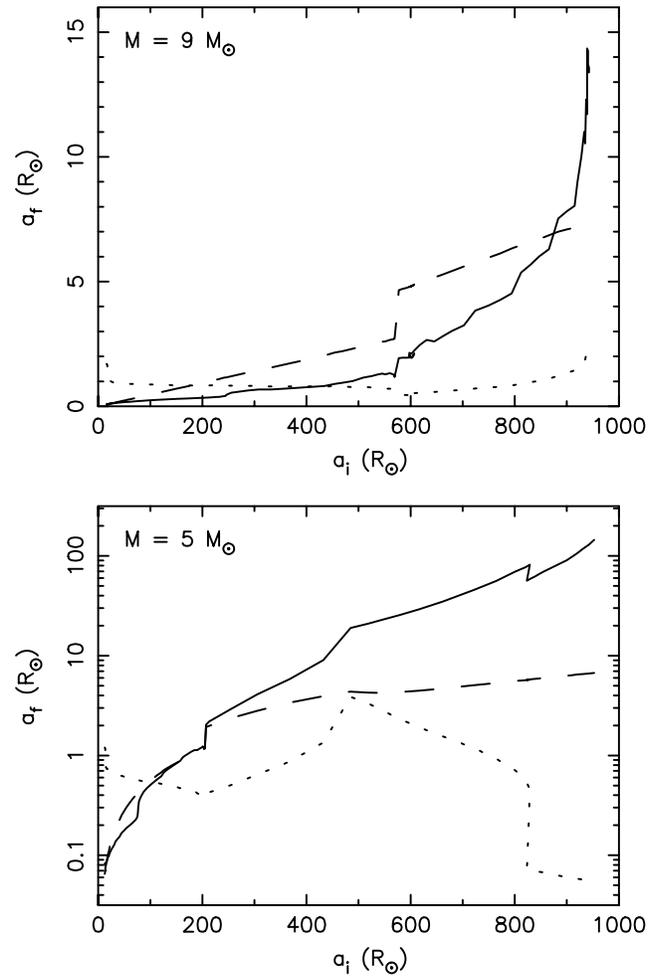

Fig. 4. Final orbital separations after a CE-phase for systems with an initial donor star of mass 9 $M_\odot$ and 5 $M_\odot$ (upper and lower panel, respectively). The companion is assumed to be a 1.3 $M_\odot$ neutron star. The full line is the final separation we obtain when the parameter λ was calculated as a function of stellar radius and the dashed line is for a constant $\lambda = 0.5$. The dotted line is the critical final separation at which the helium star (the core of the stripped donor) immediately fills its Roche-lobe after the CE and spiral-in phase. If the final separation, a_f is smaller than this value the system is most likely to merge and hence the binary will not survive. The radius of the helium star was calculated with our numerical stellar evolution code.

tive shell at the onset of the mass transfer. Therefore the CE is formed only as a result of orbital shrinkage due to mass transfer with a high mass ratio between the donor and the neutron star. In the former case, the donor star had time to develop a deep convective envelope and it will therefore expand in response to mass loss due to its isentropic entropy profile. This expansion, in combination with the orbital shrinkage mentioned above, will immediately lead to a runaway mass transfer, and hence a CE will be formed easily.

At late stages of stellar evolution, the value of λ_b has an important effect on the final orbital period. This effect is more significant in binaries with less massive donors as can be seen in the lower panel of Fig. 4 for the case of a 5 $M_\odot$ donor.
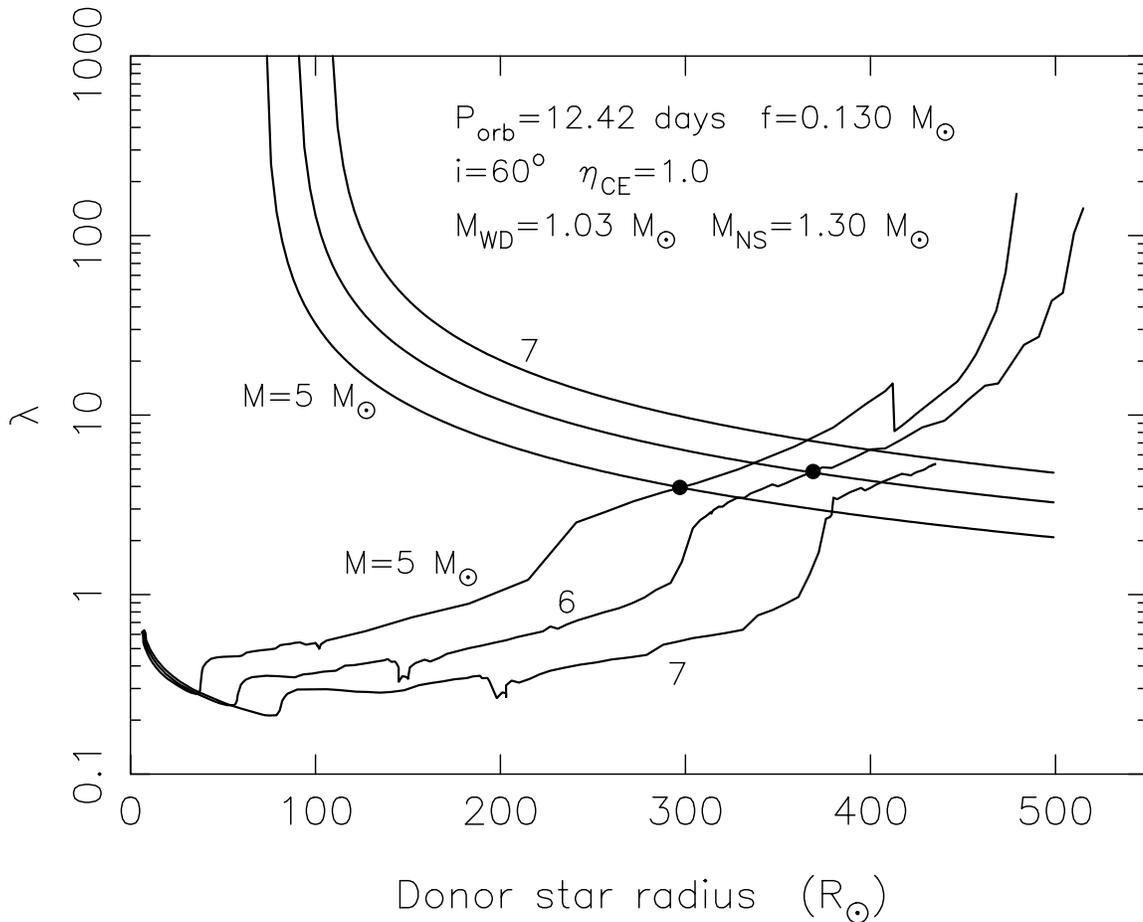
PSR J1454−5846

Fig. 5. The parameter $\lambda = \lambda_{\mathrm{b}}$ as a function of stellar radius, for donor stars with initial mass of 5–7 $M_\odot$. Also shown are curves for Eq. (4) calculated with the observed parameters given for PSR J1454–5846 and assuming $\eta_{\mathrm{CE}} = 1.0$. The intersection of these curves yields a unique solution for the properties of the original donor star (the progenitor of the white dwarf) and the pre-CE orbital period – see text for a discussion.

5.5. A new approach for tracking the evolutionary history of a system evolving through a CE

As we have demonstrated in this paper, λ is a function of stellar radius for any given mass. Combining this fact with Eq. (4), which relates the observed orbital period to the original radius of the Roche-lobe filling donor star as a function of mass and λ (or equivalently: $a_{\mathrm{f}}/a_{\mathrm{i}}$), we are able to place severe constraints on the mass and radius (and hence age) of the original donor.

An example of this approach is shown in Fig. 5 where we have used the parameters for the recently discovered binary pulsar PSR J1454–5846 (Camilo F., private communication). This system has an orbital period of $P_{\mathrm{orb}} = 12\overset{\mathrm{d}}{.}42$ and a mass function, $f = 0.130\ M_\odot$. The exact mass of the white dwarf in this system is unknown; but as an example we shall assume an orbital inclination angle, $i = 60°$ yielding $M_{\mathrm{WD}} = 1.03\ M_\odot$ (for $M_{\mathrm{NS}} = 1.30\ M_\odot$). Hence we must require that the original giant donor star had a core mass of this value by the time its envelope was ejected in the CE-phase. In Fig. 5 the intersection points for a 5 $M_\odot$ and 6 $M_\odot$ donor yield core mass values of

0.93 $M_\odot$ and 1.10 $M_\odot$, respectively. To obtain a core mass of 1.03 $M_\odot$ we must therefore search for a solution in-between. We find $M_{\mathrm{donor}} = 5.6\ M_\odot$ and the radius at the onset of the RLO is 340 $R_\odot$. The corresponding age of the donor is 93 Myr and the pre-CE orbital period is 760^{d}.

The above solution was found for an inclination angle, $i = 60°$. In Fig. 6 we have plotted the solutions for M_{donor} and R_{L} as a function of inclination angle for two different values of the efficiency parameter, η_{CE}. It is interesting to notice that M_{donor} is more or less independent of η_{CE} whereas R_{L} gets larger for smaller values of η_{CE}. This result is expected theoretically since a low value of η_{CE} requires either more orbital energy released and/or less binding energy of the envelope, and thus a larger R_{L}, in order to eject the envelope successfully. An upper limit to the mass of the white dwarf is about 1.3 $M_\odot$ which yields $i > 47°$. For $\eta_{\mathrm{CE}} = 0.5$ no solution is found for $i < 59°$. Hence it would be interesting if a future detection of a spectroscopic line from the white dwarf would yield an orbital inclination angle less than this value, since small values of η_{CE} would be excluded that way.

It should also be noted from Fig. 6 that the binary pulsar must have had an AGB radius at the onset of the RLO. Many of the observed binary pulsars which have survived a common envelope evolution also have long orbital periods. Hence, the density gradient was fairly steep at the onset of the Roche-lobe overflow and therefore the uncertainties regarding the core mass definition as already discussed in Sect. 5.2 are less important in this context.

6. Conclusions

We have adapted a numerical computer code to study the stellar structure of stars with an initial mass of 3–10 $M_\odot$ in order to evaluate the binding energy of the envelope to the core which determines the parameter λ in the energy equation of the common envelope evolution. We have presented evidence that the parameter λ depends strongly on the evolutionary stage. Hence the value of λ, at the onset of the mass transfer and CE-phase, is unique for binaries with different initial orbital periods and donor masses. Taking advantage of this, we have demonstrated a new approach for finding a unique solution to the energy equation and the original mass and radius of the donor star. An application of this approach on PSR J1454–5846 yields a constraint for the donor star and the pre-CE orbital period as a function of the unknown orbital inclination angle.

We have demonstrated that in order to obtain a long final orbital period after a CE-phase (as observed in some BMSPs), we do not require an efficiency parameter larger than unity, if the internal energy can be used efficiently to eject the envelope.

In order to obtain the final orbital separation of a binary system after a CE and spiral-in phase, we advise one to calculate the binding energy from the stellar structure by means of Eq. (6) with $0 \le \alpha_{th} \le 1$. In case the detailed structure of the donor is not available (e.g. for a quick back-of-the-envelope-calculation), Eq. (4) can be used with $\eta_{CE} = 1$ in combination with $\lambda_g \le \lambda \le \lambda_b$. The value of λ can be found in our Table 1 for a given donor star mass and radius.

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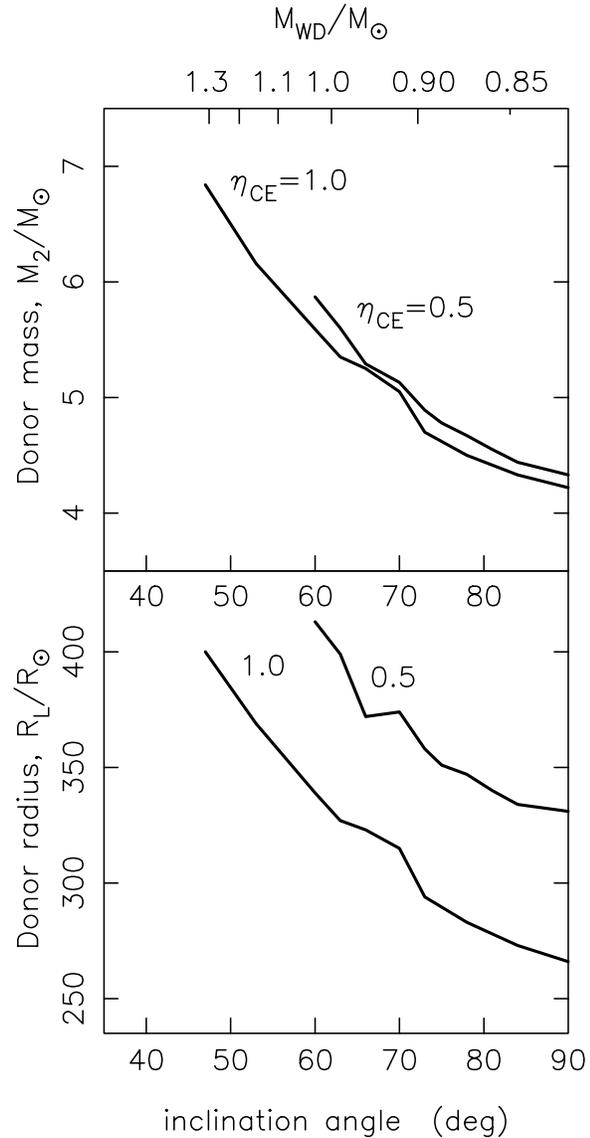Han Z., Podsiadlowski P., Eggleton P.P., 1995, MNRAS 272, 800

Fig. 6. Solutions for the mass (top panel) and radius (bottom panel) of the giant progenitor of the massive white dwarf observed in PSR J1454–5846, as a function of orbital inclination angle, i and CE efficiency parameter, η_{CE}. On the top is shown M_{WD} as a function of the inclination angle – see text for further details.

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