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A model of a steadily rotating prolate galaxy in Newtonian theory

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Abstract. A model of a prolate galaxy in constructed within the framework of Newtonian theory of gravity. The model consists of a central perfect-fluid gravitating prolate spheroid rotating steadily about its axis of symmetry and two galaxies of equal masses, which are situated on the prolate spheroid's axis of symmetry and at equal distances from its centre on either of its sides. It is shown that for physically acceptable ranges of the values of the masses of the two companion galaxies and of their distances from the prolate spheroid's centre such a model is possible in Newtonian theory. The astrophysical significance of the model is discussed as well as the fact that its existence can be verified observationally.

Key words: hydrodynamics – galaxies: kinematics and dynamics

1. Introduction

The existence of rotating *prolate elliptical galaxies* has been known for the last twenty years (Bertola & Galletta 1978). Yet, a *theoretical* model, based on Newton's or Einstein's gravitational theories, remains elusive. An attempt to construct such a theoretical model within the framework of Newtonian theory of gravity was recently made by the present authors (Florides & Spyrou 1993; in the sequel this paper will be refered to as Paper I). We considered an isolated perfect-fluid prolate spheroid rotating steadily about its axis of symmetry and showed that, at least in the case when the fluid density is constant, such a model is *inadmissible* in Newtonian theory.

More recently Spyrou, Kazanas & Esteban (1997) examined the possibility of contructing a theoretical model similar to the one in Paper I, using a *modified* Newtonian gravitational potential. More precisely they used a generalized form of the gravitational potential of a bounded perfect-fluid source based on the formula

$$V(r) = \frac{\beta}{r} - \gamma r \qquad (1)$$

where r denotes radial distance from the gravitational centre and β, γ are positive constants. We recognize the first term of the right hand side of Eq. (1) as the usual Newtonian gravitational potential due to a point source. The inclusion of the second term, $-\gamma r$, was motivated by the theory of *conformal gravity* (Mannheim & Kazanas 1989, 1990, 1994; Mannheim 1993, 1994). Spyrou, Kazanas & Esteban (1997) have shown that such a model is just as *inadmissible* in this case as in the usual Newtonian theory.

In the present paper we return to the construction, within the framework of Newtonian theory of gravity, of a theoretical model of a prolate galaxy rotating steadily about its axis of symmetry. *Unlike* Paper I, in which the galaxy was *isolated*, we now consider the possibility that the prolateness of the galaxy may be accounted for by *the tidal forces* due to neighbouring galaxies[1]. More specifically, we consider a model consisting of

(i) a prolate spheroid of perfect fluid bounded by the surface

$$B : \frac{x_1^2 + x_2^2}{a_1^2} + \frac{x_3^2}{a_3^2} = 1, \ (a_3 > a_1) \qquad (2)$$

where $(x_a) = (x_1, x_2, x_3)$ are Cartesian coordinates with origin at the center O of the spheroid and the x_3-axis is the axis of symmetry. The spheroid rotates *steadily* with angular velocity ω about the x_3-axis. From symmetry arguments ω may be a function of $r = (x_1^2 + x_2^2)^{1/2}$ and x_3, but the (Newtonian) equations of motion imply that, as in Paper I, ω is in fact a function *of r only*. We shall also assume, as in Paper I, that the perfect fluid is *incompressible* so that the matter density, ρ, is *constant*;

(ii) two galaxies B_1 and B_2 situated along the x_3-axis at the "points" $B_1 = B_1(0, 0, b)$ and $B_2 = B_2(0, 0, -b)$.

The two galaxies B_1 and B_2 cannot, of course, remain permanently at rest and, strictly speaking, we should consider this model as a three-body problem, a daunting problem indeed. To a first approximation, however, we shall assume that b is very much greater than a_3, ($b \gg a_3$), so that, at least at some initial stage, B_1 and B_2 move so slowly that they can be considered at rest. Finally, we shall assume, for simplicity, that B_1 and B_2 are spherical and each of mass M. This ensures that the centre of the prolate spheroid will remain permanently at rest at the origin O.

[1] This possibility arose from a brief *discussion* that one of us (P.S.F.) had with Professor Werner Israel about two years ago and for which we would like to thank him warmly.

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In the next Sect. 2, the construction of the model is described. In Sect. 3, the astrophysical significance of the model is discussed, and it is emphasized that the model's *physical existence* can be verified observationally. The results are discussed in the final Sect. 4. The paper ends with an Appendix, in which the time interval is evaluated theoretically, required for each of the companions to reduce its distance from the central galaxy only slightly, so that it be considered at rest for some considerable time.

2. Theoretical description of the model

In constructing our model, we shall follow the notation and approach used in Paper I. We use cylindrical polar coordinates (r,θ,z) with origin the centre, O, of the perfect-fluid prolate spheroid and the z-axis as the axis of symmetry. The boundary, B, of the spheroid is, then, given by

$$z = \pm\zeta(r), \text{ where } \zeta(r) = a_3\left(1-\frac{r^2}{a_1^2}\right)^{\frac{1}{2}}, \ (a_3 > a_1 \geq r). \quad (3)$$

As in Paper I, and in Florides & Synge (1962), the equations of motion are

$$p_{,z} - \rho U_{,z} = 0, \quad (4)$$

$$p_{,r} - \rho U_{,r} = \rho\omega^2 r \quad (5)$$

with the boundary condition

$$p = 0 \ on \ B. \quad (6)$$

In the above equations $p = p(r,z), \ \rho = \rho(r,z)$ denote the pressure and the density of the fluid at the point $P(r,\theta,z)$, respectively, and a comma (,) indicates partial differentiation; ω is the angular velocity of the fluid and, as pointed in Sect. 1, is a function of r only. *Unlike* Paper I the function U=U(r,z) now denotes the *total* gravitational potential of the system at $P(r,\theta,z)$; that is

$$U = V + W \quad (7)$$

where V=V(r,z) is the self gravitational potential of the central spheroid and W is the gravitational potential of the galaxies B_1 and B_2 situated on the z-axis at distances b and -b, respectively, from O. Since we have assumed B_1 and B_2 to *be spherical*, the potential W at the point $P(r,\theta,z)$ inside the prolate spheroid in given by[2]

$$W = \frac{GM}{B_1P} + \frac{GM}{B_2P} = GM\left\{[r^2+(b-z)^2]^{-\frac{1}{2}} + [r^2+(b+z)^2]^{-\frac{1}{2}}\right\} \quad (8)$$

where G is the gravitational constant. Also, since we are assuming that ρ is constant, the self gravitational potential V inside the prolate spheroid is given by (Chandrasekhar 1969)

$$V(r,z) = \pi G\rho[A_1(2a_1^2 - r^2) + A_3(a_3^2 - z^2)], \quad (9)$$

[2] The galaxies B_1 and B_2 can, in fact, be considered of any sphape for, since we are assuming $b \gg a_3$, the dominant part of their potential inside the prolate spheroid is as in Eq. (8).

where A_1 and A_3 are the constants

$$A_1 = \frac{1}{e^2} - \frac{1-e^3}{2e^3}\log\left(\frac{1+e}{1-e}\right), \ \ A_3 = 2 - 2A_1, \quad (10)$$

e being the eccentricity of the spheroid given by

$$e = \left(1 - \frac{a_1^2}{a_3^2}\right)^{\frac{1}{2}}. \quad (11)$$

As in Paper I, integrating Eq. (4) with respect to z (keeping r constant) and using the boundary condition (6), we get, for constant ρ,

$$p(r,z) = \rho[U(r,z) - U(r,\zeta(r))] \quad (12)$$

for the pressure of the fluid. Using this equation in Eq. (5) we get, for constant ρ,

$$\omega^2 r = -(U_{,r} + U_{,z}\zeta')_B \ \ everywhere \quad (13)$$

where

$$\zeta' = \frac{d\zeta}{dr} = -\frac{ra_3}{a_1^2}\left(1-\frac{r^2}{a_1^2}\right)^{-\frac{1}{2}} = -\left(\frac{a_3}{a_1}\right)^2\frac{r}{\zeta};$$

the subscript B in Eq. (13) indicates that the quantities inside the brackets are evaluated on the boundary B. Evidently, ω is a function of r only, as expected.

Using Eqs. (7),(8),(9) and (11) in Eq. (13) we get

$$\begin{aligned}\omega^2 &= 2\pi G\rho(A_1 - \frac{A_3}{1-e^2}) \\ &+GM\left\{[r^2+(b-\zeta)^2]^{-\frac{3}{2}} + [r^2+(b+\zeta)^2]^{-\frac{3}{2}}\right\} \\ &+\frac{GM}{(1-e^2)\zeta}\left\{(b-\zeta)\left[r^2+(b-\zeta)^2\right]^{-\frac{3}{2}}\right. \\ &\left.-(b+\zeta)\left[r^2+(b+\zeta)^2\right]^{-\frac{3}{2}}\right\}. \end{aligned} \quad (14)$$

We note that when $M = 0$ this equation reduces to $\omega = \omega_0$ where

$$\omega_0^2 = 2\pi G\rho\left(A_1 - \frac{A_3}{1-e^2}\right) \quad (15)$$

in agreement with the result of Paper I for an isolated steadily rotating prolate perfect fluid spheroid. As was proved in Paper I, the right-hand side of Eq. (15) is negative, thus making ω_0 imaginary and, therefore, the existence of such an isolated prolate spheroid impossible.

Before proceeding any further it will be useful to express the first term on the right hand side of Eq. (14), that is ω_0^2, in terms of the mass of the central spheroid. Since the volume of the spheroid (3) is given by

$$\Omega = \frac{4}{3}\pi a_1^2 a_3 = \frac{4\pi}{3}\left(\frac{a_1}{a_3}\right)^2 a_3^3 = \frac{4\pi}{3}(1-e^2)a_3^3,$$

and ρ is constant, the mass, m, of the spheroid is given by

$$m = \rho\Omega = \frac{4\pi\rho}{3}(1-e^2)a_3^3. \quad (16)$$

Thus, in terms of m, Eq. (15) becomes

$$\omega_0^2 = \frac{3Gm}{2(1-e^2)a_3^3}\left(A_1 - \frac{A_3}{1-e^2}\right). \tag{17}$$

Let us return to our present model. The crucial question is whether ω^2 in the general Eq. (14) *is positive* for all values of $r \leq a_1$. The last two terms on the right-hand side of Eq. (14) are certainly positive. But is their magnitude sufficiently large to prevail over the *negative* term ω_0^2? To answer this question we simplify Eq. (14) as follows: We have already assumed that $b \gg a_3$ (b very much greater than a_3). Remembering that $a_3 > a_1,\ r \leq a_1$, the definition of $\zeta(r)$ (Eq. (3)), and the fact that $\omega(r)$ is evaluated inside the spheroid, the following inequalities are easy to establish

$$r \leq a_1 < a_3 \ll b,\ z \leq \zeta(r) \ll b. \tag{18}$$

The quantities r/b, z/b and ζ/b are all small of order λ, say, where λ is a small dimensionless constant. Expanding the right-hand side of Eq. (14) in powers of λ we get

$$\omega^2 = \omega_0^2 + \frac{2GM}{b^3}\left(1 + \frac{2}{1-e^2}\right) + O(\lambda^2) \tag{19}$$

$O(\lambda^2)$ indicating terms of order two (and higher) in λ. We display the next term, of order two, in the expansion (14), merely to indicate the type of terms we shall be neglecting in the sequel

$$\frac{GM}{b^3}\left[4\left(\frac{\zeta}{b}\right)^2\left(3 + \frac{2}{1-e^2}\right) - 3\left(\frac{r}{b}\right)^2\left(1 + \frac{4}{1-e^2}\right)\right].$$

Thus, as it can readily be seen, the neglected terms in Eq. (19) are of the order $(a_1/b)^2$ smaller that the second term on its right-hand side.

The last term in Eq. (19) is the *dominant* contribution of the galaxies B_1 and B_2 to ω^2. We note that, if we neglect the $O(\lambda^2)$ terms, ω^2 is constant and the central spheroid is, therefore, rotating *rigidly*.

The importance of the last term in Eq. (19) is that it is positive and that, for a given eccentricity e, it depends entirely on the constants b and M. It is, therefore, possible that with a judicious choice of b and M this term can be made sufficiently large to prevail over the first negative term, ω_0^2, thus making ω *real*. This would mean that the present model is, indeed, admissible in Newtonian theory.

The precise condition on the values of M and b necessary to make ω real, follows from Eqs. (17) and (19). It is given by

$$\frac{M}{m} > \frac{3}{4}\left(\frac{b}{a_3}\right)^3\left(\frac{A_3}{1-e^2} - A_1\right)(3-e^2)^{-1}. \tag{20}$$

The requirement that ω^2 must be positive, although essential, is not *sufficient* to ensure the existence of our model. We must also show that the pressure of the fluid is well- behaved everywhere inside the spheroid. In addition, it is desirable that p(r,z) is *positive* everywhere. By Eqs. (12), (8), (7) and (9) we get, neglecting terms of order λ^2 and higher,

$$p(r,z) = G\rho(\zeta^2 - z^2)\left\{\frac{3}{4}\frac{mA_3}{(1-e^2)a_3^3} - \frac{2M}{b^3}\right\}. \tag{21}$$

Thus p(r,z) is well-behaved everywhere and, since $\zeta \geq z$ at any point (r,z) inside the fluid, it is *positive* everywhere if the term in curly brackets is positive, that is if

$$\frac{M}{m} < \frac{3}{8}\frac{A_3}{(1-e^2)}\left(\frac{b}{a_3}\right)^3. \tag{22}$$

It follows from Eqs. (20) and (22) that, neglecting terms of order λ^2 and higher, the model exists in Newtonian theory if observationally acceptable values of M and b exist such that

$$\frac{3}{4}\left(\frac{A_3}{1-e^2} - A_1\right)(3-e^2)^{-1} < \left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3 < \frac{3}{8}\frac{A_3}{(1-e^2)} \tag{23}$$

or, for e small enough so that terms of order e^4 can be neglected,

$$\frac{e^2}{15} < \left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3 < \frac{1}{4} + \frac{2e^2}{5}. \tag{24}$$

We note the equivalent form of Eq. (19)

$$\left(\frac{\omega}{\omega_N}\right)^2 = \frac{2(3-e^2)}{(1-e^2)}\left[\left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3 - \frac{3}{4}\left(\frac{A_3}{1-e^2} - A_1\right)(3-e^2)^{-1}\right] \tag{25}$$

or, again for small e,

$$\left(\frac{\omega}{\omega_N}\right)^2 = 6\left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3 + \left[4\left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3 - \frac{2}{5}\right]e^2 \tag{26}$$

where

$$\omega_N = \frac{2\pi}{P_N} = \left(\frac{Gm}{a_3^3}\right)^{\frac{1}{2}} \tag{27}$$

is the equatorial Keplerian angular frequency of a single galaxy. The single galaxy's Keplerian, equatorial rotational period (or Keplerian dynamical time), P_N, is in general different from the equatorial rotational period (or dynamical time)

$$P = \frac{2\pi}{\omega} \tag{28}$$

of the composite galaxy considered here.

We end this section by making the interesting observation that, if $e = 0$, that is if the central galaxy is a rotating sphere of radius $a_1 (= a_3)$, then Eq. (19) reduces to

$$\omega^2(e=0) = \frac{6GM}{b^3}. \tag{29}$$

Thus the tidal forces due to B_1 and B_2 are responsible for maintaining the spherical configuration of the rotating perfect fluid. We must emphasize that this result, together with *all* the results in this section, are, of course, correct to order $\lambda(\sim a_3/b)$; that is, as long as $b \gg a_3$. In the next Sextion we examine the astrophysical siqnificance the model and the observational verification of its existence.

3. Astrophysical significance of the model

The model's physical existence can be verified observationally by recalling that the formulation of our model in the previous section was made possible under the assumption that b is sufficiently larger than a_3 so that, at least at some initial stage, the companion galaxies $B_1(0,0,b)$ and $B_2(0,0,-b)$ move so slowly that they can be considered at rest. This assumption can be justified as follows: Let us assume that, initially (t=0), B_1 and B_2 are at rest at z=b and z=-b, respectively Then, according to Eq. (AVI) in the Appendix, the time, t_k taken by B_1 (and, by symmetry, by B_2) to reduce its distance from z=b to $z = kb$, $k \leq 1$ is, in units of P_N,

$$\frac{t_k}{P_N} = f(k)[8\pi^2\left(4+\frac{M}{m}\right)\left(\frac{{a_2}^3}{b}\right)]^{-\frac{1}{2}} \tag{30}$$

where the function

$$f(k) = cos^{-1}(2k-1) + 2\sqrt{k-k^2} \geq 0 \tag{31}$$

ranges between π and 0, for k ranging between 0 and 1.

The validity of our model requires t_k to be not smaller than the galaxy's dynamical time, P_N,

$$t_k \geq P_N \tag{32}$$

or, equivalently,

$$f(k) \geq [8\pi^2\left(4+\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3]^{\frac{1}{2}} \tag{33}$$

so that, for a considerable period, the two galaxies B_1 and B_2 can, indeed, be considered at rest (assuming that they are at rest initially).However, the question remains as to whether acceptable values of the parameters M, b and e can be found, which give physically meaningful value to the angular velocity, ω, of the central galaxy, and at the same time guarantee the validity of the condition (32).

So we find the conditions under which, for values of k close to 1, t_k is comparable to P_N. We assume that the each companions mass, M, is not necessarily small compared to the central galaxy's mass m, namely

$$m \sim M, \tag{34}$$

whence the condition (33) becomes

$$f^2(k) \geq 40\pi^2\left(\frac{a_3}{b}\right)^3. \tag{35}$$

Let us consider the case $k = 0.99$. Then $t_{0.99}$ is the time interval taken by B_1 (or B_2) to reduce its distance from (the centre of) the central galaxy by 1%. In this case, $f(0.99) \sim 0.399$ and it is easily verified that the condition (35) can be satisfied for

$$\frac{a_3}{b} \leq 0.0739 = \frac{1}{13.535}. \tag{36}$$

The above values of $\frac{a_3}{b}$ are in agreement with the virialized results, in the case of clusters of galaxies, for the cluster's typical linear dimensions ($\sim 10Mpc$; Deckel, Burstein & White 1997), and its neighbouring members' linear dimensions (ranging between 0.1 kpc and 1 Mpc, depending on the type of the elliptical galaxy considered; Gerhard 1994, Carroll & Ostlie 1996) and mutual distances ($5 \leq \frac{b}{a_3} \leq 10^3$; Binney & Tremaine 1987, Contopoulos & Kotsakis, 1984, Deckel, Burstein & White 1997, Spyrou 1977).

Now it is possible to derive an upper bound for the central galaxy's acceptable eccentricity. Assuming small values of e, in order to avoid all sorts of instabilities of highly- eccentric spheroids, and applying the conditions (26) and (34), we see that ω is real, if

$$15\left(\frac{a_3}{b}\right)^3 \geq \left[1-\frac{1}{10}\left(\frac{a_3}{b}\right)^3\right]e^2, \tag{37}$$

and so, for a typical value

$$b = 20a_3, \tag{38}$$

we find

$$e \leq 0.043. \tag{39}$$

Furthermore, for the same values ($b = 20a_3,\ k = 0.99$) and for e=0.040, we can easily verify that in general ω is smaller than a few percent of ω_N, P is at least one order of magnitude larger than $P_N, t_{0.99}$ is at least a few times larger than P_N, and the rotational equatorial velocity is smaller than a few percent of $\omega_N a_3$.

As an indicative example we consider an elliptical galaxy with $m \sim 10^{12} m_{\odot}, a_3 = 50kpc$ for which

$$\omega_N = 6.000\times10^{-9}y^{-1},\ \ \omega_N a_3 = 2.933\times10^2\,\frac{\mathrm{km}}{\mathrm{s}} \tag{40}$$

$$P_N = 1.047\times10^9 y. \tag{41}$$

Therefore this composite galaxy's dynamical time, P, is of the order of magnitude of the age of the universe ($\sim 10^{10}y$), and, for exactly this reason, we preferred to compare $t_{0.99}$ with P_N, not with P. Similarly the equatorial velocity of the galaxy equals a few km s^{-1}, in agreement with the observational results that the rotational velocities of elliptical galaxies are less than 150 km s^{-1} and are often barely non-zero (Gerhard 1994). Of course, similar results are obtained from Eq. (29), written, in the form

$$\frac{\omega(e=0)}{\omega_N} = \frac{P_N}{P} = \left[6\left(\frac{M}{m}\right)\left(\frac{a_3}{b}\right)^3\right]^{-\frac{1}{2}}. \tag{42}$$

4. Discussion and outlook

After two attempts with negative results (Florides & Spyrou 1993; Spyrou, Kazanas & Esteban 1997), we have shown for the first time how to construct a model of a prolate elliptical galaxy within the framework of Newtonian theory of gravity. The model is well defined and it consists of a central perfect-fluid gravitating body (galaxy), accompanied by two equal masses (galaxies) located along its major axis and acting on it tidally. By direct computation of the force balance between the combined gravitational, pressure gradient and centrifugal forces, we evaluated the central spheroid's angular velocity of axial rotation. The precise conditions necessary for the angular velocity of rotation to be real and the pressure inside the fluid spheroid to be positive (see condition 23) bound the value of the quantity (M/m) $(a_3/b)^3$.

Our model is amenable to observations, namely its physical existence can be verified by observations. It seems that slowly rotating prolate galaxies of very small eccentricities, in cluster of galaxies, fit comfortably to our model. Therefore the simple suggestion to the observers is to search along the major axis of a (low-eccentricity) prolate galaxy for identifying the two companions.

The above suggestion to observers is furthermore supported by the observation, in clusters of galaxies, of alignment of galaxies-members preferentially with the cluster's major axis, or with the radial direction in the case of nearly spherical clusters. Thus, as reported by Hawley & Peebles (1975) and confirmed by Thompson (1976), there are indications that such a preferential alignment occurs in the nearly spherical Coma cluster. An analogous alignment of ellipticals was observed in the cluster centred on NGC 439 (MacGillivray & Dodd 1979a,b) (with some of the ellipticals, however, having their major axes perpendicular to the radial direction of the cluster). Analogous results were obtained by Fong, Stevenson & Shanks (1990). The best evidence so far for alignment is for the brightest cluster ellipticals (Trevese, Cirimele & Flin 1992). It is worth mentioning that, according to Ciotti & Giampieri (1998), the radially aligned configurations found through N-body simulations (Ciotti & Dutta 1994) are indeed equilibrium positions, in complete accordance with the results of our Appendix at the end.

Our proposed model of prolate galaxies could be taken into account in many cases of current interest. First of all is the construction itself of theoretical models of prolate galaxies. As emphasized already in Paper I, this is necessary in view of the many interesting properties of the prolate galaxies. Such properties are a) the presence of almost equatorial dust lanes and their formation's explanation (Van Albada & Sanders 1982), b) the galaxy's stability as deduced from the various kinds of Newtonian geodesic motions (Bohn 1983; Vandervoort 1980; Vandervoort & Welty 1982; Benacchio & Galletta 1980) or relativistic geodesic motions in them (Spyrou & Varvoglis 1982), and their use in explaining at least part of the observed ellipticity of prolate galaxies (Spyrou & Varvoglis 1987), and c) the fact that until recently we knew only one astronomical object with large-scale prolate structure, namely the bars along the (barred) spiral galaxies.

Beyond the collapse of rotating perfect-fluid configurations (Shapiro & Teukolsky 1992) it is only recently that the interest arose in constructing theoretically models of prolate disk-like spheroidal shells (Gonzales & Letelier 1998). In relation to this it would be interesting to extend our results in the context of the comformal Weyl gravity, along the lines of Spyrou, Kazanas & Esteban (1997), for studying the stability of necessarily differentially rotating stellar systyems. Preliminary results in this case, show that the inclusion of the Weyl's gravity linear potential seems to force the equilibrium configurations to become cigar-like rotating about the small axis (Spyrou, Kazanas and Esteban, under preparation). Moreover, our results might be of interest in the effort of physically identifying known vacuum solutions of the Einstein vacuum equations with the spacetime of a prolate perfect-fluid galaxy (Spyrou & Papadopoulos 1993). Finally, the existence of such composite prolate models could be used in checking the validity of the usual assumption that the circumnuclear galactic masering sources, used in the determination of the nuclear galactic masses (Kormendy & Richstone 1995, Maoz 1995, Miyoshi, Moran, Herrnstein et al. 1995), follow geodesic motions (Rees 1995). Recently, Kleidis & Spyrou (2000b; see also Spyrou 1997a,b,c,d; 1999; and Kleidis & Spyrou 2000a) examined, in the framework of the full theory of general relativity, the functional similarity, through conformal transformations, on the one hand of the hydrodynamical flow motions (like e.g. of a masering source) in the interior of a perfect-fluid gravitating source and on the other hand of the geodesic motions in the same source. Beyond giving a very precise physical meaning to the conformal factor, they also found that the above mentioned functional similarity imposes certain conditions on the nature and distribution of mass, pressure and energy of the fluid source, which affect the observationally determined nuclear mass of the galaxy - fluid source, in the sense that this nuclear mass is underestimated in comparison to the corresponding real physical quantity. In concluding we point out that there exist some indications that rigidly rotating heterogeneous (and/or anisotropic) perfect-fluid prolate spheroids could exist (Florides and Spyrou, in preparation).

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Appendix

Our assumption that the companion galaxies B_1 and B_2 can, for sufficiently large values of the ratio b/a_3, be considered at rest can be justified as follows: Let B_1 and B_2 be at rest at time t=0 at the points $B_1(0,0,b)$ and $B_2(0,0,-b)$. Due to the gravitational attraction of the central galaxy on B_1 and B_2 and the mutual gravitational attraction between B_1 and B_2, B_1 and B_2 will start moving towards the origin O; from symmetry the central galaxy (centre O) will remain at rest (assuming that it is at rest at t=0). Also, because of symmetry, the motion of B_1 and of B_2 towards O will be identical.

Let z be the distance of B_1 say, from O at time $t(>0)$. Then Newton's equation of motion gives

$$M\ddot{z} = -\frac{GMm}{z^2} - \frac{GM^2}{(2z)^2}, \left(\dot{z} = \frac{dz}{dt}, etc\right) \tag{A.I}$$

or

$$\ddot{z} = -\frac{R}{z^2} \tag{A.II}$$

where

$$R = G\left(m + \frac{M}{4}\right).$$

Writing $\upsilon = \dot{z}$, so that $\ddot{z} = \upsilon\frac{d\upsilon}{dz}$, Eq. (Al) becomes

$$\upsilon\frac{d\upsilon}{dz} = -\frac{R}{z^2}.$$

Since $\upsilon = 0$ when z=b, this integrates to

$$\upsilon = -\sqrt{2R}\left(\frac{1}{z} - \frac{1}{b}\right)^{\frac{1}{2}} \tag{A.III}$$

the negative sign indicating inward motion. With $\upsilon = \dot{z}$ Eq. (AIII) becomes

$$\frac{dz}{\left(\frac{1}{z} - \frac{1}{b}\right)^{\frac{1}{2}}} = -\sqrt{2R}dt.$$

To integrate this equation we use the standard substitution

$$z = bcos^2\theta \equiv \frac{b}{2}(1 + cos2\theta) \tag{A.IV}$$

where $\theta = 0$ at t=0. We get

$$\sqrt{2R}t = \frac{b^{\frac{3}{2}}}{2}(2\theta + sin2\theta), \tag{A.V}$$

where we have used the initial conditions z=b, $\theta = 0$ at t=0. Thus the motion of B1 is given by the parametric equations (AIV) and (AV).

We can now ask the question: How long does it take for z to be reduced from z=b to $z = kb, k \leq 1$ During this time θ changes from $\theta = 0$ to θ, where, by Eq. (AIV),

$$cos2\theta = 2k - 1.$$

Hence, by Eqs. (AV) and (AII), the required time is given by,

$$t = \frac{1}{2}\left[\frac{b^3}{2G\left(m + \frac{M}{4}\right)}\right]^{\frac{1}{2}} \times \left[cos^{-1}(2k - 1) + 2\sqrt{k - k^2}\right] \tag{A.VI}$$

which is practically Eq. (30) in the text.

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