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Relation between energy transfer and evolution of W UMa-type binaries

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Abstract. In the present article the relation between the energy transfer rate from the primary to the secondary and the evolutionary degree of W UMa-type binaries is given and the energy transfer rates and the evolution degrees of three W UMa-type contact binaries (TV Mus, SS Ari and RZ Com) are calculated. The analyzed results show that the maximum value of the energy transfer rate is basically determined by the mass ratio. The greater the mass ratio, the greater the maximum possible value of the energy transfer rate. The energy transfer rate is related to the evolutionary degree of the primary, and the greater the evolutionary degree of the primary, the smaller the energy transfer rate. The energy transfer from the primary to the secondary could stop when the evolutionary degree of the primary reaches a specific value which is determined by a property of the system.

Key words: stars: binaries: close – stars: evolution – stars: variables: general

1. Introduction

For more than the past thirty years, the explanation of the light curve of W UMa-type binaries has been one of the research subjects in which lots of researchers are interested. However, so far, at least three observational phenomena of the W UMa-type binaries are still not well understood. 1. The Lucy paradox; as a consequence of their Roche geometry and similar effective temperature, the components of late-type contact binaries have luminosity ratios roughly equal to the first power of their mass ratios, rather than the approximately fourth power observed for a single main-sequence star (Kähler, 1989; Wang, 1994). 2. Binnedijk paradox. W UMa-type contact binaries have been divided by Binnendijk (1970) into A- and W-subtypes. The primary minimum corresponds to transit eclipse of the larger, more massive component for the A-subtype and occultation of eclipse of the smaller, less massive component for the W-subtype. The effective surface temperature of the secondary in a W-subtype is higher than that of the primary. This is also called the W-phenomenon. 3. The O'Connell effect. The two maxima of the light curve of many W UMa-type contact binaries have different heights, showing that the brightness of the binary system at the 0.25 orbital phase is different from that at the 0.75 orbital phase (Linnell, 1986, 1987).

It proved to be very difficult to construct theoretical models of the contact binaries that are able to explain the characteristics of the light curves of these systems (Kähler 1989). Lucy (1968) published the first theoretical model concerning the structure of a contact binary in 1968, and in principle the light curve of the contact binary can be simulated with the model. The model of large-scale energy transfer from the primary to the secondary, roughly equalizing surface temperatures over the entire systems, has solved several outstanding problems. However, the Lucy's model can not produce the W-subtype light curves, nor can this theoretical model explain the observed O'Connell effect. Although the other models were suggested by Biermann & Thomas (1972), Whelan (1972), Shu et al. (1976), Lucy (1976), Flannery (1976), and Robertson & Eggleton (1977), they fail to provide satisfactory agreement with the observations (Kähler 1989). The mass and energy transfer between the two components of the systems may still be an unsolved problem. The problem of energy transfer involves not only the drive mechanism but the evolution effect of the systems. Numerous studies of these problems have been carried out by many investigators (Mochnacki 1981, Smith 1984, Kaluzny 1985, Hilditch et al. 1988).

In the present article, we adopted some simple experimental approximations to analyze energy transfer and the evolution of contact binary systems. The result obtained was applied to three typical W UMa-type contact binaries selected as examples. Finally, some conclusions and discussions are given.

2. The energy transfer rate and the evolution degree

Unless otherwise specified, the symbols used in this present article have the same meanings as those in common use. As far as the parameters of the components of a binary are concerned, the corresponding solar parameters are used as the units. The subscripts 1 and 2 respectively stand for the primary component and secondary component, the parameters with the subscripts being the asterisks, express the relevant parameters of the main

Send offprint requests to: Qingyao Liu sequence stars with a zero age and those with no subscript are suitable for the two components.

The definition is given as follow:

$$U = \frac{\Delta L}{L_1^o} = \frac{L_2^o - L_2}{L_1^o}, \tag{1}$$

where L_1^o and L_2^o are the observed luminosities of the primary and secondary, respectively, Δ L the luminosity which comes from the primary and is radiated by the secondary and U the rate of energy transfer of which the physical significance is the ratio of the energy transferred from the primary to the secondary per unit time to the energy radiated by the primary per unit time. Innate luminosities, L_1 and L_2, can be obtained from Eq. (1):

$$L_2 = L_2^o - L_1^o U = L_1^o \left(\frac{L_2^o}{L_1^o} - U \right) \tag{2}$$

and

$$L_1 = L_1^o (1 + U), \tag{3}$$

where the assumption of thermal equilibrium is adopted. Although in general it is not possible for a contact binary to simultaneously maintain hydrostatic and thermal equilibrium, but the approximation in the Eqs. (1) and (3) may not be far off (Hazlehurst 1993). From Eqs. (2) and (3), one may obtain

$$\frac{L_2}{L_1} = \frac{\frac{L_2^o}{L_1^o} - U}{1 + U} \tag{4}$$

and Eq. (4) can be rewritten as

$$U = \frac{\frac{L_2^o}{L_1^o} - \frac{L_2}{L_1}}{1 + \frac{L_2}{L_1}}. \tag{5}$$

From the black body radiation law

$$L^o = T^4 R^2 \tag{6}$$

and the mass-luminosity relation of the late-type main sequence stars

$$L = m_*^4, \tag{7}$$

where m_* is the mass of a component in main sequence, Eq. (5) may be written as

$$U = \frac{\left(\frac{T_2}{T_1}\right)^4 \left(\frac{R_2}{R_1}\right)^2 - q_*^4}{1 + q_*^4}, \tag{8}$$

where

$$q_* = \frac{m_{*2}}{m_{*1}}. \tag{9}$$

The change in the radius of a star is one of the principal effects of the evolution of the star. According to Webbink (1976), the evolution of a contact system is controlled by the primary and the radius of the secondary is controlled by its contact with the primary. Therefore, the evolution degree of the contact system may be described by means of the change in the radius of the primary. The definition of the radius of the primary of a binary may be given as follows:

$$R_1 = E R_{1*}, \tag{10}$$

where R_1 is the observed radius of the primary of the binary, E the evolutionary degree of the primary and R_{1*} the main sequence radius of the primary. According to Lacy (1977), the relation between the main sequence radius and the mass of the primary is

$$R_{1*} = C m_{1*}^{\beta}. \tag{11}$$

As for the late-type stars, C=0.955 and β= 0.917. From Eqs. (10) and (11) one may have

$$m_{1*} = \left(\frac{R_1}{EC} \right)^{1/\beta}. \tag{12}$$

According to the Third Law of Kepler, one may obtain

$$A^3 = 74.5 p^2 m_1 (1 + q), \tag{13}$$

where A is the distance between the two components with the solar radius as its unit and p the orbital period with a day as its unit. Based on Binnendijk (1970), one may know

$$r_1 + r_2 = 0.76, \tag{14}$$

where r is the relative radius of one of the two components and its definition is given as r=$\frac{R}{A}$, and therefore, from Eq. (14), one may have

$$A = \frac{R_1 (1 + q^{0.46})}{0.76}, \tag{15}$$

where the Kuiper approximation (1941),

$$\frac{R_2}{R_1} = q^{0.46}, \tag{16}$$

is adopted where q is the observed mass ratio

$$q = \frac{m_2}{m_1}. \tag{17}$$

Assuming that the system has the conservation of the total mass in the course of the evolution of a binary system, then one may obtain

$$m_1 (1 + q) = m_{*1} (1 + q_*). \tag{18}$$

Substituting Eqs. (18), (12) and (15) into Eq. (13), one may have

$$q_* = \frac{E^{1.09} R_1^{1.91} (1 + q^{0.46})^3}{34.38 p^2} - 1 \tag{19}$$

and substituting Eqs. (16) and (19) into Eq. (8), one may have

$$U = \frac{\left(\frac{T_2}{T_1}\right)^4 q^{0.92} - \left(\frac{E^{1.09} R_1^{1.91} (1+q^{0.46})^3}{34.38 p^2} - 1 \right)^4}{1 + \left(\frac{E^{1.09} R_1^{1.91} (1+q^{0.46})^3}{34.38 p^2} - 1 \right)^4}. \tag{20}$$

The relation between the energy transfer rate and the evolutionary degree of the primary of a W UMa-type contact binary is described by Eq. (20).

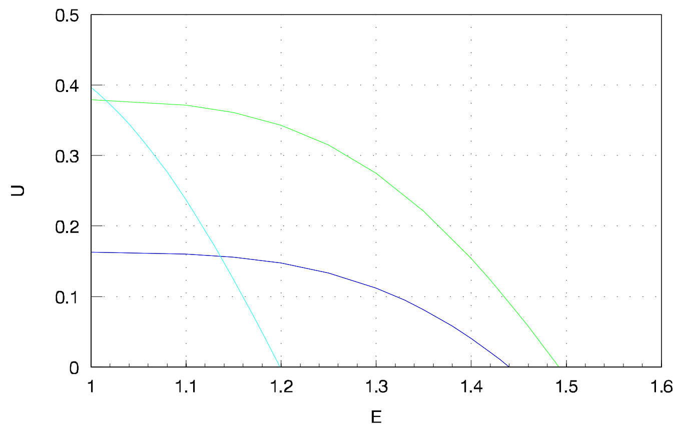

Fig. 1. The relation between the energy transfer rate U and the evolutionary degree E for the late-type contact binaries RZ Com, SS Ari and TV Mus.

3. Application to a W UMa-type contact binary

The W UMa-type contact binaries are divided into two subtypes, i.e. A-subtype and W-subtype. Studies show that all the A-subtype contact binaries have evolved and part of W-subtype contact binaries have already evolved while another part of the W-subtype contact binaries have not evolved yet (Hilditch et al., 1988). A typical binary from each of the three cases (i.e. the A-subtype binaries, the evolved W-subtype ones and the unevolved ones) is selected and the relation between its energy transfer and evolutionary degree is analysed. The parameters of the discussed W UMa-type systems are listed in Table 1.

1. TV Mus, W Ursa Majoris contact binary with the spectral type of F2, its apparent brightness at the light maximum $m_v = 10.8$ and its orbital period 0.4457 day. The research based on the photoelectric photometry carried out by Hilditch et al. (1989) shows that TV Mus is an A-subtype contact binary with the photometric mass ratio q=0.15. Hilditch et al. (1989) have also carried out the observations and research of the radial velocity of the system, giving the spectroscopic orbit parameters. The spectroscopic mass ratio is 0.12, slightly smaller than the photometric mass ratio. According to the study of Hilditch et al. (1988), TV Mus is an evolved contact binary nearby the main sequence. In order to study the relation between the energy transfer and component evolutionary degree of the system, the authors take the parameters of the binary to be the radius of the primary $R_1 = 1.66R_\odot$, q=0.15 and the effective surface temperature ratio $\frac{T_2}{T_1} = 1.018$ (Maceroni & Van't Veer 1996). From Eqs. (20) in the above-mentioned section one may obtain the relation between the energy transfer rate U and primary evolutionary degree E of TV Mus, as follows:

$$U = \frac{0.163 - (1.099E^{1.09} - 1)^4}{1 + (1.099E^{1.09} - 1)^4}. \quad (21)$$

The relation curve U=f(E) is given in Fig. 1, from which one can see that when the evolutionary degree of the primary changes from 1 to 1.2, i.e. at the beginning of the evolution, the energy transfer rate reaches its maximum, only with very slow decrease. The evolutionary degree of the primary is greater than 1.2, the energy transfer rate rapidly decreases with the further evolution of the primary and when the evolutionary degree of the primary reaches 1.44, the energy of the primary stops transferring to the secondary.

The mass exchange between the two components in the course of evolution of a binary is unavoidable and then certainly causes the changes in the parameters of the system, especially the change in the mass ratio. Fig. 1 shows that the evolution of the primary also influence the energy transfer, though the mass ratio is one of the principal factors having effects on the energy transfer of a contact binary. As far as TV Mus, the evolved A-subtype contact binary, is concerned, the estimate of its energy transfer rate is possible, owing to its mass ratio q=0.15. When the evolutionary degree of the primary is 1, i.e. a main sequence star with zero age, the energy transfer rate is 0.162. The mass of the primary of this binary is 1.32 $m_\odot$, corresponding to the main sequence radius 1.23 $R_\odot$, and its present radius is 1.66$R_\odot$, and therefore its evolutionary degree should be 1.35. From Eq. (20) it can be obtained that the energy transfer rate from the primary to the secondary of the binary system is 0.081. This value is only 0.5 of the value 0.162 of the energy transfer rate at the zero age main sequence.

2. SS Ari, a W-subtype evolved W UMa-type contact binary with the spectral type of F8, the visual magnitude during its maximum is $10.^m0$ and the orbital period is 0.4060 days. The photometric studies carried out by Lu (1991) show that it is a W-subclass contact binary, with the photometric mass ratio q=0.302. The mass ratio determined from the measurement of the radial velocity by Lu (1991) is 0.295. In present article, the authors adopt the mass ratio q=0.300 and the other adopted parameters are the radius of the primary $R_1 = 1.31R_\odot$ and the mass of the primary $m_1 = 1.16m_\odot$ and the effective surface temperature ratio of the two components $T_2/T_1 = 1.036$.

From Eq. (20), one may obtain the relation between the energy transfer rate U and primary evolution degree E of SS Ari

$$U = \frac{0.380 - (1.154E^{1.09} - 1)^4}{1 + (1.154E^{1.09} - 1)^4}. \quad (22)$$

The relation curve of U=f(E) is also given in Fig. 1, from which one can see that the energy transfer rate from the primary to the secondary decreases with the increase in the evolutionary degree of the primary. When the evolutionary degree of the primary is 1, i.e. a main sequence star with zero age, the energy transfer rate is 0.379. After the primary starts to evolve, the energy transfer rate decreases promptly and when the evolutionary degree reaches 1.49, the primary stops transferring the energy to the secondary.

The energy transfer rate can be also estimated based on the present parameters of SS Ari. The main sequence radius of its zero age is 1.094 $R_\odot$, corresponding to the mass of the primary 1.16 $M_\odot$. The present radius of the primary is 1.31 solar radius and according to the definition, the evolutionary degree of the primary is 1.20. From this it can be obtained that the energy transfer rate is 0.343.

Table 1. The parameters of discussed W UMa-type systems

star name	period	q	M_1	M_2	R_1	R_2	T_1	T_2
TV Mus	0.4457	0.15	1.32	0.20	1.66	0.75	5980	6088
SS Ari	0.4060	0.300	1.16	0.35	1.31	0.74	5745	5950
RZ Com	0.3385	0.436	1.108	0.45	1.122	0.71	5500	5564

3. RZ Com, an unevolved W-subtype W UMa-type contact binary with the spectral type of F7, its visual magnitude at the light maximum phase is 10.8 mag. and its orbital period is 0.3385 day. The photometric study of the binary carried out by Wilson and Devinney (1973) shows that the photometric mass ratio is 0.436. The spectroscopic mass ratio determined from the measurement of the radial velocity by Mclean et al. (1983) is 0.43. In the present article, the parameters adopted are the radius of the primary $R_1 = 1.122R_\odot$, the mass of the primary $m_1 = 1.108m_\odot$, the mass ratio q=0.436 and the effective surface temperature ratio of the two components $T_2/T_1 = 1.012$.

From Eq. (20), one may obtain the relation between the energy transfer rate U and primary evolution degree E of RZ Com

$$U = \frac{0.489 - (1.057E^{1.09} - 1)^4}{1 + (1.057E^{1.09} - 1)^4}. \quad (23)$$

The relation curve U=f(E) is also given in Fig. 1, from which one can see that the energy transfer rate from the primary to the secondary decreases with the increase in the evolutionary degree of the primary. When the evolutionary degree of the primary is 1, i.e. a main sequence star of zero age, the energy transfer rate is 0.397. After the primary starts to evolve, the energy transfer rate decreases rapidly and when the evolutionary degree reaches 1.198, the primary stops transferring energy to the secondary.

The energy transfer rate can also be estimated according to the present parameters of RZ Com. Its zero age main sequence radius is 1.120 $R_\odot$, corresponding to the mass of the primary 1.190 $M_\odot$. The present radius of the primary is 1.122 solar radius and based on the definition, the evolutionary degree of the primary is 1.002, i.e. it is basically a zero age main sequence star, from which one can obtain that the energy transfer rate is 0.394 which is very close to the value of 0.397 at the zero main sequence.

4. Conclusions and discussions

From the above-mentioned analyses one may draw conclusions: The energy transfer rate between the two components of a W UMa-type contact binary has relation not only to the mass ratio but also to the evolutionary degree of the primary. The maximum value of the energy transfer rate fundamentally depends on the mass ratio and therefore, the larger the mass ratio of the system, the greater the possible maximum value of the energy transfer rate. Under the condition of a given mass ratio, the energy transfer rate changes with the evolutionary degree of the primary and the greater the evolutionary degree, the smaller the value of the energy transfer. The energy transfer could stop when the evolutionary degree of the primary reaches a specifical value determined by a property of the systems.

When the energy transfer from the primary to the secondary stops, i.e.U=0, from the Eq. (20), one may have

$$E_s = \frac{25.63p^{1.83}(q^{0.23}\frac{T^2}{T^1} + 1)^{0.92}}{(R_1)^{1.75}(1 + q^{0.46})^{2.75}}, \quad (24)$$

where E_s indicates the evolutionary degree of the primary. Once the energy transfer from the primary to the secondary stops, we do not know what happens with the further evolution of the primary but it may be interesting to note that the energy transfer in the W UMa-type contact system is not always along one direction from the primary to the secondary.

Though the Binnendijk's classification was only based on light-curve properties, it has often been suggested that A-subtype systems could be considered as later evolutionary stages of W-subtypes (Wilson 1978, Mochnacki 1981). However, Maceroni & Van't Veer (1996) suggested that most A-subtype systems have no evolutionary link with the W-subtypes, as they have a total mass and/or total angular momentum which is too large, resulting from evolution of W-subtype towards smaller mass ratios. The hotter secondary of the W-subtype could be caused by the specific properties of W UMa-type contact systems. The effective surface temperature of the secondary of the W-subtype contact binary is higher than that of the primary, i.e. T_2/T_1 is greater than 1 and therefore, the following formula can be inferred from Eq. (20):

$$U\rangle \frac{q^{0.92} - \left(\frac{E^{1.09}R_1^{1.91}(1+q^{0.46})^3}{34.38p^2} - 1\right)^4}{1 + \left(\frac{E^{1.09}R_1^{1.91}(1+q^{0.46})^3}{34.38p^2} - 1\right)^4}. \quad (25)$$

A-subtype late-type contact binaries do not obey this relation.

According to Mochnacki (1981), the rate of energy transfer of a W UMa-type contact system may be approached with a simple equation,

$$U = \frac{\left(\frac{T_2}{T_1}\right)^4 q^{0.92} - q^4}{1 + q^4}, \quad (26)$$

where q is the mass ratio of the systems. The Mochnacki's approach reveals that the energy transfer rate is determined by the mass ratio of the systems. The present study indicates that the mass ratio is a main factor in determining the energy transfer rate, but the evolutionary degree of the primary may also affect the energy transfer. The Mochnacki's approach gives a maximum possible value of the energy transfer rate because of his assumption of zero-age main sequence components of the systems. If we let E=1, i.e. a unevolved system, Eq. (20) is in agreement with Mochnacki's result. As the evolutionary effect of the primary is considered in the present study, the energy transfer rate obtained from the present approach is less than that of the Mochnacki's one. In order to calculate the energy transfer rate, it is necessary to count the evolutionary effect of the primary of the system.

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