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LETTER

Letter to the Editor

Are passive protostellar disks stable to self-shadowing?

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Abstract. The uniqueness and stability of irradiated flaring passive protostellar disks is investigated in the context of a simplified set of equations for the vertical height H as a function of radius R. It is found that the well-known flaring disk solution with $H \propto R^{9/7}$ is not unique. Diverging solutions and asymptotically conical ($H \propto R$) solutions are also found. Moreover, using time-dependent linear perturbation analysis, it is found that the flaring disk solution may become unstable to self-shadowing. A local enhancement in the vertical height alters the functional form of irradiation grazing angle, and causes the 'sunny side' of the enhancement to grow and the 'shadow side' to collapse in a run-away fashion. This instability operates in regions of the disk in which the cooling time is much shorter than the vertical sound crossing time, which may occur in the outer regions of the passive irradiated disk if dust and gas are sufficiently strongly thermally coupled. Processes that may stabilize the disk, which include active accretion, irradiation from above (e.g. a scattering corona) and low disk optical depth, are likely to operate only at small or at large radius. The simple analysis of this Letter therefore suggests that the instability may alter the flaring disk structure at intermediate radii (between the actively accreting and fast rotating inner regions and the optically thin outer regions).

Key words: accretion, accretion disks – circumstellar matter – stars: formation, pre-main-sequence – infrared: stars

1. Introduction

It has become widely accepted that the infrared excess observed in the spectral energy distribution of many T-Tauri stars is caused by thermal emission from a circumstellar dusty disk (Adams & Shu, 1986; Adams, Lada & Shu, 1987). However, the spectra predicted by theoretical models of both accretion-powered disks (Lynden-Bell & Pringle, 1974) and flat reprocessing disks (Friedjung, 1985) have an infrared spectrum that goes as $\lambda F_\lambda \propto \lambda^{-1.33}$. This is steeper than the $\lambda F_\lambda \propto \lambda^{-0.5 \text{ to } -1.0}$ typically observed from T-Tauri stars (Rucinski, 1985; Rydgren & Zak, 1987). A natural way to explain the relatively warm dust at large radii (which is needed to increase the slope of the model spectrum), is to invoke a flaring disk geometry, in which $H/R \propto R^\gamma$, with $\gamma > 0$ (Kenyon & Hartmann, 1987). Because of the angle between the disk surface and the star, the disk intercepts considerable stellar flux even at large radii, and therefore acquires a shallower temperature profile than the $T \propto R^{-3/4}$ found from the accreting and/or flat disk models. The flaring index γ can be determined self-consistently by equating the intercepted flux with the emitted blackbody flux, under the assumption that the disk is vertically isothermal. These models yield $\gamma = 2/7$ (Chiang & Goldreich, 1997, henceforth CG97; D'Alessio et al. 1999, henceforth DCHLC99). Apart from producing the flatter SEDs, it was recognized that the optically thin surface layers of the disk will have much higher temperatures than the interior (Calvet et al.1991; Malbet & Bertout 1991; CG97). In addition to producing an extra component in the SED, this optically thin layer is also able to explain certain dust features in emission which would otherwise be expected in absorption. A more direct piece of evidence for the flaring nature of protostellar disks was provided by the HST images of HH30 (Burrows et al. 1996).

Despite the successes of the flaring disk model in explaining observations of T-Tauri stars, there are still some theoretical issues that remain to be addressed. In this paper we address two questions: are the flaring disk models stable against perturbations in vertical height and temperature, and are they unique solutions to which a time-dependent evolution of the passive disk settles down? The issue of stability has been addressed before by DCHLC99. They considered perturbations in the temperature of the disk, and found that they damp out as they propagate inward. However, their analysis is based on the assumption that the disk vertical height quickly responds to changes in the temperature, or in other words that the vertical sound crossing time t_{dyn} is much smaller than the cooling time t_{therm}. This is true for the inner regions of the disk, but breaks down at large radii. In this Letter we study the other extreme case: the case of $t_{\text{dyn}} \gg t_{\text{therm}}$, which may occur in the outer regions of the disk. We start from the same equations as DCHLC99 and CG97, but replace the static equation for the vertical disk height with a simplified one-zone dynamic model. We derive the pertubation equations and dispersion relations from them, and find that under these circumstances the disk becomes unstable.

We will start the analysis in Section 2 with a description of the family of solutions of a static irradiated optically thick disk. The flared disk model mentioned above is just one of these solutions. The other solutions either diverge at, or are conical ($H \propto R$) beyond some radius. In Section 3 we will discuss the stability under the two extreme conditions ($t_{\rm therm} \ll t_{\rm dyn}$ and $t_{\rm therm} \gg t_{\rm dyn}$). In Section 4 we discuss the consequences of this instability, and when it is expected to show up.

2. Static equations for an irradiated disk

A disk with flaring angle α intercepts a flux of $F_{\rm irr} = \alpha(R_*/R)^2\sigma_{\rm R}T_*^4$ ergcm^{-2}s^{-1}. The flaring angle α is defined as the angle between the grazing incident stellar light and the surface of the flaring disk, and is assumed to be small. If we assume the disk to be vertically isothermal, the emitted flux is $F_{\rm emit} = \sigma_{\rm R}T_{\rm e}^4$, where $T_{\rm e}$ is the temperature of the disk. Equating $F_{\rm irr}$ with $F_{\rm emit}$ yields

$$T_{\rm e} = \alpha^{1/4}\left(\frac{R_*}{R}\right)^{1/2} T_* \,. \tag{1}$$

When $R \gg R_*$ the flaring angle is given by

$$\alpha = R\frac{d}{dR}\left(\frac{H_{\rm s}}{R}\right) , \tag{2}$$

where $H_{\rm s}$ is the height of the surface of the disk above the midplane. For a static solution the vertical pressure balance equation for an isothermal disk yields a Gaussian density profile of the form

$$\rho = \frac{\Sigma}{H_{\rm p}\sqrt{2\pi}}\exp\left(-\frac{z^2}{2H_{\rm p}^2}\right) , \tag{3}$$

where Σ is the surface density, and $H_{\rm p}$ is the pressure scale height

$$H_{\rm p} = \sqrt{\frac{kT_{\rm e}R^3}{\mu_{\rm g}m_{\rm u}GM_*}} , \tag{4}$$

where $\mu_{\rm g}$ is the mean molecular weight and $m_{\rm u}$ is the unit atomic mass. The surface height $H_{\rm s}$ and the pressure scale height $H_{\rm p}$ are related by a number of the order of a few, dependent on the opacity. We assume this to be approximately a constant $\chi \equiv H_{\rm s}/H_{\rm p}$. CG97 take $\chi \simeq 4$ for their analytic calculation of the disk structure. If we eliminate $T_{\rm e}$ from the above set of equations we arrive at

$$\frac{1}{\chi}\left(\frac{\mu_{\rm g}m_{\rm u}GM_*}{kT_*R_*}\right)^4\left(\frac{R_*}{R}\right)^2\left(\frac{H_{\rm p}}{R}\right)^8 = R\frac{d}{dR}\left(\frac{H_{\rm p}}{R}\right) . \tag{5}$$

Defining the dimensionless quantities $r \equiv R/R_*$ and $h \equiv H_{\rm p}/R$, one arrives at

$$r\frac{dh(r)}{dr} = \frac{C}{r^2}h(r)^8 , \tag{6}$$

where C is

$$C = \frac{1}{\chi}\left(\frac{\mu_{\rm g}m_{\rm u}GM_*}{kT_*R_*}\right)^4 . \tag{7}$$

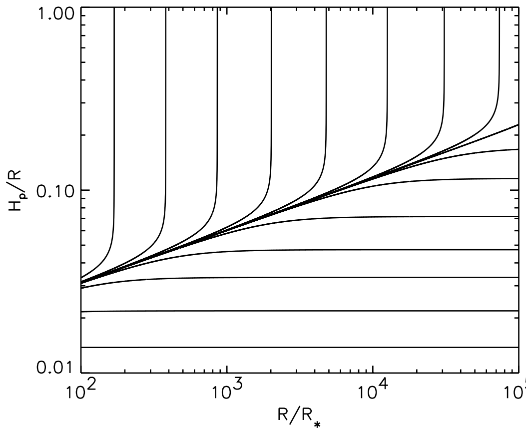

Fig. 1. The family of static irradiated passive disk solutions according to Eq.(6) for $C = 1\times10^{14}$.

This ordinary differential equation (ODE) has a 1-parameter family of solutions:

$$h(r) = \left(\frac{7C}{2}\frac{1}{r^2} + K\right)^{-1/7} , \tag{8}$$

where K is an arbitrary real constant. For $K < 0$ the solutions diverge at $r = \sqrt{-7C/2K}$, for $K > 0$ the solutions become constant beyond $r = \sqrt{7C/2K}$, and for $K = 0$ one obtains a powerlaw solution. These three classes of solutions are shown in Fig.(1).

The power law solution is given by

$$h(r) = \left(\frac{2}{7C}\right)^{1/7} r^{2/7} , \tag{9}$$

which is the solution given by CG97. All the other solutions asymptotically tend to this power law solution, Eq.(9), as $r \downarrow 0$. But as r goes to infinity the solutions deviate from this power law at some radius, and either diverge to $h \to \infty$ at a particular radius, or asymptotically approach a conical geometry $h(r) \to$ const, i.e. $H_{\rm s} \propto R$.

The direction in which the solutions deviate from each other is interesting. Since the inner regions of the disk (small radii) cast their shadow over the outer regions (large radii), the "causality" of the system is pointing outwards. A change in the structure of the disk at small radii has its repercussions on the disk at large radii, but not vice versa. In principle this means that the (numerical) integration of the ODE (Eq.5) must be done from inside-to-outside (i.e. in causal direction). However, in this direction the solutions tend to strongly deviate from each other. A slight change in integration constant at the start of the integration causes a very large difference at large radii. For stable integration of the ODE (Eq.5) one should place the boundary condition at the outside and integrate inwards. Though mathematically correct, it is physically meaningless, and may hint to an intrinsic instability of the time-dependent equations.

3. Time-dependent equations for irradiated disk

The stability of the flaring (power law) solution can be studied analytically using time-dependent linear perturbation analysis. We study two regimes, one of which has been studied before by DCHLC99.

3.1. Temperature perturbations at hydrostatic equilibrium

In the analysis of DCHLC99 the cooling time scale of the disk was assumed to be much larger than the orbital time scale, which is true for the inner regions of the disk. We may then assume that the disk will always be in hydrostatic equilibrium, and we follow perturbations of the temperature. Following DCHLC99 we write

$$\Gamma \frac{dT_{\mathrm{e}}}{dt} = F_{\mathrm{irr}} - F_{\mathrm{emit}}\,, \tag{10}$$

where Γ is the thermal heat capacity. After substituting Eqs.(2,4,7), the definitions of F_{irr} and F_{emit}, and defining the perturbation $T_{\mathrm{e}}(r,t) = T^0_{\mathrm{e}}(r)(1+\tilde{\psi}(r,t)r^2)$, where $T^0_{\mathrm{e}}(r)$ is the temperature belonging to the static flaring disk solution Eq.(9), one finds the following perturbation equation (see DCHLC99 for details):

$$\frac{\partial\tilde{\psi}}{\partial\bar{t}} = \frac{7}{4}\left(\frac{T^0_{\mathrm{e}}}{T_*}\right)^3 r\frac{\partial\tilde{\psi}}{\partial r}\,, \tag{11}$$

where $\bar{t} \equiv (\sigma_{\mathrm{R}} T_*^3/\Gamma)t$. Eq.(11) is an advection equation. Apparently a perturbation moves inwards, and its amplitude damps, since $\tilde{\psi}$ is conserved in amplitude along the inward moving characteristic, and therefore the comoving amplitude of $\tilde{\psi}r^2$ becomes smaller (DCHLC99).

3.2. Hydrodynamic perturbations at thermal equilibrium

When the cooling time of the disk is much shorter than the orbital time, one can assume that the disk is always in thermal equilibrium, and the disk height will react to changes in the irradiation flux in a hydrodynamic way, instead of being in vertical hydrostatic balance. In principle one should solve the time-dependent vertical hydrodynamics of the disk (1-D hydrodynamics). But for the present analysis we simplify the picture, and *assume* that the density profile is always given by Eq.(3), but with H_{p} a function of time. This assumption can be justified, since in the early (linear) stages of a possible instability one has $dv_z/dt \propto z$, which preserves the Gaussian shape of the density profile. By neglecting quadratic terms in the vertical velocity, one can then derive

$$\frac{d^2H_{\mathrm{p}}}{dt^2} = \frac{kT_{\mathrm{e}}}{\mu_{\mathrm{g}} m_{\mathrm{u}}}\frac{1}{H_{\mathrm{p}}} - \frac{GM_*}{R^3}H_{\mathrm{p}}\,. \tag{12}$$

When we define the dimensionless time $\tilde{t} \equiv \sqrt{GM_*/R_*^3}t$, a dimensionless form of the above equation is

$$\frac{\partial^2 h(r,\tilde{t})}{\partial\tilde{t}^2} = \frac{C^{-1/4}}{r^{5/2}h(r,\tilde{t})}\left(r\frac{\partial h(r,\tilde{t})}{\partial r}\right)^{1/4} - \frac{h(r,\tilde{t})}{r^3}\,, \tag{13}$$

with C as defined in Eq.(7)

We now analyze the time-dependent behavior of linear perturbations of the power law solution (Eq.9). We take

$$h(r,\tilde{t}) = \left(\frac{2}{7C}\right)^{1/7} r^{2/7}\left[1+\psi(r,\tilde{t})\right]. \tag{14}$$

Substituting this in Eq.(13) yields (exactly):

$$\frac{\partial^2\psi}{\partial\tilde{t}^2} = \frac{1}{r^3}\left\{\frac{1}{(1+\psi)}\left(1+\psi+\frac{7}{2}r\frac{\partial\psi}{\partial r}\right)^{1/4} - (1+\psi)\right\}. \tag{15}$$

For $\psi \ll 1$ this becomes

$$\frac{\partial^2\psi}{\partial\tilde{t}^2} = -\frac{7}{8r^3}\left\{2\psi - r\frac{\partial\psi}{\partial r}\right\}. \tag{16}$$

This equation has solutions of the form

$$\psi = A\, r^2 e^{\sigma\tilde{t}}\sin\left(\omega\tilde{t} - kr^3 + \phi_0\right)\,, \tag{17}$$

with the following dispersion relations:

$$\omega^2 = \sigma^2\,, \tag{18}$$

$$k = -(16/21)\,\omega\sigma\,. \tag{19}$$

For inwards propagating waves (i.e. $k/\omega < 0$) one has $\sigma > 0$, and hence an exponentially growing mode[1]. The instability growth rate is inversely proportional to the wavelength of the perturbation, which indicates that the instability will be dominated by the shortest wavelengths. It should be noted that the validity of the equations breaks down at the scale of the disk vertical height. For modes with larger k one should take radial radiative diffusion into account. This is, however, beyond the scope of the present set of equations.

4. Conditions for instability

The instability only sets in under certain physical conditions. The most obvious conditions for this instability to occur are:

1. Radiative cooling/heating time scale short compared to vertical sound crossing time, i.e. $t_{\mathrm{dyn}} \gg t_{\mathrm{therm}}$.
2. Thermal dust-gas coupling is strong enough to keep the *gas* cooling time shorter than the vertical sound crossing time.
3. The equatorial temperature is dominated by irradiation, i.e. viscous dissipation by accretion is small in comparison.
4. The disk is optically thick to starlight in radial (grazing) direction.
5. The irradiation occurs through flaring; i.e. the reflected stellar flux from a scattering corona is weak in comparison to the direct interception of stellar light through flaring.

We have not investigated whether disk models that obey all the above points, but are vertically optically thin in the near/mid infrared, are also unstable.

[1] After submission the author became aware of work by E. Chiang (to be published) which reports on similar results.

Condition 1 is fulfilled for $R \gg R_{\rm inst}$, where $R_{\rm inst}$ can be derived by equating the vertical sound crossing time with the thermal time: $t_{\rm dyn} = t_{\rm therm}$. We have

$$t_{\rm dyn} \simeq \sqrt{\frac{R^3}{GM_*}}\,, \tag{20}$$

$$t_{\rm therm} \simeq \frac{k\Sigma}{2m_{\rm u}\sigma_{\rm R}T_{\rm e}^3}\,. \tag{21}$$

The surface density $\Sigma(R)$ (in units of $\mathrm{g/cm^2}$) is a free function of the model, as long as the disk remains optically thick to the incident stellar radiation. Equating $t_{\rm dyn} = t_{\rm therm}$ yields the radius $R_{\rm inst}$ beyond which the instability may set in:

$$\frac{R_{\rm inst}}{\rm AU} = 3\times 10^{-5}\left(\frac{M_*}{M_\odot}\right)^{13/3}\left(\frac{L_*}{L_\odot}\right)^{-4}\left(\frac{H_{\rm s}}{H_{\rm p}}\right)^{-4}\Sigma^{14/3}, \tag{22}$$

where L_* is the luminosity of the star.

The instability found here is an instability of the highly simplified analytic flaring disk models. Including more realistic physics could perhaps stabilize the disk. Some elements of realistic physics that should be considered in future work before one can claim that the instability is real are:

1. 2-D axisymmetric hydrodynamics.
2. Radial radiative transfer of stellar radiation instead of the inclination angle formula.
3. 1-D vertical radiative transfer, because the top layers have a higher temperature (see CG97).
4. Radial diffusion of radiation, or even fully 2-D radiative transfer, because radial exchange of energy might counteract the shadowing effect which causes the instability.

5. Discussion

This Letter presents an analysis of the structure and stability of irradiated non-accreting protostellar disks based on a highly simplified set of equations. In the context of these equations it is concluded that the flaring non-accreting reprocessing disk model is unstable when the vertical sound crossing time is larger than the heating/cooling time scale. It should be noted, however, that detailed multidimensional numerical modeling is required to verify whether this instability is real or merely an artifact of the over-simplified equations.

The instability has a simple physical interpretation. Consider a flaring disk solution, and perturb it with an enlargement of the scale height at some point. The ‘sunny side’ of the hill will get overheated and expand vertically in a run-away fashion. The ‘shadow side’ will receive insufficient radiation and collapse. As the perturbations grow into the non-linear regime they start to cast a ‘real’ shadow ($\alpha < 0$) over the disk at larger radii, thus completely depriving this part of the disk of irradiation. At this point the validity of the simplified equations breaks down, and what happens in this non-linear regime is unclear. Disk self irradiation (e.g. Bell 1999) might become important at this stage.

The reason why we find an instability, while DCHLC99 found only damping modes is because they considered only perturbations in the disk where the thermal time scale exceeds the vertical sound crossing time scale. While for actively accreting disks this condition is always satisfied, for passive disks the reverse may be true. This was recognized by DCHLC99, but an analysis of modes on a dynamic time scale in the outer regions of the protostellar disks was not carried out, hence their different conclusion.

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