

Centrifugal buoyancy as a mechanism for neutron star glitches

B. Carter¹, D. Langlois¹, and D.M. Sedrakian^{1,2}

¹ Département d’Astrophysique Relativiste et de Cosmologie, C.N.R.S., Observatoire de Paris, 92195 Meudon, France

² Erevan State University, Erevan, Republic of Armenia

Received 23 April 1999 / Accepted 20 March 2000

Abstract. The frequent glitches (sudden increases of the apparent angular velocity) observed in certain pulsars are generally believed to be attributable to discontinuous angular momentum transfer to the outer neutron star crust from a differentially rotating superfluid layer, but the precise mechanism is not quite elucidated. Most explanations invoke vortex pinning as the essential mechanism responsible for the build up of strain in the crust that is relaxed, either by fracture of the solid structure or by discontinuous unpinning, during the glitch. It is shown here that there is another mechanism that could give rise to strain, and subsequent fracture, of the solid crust, even if vortex pinning is ineffective: this is the effective force arising from the deficit of centrifugal buoyancy that will be present whenever there is differential rotation. This centrifugal buoyancy deficit force will be comparable in order of magnitude, but opposite in direction, to the force that would arise from vortex pinning if it were effective.

Key words: stars: neutron – stars: pulsars: general – stars: rotation – hydrodynamics

1. Introduction

The ultimate motivation for this article is the problem of explaining one of the salient observational features of isolated (non-binary) pulsars, which is that comparatively long periods of continuous “spin down” of the observed frequency Ω are occasionally interrupted by small “glitches”. Such a glitch consists of a sudden small increase, $\delta\Omega$ say, that partially cancels the continuous negative variation $\Delta\Omega$ that has been accumulated since the preceding glitch.

Since very soon after its discovery in 1968, it has been generally agreed that the pulsar phenomenon is attributable to a strong magnetic field anchored in the outer crust layers of a central neutron star. The observed frequency Ω is to be interpreted as the rotation frequency of the outer crust layer, whose continuous spin down is evidently due to the continuous decrease of the angular momentum J due to radiation from the external magnetosphere. After thirty years of work, two basic problems remain.

The first is to account for the spectrum (from radio to X-ray and beyond) and the detailed pulse structure of the radiation, which are presumed to depend on the still very poorly understood workings of the magnetosphere.

The second problem – the one with which the present article is concerned – is to account for the frequency “glitches”. It is generally recognised that the glitches must be explained in terms of what goes on in the interior of the neutron star, and it is also generally believed that the glitch phenomenon is essentially dependent on the property of solidity that is predicted (on the basis of simple, generally accepted theoretical considerations) to characterise the crust of the neutron star after it has fallen below the relevant extremely high melting temperature, which occurs very soon after its formation.

While there is agreement in presuming that discontinuous changes in the solid crustal structure must be responsible for the observed frequency glitches, what remains to be established is the qualitative nature of the process that is primarily responsible for such “crustquakes”.

The purpose of this article is to draw attention to the potential importance, as a mechanism for this process, of the stresses induced in the crust just by the effective force arising from the deficit of centrifugal buoyancy that will be present whenever there is differential rotation.

It is to be noticed that centrifugal buoyancy is a phenomenon that has been previously considered in the context of neutron stars, at least with reference to one of its possible consequences, namely Ekman pumping. This is a mechanism that can considerably shorten the timescale needed for the redistribution of angular momentum (in comparison with viscous diffusion characterized by the timescale given by $\tau_{\text{visc}} \approx R_*^2/\nu_*$ where ν_* is the typical kinetic viscosity coefficient and R_* is the relevant stellar radial length scale) and thus the damping of differential rotation in cases for which (as will be the case in a typical pulsar) the star is rotating fast enough for the corresponding rotation timescale $\tau_{\text{rot}} = 2\pi/\Omega$ to be short compared with τ_{visc} . In such circumstances, “Ekman pumping” will supplement the very slow diffusive transport by more rapid convective transport propelled by centrifugal buoyancy forces. The ensuing “Ekman timescale” τ_{E} for the effective damping of differential rotation in such cases will be given roughly by the geometric mean of the pure diffusion and rotation timescales, i.e. $\tau_{\text{E}} \approx \sqrt{\tau_{\text{rot}}\tau_{\text{visc}}}$.

Send offprint requests to: D. Langlois (langlois@obspm.fr)

While it has been suggested that either Ekman pumping or magnetic coupling may be able bring the *core plasma* into corotation with the crust (Easson 1979), it has since been pointed out that Ekman pumping will be inhibited by density stratification (see e.g. Abney & Epstein 1995) and will therefore be inefficient. It is plausible that it also applies to the uncharged *crust neutron superfluid* that is believed (see e.g. Sauls 1988) to permeate the lower layers of the crust in the density range from 10^{11} to about 10^{14} gm/cm³. This means that the convectively accelerated Ekman timescale, $\tau_E \approx R_* \sqrt{2\pi/\nu_* \Omega}$, is too long to prevent the development of significant differential rotation. The negligibility, in such cases, of Ekman pumping is attributable to the effective negligibility of viscosity, but should not be construed as implying the negligibility of centrifugal buoyancy forces. In previous discussions of such scenarios – and in particular of the simplified strictly stationary limit in which the effective viscosity is neglected, so that no possibility of Ekman pumping can arise at all – the role of centrifugal buoyancy forces has been rather generally overlooked. The upshot of the present investigation of stationary differentially rotating configurations is to show that in such cases the general neglect of the centrifugal buoyancy effect is quite unjustified, and that on the contrary this effect is potentially capable by itself of providing the dominant contribution to the crust stresses that are ultimately released in “glitches”.

2. Glitches driven by the spheroidality mechanism

In the first years after the problem of accounting for neutron star “crustquakes” was posed, attention was concentrated on what is describable as the “spheroidality mechanism” (Ruderman 1969). This mechanism depends on the supposition that the solidity forces will not be strong enough to allow the stellar equilibrium configuration to differ very much from a perfectly fluid equilibrium state, which would be spherical in the absence of rotation, but which will actually have the form of an oblate spheroid with ellipticity proportional to Ω^2 . The moment of inertia, defined as the ratio $I = J/\Omega$, will be given for a slowly rotating fluid by an expression of the form

$$I = I_0(1 + \Omega^2/\Omega_*^2), \quad (1)$$

where I_0 is its spherical limit value and Ω_* is a constant characterising the rather high angular frequency needed for relative deviations from spherical symmetry to be of order unity. A more accurate formula involving higher order corrections would be needed for a star with angular velocity near the critical value $\Omega^2 \simeq \Omega_*^2$, but the cases in which glitches have been observed so far are all characterised by

$$\Omega^2 \ll \Omega_*^2. \quad (2)$$

For a perfectly fluid star model, a continuous angular momentum variation $\Delta J < 0$ would bring about a corresponding momentum of inertia variation ΔI that would be given by

$$\frac{\Delta I}{I} \approx 2 \frac{\Omega^2}{\Omega_*^2} \frac{\Delta \Omega}{\Omega} < 0. \quad (3)$$

Due to the solidity of the crust, which tends to preserve the more highly elliptic initial configuration, the actual change in the moment of inertia will fall short of what is predicted by this formula, but at some stage the strain will build up to the point at which the solid structure will break down (see Fig. 1). It is predicted that there will then be a “crustquake” in which the solid structure suddenly changes towards what the perfect fluid structure would have been, thereby changing the moment of inertia by an amount

$$\delta I = \varepsilon \Delta I, \quad (4)$$

where ε is an efficiency factor that should presumably lie somewhere in the range

$$0 < \varepsilon \lesssim 1. \quad (5)$$

Since the amount of angular momentum loss during the very short duration of the glitch will be negligible, the corresponding discontinuous angular velocity change will be given by

$$\frac{\delta \Omega}{\Omega} = -\frac{\delta I}{I}. \quad (6)$$

Its value will therefore be expressible in terms of the order of unity efficiency factor ε by

$$\delta \Omega = -2\varepsilon \frac{\Omega^2}{\Omega_*^2} \Delta \Omega, \quad (7)$$

in which it is to be recalled that $\Delta \Omega$ denotes the continuous (negative) change in angular velocity since the preceding glitch. This mechanism must presumably operate, and may account for some observed glitches, but it soon became clear (Baym & Pines 1971) that even if this mechanism is maximally efficient, with

$$\varepsilon \simeq 1, \quad (8)$$

the magnitude predicted by (7) is much too low for such a mechanism to be able to account for the comparatively large glitches that are frequently observed in cases such as that of the Vela pulsar.

3. Glitches driven by differential rotation

Soon after the empirical discovery of glitches too large to be accounted for by the “spheroidality” mechanism, it came to be recognised by theorists (Anderson & Itoh 1975) that a plausible explanation involved the superfluid property of the deeper layers of sufficiently cool neutron stars. This property makes it possible to conceive that an interior neutron superfluid layer with moment of inertia, I_n say, can rotate with an angular velocity, Ω_n say, that may differ from the externally observable angular velocity Ω that characterises the part of the star that corotates with the crust, with its own moment of inertia

$$I_c = I - I_n. \quad (9)$$

In such a case it can be supposed that when an external braking mechanism causes the corotating crust component to undergo an angular velocity change $\Delta \Omega$, the angular velocity Ω_n of the

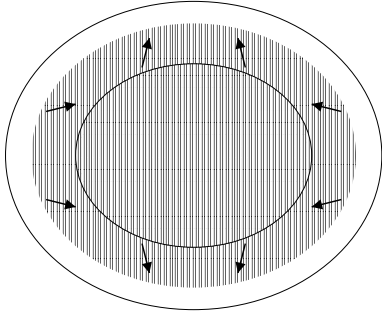


Fig. 1. Qualitative sketch indicating direction of force expected to act on (magnetically slowed down) crust due to *spheroidality mechanism*, in absence of differential rotation. Solid lines indicate outer and inner boundaries of crust. Vertical shading indicates the alignment of the vortices in the region occupied by neutron superfluid, which is not confined to the core but interpenetrates the greater part of the solid crust as well. Note that the vortices represented here are not physically relevant in this particular mechanism.

independently rotating neutron superfluid layer may in the short run be unaffected, with negligible variation expressible by

$$\Delta\Omega_n = 0, \quad (10)$$

but that, when the ensuing angular velocity difference between the corotating crust and the neutron superfluid layer exceeds some critical value there will be a discontinuous adjustment whereby this angular velocity difference is reduced by some process involving a transfer of angular momentum between the two components. Such a process will evidently entail a negative adjustment $\delta\Omega_n$ of the angular velocity of the neutron superfluid layer and an accompanying positive adjustment $\delta\Omega$ of the (observable) angular velocity of the corotating crust component, whereby the latter increases its angular momentum by an amount $I_c\delta\Omega$ that is equal to the amount $-I_n\delta\Omega_n$ that is lost by the neutron superfluid component, so that the total angular momentum change during the discontinuous ‘glitch’ process is zero, i.e.

$$I_c\delta\Omega + I_n\delta\Omega_n = 0. \quad (11)$$

If this adjustment process were a hundred per cent efficient, the net variation $\Delta\Omega + \delta\Omega$ of the corotating crust angular velocity would be exactly matched by the net neutron superfluid angular velocity variation, which by (10) will be simply given by $\delta\Omega_n$, so that one would have

$$\delta\Omega_n = \varepsilon(\Delta\Omega + \delta\Omega), \quad (12)$$

with $\varepsilon \simeq 1$. In practice one would expect that there would typically be an incomplete adjustment, still expressible by a relation of the form (12), but with an efficiency factor ε having some lower value in the range (5). By substituting (11) in (12) it can be seen that the observable glitch magnitude will be given by

$$\delta\Omega = \frac{-\varepsilon I_n \Delta\Omega}{I_c + \varepsilon I_n}, \quad (13)$$

and hence, by (9), that for an efficiency factor ε with any value in the range (5) the glitch magnitude will satisfy the inequality

$$\delta\Omega \gtrsim -\varepsilon \frac{I_n}{I} \Delta\Omega. \quad (14)$$

By comparing (14) with (7), it can be seen that, for a given assumed value of the efficiency factor ε , the differential rotation adjustment mechanism characterised by (14) can give rise to a much larger glitch magnitude $\delta\Omega$ than is possible by the spheroidality adjustment mechanism characterised by (7), because the factor I_n/I in (14) can be of order unity, whereas the corresponding factor in (7), namely $2\Omega^2/\Omega_*^2$ is very small compared to unity in even the most rapidly rotating pulsars. Thus, unlike the spheroidality mechanism, mechanisms involving angular momentum transfer between differentially rotating components can plausibly be considered as candidates for explaining the frequent large glitches observed in the Vela pulsar.

4. Glitch mechanisms due to the vortices

In the context of a glitch due to differential rotation, the question that arises is what physical mechanism can increase the effective coupling between the superfluid component and the crust, in order to generate a transfer of angular momentum.

The explanations that exist in the literature are based on an important property of a superfluid neutron star, which we have not yet mentioned in this article: the existence of an array of vortex lines in the rotating neutron superfluid component, each vortex carrying a quantum of vorticity $\kappa = h/(2m_n)$ (where m_n is the neutron mass). The vortex number density (per unit area) n_v is directly related to the superfluid angular velocity Ω_n by the expression

$$n_v = \frac{2\Omega_n}{\kappa} \quad (15)$$

(for uniform rotation).

The kind of angular momentum transfer mechanism that has for many years been generally considered to offer the most likely explanation for large glitches is based on the supposition that these vortices will be ‘pinned’ in the sense of being effectively anchored in the lower crust, either by pinning in the strict sense (Anderson & Itoh 1975) or by a sufficiently strong friction force (Alpar et al. 1984). The braking of the crust will thus have the effect of slowing down the vortices relative to the underlying superfluid, thereby giving rise to a Magnus force tending to move them out through the superfluid layer and thus slow it down as well. However this tendency to move out will be thwarted by the same anchoring effect that gave rise to it in the first place. This conflict will cause the pinning forces to build up to a critical point at which there will be a breakdown bringing about a discontinuous readjustment of the kind described by the analysis of the preceding section, and in particular by the formula (14).

The breakdown can occur in two different manners:

(a) There can be a sudden unpinning of many vortices, due to the breaking of the pinning bonds (Anderson & Itoh 1975; Link & Epstein 1991).

(b) Another possibility is that the crust lattice breaks before vortex lines can unpin from it, as suggested by Anderson & Itoh (1975) and studied in detail by Ruderman (1976).

Finally, we would like to mention another interesting glitch mechanism due to Link & Epstein (1996), which may be relevant for the present work:

(c) their thermally driven glitch mechanism is based on the so-called vortex creep model (Alpar et al. 1984), in which the coupling between the vortices and the crust is strongly temperature dependent. A sudden local increase of the inner crust temperature, such as may be due to a crustquake, can then be shown to induce a glitch.

It must be emphasized that all these three mechanisms, even if corresponding to some breaking of the crust as in the scenarios (b) and (c), are very different from the mechanism of Sect. 2, in the sense that they all are in the context of a two-component star, with the neutron superfluid rotating faster than the crust and thus acting as a reservoir of angular momentum. In the following sections, we will consider a mechanism which is not based on the presence of vortices, but still in the context of differential rotation.

Finally, let us mention the question of how big is I_n compared with I , in other words how much of the neutron fluid is effectively free to rotate independently of the rest? In the unpinned part the vortices can move out freely so as to establish corotation, so I_n may be relatively small (Jones 1991), representing the moment of inertia just of the small fraction of the neutron fluid that interpenetrates the deeper layers of the solid crust where pinning is expected to be most effective. However effective pinning may not be confined to the solid crust: it may also be achieved by forces exerted by quantised magnetic field lines (resulting from superfluidity of the protons) in the layers below the crust, in which case the relevant value of I_n might be much larger (Sedrakian & Sedrakian 1995). Another question (which applies also to the less important spheroidality mechanism discussed above) is that of the absolute values of the discontinuous changes. The foregoing reasoning is concerned just with the ratio of $\delta\Omega$ to $-\Delta\Omega$ but does not tackle the harder problem of their absolute values.

5. Potential importance of the centrifugal buoyancy mechanism

So far we have only been summarising what has long been well known to workers in this field. We now come to what seems to us to be an important point that has been overlooked, which is that independently of vortex pinning there is another, comparably powerful mechanism, that can also cause discontinuous angular momentum transfer to a solid crust from an independently rotating superfluid layer. This mechanism does not depend on superfluidity in the strict sense but merely requires perfect fluidity in the sense that the effective viscosity should be low enough for the slowdown of the neutron fluid to lag behind the slowdown (due to its coupling with the radiating magnetosphere) of the solid outer layers. The point is that if the outer layers were also effectively fluid, there would be a convective read-

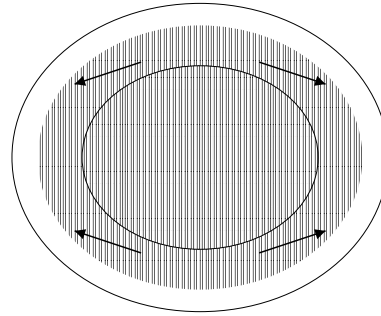


Fig. 2. Qualitative sketch indicating direction of force expected to act on (magnetically slowed down) down on crust due to *vortex pinning mechanism*, if it is effective, when the (interpenetrating) neutron superfluid retains a higher rotation rate.

justment, in which annular rings of fluid would change their relative positions, each retaining its separate angular momentum, in such a way that those with less angular momentum per unit mass, and thus with less “centrifugal buoyancy” would move towards the axis while those with more would move out so as to establish a state of equilibrium in which, provided the pressure depends only on the density, the angular velocity would decrease outwards as a function just of cylindrical radius, in accordance with the well known Taylor-Proudman theorem (see, e.g., Greenspan 1968). The effect of crust solidity will be to temporarily postpone such readjustments, by the development of the anisotropic stresses needed to balance the centrifugal buoyancy forces. However when such stresses have built up to the critical point at which the solid structure breaks down, the pent up centrifugal buoyancy forces will produce a “starquake” in which the convective readjustment that would have occurred continuously in the fluid case, is finally achieved in a discontinuous transition.

It is to be noticed that in contrast with the vortex pinning effect (in the following, for easier comparison, we will have in mind the scenario (b) of Sect. 4), which tends to pull the more slowly rotating crust material outwards from the axis towards the equator (see Fig. 2), the effect of the centrifugal buoyancy deficit in the crust is to pull the crust material inwards towards the axis of the star, where it will finally be subducted into the fluid interior (see Fig. 3). Although the centrifugal buoyancy effect produces convective circulation in just the opposite direction to that produced by vortex pinning (which if it were strong enough would lead to subduction at the equator rather than the axis (Ruderman 1991)) its effect on the angular momentum distribution would be similar, i.e. the net effect of a centrifugal buoyancy crustquake will be a discontinuous transfer of angular momentum to the crust from the more rapidly rotating fluid layer. This means that the crude quantitative estimate given by Eq. (14) is applicable just as well to the effect of a centrifugal buoyancy crustquake as to a vortex pinning crustquake.

The main point we want to emphasise is that whereas vortex pinning may indeed be the main driving force for the build up of the stress that is relaxed in crustquakes, the extent to which it really is depends on detailed considerations about the strength

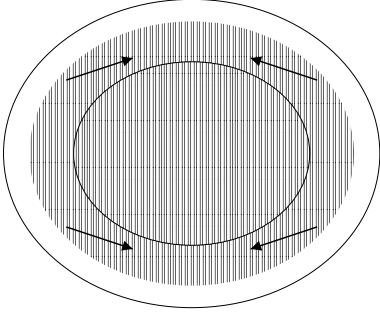


Fig. 3. Qualitative sketch indicating direction of force expected to act on (magnetically slowed down) crust, even if vortex pinning is ineffective, due to the *centrifugal buoyancy mechanism* when the (interpenetrating) neutron superfluid retains a higher rotation rate.

of vortex pinning. On the other hand the opposing centrifugal buoyancy mechanism will always function whenever there is differential rotation. It will be seen in the next section that when it is fully effective the oppositely directed pinning mechanism will be strong enough to overwhelm (i.e. to more than cancel) the buoyancy mechanism, but the latter mechanism is more robust in the sense that it will always make a significant contribution.

Our tentative conclusion – which we are proposing as a subject for debate and further investigation – is that the hitherto neglected centrifugal buoyancy effect may be the dominant cause of the crustquakes that are observed as pulsar glitches, while vortex pinning crustquakes, if they occur at all, are relatively rare. This does not mean that vortex pinning is unimportant for the phenomenon, because it is likely to be what determines the magnitude of the relevant moment of inertia contribution I_n in the estimate (14) for the ratio of $\delta\Omega$ to $-\Delta\Omega$. However what it means is that the vortex pinning stresses are not what is immediately responsible for the discontinuous breakdown, and hence not what is of dominant relevance for estimating the absolute values of $\Delta\Omega$ at which it is likely to occur.

6. The working of the centrifugal buoyancy deficit mechanism

An accurate treatment of neutron star would of course require a general relativistic analysis (Langlois et al. 1998; Comer et al. 1999), but as a first step towards the estimation of the stress forces needed to maintain equilibrium where the crust constituent is interpenetrated by an independently rotating fluid constituent, it will suffice for our present purpose to work in a Newtonian framework, using a highly idealised two-constituent model in which the corotating crust component (including the protons and electrons, as well as a fraction of the neutrons that is bound into atomic type nuclei) and the neutron superfluid are considered as independent material media having respective mass densities

$$\rho_c = mn_c, \quad \rho_n = mn_n \quad (16)$$

and spatial velocity components v_c^i and v_n^i ($i = 1, 2, 3$) where m is the proton mass and n_c and n_n are the corresponding baryon number densities. For an approximate description of the kind

of scenario envisaged by Alpar et al (1988) in which the rigidly corotating constituent consists not just of the crust lattice but also of the proton superfluid in the core which will be locked to the crust by electromagnetic interactions we adopt a simplified treatment in which it is postulated that the dynamics is governed by Euler type equations of motion of the familiar form

$$\rho_c(\partial_0 v_c^i + v_c^j \nabla_j v_c^i) = -\nabla^i P_c - \rho_c \nabla^i \phi + f_c^i, \quad (17)$$

$$\rho_n(\partial_0 v_n^i + v_n^j \nabla_j v_n^i) = -\nabla^i P_n - \rho_n \nabla^i \phi + f_n^i, \quad (18)$$

using ∂_0 to denote partial differentiation with respect to Newtonian time, where ϕ is the Newtonian gravitational potential, and where P_c , P_n and f_c^i , f_n^i respectively denote the relevant pressure scalars and force density vectors. In a lowest order approximation in which both components can be considered to obey barotropic equations of state giving their energy densities ε_c and ε_n as functions respectively of n_c and of n_n , they will be characterised by corresponding chemical potentials

$$\mu_c = \frac{d\varepsilon_c}{dn_c}, \quad \mu_n = \frac{d\varepsilon_n}{dn_n}, \quad (19)$$

from which the associated pressure contributions can be evaluated as

$$P_c = \mu_c n_c - \varepsilon_c, \quad P_n = \mu_n n_n - \varepsilon_n. \quad (20)$$

This implies that the required gradient terms will be given by

$$\nabla^i P_c = n_c \nabla^i \mu_c, \quad \nabla^i P_n = n_n \nabla^i \mu_n. \quad (21)$$

(It is to be remarked that in a more detailed analysis the baryon chemical potential μ_c in the component corotating with the crust would be interpretable as the sum of proton and electron contributions, $\mu_c = \mu_p + \mu_e$.)

Although adequate for the fluid constituent, a purely barotropic description will not be sufficiently accurate for the crust constituent in which we want to allow for the effects of solidity. The usual way to do this is to replace the isotropic pressure gradient term $\nabla_i P_c$ by a stress gradient term of the form $\nabla_j T_{c i}^j$ where $T_{c i}^j$ is the total stress tensor. It will be convenient for our purpose to decompose the latter in the form

$$T_{c i}^j = P_c \delta_i^j - s_i^j, \quad (22)$$

where the extra anisotropic stress contribution s_i^j is a correction term that will be small compared with the dominant isotropic contribution $P_c \delta_i^j$. This means that while f_n^i is to be interpreted as the interaction force density, if any, exerted on the neutron superfluid component by effects such as vortex pinning, on the other hand the term f_c^i in (17) will consist, not just of the equal and opposite interaction term $-f_n^i$ but also of an extra correction term f_s^i due to the anisotropic stress correction representing the effect of the solidity property, i.e. we shall have

$$f_c^i = f_s^i - f_n^i, \quad f_s^i = -\nabla_j s_i^j. \quad (23)$$

The anisotropic stress contribution s_i^j and the associated force density f_s^i might also include an allowance for magnetic effects,

such as are ultimately responsible for the external braking mechanism and for locking the proton superfluid in the core to the outer crust lattice. However for the equilibrium of the strictly stationary states with which we shall be concerned here such magnetic effects are not important, so it may be considered that the stress force density f_s^i arises just from the Coulomb lattice rigidity in the crust, and that it vanishes in the high density core.

Let us now restrict our attention to configurations that are stationary, so that the terms acted on by ∂_0 will vanish, and let us suppose the motion consists just of a circular motion about the x^3 axis, so that each comoving particle moves with a fixed value of the cylindrical radius $\varpi = (x^1{}^2 + x^2{}^2)^{1/2}$. This means that the velocity gradient terms in the equations of motion will be given by

$$v_c^j \nabla_j v_c^i = -\frac{1}{2} \Omega_c^2 \nabla^i \varpi^2, \quad v_n^j \nabla_j v_n^i = -\frac{1}{2} \Omega_n^2 \nabla^i \varpi^2, \quad (24)$$

where Ω_c is the local angular velocity of the crust constituent and Ω_n is the local angular velocity of the superfluid constituent. Under these conditions the Euler Eqs. (17) and (18) can be rewritten in the form

$$\frac{1}{2} \Omega_c^2 \nabla^i \varpi^2 - \nabla^i (\phi + m^{-1} \mu_c) = \rho_c^{-1} (f_n^i - f_s^i), \quad (25)$$

and

$$\frac{1}{2} \Omega_n^2 \nabla^i \varpi^2 - \nabla^i (\phi + m^{-1} \mu_n) = -\rho_n^{-1} f_n^i. \quad (26)$$

If vortex pinning were effective, it would contribute to f_n^i the force density needed to counteract Joukowski-Magnus type lift force density f_J^i that would be exerted on the vortices by the Magnus effect, which would be given by

$$f_J^i = \rho_n (\Omega_n - \Omega_c) \Omega_n \nabla^i \varpi^2, \quad (27)$$

but in the absence of vortex pinning or other coupling forces, the right hand side of (26) will simply vanish, in which case it can be seen that the fluid will satisfy the Taylor-Proudman condition, meaning that its angular velocity Ω_n and also the combination $m\phi + \mu_n$ must vary as a function only of the cylindrical radius ϖ .

Since the interaction force density f_n^i will cancel out of the linear combination of (25) and (26) obtained from the direct sum of (17) and (18), it follows that this combination will take a simple form that is conveniently expressible – independently of whether vortex pinning is actually effective or not – in terms of the “would-be” Joukowski force density (27) as

$$\begin{aligned} \nabla^i P + \rho (\nabla^i \phi - \frac{1}{2} \Omega_c^2 \nabla^i \varpi^2) \\ = f_J^i + f_s^i - \frac{1}{2} \rho_n (\Omega_n - \Omega_c)^2 \nabla^i \varpi^2, \end{aligned} \quad (28)$$

in which the total pressure P and mass density pressure ρ are defined in the obvious way as

$$P = P_c + P_n, \quad \rho = \rho_c + \rho_n. \quad (29)$$

In a systematic calculation by successive approximations, the first stage would be to obtain a zeroth order solution of

the stellar equilibrium problem in which the (first order) crust rigidity and differential rotation contributions on the right hand side of (28) would simply be neglected. What we are interested in here is the next stage, which involves the first order equation (from which the zeroth order part has cancelled out) that is obtainable by taking the difference of (25) and (26).

Before going ahead it is necessary to stress that, since only weak interactions are involved, it cannot be taken for granted that the relevant nuclear transitions involved in the “neutron drip” process whereby matter is transferred between the ionic crust material and the interpenetrating neutron superfluid will be very rapid compared with the “secular evolution” timescales on which the state under consideration is significantly modified. If the “neutron drip” process were sufficiently rapid one would obtain not just mechanical equilibrium, such as expressed by Eqs. (25) and (26), but also thermodynamical equilibrium in the rest frame of the crust, in the sense that the energy per baryon of the “normal” matter corotating with the crust, which is just μ_c , would be the same as the energy per baryon of the neutron fluid with respect to the crust corotating frame, which has the value $\mu_n + \frac{1}{2} m (\Omega_n - \Omega_c)^2 \varpi^2$. In practice however, due to the slowness of the relevant nuclear transitions (Haensel 1992), it is necessary to allow for the possibility of a finite deviation,

$$\Delta\mu = \mu_c - \mu_n - \frac{1}{2} m (\Omega_n - \Omega_c)^2 \varpi^2, \quad (30)$$

from exact thermodynamic equilibrium. Estimates of the likely values for such a chemical potential excess due to the simple spheroidality adjustment mechanism, discussed above in Sect. 2, have been provided by the recent work of Reisenegger (1995). Significantly larger values are likely to arise from the differential rotation mechanisms considered here due to the resulting tendency for the crust constituent to be convected relative to the neutron fluid constituent.

Including allowance for the possibility of a neutron drip delay contribution

$$f_x^i = n_c \nabla^i (\Delta\mu), \quad (31)$$

representing the force density due to the chemical potential excess (30) if any, the solid stress force density f_s^i ultimately responsible for the glitches in which we are interested can be seen to be given by the first order equation obtained by subtracting (26) from (25), which will be expressible in the form

$$f_s^i = f_x^i + \frac{\rho}{\rho_n} f_n^i + f_b^i. \quad (32)$$

The final term in the above equation is what can be interpreted as the extra force needed to compensate for the buoyancy deficit of the crust due to its lack of rotation velocity relative to the neutron superfluid, and is given by

$$f_b^i = \rho_c (\Omega_n - \Omega_c) (\nabla^i (\varpi^2 \Omega_n) - \varpi^2 \nabla^i \Omega_c). \quad (33)$$

7. Estimation of the centrifugal buoyancy deficit force density

The solidity property of the crust implies that, in a stationary state, its rotation must be rigid, i.e.

$$\Omega_c = \Omega, \quad \nabla^i \Omega = 0, \quad (34)$$

where Ω is a uniform angular velocity value (the one that is actually observable from outside), so the formula (33) for the buoyancy deficit force density can be immediately simplified to the form

$$f_b^i = \rho_c (\Omega_n - \Omega) \nabla^i (\varpi^2 \Omega_n). \quad (35)$$

If the superfluid were macroscopically irrotational, i.e. if there were no vortices present, then $\varpi^2 \Omega_n$ would have a uniform value so the right hand side of (35) would also vanish, i.e. the effective buoyancy deficit force density f_b^i would be zero.

What we actually anticipate in the context of the pulsar slow-down problem is that Ω_n will be approximately uniform (representing rigid rather than irrotational motion) with a value equal to that of the crust component at a rather earlier stage, perhaps just after the previous glitch, and that the velocity difference will therefore be small compared with the total angular velocity

$$|\Omega_n - \Omega| \ll |\Omega|. \quad (36)$$

Thus, by neglecting corrections of quadratic order in this velocity difference, we see that (35) can be conveniently approximated by the simpler formula

$$f_b^i \simeq \rho_c (\Omega_n - \Omega) \Omega_n \nabla^i (\varpi^2), \quad (37)$$

which will be accurate to linear order in the difference $\Omega_n - \Omega$. It is to be remarked that, to the same order of accuracy, the neutron drip delay force contribution (31) will be given by the approximation

$$f_x^i \simeq n_c \nabla^i (\mu_c - \mu_n). \quad (38)$$

It is to be observed that the formula (37) for the buoyancy deficit force density closely resembles the Joukowski formula (27) for the lift force density f_J^i that would be exerted on the vortices by the Magnus effect if they are pinned to the crust: this Joukowski force density is evidently related to the buoyancy deficit force density by the simple proportionality relation

$$f_J^i \simeq \frac{\rho_n}{\rho_c} f_b^i. \quad (39)$$

It follows that, in terms of the effective (centrifugally adjusted) gravitational potential

$$\psi_c = \phi - \frac{1}{2} \Omega_c^2 \varpi^2, \quad (40)$$

the basic stellar equilibrium Eq. (28) will reduce to the form

$$\nabla^i P + \rho \nabla^i \psi_c \simeq f_J^i + f_s^i, \quad (41)$$

in which the zeroth order terms are grouped on the left and the first order terms are on the right (while the final second order term on the right of (28) has been neglected). Since the left

hand side consists just of the small difference left over after the approximate cancellation of the dominant zeroth order terms, this equation does not provide any utilisable information about the solid force density f_s^i in which we are interested: on the contrary, after f_s^i has been evaluated by other means, (41) can be used to calculate the corresponding first order adjustments to the zeroth order pressure and density distributions.

The equation that does supply the relevant information about the solid stress force density f_s^i in which we are interested is the first order equilibrium condition (32), whose terms can be instructively regrouped in the form

$$f_s^i - f_x^i = f_b^i + \frac{\rho}{\rho_n} f_n^i, \quad (42)$$

in which it can be seen from (37) that the right hand side will always be approximately proportional to the first order difference $\Omega_n - \Omega$ whether or not pinning is effective. (This shows incidentally that differential rotation would be impossible if both the rigidity force f_s^i and the chemical delay contribution f_x^i were negligible.)

In particular, the relation (42) shows, by (39) that in the pinned case, i.e. when the superfluid is submitted to a force density

$$f_n^i \simeq -f_J^i \quad (43)$$

on the crust, the stress force density f_s^i necessary for equilibrium will be given by the simple formula

$$f_s^i = -f_J^i + f_x^i, \quad (44)$$

in which the term on the right is just the Joukowski-Magnus contribution, as is assumed in the conventional presentation of the vortex pinning theory of pulsar glitches.

The formula (44) is potentially misleading in that it gives the false impression that if the pinning were ineffective, so that instead of being given by (43) the force exerted on the superfluid by the crust were simply zero,

$$f_n^i = 0, \quad (45)$$

then the term on the right of the stress force density formula would similarly disappear, whereas in fact substitution of (45) in (32) leads to the replacement of (44) by the formula

$$f_s^i = f_b^i + f_x^i, \quad (46)$$

in which, instead of the Joukowski-Magnus contribution $-f_J^i$, the right hand side is now given by the oppositely directed buoyancy deficit force contribution f_b^i .

Our reasoning so far does not make it obvious whether or not the crust will develop a sufficiently non-uniform chemical potential excess $\Delta\mu$ to provide a significant chemical excess force f_x^i . If it is a good approximation to suppose that chemical excess force in the crust vanishes,

$$f_x^i = 0, \quad (47)$$

(as seems to have been implicitly assumed in most previous works but which needs to be confirmed or infirmed quantitatively) then, in the case where pinning would not be effective

(as has been advocated by Jones (1991) contrarily to earlier works), it follows from (46) that there will still be a solid stress force density given by

$$f_s^i \simeq f_b^i, \quad (48)$$

in which the centrifugal buoyancy deficit force density on the right is given by Eq. (37). This formula can be seen to differ from the (alternative) well known formula – for the stress due to pinning –, deduced from (44) by the same assumption (47),

$$f_s^i \simeq -f_J^i, \quad (49)$$

with the Joukowsky-Magnus term on the right hand side given by (27), only by having the opposite sign and by having a proportionality factor given by the density ρ_c of the corotating crust component instead of the density ρ_n of the differentially rotating neutron superfluid component.

8. Discussion and conclusions

In the lower crust region that seems most likely to be relevant for the explanation of the large glitches observed in the Vela pulsar one would expect the corotating constituent to be characterised by a density ρ_c (attributable mainly to protons and bound neutrons in the atomic type ions forming a solid lattice) having a range of values that is roughly comparable with that of the corresponding neutron superfluid density ρ_n (quantitatively round about 10^{13} g/cm³). Thus although they are of opposite sign (tending to push the crust material outward in the case (43) of vortex pinning, but to push it inwards in the case (45) for which pinning is absent) the alternative formulae (48) and (49) both predict the same rough order of magnitude for the stress induced on the crust by the existence of a difference between the angular velocity Ω_n of the neutron superfluid constituent and the (externally observable) angular momentum Ω characterising the crust.

The implication is that, as a candidate for explaining the large magnitude of the discontinuous changes $\delta\Omega$ that are commonly observed in a pulsar such as Vela, the previously overlooked buoyancy deficit mechanism characterised by the formula (48), i.e.

$$f_s^i \simeq \rho_c (\Omega_n - \Omega) \Omega_n \nabla^i \varpi^2, \quad (50)$$

(pushing outward along the cylindrical radial direction) seems at first sight to be just as promising as the more thoroughly investigated vortex pinning mechanism, which, if the chemical contribution f_x^i were unimportant, would be given according to (49) by

$$f_s^i \simeq -\rho_n (\Omega_n - \Omega) \Omega_n \nabla^i \varpi^2, \quad (51)$$

(pushing inward along the cylindrical radial direction). In order to obtain definitive conclusions it is clear however that much more work on both kinds of mechanism will be needed. In particular it will be necessary to pay more attention than hitherto to the role of the chemical excess force (31) and other potentially relevant mechanisms such as dynamical drag on vortices.

The present situation can be summarised by the statement that the large magnitude of the observed glitches in Vela provides strong evidence for the existence of angular velocity differences – and hence for the existence of superfluidity – in the pulsar interior, but that it is premature to claim it also provides strong evidence for vortex pinning because stresses of comparable magnitude could be produced in the absence of pinning by the centrifugal buoyancy deficit mechanism.

Acknowledgements. We wish to thank R. Prix and P. Hansel for very valuable discussions. One of us (D.S.) would like to acknowledge “Jumelage France-Arménie” exchange programme for financial support.

References

- Abney M, Epstein R.I., 1995, *J. Fluid. Mech.* 312, 327
 Alpar M.A., Anderson P.W., Pines D., Shaham J, 1984, *ApJ* 276, 325; *ApJ* 278, 791
 Alpar MA, Langer S.A., Sauls J.A., 1984, *ApJ* 282, 533; Alpar M.A., Sauls J.A., 1988, *ApJ* 327, 723
 Anderson P.W., Itoh N., 1975, *Nat* 256, 25
 Baym G., Pines D., 1971, *Ann. Phys.* 66, 816
 Comer G.L., Langlois D., Lin L.M., 1999, *Phys. Rev D* 60, 104025
 Easson I., 1979, *ApJ* 233, 711
 Greenspan H.P., 1968, *The theory of rotating fluids*, Cambridge University Press
 Haensel P., 1992, *A&A*, 262 131
 Jones P.B., 1991, *ApJ* 373, 208
 Langlois D., Sedrakian D.M., Carter B., 1998, *MNRAS* 297, 1189
 Link B.K., Epstein R.I., 1991, *ApJ* 373, 592
 Link B.K., Epstein R.I., 1996, *ApJ* 457, 844L
 Reisenegger A., 1995, *ApJ* 442, 749
 Ruderman M., 1969, *Nat* 223, 597
 Ruderman M., 1976, *ApJ* 203, 213
 Ruderman M., 1991, *ApJ* 366, 261
 Sauls J.A., 1988, in: Ogelman H., Van Heuvel E.P.J. (eds.), *Timing Neutron Stars*, Dordrecht: Kluwer, pp. 457
 Sedrakian A.D., Sedrakian D.M. 1995, *ApJ* 447, 305