

# Spatial structure and periodicity in the Universe

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**Abstract.** We analyze the possibility that the spatial periodicity of  $128h^{-1}$  Mpc in the galaxy count number observed recently in deep pencil-beam surveys could be explained by the mere existence of structure with the appropriate scale in the galaxy distribution in the Universe. We simulate a universe where the distribution of galaxies has an intrinsic length scale and then investigate the probability of observing in a given direction a spatial periodicity similar to that observed in the pencil-beam surveys of the galactic polar regions. A statistical analysis shows that this probability is of the order of  $10^{-8}$ , a value which excludes the afore mentioned explanation. This result contrasts with the estimates based on Voronoi foam models which yield probabilities of the order  $10^{-1} - 10^{-2}$ . The conclusion that emerges from these studies put together with the present one, is that although they can be obtained in some models, the presence of an intrinsic scale in the structure is not sufficient to achieve reasonable probabilities for observation of the detected periodicities.

**Key words:** cosmology: large-scale structure of Universe – cosmology: distance scale – galaxies: distances and redshifts – galaxies: statistics – methods: statistical

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## 1. Introduction

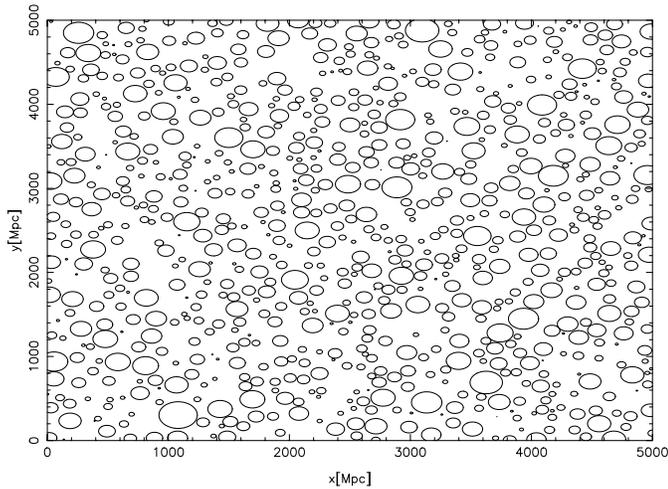
The cosmic microwave background radiation shows a remarkable isotropic structure of the order  $\delta T/T \approx 10^{-5}$ , which is taken as an evidence for the smoothness of the mass density of the Universe on the largest scales comparable to the Hubble radius (Wilkinson 1987),  $H_0^{-1} \approx 3000h^{-1}$  Mpc ( $H_0 = 100h$  km s $^{-1}$  Mpc $^{-1}$ ). On the other hand, the existence of galaxies and clusters of galaxies as well as the (larger than unity) amplitude of the galaxy-galaxy correlation function (Peebles 1980) indicate that the Universe possesses a highly irregular structure on scales less than  $10h^{-1}$  Mpc.

On intermediate scales, ranging from about  $30h^{-1}$  Mpc to around  $2000h^{-1}$  Mpc, the existing data do not allow a conclusive interpretation about the mass density distribution of the

Universe. Nevertheless, there are a number of different observations suggesting that the structure on these scales is quite varied. The most interesting of these observations include the voids with sizes greater than  $30h^{-1}$  Mpc observed in the distribution of bright galaxies (Kirshner et al. 1981; de Lapparent et al. 1986) the (larger than unity) amplitude of the cluster-cluster correlation function on scales up to  $30h^{-1}$  Mpc, the peculiar velocity field in our local vicinity on a scale of about  $50h^{-1}$  Mpc, and the “Great Wall” at a distance of approximately  $100h^{-1}$  Mpc (Geller & Huchra 1990). Also, noteworthy is the analysis of the spatial distribution of superclusters on the intermediate scales (Einasto et al. 1997; Einasto 1999; Abell et al. 1989). For this analysis a sample of more than 200 superclusters (systems with more than 8 clusters where the distances between nearest-neighbors among member clusters do not exceed  $24h^{-1}$  Mpc) was used, where the estimated redshift of simple members reaches up to  $z = 0.12$  (around 1/4 of the redshift reached in the pencil-beam galactic surveys). These three-dimensional data of the distribution of superclusters has been used to calculate the correlation function as well as the power spectrum. In the analysis of the power spectrum it was found that it is characterized by a peak with a wavelength of  $(120 \pm 15)h^{-1}$  Mpc.

But probably the most puzzling discovery concerns the recent observations which show an apparent periodicity in the distribution of redshifts in pencil-beam surveys of the north and south galactic polar regions (Broadhurst et al. 1990). These surveys extend to a redshift of around  $z \approx 0.5$  in both directions, and there is a strong evidence for a periodicity of  $128h^{-1}$  Mpc which extends for over 13 periods. Although this result was considered initially as a statistical anomaly (Kaiser & Peacock 1991), later and more detailed studies (Kopylov et al. 1988; Mo et al. 1992; Fetisova et al. 1993; Einasto & Gramann 1993; Szalay et al. 1993; Einasto et al. 1997) provided new evidence for the alluded periodicity on the same scales.

If this periodicity would be established in most directions, a naive interpretation would imply that galaxies and clusters are distributed on concentric shells centered around our galaxy, clearly a blow to our cosmological conceptions. Another more plausible explanation is based upon the idea that the observed periodicity is just a visual effect induced by a periodic oscillation of the gravitational constant (Morikawa 1990; Hill et al. 1990; Salgado et al. 1996, 1997; Quevedo et al. 1997, 1999;



**Fig. 1.** Two-dimensional section of a universe filled with a random smearing of three-spheres with a Gaussian distribution in radii. A constraint is imposed in order that the spheres do not intersect with each other. Notice that the distortion of the circles into ellipses is only due to the difference of scales on the  $x$ – $y$  axes.

Sudarsky et al. 1998), and that the mass density of the Universe on intermediate scales remains homogeneous and isotropic as on the largest scales. This explanation requires the introduction of a time-dependent scalar field into the model (the oscillating  $G$  model) which is non-minimally coupled to gravity.

In view of the remarkable conclusions that would derive from any of these explanations it seems imperative to consider whether the observations are likely to occur in the absence of these scenarios, and can be the result of chance augmented with the assumption of the presence of structure at the appropriate scale but with an otherwise random distribution. We will study the plausibility of this latter explanation inspired on the idea, suggested by Szalay et al. (1993), that the structure of the Universe on these scales consists of a regular network of superclusters and voids with a step size of the order of the wavelength discovered in the power spectrum. The specific structure that is studied by Van de Weygaert (1991) and Subba Rao & Szalay (1992) resembles a collection of honeycombs or a three-dimensional chess-board. Such structures known as Voronoi foams result in a reasonable probability for the observation of a periodicity similar to that in the actual observations.

The aim of this work is to investigate whether it is sufficient to have a universe with an intrinsic length scale in its large scale structure in order to explain the observation of the periodicity mentioned above. In particular, the present work can be seen as a test of the suggestion that a hypothetical “built-in scale in the initial spectrum” arising from double inflation models and supported by cosmic microwave background radiation data (Einasto 1997), could be the explanation of the above mentioned periodicity. Therefore, we will consider a model that does not introduce any other large scale pattern besides the existence of the above mentioned characteristic scale.

Needless is to say that neither our model nor the Voronoi foam models are anything more than “toy” models as they lack,

for example, a proven dynamical scenario for their formation starting from reasonable initial data. We want to construct a simple realization of such a model. We will do this by assuming that the galaxies are evenly distributed over the surface of three-dimensional spheres with an average radius comparable with the appropriate scale, with the spheres themselves randomly distributed in the Universe. In a universe filled by these spheres we choose a point as the “observational site” and perform a statistical analysis to investigate the probability of observing a periodic distribution with the characteristics of those reported in (Broadhurst et al. 1990) in a particular direction. We obtain an upper bound for the value of such probability of the order  $10^{-8}$ , a value that obviously excludes the alluded structure as an acceptable explanation of the observations. The assumption that the galaxies are homogeneously distributed on the surface of the spheres is not expected to affect the main result of our numerical analysis in the direction that would invalidate this upper bound.

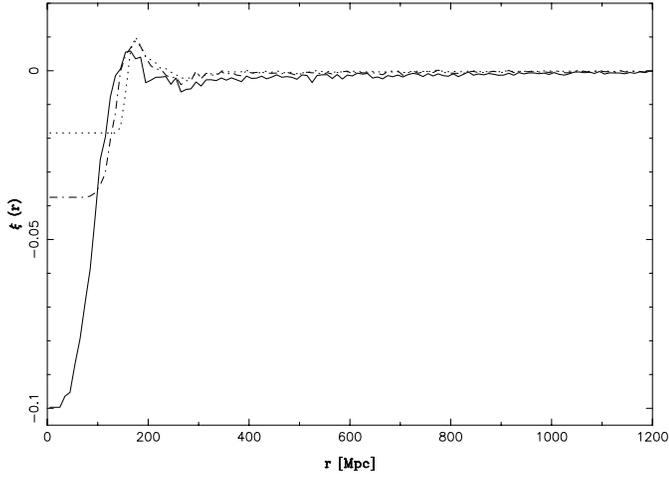
## 2. The model

We have taken the radii of the spheres to have a Gaussian distribution centered around the mean value of  $80h^{-1}$  Mpc and analyzed the results for different choices of the standard deviation  $\sigma$  of the distribution (namely with  $\sigma = 5, 15, 30h^{-1}$  Mpc). We have implemented a code that simulates the universe as a three-dimensional volume of  $(5000 h^{-1}\text{Mpc})^3$  filled by these spheres subjected to the condition that the spheres do not intersect with each other. In Fig. 1, a two-dimensional section of this volume is depicted. We emphasize that the above condition of no-sphere overlapping and the Gaussian distribution on the spheres radii imply that the numerical process of filling the toy universe stalls after a certain number of spheres. After that stage, the inclusion of more spheres demands a very long computing time. The adequate number of spheres needed to fill the universe is not fixed in advance, but the important point is that such a number must be so that the correlation function of the matter in the universe exhibits a characteristic scale, which is the most important aspect in the scenario that we are testing. We have confirmed that once correlations are present the results become insensitive to a further increase in the number of spheres.

In order to check the appearance of scale, we have computed a two-point correlation function defined in the following way. Let  $N$  be the total number of spheres in the toy universe, and let  $n_i(r)$  be the number of sphere centers contained between two concentric shells with radii  $r$  and  $r + \delta r$  around the center  $i$  (with  $\delta r \sim 10 \text{ Mpc } h^{-1}$ ). The number density of sphere centers within  $r$  and  $r + \delta r$  is then given by

$$C(r) = \frac{1}{N} \sum_{i=1}^N \frac{n_i(r)}{r^2}. \quad (1)$$

This is to be constant if correlations between neighbors at all scales are absent. It is customary to extract from this, the excess density number by defining the two-point correlation function



**Fig. 2.** Two-point correlation function of the matter content as a function of the distance. The dotted, solid and dash-dotted lines corresponds to the values  $\sigma = 5$ ,  $\sigma = 30$  and  $\sigma = 15$  respectively. Notice the presence of correlation at scales of  $\sim 160$ – $180$  Mpc and anticorrelations at shorter scales.

as the difference between the above density distribution and the mean density of sphere-center number:

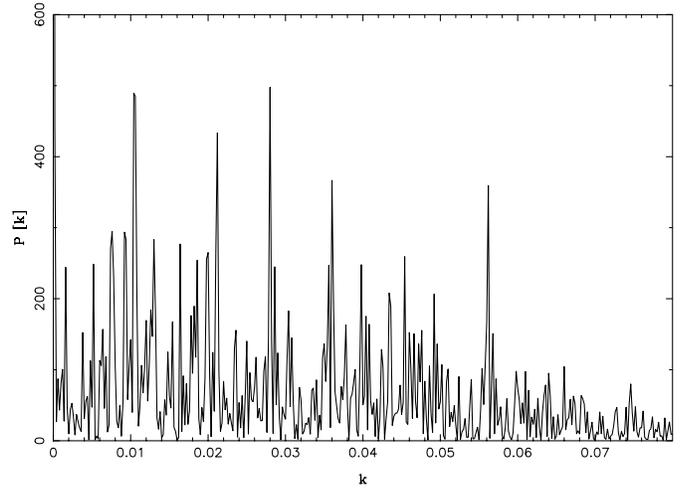
$$\xi(r) = C(r) - \langle C \rangle. \quad (2)$$

More specifically,  $\langle C \rangle$  corresponds to the mean of  $C(r)$  at scales for which  $C(r)$  is nearly constant (i.e.,  $\xi \sim 0$  at those scales). If the Universe has a characteristic length scale we expect it will be evident in the plot of the correlation function.

The results are shown in Fig. 2 with a typical number  $N \approx 4 \times 10^4$ . We can appreciate the presence of a characteristic scale at distances which are comparable to the mean value of the spheres radii, and stress that such scale is already present when the spheres fill a volume of only about 25% of the total volume of the toy universe. We do not expect any drastic changes in the qualitative as well as in the quantitative behavior of the correlation function if we increase the number of spheres in the universe. Moreover, as shown in Fig. 2, the resulting length scale is almost independent of the value of  $\sigma$  considered.

Now we will estimate the probability of observing in a specific direction a periodicity like the one seen in the real observations. To this end we note that the power spectrum of the observations in Szalay et al. (1993) revealed a peak at a frequency corresponding to  $128h^{-1}$  Mpc which contained 23% of the total integrated power. Thus we need to estimate the probability of observing in a specific direction a matter distribution with a power spectrum containing a peak with approximately the same percentage of the total power.

The “observational site” has been chosen to be a vertex of the box which in fact represents an octant of a larger toy-universe. This particular choice allows to consider the longest possible extent for the line of sight in each direction, but it does not affect the result of the statistical analysis which is performed for about  $10^5$  directions and for “pencil-beams” extending approximately to  $5000h^{-1}$  Mpc.



**Fig. 3.** Example of a power spectrum (Fourier analysis) of the resulting distribution of matter along a line of sight.

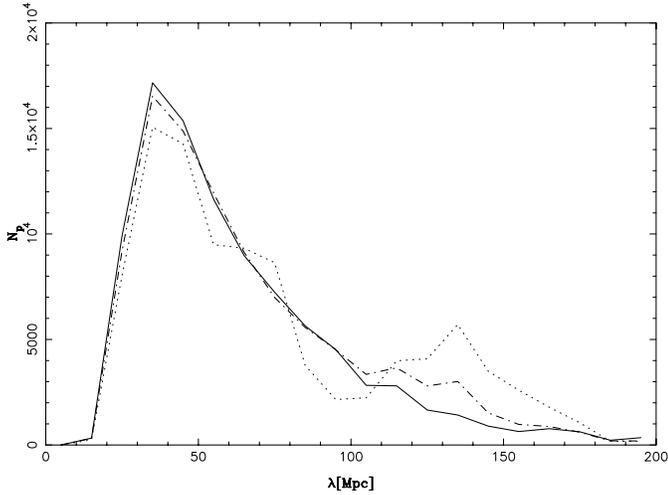
In this model each direction corresponds to a numerical pencil-beam, that is, a straight line which intersects several spheres, each intersection thus representing high-density spots. The ensemble of the distance separations between the spots results thus in a one-dimensional distribution of matter along the line of sight, which we represented mathematically by an ensemble of step functions of width  $10 \text{ Mpc } h^{-1}$  with height normalized to unity centered on each intersection of the line of sight with the surface of a sphere. We have also consider the effect of replacing step functions by Gaussian functions and showed that this does not change the probability estimates beyond the statistical error bars. Finally, we have performed a Fourier analysis of this distribution and confronted the resulting power spectrum with the observational one. Fig. 3 shows the resulting power spectrum of the matter distribution in a representative direction. We note that there is a substantial power at high frequencies, a feature associated with the use of step functions that is slightly suppressed by the use of Gaussian functions.

In order to compare with the real observations we look in each direction for the peak containing most of the power. We then calculate the percentage  $\Pi$  of the power-content within the peak:

$$\Pi = \frac{\text{Power content within the peak}}{\text{Total power}} \times 100. \quad (3)$$

As we mentioned, this procedure has been applied to  $10^5$  different directions starting from the specific site described above, obtaining representative peaks with values  $\Pi$  at different frequencies in each case. The distribution of the number of peaks with a given  $\Pi$  is plotted in Fig. 4 as a function of its corresponding wavelength. We note that most of the highest peaks cluster around a specific wavelength of  $\sim 40 \text{ Mpc } h^{-1}$ , however the power contained in most of them represents barely 2% of the corresponding total integrated power (see for instance the Fig. 3).

Since one can define the probability of observing a periodicity in a certain direction as the number of directions in which



**Fig. 4.** Distribution of the number of peaks with a given  $\Pi$  (maximum relative power content) associated to each direction of sight as a function of their corresponding wavelength in the Fourier space. The dotted, solid and dash-dotted lines corresponds to the values  $\sigma = 5$ ,  $\sigma = 30$  and  $\sigma = 15$  respectively.

the periodicity is present over the total number of directions, one should analyze an adequate sample in which one observes a sufficient number of cases with the desired periodicity. Since the sample of  $10^5$  directions showed no periodicity with a peak having the required power  $\Pi \sim 23\%$ , one should be tempted to give only an upper bound of  $10^{-5}$  for the probability of observing that value. This is not the best upper bound that can be obtained and it is only determined by the finiteness of the computing time. We have been able nevertheless to improve on this bound by the following procedure: We have fitted our data for the probability  $p$  of a peak with a given percentage  $\Pi$  of the power as a function of that percentage to various analytical expressions and found that this function is very well fitted by a power law:

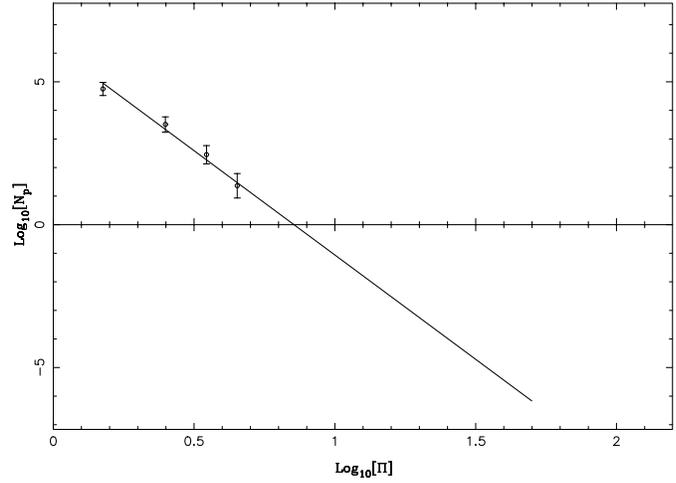
$$p(\Pi) = c\Pi^\alpha \quad (4)$$

$$p(\Pi) = \frac{N_p}{\text{total number of directions of sight}}, \quad (5)$$

where  $c$  and  $\alpha$  are constants with values depending on the chosen  $\sigma$  (see Table 1). This allows us to extrapolate the probability of finding the observed periodicity with  $\Pi = 23\%$  in a given direction.

An example of this law for  $\sigma = 30$  Mpc is shown in Fig. 5. A similar fit was made for the cases with other values of  $\sigma$  with the remarkable result that the power law is practically unaffected. This is a strong evidence for the robustness of our results.

We also stress that the data-set fitted to the above power law has values of  $\chi^2$ , which according to the degrees of freedom of the data ( $\sim 2$ ), indicates the good quality of the fits (see Table 1). This allows to make confident predictions for the probability of observing the value of  $\Pi = 23\%$  which corresponds to the observations of Szalay et al. (1993). From (5) one easily concludes that the value of  $p(23\%)$  ranges in the interval  $10^{-8} - 10^{-10}$ . This interval is determined by taking into account the statistical



**Fig. 5.** Power law probability (not normalized) of obtaining a peak with a corresponding value  $\Pi$ . Note that the higher the relative power content of the representative peak the lower the number of directions of sight where such a peaks is present. In particular the probability of observing a peak containing 23% of the total integrated power is  $\sim 10^{-8}$ .

**Table 1.** Parameters of the power-law (5) for the probability of finding a spatial periodicity in a given direction. Here the statistical uncertainties of data are taken into account for determining the ranges of the parameters.

$\sigma$	$c$	$\alpha$	$\chi^2$
30	$2.446564 \times 10^6 \pm 1.722128 \times 10^6$	$-7.30 \pm 0.20$	0.401
15	$2.390874 \times 10^6 \pm 1.682928 \times 10^6$	$-7.22 \pm 0.20$	0.377
5	$1.3196748 \times 10^7 \pm 1.0245539 \times 10^7$	$-8.81 \pm 0.31$	2.468

uncertainties of the fits. This implies that the probability of obtaining a peak with the observational power-content is at most  $10^{-8}$ . Needless to say that this result completely discards the possibility of detecting the observed periodicity in a universe with this kind of structure.

As we have stressed, we have performed a similar analysis replacing the step functions by Gaussian functions with the result that the upper bound for the probability  $p$  remains essentially unaffected and all the values of  $p$  remain within the interval determined by the statistical uncertainties.

It should be emphasized that in this work we are considering the possibility of detecting periodicity in a particular direction under the *sole* assumption that there exists an intrinsic length scale in the galaxy distribution. Our results show that this probability is completely negligible. However, this result could drastically be changed within a different model which would be characterized not only by a length scale but also by the presence of a specific correlation in the spatial distribution of galaxies. A concrete example of a model with this type of additional structure has been investigated (Van de Weygaert, 1991; Subba Rao & Szalay, 1992). In these works, dynamical Voronoi foams (Icke & Van de Weygaert, 1987; Van de Weygaert & Icke, 1989) with a controlled amount of randomness are proposed as describing

the spatial galactic distribution. The probability for finding periodicity in these models depends on the amount of randomness and can drastically be increased from 3% up to 15%. This probability can reach even higher values (10% to 40%) in the case of a regular lattice (Kurki-Suonio et al., 1990). In the extreme and naive model of galaxies distributed on the surfaces of concentric shells, one obviously would obtain a 100% probability for an observer at the center. We want to stress that the models that are more easily reconciled with acceptable cosmological scenarios are those that introduce a priori the least amount of scale order, and such that the order introduced corresponds to the smallest possible scale. In this sense the models that introduce just a characteristic length scale and no other large scale order should be considered as constituting the most viable explanations for the observations.

The conclusion and the main result of our analysis is that the existence of a specific scale in the distribution of mass density of the universe, as showed by a two-point correlation function, does not by itself provide a satisfactory explanation for the spatial periodicity of  $128h^{-1}$  Mpc in the galaxy distribution of deep pencil-beam surveys as it results in a negligible probability for such observation to be obtained in a particular direction. This result seems to force us again to look for alternative explanations for the observations of periodicity in deep pencil-beam surveys. On the other hand, it might be interesting to consider variations of this type of model (for example, changing the intersection rule) to identify the features that lead to an enhanced probability for obtaining the desired periodicity.

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