Models of universe with a delayed big-bang singularity

III. Solving the horizon problem for an off-center observer

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Abstract. This paper is the third of a series dedicated to the study of the Delayed Big-Bang (DBB) class of inhomogeneous cosmological models of Lemaître-Tolman-Bondi type. In the first work, it was shown that the geometrical properties of the DBB model are such that the horizon problem can be solved, without need for any inflationary phase, for an observer situated sufficiently near the symmetry center of the model to justify the "centered earth" approximation. In the second work, we studied, in a peculiar subclass of the DBB models, the extent to which the values of the dipole and quadrupole moments measured in the cosmic microwave background radiation (CMBR) temperature anisotropies can support a cosmological origin. This implies a relation between the location of the observer in the universe and the model parameter value: the farther the observer from the symmetry center, the closer our current universe to a local homogeneous pattern. However, in this case, the centered earth approximation is no longer valid and the results of the first work do not apply. We show here that the horizon problem can be solved, in the DBB model, also for an off-center observer, which improves the consistency of this model regarding the assumption of a CMBR large scale anisotropy cosmological origin.

Key words: cosmology: theory

1. Introduction

The large scale homogeneity of the universe has been recently questioned in an increasing number of works. For instance, the controversy over whether the universe is smooth on large scale or presents an unbounded fractal hierarchy is not yet ended, and its final resolution requires the next generation of galaxy catalogs (Martinez, 1999). From another point of view, a direct test of the Cosmological Principle on our past light cone, up to redshifts approaching unity, has been recently proposed, using type Ia supernovae as standard candles (Célérier, 2000). If such tests were to exclude the universe homogeneity assumption up to such large scales, and even beyond, we would need alternative inhomogeneous models to describe its evolution.

This article is the third in a series dedicated to the study of a new cosmological application of the inhomogeneous Lemaître-Tolman-Bondi (Lemaître, 1933; Tolman, 1934; Bondi, 1947) spherically symmetrical dust models.

In a first work (Célérier & Schneider, 1998, hereafter refered to as CS), a subclass of these models which solves the standard horizon problem without need for any inflationary phase has been identified. This subclass exhibits spatial flatness and a conic Big-Bang singularity of "delayed" type. In this preliminary approach, the observer has been assumed located sufficiently near the symmetry center of the model as to justify the "centered earth" approximation. The horizon problem has thus been solved using the properties of the null-geodesics issued from the last-scattering surface and propagating in a matter dominated region of the universe, as seen from a centered observer.

However, we stressed in CS a potential difficulty, namely the observer at the center. Although such a location is not forbidden by scientific principles, it does not account for the observed large scale temperature anisotropies of the cosmic microwave background radiation (CMBR).

The dipole moment in the CMBR anisotropy is usually considered to result from a Doppler effect produced by our motion with respect to the CMBR rest-frame (Partridge 1988). In the second work of the series (Schneider & Célérier 1999, hereafter refered to as SC), we assumed that the measured CMBR dipole and quadrupole moments can have, totally or partially, a cosmological origin, and we studied to which extent they can be reproduced, in a peculiar example of our Delayed Big-Bang (DBB) class of models, with no *a priori* assumption of the observer's location. We have shown that this implies a relation between this location and the model parameter value, namely the increasing rate b of the Big-Bang function. The farther the observer is from the symmetry center, the smaller is the value of b.

The purpose of this present work is therefore to prove that the geometry of the DBB model is such that the horizon problem can be solved, in principle, with no *a priori* constraint on the location of the observer. By "in principle", we mean: provided the null-geodesics are not too distorted in the radiation dominated area, such as to prevent them from reconnecting before the Big-Bang surface. Were this not the case, it would yield a constraint on the model parameters, as discussed in CS. This allows us to generalize the previous results of CS to a model

with an off-center observer, thus improving the consistency of the assumption of a possible cosmological origin of the large scale features of the CMBR temperature anisotropies.

The above property will have however to be interpreted with care, as regards the Ehlers-Geren-Sachs (1968) and almost Ehlers-Geren-Sachs (Stoegger et al. 1995) theorems, which are usually considered as robust supports for the Cosmological Principle. As was stressed in the preceding works, this Principle is not mandatory and does not apply to the DBB model. This issue is discussed below in Sect. 4.

The present paper is organised as follows: a brief reminder of the characteristics of the DBB model is given in Sect. 2. Arguments and proofs are developed in Sect. 3. The discussion and conclusion appear in Sect. 4.

2. The inhomogeneous delayed big-bang model

The main features and properties of the model are here briefly mentioned. For a more detailed presentation and a discussion of the assumptions retained, the reader is referred to CS and SC.

The proposed DBB model is a subclass of the LTB flat model. Its line-element, in comoving coordinates (r, θ, φ) and proper time t, is

$$ds^2 = c^2 dt^2 - R'^2(r,t)dr^2 - R^2(r,t)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (1)$$

An appropriate choice of the radial coordinate r yields

$$R(r,t) = \left(\frac{9GM_0}{2}\right)^{1/3} r[t - t_0(r)]^{2/3}. \quad (2)$$

The Big-Bang function, $t_0(r)$, verifies[1]

$$\begin{aligned} t_0(r=0) &= 0, \\ t_0'(r) &> 0 \qquad \text{for all} \qquad r, \\ 5t_0'(r) + 2rt''_0(r) &> 0 \qquad \text{for all} \qquad r, \\ rt_0'|_{r=0} &= 0. \end{aligned}$$

The physical singularity of the model - i.e. the first surface, encountered on a backward path from "now", where the energy density and the invariant scalar curvature go to infinity - is the shell-crossing surface, represented in the (r, t) plane by the curve:

$$3t - 3t_0(r) - 2rt_0'(r) = 0.$$

As the energy density increases approaching this surface, radiation becomes the dominant component in the universe, pressure can no longer be neglected, and the LTB model no longer holds. The region between the Big-Bang surface $t = t_0(r)$ and the shell-crossing one is thus excluded from the part of the model retained to describe the matter dominated region of the universe, which we discuss here.

The optical depth of the universe to Thomson scattering is approximated by a step function (see SC). The last-scattering

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[1] An interesting example, studied in CS and SC, is the subclass with $t_0(r) = br^n$, $b > 0$, $n > 0$.

surface is thus defined, in the local thermodynamical equilibrium approximation (see CS), by its temperature, $T = 4000K$, as is the now-surface, $T = 2.73\ K$, where the observer is located. The equal temperature surfaces verify

$$t = t_0(r) + \frac{r}{3}t_0'(r) + \frac{1}{3}\sqrt{r^2 t_0'^2(r) + \frac{3S(r)}{2\pi G k_B a_n m_b T^3}}. \quad (3)$$

The value of the entropy function $S(r)$ is assumed to be constant with r. The shell-crossing surface is thus asymptotic to every (monotonically increasing with r) $T = const.$ curve.

An observer located at a distance from the center sees an axially symmetrical universe in the center direction. In the geometrical optics approximation, the light travelling from the last-scattering surface to this observer follows null-geodesics, which is thus legitimate to consider in the meridional plane. The photon path is uniquely defined by the observer's position (r_p, t_p) and the angle α between the direction from which the light ray comes and the direction of the center of the universe, C.

The microwave radiation observed in the direction of this symmetry center follows a light-cone issued from point D on the last-scattering surface, then passes through the center ($r = 0$) and reaches the earth at point O. Observed in the opposite direction, it starts from point E, and travels on the EO null-geodesic to the observer (see Fig. 2).

The radial null-geodesic equation, as established in CS, is

$$\frac{dt}{dr} = \pm\frac{1}{3c}\left(\frac{9GM_0}{2}\right)^{1/3}\frac{3t - 3t_0(r) - 2rt_0'(r)}{[t - t_0(r)]^{\frac{1}{3}}}. \quad (4)$$

It is easy to see that, with a function $t_0(r)$ verifying conditions (3), for a fixed t, $\left|\frac{dt}{dr}\right|$ decreases with increasing r, and thus: $r_D < r_E$.

If observed with an angle α in the inward direction, a light beam issued from point A on the last-scattering surface approaches C to a comoving distance r_{min}, then proceeds toward O. In the outward (opposite) direction, it follows the BO geodesic (see Figs. 1 and 2).

The corresponding null-geodesics are solutions of the system of differential equations established in SC (in units $c = 1$):

$$\frac{dt}{d\lambda} = k^t, \quad (5)$$

$$\frac{dr}{d\lambda} = \pm\frac{1}{R'}\left[(k^t)^2 - \left(\frac{R_p\sin\alpha}{R}\right)^2\right]^{1/2}, \quad (6)$$

$$\frac{dk^t}{d\lambda} = -\frac{\dot{R'}}{R'}(k^t)^2 + \left(\frac{\dot{R'}}{R'} - \frac{\dot{R}}{R}\right)\left(\frac{R_p\sin\alpha}{R}\right)^2. \quad (7)$$

with a plus sign in Eq. (6) from O to r_{min} ($\frac{dr}{d\lambda} > 0$), and a minus sign from r_{min} to A ($\frac{dr}{d\lambda} < 0$). The equations corresponding to the observer looking outward (OB curve) require a minus sign.

If one considers Eq. (6), for a fixed t and for a same variation $d\lambda$ of the λ affine parameter, the absolute value of the radial
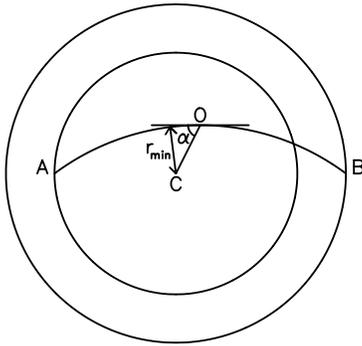
Fig. 1. The CMBR observed from O with an angle α. Schematic illustration of the trajectory of two CMBR light beams received by the observer O and making an angle α with the direction of the symmetry center C of the universe. The inner circle is the 2-sphere on the last-scattering surface from which the beam issued from point A is emitted (the observer looks inward). The outer circle is the one from which the beam starting from B is emitted (the observer looks outward).
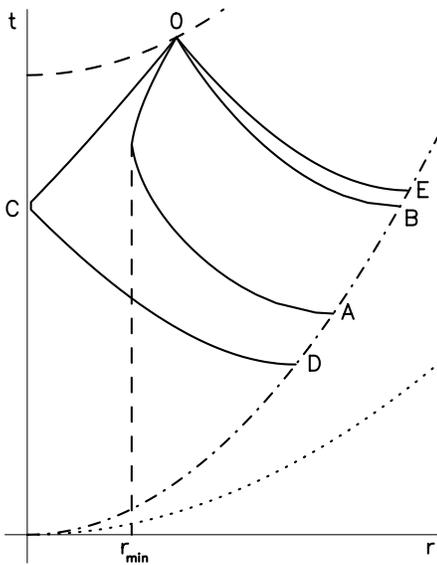

Fig. 3. Penrose-Carter diagram sketching the permanent solution of the horizon problem by the DBB model. The current observer O sees, on the last-scattering surface, a causally connected (DE) region, included in the forward light cone of P. The same will hold when the O' point is reached, and from any other time in the observer's past or future history.

Fig. 2. The CMBR observed from O in the (r, t) plane. The dotted curve represents the Big-Bang surface $t = t_0(r)$. The broken line with dots represents the shell-crossing singularity and the last-scattering surfaces that cannot be resolved at the scale of the figure. The broken curve is the now-surface $T = 2.73\,K$. The solid lines are the light-cones.

3. Solving the horizon problem

An inspection of Eqs. (1) and (4), as done in CS, shows that the shell-crossing singularity surface is null: it cannot be crossed by any ingoing null geodesic.

The solution of the horizon problem, for an off-center observer in a DBB model, can thus proceed from its representation using a Penrose-Carter conformal diagram (see Fig. 3)[2].

The light-like character of the shell-crossing surface forces all matter to be causally connected at $t = 0$, and any finite region to have been in causal contact at some $t > 0$.

The horizon problem is thus solved permanently in this model, *a priori*, for any location of the observer[3].

It is worth emphasizing that if the inflationary assumption also solves the horizon problem, it does so only temporarily. In effect, if one considers the horizon problem in a standard FLRW universe, as sketched in Fig. 4, the Big-Bang surface is space-like. It thus implies the existence of a limiting point L, in the history of the observer O, beyond which the observer sees, on the last-scattering surface, some no causally connected points. The current observer, being located above L, is confronted with this horizon problem.

The solution proposed by the inflationary assumption is presented in Fig. 5. Adding an inflationary phase in the early history of the universe amounts to adding a slice of de Sitter space-time between the Big-Bang and the last-scattering, thus postponing the limit L when the observer can see non-causally connected points in the CMBR. Inflation thus solves the horizon problem, but only temporarily.

coordinate variation dr is smaller with $\alpha \neq 0$ or π than with $\alpha = 0$ or π. It follows that $r_D < r_A < r_B < r_E$.

With α taking every value between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and considering the two oppposite directions of sight, the CMBR is observed from O as coming from a set of points each located on a 2-sphere belonging to the the subset $\{r = const., r_D < r < r_E\}$ on the last-scattering surface $T = 4000\,K$.

To prove that this set of points can be causally connected, it is sufficient to show that there is at least one forward radial light-cone, i.e. issued from a point $(r = 0, t > 0)$ including the DE subset.
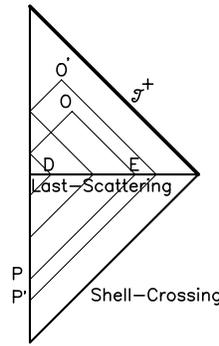
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[2] Only half of the diagram is drawn, as permitted by spherical symmetry.

[3] As pointed out in Sect. 4 (below), constraints yielded by observational data can be imposed upon the observer location, but solving the horizon problem, in the dust approximation, does not provide any constraint of this kind.
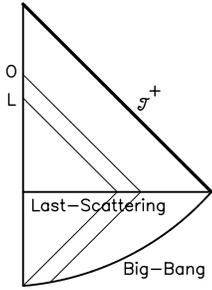
Fig. 4. Penrose-Carter diagram showing the horizon problem in a FLRW universe. The thin lines represent the light-cones. The CMBR, as seen by the observer located on the vertical axis, corresponds to the intersecting point of the observer backward light-cone and the last-scattering line. For a complete causal connection between every point seen in the CMBR, the backward light-cone issued from this intersecting point must reach the vertical axis before the Big-Bang curve. L is thus the limiting time beyond which the observer O experiences the horizon problem.
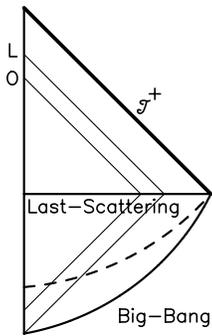

Fig. 5. Penrose-Carter diagram showing the temporary solution of the horizon problem by inflation. The slice of de Sitter space-time corresponding to an inflationary phase, and added to Fig. 4, is shown between the Big-Bang and dashed lines. L is thus postponed, allowing the current observer O to see a causally connected CMBR. When time elapses and the observer reaches the above L region, the horizon problem returns.

4. Discussion and conclusion

In CS we identified a subclass of the LTB models with a Big-Bang of "delayed" type which solves the standard horizon problem without need for any inflationary phase. In this preliminary approach, the observer was assumed located sufficiently near the symmetry center of the model as to justify the "centered earth" approximation.

Here, we report a further analysis of the properties of the DBB model to show that this model solves the horizon problem even with an off-center observer. The model is thus relieved of a prescription that could be considered as "unnatural".

The model is also provided with a new free parameter, the spatial location r_p of the earth in the universe, which accounts for the large scale inhomogeneities observed in the CMBR temperature anisotropies. The measured dipole and quadrupole moments of these anisotropies set bounds on the correlated values of this r_p parameter and of the local deviation of the model from

homogeneity, accounted for by the slope of the Big-Bang function. A possible cosmological part of these large scale features seen in the CMBR, if once observationally identified, would select an even narrower curve in the parameter space of the model, as shown in SC.

It is of the utmost importance to stress that, as was the case with a centered observer, these results hold for any universe arbitrarily *locally* close to the FLRW $t_0(r) = const.$ asymptotic model. The only requirements to be fulfilled are conditions (3) which are obviously compatible with an almost "flatness" of the Big-Bang function up to comoving shells arbitrarily far out the r_p shell where the observer is located. The properties of the light-cones are preserved as long as this function does not reduce to a mere constant. For instance, the subclass retained in SC, with $t_0(r) = br$, reduces to a FLRW model for b equal to zero, but fulfills the conditions (3) for b as small as one wishes, provided b does not vanish. No bound can therefore be *a priori* inferred on the observer location, as, according to SC, an arbitrarily small value of b corresponds to an arbitrarily large value of r_p, and conversely[4].

A point worth discussing here is the validity of this claim as regards the almost Ehlers-Geren-Sachs (AEGS) theorem (Stoegger et al. 1995). This theorem states that "if all fundamental observers measure the cosmic background radiation to be almost isotropic in an expanding universe region, then that univere is locally almost spatially homogeneous and isotropic in that region." The U region considered by the AEGS authors is "the region within and near our past light cone from decoupling to the present day". It is easy to see that small b DBB models fulfill the AEGS prescription, as they can remain "close" to FLRW models for shells located between the center and an arbitrarily large value of the comoving radial coordinate, including the AEGS region. But the further away part of these small b DBB models infinitely diverges from homogeneity. On the contrary, the AEGS theorem does not apply to large b DBB models, implying an observer close to the center. The founding assumption of this theorem, namely the local Copernican principle applied to the U region, is not retained in this case. As was discussed in CS, such a choice is perfectly compatible with all available observational data and scientifically grounded principles.

It is also interesting to note that, contrary to the inflationary assumption which restores causality between the different points seen in the CMBR, but only temporarily, the DBB model provides a permanent solution to the horizon problem, whatever the position of the observer on his world line.

In the prospect of future observational tests of the large scale (in)homogeneity of the universe, the development of other interesting inhomogeneous models must be regarded as an important issue. However, this presented result is only a first improvement in the release of the simplifying assumptions (retained in CS) for a preliminary study of the properties

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[4] Such bounds could in fact proceed from further analyses of the model in the light of other theoretical consideratioins or observational data, but solving the horizon problem does not yield any constraint of this kind.

induced by a "delayed Big-Bang". Other analyses are still needed, among which the release of the spatial spherical symmetry of the model and of the dust approximation should be considered as priorities.

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