A toroidal black hole for the AGN phenomenon

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Abstract. A new approach to the study of the AGN phenomenon is proposed, in which the nucleus activity is related to the metric of the inner massive black hole. The possibility of a Toroidal Black Hole (TBH), in contrast to the usual Spherical Black Hole (SBH), is discussed as a powerful tool in understanding AGN related phenomena, such as the energetics, the production of jets and the acceleration of particles, the shape of the magnetic field and the lifetime of nucleus activity.

Key words: acceleration of particles – black hole physics – galaxies: nuclei

1. Introduction

Huge energetics of Active Galactic Nuclei (AGN) are usually interpreted in terms of accretion around a massive black hole (Frank et al. 1985). So far, the standard assumption has been that the black hole is a spherical (SBH), static (i.e. described by the Schwarzschild metric) or else rotating around its axis (i.e. described by the Kerr metric) one. However, models of collapse leading to different topologies of the resulting object have been investigated (Smith & Mann 1997) and a mathematical description of their properties in the framework of general relativity has been developed by several authors, e.g. Vanzo (1997), Brill et al. (1997). Recently, a few hints about the possible connection between topological models of black holes and high-energy astrophysical phenomena like AGNs have been claimed (Spivey 2000). More specifically, the possibility points toward a Toroidal Black Hole (TBH), in which the event horizon is topologically equivalent to a torus, to explain the AGN observations.

In this paper, we consider such a situation and evaluate the influence of this topology in accounting for it.

First, we give a basic description of the conditions needed to obtain a TBH and the possibilities of matching them. Second, the geometry of a TBH is characterized through a dynamical treatment of particle motion around it. Then, some hints about the shape and intensity of magnetic field are provided, yielding the emission properties of such a structure; along this route, a particles acceleration and jet production mechanism is pro-

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posed. Last, an interpretation of the finite lifetime of an AGN is deduced from the whole picture.

2. The toroidal structure for a black hole

2.1. The spacetime metric

A generalization of the black hole metrics can be given as follows (Smith & Mann 1997):

$$ds^2 = -\left(V + b - \frac{2M}{R}\right) dt^2 + \frac{dr^2}{\left(V + b - \frac{2M}{R}\right)}$$
$$+ R^2[d\theta^2 + c \, sinh^2(\sqrt{a}\theta)d\phi^2], \tag{1}$$

where t, r are the time and radial coordinates, θ and ϕ are coordinates on a two-surface of constant curvature and V is a potential term, whose meaning will be discussed later. The parameters b, c, a fix the topology of the structure: in particular, if $b = -a = 0$ ($b = -a$ following the solution of Einstein field equations in empty space) and $a \to 0$, $c = +\frac{1}{a}$, then the topology is that of a torus and the space-time is asymptotically anti-de Sitter (AdS). We recall that a Schwarzschild metric for an asymptotically flat spacetime exhibits $b = 0$ and +1 instead of the potential term V. Therefore, such a configuration is bounded to the presence of V. A standard expression for the potential would be of the form:

$$V = \frac{\Lambda}{3}R^2, \tag{2}$$

where Λ is a negative constant in order to provide an AdS spacetime. However, this cannot be the cosmological constant, which is known to have a positive value (Perlmutter et al. 1997, 1999; Riess et al. 1998; Roos & Harun-or-Rashid 2000) and which is a few orders of magnitude smaller than needed to act as an effective AdS term. That is why another physical reasoning must be found for V and it must be of astrophysical nature (e.g. gravitational potential of the surrounding galaxy); anyhow, some ideas about its origin will be further given.

2.2. The geodesics

A simplified picture of the gravitational potential of a rotating TBH can be gained by the motion of particles around it. Actually, particle dynamics is the best tool to clarify the physical
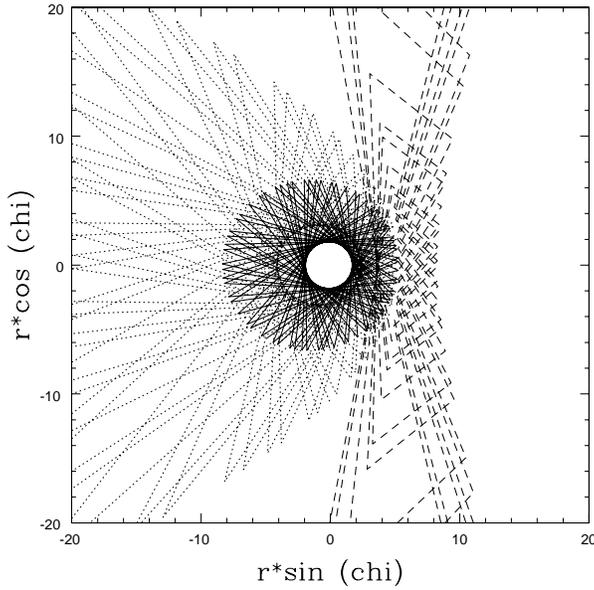
Fig. 1. Particle trajectories in the equatorial plane, assuming they are experiencing just the TBH gravitational potential; the orbits are parametrized as elliptical with $p = 5.6$ and $V \simeq const$ is assumed. Different values for the e parameter have been assumed: (*solid line*) $e = 0.2$, (*short-dashed line*) $e = 1.0$, (*long-dashed line*) $e = 4.5$.

phenomena happening around the hole. Such a task can be pursued by solving the geodesics equations (Lawden 1982):

$$\frac{d}{ds}\left(2\,g_{ri}\frac{dx^i}{ds}\right) - \frac{\partial g_{jk}}{\partial x^r}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \tag{3}$$

for the different components g_{ri} of the metric tensor. The corresponding radial equation can be stated as follows:

$$\frac{dr}{dt} = \left[c^2(k^2 - V) + \frac{2MGc^2}{r} + \frac{2MGh^2}{c^2r^3} - h^2V\right]^{1/2}, \tag{4}$$

where h is the specific angular momentum along the z-axis and k is a constant of integration. Therefore, the energy conservation equation can be derived:

$$\left(\frac{dr}{dt}\right)^2 - \frac{2MGc^2}{r} - \frac{2MGh^2}{c^2r^3} = c^2k^2 + V(k^2 - h^2), \tag{5}$$

which can be compared with the SBH situation:

$$\left(\frac{dr}{dt}\right)^2 + \frac{h^2}{r^2} - \frac{2MGc^2}{r} - \frac{2MGh^2}{c^2r^3} = const. \tag{6}$$

The basic difference is based on the centrifugal term $\frac{h^2}{r^2}$: indeed, if V has no radial dependence, then the TBH *has no centrifugal barrier* and, in order to reach the black hole, the accreting matter will just need to overwhelm a certain threshold value independent of r and fixed by the right-hand side of the above equation. We stress that this trend is of fundamental relevance for the amount of angular momentum which must be exchanged by the infalling gas before accreting.

To depict particle trajectories and in order to make a comparison with the SBH result, we followed the standard approach

of parametrizing the orbit in a general elliptical way (Chandrasekhar 1983):

$$r(\chi) = \frac{p}{(1 + e\,cos\chi)}, \tag{7}$$

in terms of a dimensionless parameter χ along the orbit, then we solved the time ($\frac{dt}{d\chi}$) and angular ($\frac{d\phi}{d\chi}$) components of the geodesics, with zero initial conditions. In Fig. 1 the trajectories in the equatorial plane are sketched for $V \simeq const$ with $p = 5.6$ and different choices for the e parameter. The geodesics have been evaluated for $e = 0.2$, $e = 1.0$ and $e = 4.5$. We recall that, in general, for a central potential of the form $V = -\frac{V_0}{r}$, it holds:

$$e = \sqrt{1 + \frac{2h^2 E}{V_0^2 m}}, \tag{8}$$

for a particle with energy E and mass m. Then, the first two choices for the e parameter correspond respectively to low and mildly energetic particles. In this case, particles spiral around the torus and are focused in a circular region internal to the structure, whose extent is wider for the more energetic particles. As they turn around the torus they gain acceleration and, in particular, while approaching the centre of symmetry ($r \to 0$), their acceleration is proportional to the time they spend in the inner region, as follows:

$$\frac{dr}{dt} \sim \sqrt{2MG}\left(\frac{h}{c}\right)r^{-3/2} \Rightarrow a \propto t^{-8/5}, \tag{9}$$

so that the less they spend in the center the more they can gain and we will refer to this stage as a *gravitational kick*: eventually, when a particle reaches the matching-condition orbit, it can be kicked out through the circular inner region. We note that $e = 1.0$ provides a limiting state, because a few particles can escape the black hole through a parabolic orbit. Last, particles with e greater than unity have too much energy to get trapped by the black hole, therefore they just approach it and experience a *slingshot mechanism*, kicking them off.

If they are focused to the center, particles are likely to undergo collisions, which could be an effective way to exchange angular momentum and can provide another mechanism of acceleration by means of a Fermi-type stochastic procedure. In that framework, the energy exchanged by a particle per unit time due to collisions is proportional to the particle velocity v and to the size l of the acceleration region (Longair 1981):

$$\frac{dE}{dt} = \frac{4}{3}\left(\frac{v^2}{cl}\right)E. \tag{10}$$

The conservation equation depends on the time spent by the particle in that region t_f and if we neglect diffusion and source terms, it holds:

$$\frac{dN}{dt} = -\frac{d}{dt}[\alpha\,E\,N(E)] - \frac{N}{t_f}, \tag{11}$$

where $\alpha \equiv \frac{v^2}{cl}$. The final spectrum of the particles emitted by the core in a steady state situation ($\frac{dN}{dt} = 0$) is given by:

$$N(E) \propto E^{-\left(1 + \frac{1}{\alpha t_f}\right)}, \tag{12}$$

so that *a jet starting from the center will have a power-law distribution.*

We remind that to have an effective acceleration mechanism, the particles must have energies greater than the maximum energy loss felt during acceleration, or else the loss can be overcome through a sufficiently rapid initial acceleration mechanism: that is exactly what makes a TBH a likely candidate for an effective process.

2.3. Extended particle dynamics

The spiral-shaped trajectory is not the only motion the particles undergo. In addition to it the centrifugal and the Coriolis forces must be added; moreover, if we consider the flow around the TBH to be embedded in the magnetic field produced by the accretion disk, a Lorentz force term arises. Actually, the accretion disk can be considered to be composed of annuli of charged particles fluxes, providing azimuthal currents: this results in a magnetic field in which field lines encircle those currents. Therefore, the three forces can be evaluated as follows:

- $F_{cf} = \gamma m_0 (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{\Omega}$;
- $F_{Cor} = m_0 \left[2\gamma \frac{dr}{dt} + r \frac{d\gamma}{dt} \right] (\mathbf{e_r} \times \mathbf{\Omega})$;
- $F_B = \frac{q}{c} (\mathbf{v} \times \mathbf{B})$;

for a particle of rest-mass m_0, charge q and Lorentz factor γ. The effect of the Lorentz force is a drift in the azimuthal coordinate, adding to the rounded spiral motion or eventually against it and braking the flow; nevertheless, this motion provides a shear effect and an energy exchange between $+/-$ charged flows. On the other hand, the superposition of the centrifugal force and the Coriolis force (opposite to the direction of rotation) can allow particles to move towards other orbits and could be responsible for particles pointing to the centre of symmetry. The azimuthal drift develops a ring current associated to an encircling magnetic field, which adds to the disk component. At the end, an axisymmetric toroidal plasma configuration arises, resembling the *tokamak* device involved in nuclear fusion studies. The total magnetic field will exhibit helical field lines (Dendy 1993):

$$\mathbf{B} = \frac{1}{R} \mathbf{\nabla}\psi \times \mathbf{e_\phi} + B_\phi \mathbf{e_\phi}, \tag{13}$$

where the potential ψ can be inferred by means of the *Grad-Shafranov equations*, derived by the pressure-balance condition on the azimuthal component:

$$(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{e_\phi} = \mathbf{\nabla}p \cdot \mathbf{e_\phi} = 0, \tag{14}$$

and the radial component:

$$(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{e_r} = \frac{\partial p}{\partial r}, \tag{15}$$

and the plasma is self-confined to the tokamak shape by means of a θ-*pinch* and a z-*pinch* effects.

However, it's worth stressing that this tokamak-like plasma structure has a basic difference with respect to the real tokamak device, that is the strong gravitational field. In fact, the plasma shape is mantained by means of the TBH gravitation, in contrast to the external magnetic field involved in tokamaks. Therefore, a strong gravitational force is likely to prevent diffusive motion, which appears in magnetic confined toroidal plasma. This gives to the system enough stability to treat it by the Grad-Shafranov equations.

In conclusion, particles spiral around the torus and eventually approach the acceleration inner region, where they are kicked out by a gravitational kick or they partake in the Fermi-type power-law distribution and flow away along the TBH axis; there, they can be trapped by the magnetic field, collimating them in a jet.

There is another process acting in such a configuration related to dynamical instabilities. Indeed, particle trajectories are supposed to experience Lindblad resonances as follows from perturbations in the gravitational potential (Binney & Tremaine 1987). Both with particle collisions and energy/momentum exchange, instabilities can act as an accretion mechanism, whenever the gravitational kick conditions are not matched. Gravitational instabilities could also influence the tokamak stability, but they should face plasma instabilities and the competition between the two fixes the final configuration. Actually, to be exhaustive and more quantitative, we should perform a hard magnetohydrodynamical (MHD) treatment of the particle motion. Since it requires a hard computational work, we cannot discuss it here; nevertheless, we recall that MHD 2D simulations have shown that turbulence in the accretion disc has a fundamental role. The matter in the region of unstable orbits does not follow simple energy and angular momentum conserving free fall trajectories, but a deep exchange is involved by means of turbulence on the way to the event horizon and also the shear components are enormous (e.g. Hawley & Krolik 2000, Igumenshchev & Abramowicz 2000 and references therein). This will help putting matter onto orbits which spiral around the torus, so that it easily reaches the inner region, where it will experience the gravitational kick.

3. The emission properties of a TBH

The huge wide-band emission from AGN is mostly explained through processes involving the accretion disk (Frank et al. 1985). Nevertheless, the most energetic emission, i.e. gamma rays, is far from being self-consistently explained: the observed gamma ray energy along the jets in quasars is close to four orders of magnitude higher than expected by theory (Spivey 2000). In particular, the mechanism proposed by Blandford & Znajek (1977) cannot account for such an amount of energy, so that it cannot be the only responsible for it. As discussed in the previous section, in the framework of a TBH a natural explanation for jet production emerges, then it is important to estimate the highest values for the Lorentz factor that particles can reach. The limit to that value is attained because of Inverse Compton (IC) and synchrotron losses (Longair 1981):

- $\gamma_{max}^{IC} \simeq \sqrt{\frac{3 \times 10^7 \omega}{U_{rad}}}$;
- $\gamma_{max}^{SYNC} \simeq \frac{\sqrt{5 \times 10^8 \omega}}{B}$;

where U_{rad} refers to the radiation field particles move in. It follows that in the approximation of a keplerian orbit at a distance corresponding to the event horizon, the maximum Lorentz factor around a TBH is increased by a factor:

$$\frac{\gamma_{max}^{TBH}}{\gamma_{max}^{SBH}} \propto \sqrt{\frac{\omega_{TBH}}{\omega_{SBH}}} \sim V^{3/4}, \tag{16}$$

for fixed values of the radiation field and the magnetic field.

As concerns the spectrum, a TBH can give rise to intense bremsstrahlung and synchrotron emission for a sufficiently high angular velocity ω attained by the spiralling particles. If we consider the aforementioned power-law distribution, then the emission due to relativistic bremsstrahlung, synchrotron radiation and IC will be (Rybicki & Lightman 1979):

- $I_{brems} \propto \nu^{-\left(1+\frac{1}{\alpha t_f}\right)}$;
- $I_{SYNC} \propto B^{\left(1+\frac{1}{2\alpha t_f}\right)} \nu^{-\left(\frac{1}{2\alpha t_f}\right)}$;
- $I_{IC} \propto \nu_0^{\left(\frac{1}{2\alpha t_f}-1\right)} \nu^{-\left(\frac{1}{2\alpha t_f}\right)}$;

where B is the embedding magnetic field and ν_0 is the injection energy.

Moreover, the Blandford-Znajek mechanism is increased by the extra magnetic field added to the embedding disc field, due to the dynamics leading to the tokamak-like plasma configuration; as a result, the power extracted increases with respect to the SBH situation by a factor of (Frank et al. 1985):

$$W_{B-Z} \propto \left(\frac{B_{TBH}}{B_{SBH}}\right)^2. \tag{17}$$

In addition, a new process could be involved in emission properties of a TBH, due to the tokamak-like plasma, that is the *sawteeth emission*, i.e. an intense periodic burst arising from the centre of a tokamak device, whose energy ranges from hard X-rays up to gamma rays (Dendy 1993). Its physical origin is still debated, but it seems to depend on magneto-hydrodynamical instabilities acting on the plasma, such that a slow increase in plasma pressure is followed by an abrupt fall, the current rapidly grows and magnetic reconnection feeds the burst.

Another emission process, that is purely related to the space-time characteristics, is the *Penrose process* (Penrose 1969). It follows from the fact that in a rotating black hole the surface on which the time component of the metric tensor vanishes, i.e. $g_{tt} = 0$, differs from the event horizon: the two surfaces enclose a region called *ergosphere* where the energy of a particle as observed from far way can be negative. The original *Penrose process* follows from the decay of a particle into two photons, one of which crosses the horizon and gets lost, the other one gains energy in the range fixed by the Wald inequality (Wald 1984):

$$\gamma E - \gamma\, v(E^2 - g_{tt})^{1/2} \le \epsilon \le \gamma E + \gamma\, v(E^2 - g_{tt})^{1/2}, \tag{18}$$

then a TBH is more efficient than a Kerr black hole in producing high-energy photons in this way, provided that $\gamma_{max}^{TBH} \ge \gamma_{max}^{SBH}$.

4. Lifetime of AGN activity

If the AGN activity is related to the toroidal shape of the black hole, a transition to a quiescent state is expected as the black hole reaches SBH status. In fact, as matter accretes the hole, the event horizon increases, the torus inflates and lately looses its starting configuration turning spherical. On the above line, we can infer a more quantitative estimate for the lifetime of the activity phase by equating the metric tensor components for a TBH and a SBH. If R_{in} is an initial radial dimension for the torus, e.g. the middle value of the torus thickness with respect to the centre of symmetry, and R_{fin} is the final radius of the SBH, then the transition happens if the following condition is matched:

$$V - \frac{2MG}{R_{in}c^2} = 1 - \frac{2MG}{R_{fin}c^2}, \tag{19}$$

or else:

$$R_{fin} = \frac{r_g}{1 - V + \frac{r_g}{R_{in}}}, \tag{20}$$

where r_g is the gravitational radius. Besides, the lifetime is related to the accreted matter ΔM and to the accretion rate $\frac{dM}{dt}$ by:

$$dt(life) \sim \frac{\Delta M}{dM/dt}, \tag{21}$$

yielding:

$$dt(life) \propto \frac{1}{dM/dt} R_{fin}^3. \tag{22}$$

Since $\frac{dM}{dt}$ is observationally estimated, we could enter the lifetime debate if only we knew the potential. Lacking that, we can only study $dt(life)$ as a function of some hypothetical functional form for it:

1. $V \simeq const$;
2. $V \propto 1/R$;
3. $V \propto log R$;

where the first case could be regarded as a cosmological vacuum energy density, while the other expressions could mimic the background potential of a surrounding axisymmetric galaxy. In particular, the second functional form refers to an embedding newtonian gravitational field; on the contrary, the logarithmic shape is motivated in order to reproduce the flatness of the galactic rotation curve (Binney & Tremaine 1987). In this frame, the formation of a TBH can be deduced if a protogalaxy develops a sufficient extended background potential and the collapse to a massive black hole is forced to lead to a toroidal configuration[1].

In Fig. 2, the trend of $dt(life)$ is sketched for the above mentioned different potentials, assuming a constant accretion rate dM/dt. The $1/R$ potential gives a fixed value independent of R_{in}, while the $V \simeq const$ situation gives an increasing

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[1] We suggest that another way to obtain a seed TBH should follow the dynamical evolution of a black hole binary system; however, such a topic deserves a strict treatment.
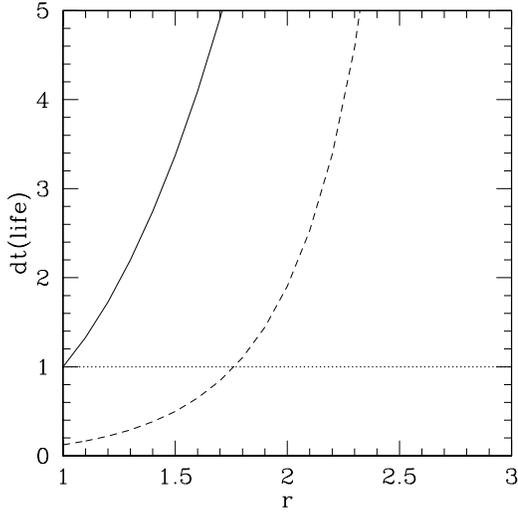
Fig. 2. AGN lifetime for a TBH model with constant accretion rate: (*solid line*) $V \simeq const$; (*short-dashed line*) $V \propto 1/R$; (*long-dashed line*) $V \propto log\ R$. The normalization has been arbitrarily fixed and $r \equiv \frac{R_{in}}{r_g}$.

function, as the $log\ R$ potential does. If the AGN lifetimes have small dispersion and are strictly focused on a single value, e.g. $10^6 - 10^7$ yr (e.g. Cavaliere & Vittorini 2000), then the $1/R$ potential seems to give a likely interpretation to it.

Last, we stress that the TBH/SBH transition is likely to be a violent event in the galaxy evolution. From a physical point of view, the galactic potential turns from being axisymmetric (TBH) and with a relevant external component (V) to a spherical configuration dominated by the central SBH. Therefore, a rapid and huge accretion should happen, the black hole swallows the most of the surrounding, so that the activity stops and a quiescent state is reached.

5. Comparison between TBH and SBH

In this section, we trace a few guidelines to emphasize why a TBH could be more convenient in explaining the AGN phenomenon than the standard SBH model.

First of all, the TBH provides a powerful mean to accelerate particles and to give them high Lorentz factors[2]; on the contrary, a SBH has more severe limitations in doing it and usually other complicated phenomena involving the surrounding galaxy and inter-stellar medium must be invoked, e.g. shocks induced by stellar ejections or hydrodynamical expansion (e.g. Rees et al. 1982). Moreover, the variety of emission, which is well described by the standard SBH model, is not restricted by a TBH model, which affects just the energetics. On the other hand, our model could explain the production of jets in a more likely way and could account for the large angle dispersion observed, as follows from the opening angle of the TBH.

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[2] For sake of completeness, we stress that such a mechanism could be efficient in other astrophysical contexts, e.g. acceleration of Ultra Relativistic Cosmic Rays.

An important new perspective with this model is the way it explains the finite lifetime of the AGN phenomenon, that is hardly interpreted by the standard scheme through a multiparameter modelling of black hole Initial Mass Function (IMF), accretion rate and other quantities (e.g. Monaco et al. 1999). Indeed, we stress the power of a TBH model in significantly simplifying the finite lifetime puzzle.

So far, we see no drawbacks for a TBH model with respect to a SBH model, but we underline how it could improve and complete the general picture of the AGN phenomenon. An argument which could be raised against our view could come from iron emission line observations, well explained in terms of being generated close to a spinning SBH, i.e. a Kerr black hole (e.g. Fabian et al. 1989, Matt et al. 1993). Nevertheless, the emission line arises several gravitational radii away from the hole, where particles could not distinguish among a SBH and a TBH. However, the difference in the relativistic gravitational shift effect should be:

$$\frac{\Delta\nu}{\nu}(TBH) = \frac{\Delta\nu}{\nu}(SBH)\left(\frac{V - 3r_g/R}{1 - r_g/R}\right)^{1/2}, \qquad (23)$$

and for $R \gg r_g$ it holds:

$$\frac{\Delta\nu}{\nu}(TBH) \propto \sqrt{V}\frac{\Delta\nu}{\nu}(SBH), \qquad (24)$$

therefore this effect is likely to be undistinguished by recent X-ray data; besides, the deep model dependence of the Kerr black hole interpretation of the iron line gives no hope for a choice among the two models, until higher resolution observation will be available by the next X-ray satellites *Chandra* and XMM.

6. Conclusions

We inferred the main properties of a TBH in contrast to the widely accepted SBH model to describe the AGN phenomenon. In particular, we found that a rapidly rotating TBH can provide more energy to feed particle acceleration and production of jets. Besides, a TBH should undergo a transition to a SBH and so to a quiescent state (if the AGN activity is a peculiarity of TBH alone), forcing nuclear activity to fade and giving a reasonable physical explanation to the lifetime of AGNs.

The only caveat of our picture is that the model relies on the presence of a potential V acting as an AdS term, whose origin cannot be due to the cosmological constant and is missing in the present discussion. The topic deserves a more strict treatment, anyhow we tend to consider it as a term due to the surrounding gravitational structure, favouring the TBH collapse. Other hints could be found in cosmology, for instance in quintessence models in which the scalar field should have non minimal coupling to gravity, or else in cosmological models allowing dark matter-dark matter coupling (e.g. Perrotta et al. 2000, Amendola 2000); nevertheless, such a cosmological origin is very unlikely, due to the extremely low value for the potential, preventing its effectiveness in acting as a collapse source.

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