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A thermokinetic study of wave–modulated solar wind electrons using truncated maxwellians

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Abstract. Classical kinetic solar wind theories reveal that the solar wind electron distribution function has a delicate influence on the acceleration of the coronal solar wind and its asymptotic velocity. In collisionless kinetics this results from the fact that no hyperbolic electrons with sunward velocities can be expected above the coronal exobase. This problem affects both the associated electron pressure and the effective electric polarisation potential helping protons to leave the solar gravitational potential. The actual electron distribution thus influences the asymptotic properties of the solar wind flow. Of importance for a comprehensive understanding of the solar wind acceleration thus is the actual mechanism to populate the sunward velocity branch of the distribution function in the hyperbolic energy regime which according to classical kinetic theory should be unpopulated. In view of the expected electron temperatures at the corona one finds that 30 to 50 percent contributions to the dynamic electron pressure results from this questionable regime. We study the influence of electron depletions in the sunward hemisphere of velocity space in terms of modified velocity moments like the resulting solar wind electron temperature, bulk velocity and heat flow. Parametrizing the electron distribution function by truncated Maxwellians we show that all higher moments of the distribution function can be generated based on knowledge of the three lowest moments. Using solar wind data on electron density, drift, and temperature, we derive an expression for the electron heat flow which perfectly fits the ULYSSES heatflow measurements both by its absolute magnitude and by its radial gradient. To justify truncated electron distribution functions by physical processes we also study the effect of an energy dissipation of fast magnetosonic waves cascading up to the range of whistler frequencies and consider the specific local heat source due to absorption of such wavepowers. An adequate representation of the electron temperature profile without the account of a heating due to wave energy transfer to solar wind electrons may not be achievable at regions beyond 1 AU. As we can also show wave-induced energy absorption occurs just with the adequate rate allowing for truncated Maxwellian electron distribution functions to be maintained in the expanding solar wind.

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1. Introduction to the exospheric view of the corona

Exospheric theories of the solar wind by Chamberlain (1960), Aamont & Case (1962), Jensen (1963), Brandt & Cassinelli (1966) revealed that above the coronal exobase a simple Maxwellian distribution can not be expected for either electrons or ions. Since in these theories the lower corona is considered to be the only source of particles, in a collisionless regime above the coronal exobase there are no particles which populate the sunward hyperbolic part of the velocity space. This part can be estimated by the so–called Pannekoek–Rosseland ambipolar polarisation potential impeding electrons from leaving the corona and given by:

$$\Phi(r) = -\frac{m_p - m_e}{2e}\frac{GM_0}{r_0}\left(1 - \frac{r_0}{r}\right). \tag{1}$$

Here m_p and m_e are the proton and electron masses, M_0 and r_0 are the mass and the radius of the sun, and G is the gravitational constant. No electrons placed in this impeded part of velocity space should exist (e.g. see Fahr & Shizgal 1983). Also their contribution to the total electron pressure, calculated on the basis of a full Maxwellian, is missing. Since these contributions amount to between 10 and 35 percent (Fahr et al. 1997), the question poses itself as to how much this pressure deficit could influence the effective solar wind acceleration and the solar wind thermodynamics. Studies of this influence were undertaken by Fahr et al. (1990), Pierrard & Lemaire (1996), and Meyer-Vernet & Issautier (1998).

The Pannekoek–Rosseland potential guarantees charge neutrality in a stationary solar corona, i.e it applies in the case of a vanishing solar wind outflow. In case of an expanding corona a selfconsistent polarisation field has to be found (see Sen 1969; Jokers 1970; Hollweg 1974, Lemaire & Scherer 1970, 1971, or for a review Fahr & Shizgal 1983). The effect of this pressure deficit on the solar wind dynamics can already be studied at least qualitatively in the work by Jokers (1970) showing that higher asymptotic solar wind velocities are associated with higher effective electron potential ramps. This forces the electron distribution function to better assimilate a full Maxwellian and thus reduces the electron pressure deficit. So far no selfconsistent polarisation field has been calculated in a satisfactorily consistent manner within exospheric corona theories with the inclusion of electrons in the hyperbolic as well as the elliptic branch, i.e. satellite particle branch. This is mainly because selfconsistency can only be achieved by inclusion of elliptic or trapped electron particle populations as was shown e.g. by Jokers (1970). These populations, however, are established by electrons suffering scattering processes above the exobase and are due to electrons which are trapped between some outer potential well and some inner magnetic mirror point. Within the frame of pure exospheric theory these elliptical electrons consequently should not exist at all, making the concept of an exospheric solar wind problematic, if not, at present, unviable.

The features of the observed electron distributions cannot be represented by collisionless exospheric concepts (Olbert 1983) since the validity of magnetic invariants induces much too anisotropic distribution functions. Introducing a collision–induced relaxation term of a BGK-type (Bhatnagar, Gross, Krook 1954) Olbert could show that the electron distribution with increasing solar distances develops into a so–called Strahl–configuration. As shown by Griffel & Davies (1969), Scudder & Olbert (1979a, 1979b) or Fahr & Shizgal (1983). Coulomb collisons cannot impede the electron distribution functions from degenerating into highly anisotropic functions. Only wave-particle interactions by electron Whistler waves (see Dum et al. 1980, Gary et al. 1994) or excited plasma instabilities of the firehose type may help as a remedy at larger distances (see e.g. Fahr & Shizgal 1983).

This is pointed out by ULYSSES measurements of electron distribution functions between 1 and 5 AU (Scime et al. 1994). As proven by these data, the inherent electron heat conduction flow drops off with increasing solar distance much faster than expected from a collisionfree expansion of solar wind electrons, although the magnitude of the heat conduction flow is much smaller than derived from the electron temperature gradient using the Spitzer-Härm value for the heat conduction coefficient (Spitzer & Härm 1953; Spitzer 1962). This also seems to prove that even at distances beyond 1 AU where Coulomb collisions are absolutely inefficient the solar wind electron plasma does not behave as “collisionfree” in a strict sense, but there seems to be an effective mechanism (like pitch–angle scattering and isotropization) operating which helps redistributing the energy in the heat conduction flow to randomized electron thermal energy. Not knowing, how to describe such a dissipation mechanism quantitatively, neither a selfconsistent polarisation potential nor an adequate electron pressure and temperature can be calculated. We therefore in the following develop an approximative method to study the effect of truncated electron distribution functions on associated electron pressure deficits and heat conduction flows

2. Solar wind electron hydrodynamics based on truncated Maxwellians

Solar wind electrons are tightly bound to interplanetary magnetic field lines. Thus their magnetic moments essentially behave as invariants of the motion since over electron gyroscales or gyroperiods fluctuating magnetic fields can hardly exert any influence. This also enforces the local electron distribution function to be closely associated to regions of magnetically conjugated footpoints in the corona. Down here of course collison–dominated conditions prevail and quasi–Maxwellian distributions are easily established. Since from such points electrons are emitted to the associated upper space point with a rate defined by the antisunward branch of the coronal Maxwellian, one may find electrons at the upper space point connected by Liouville theorem to the lower corona. (e.g. see papers by Lemaire & Scherer 1971, 1973; Lie-Svendsen et al. 1997; Pierrard & Lemaire 1996; Meyer-Vernet & Issautier 1998; Meyer-Vernet 1999). This distribution at an upper space point, however, without relaxation processes in operation above the coronal exobase, clearly has to differ from a full Maxwellian, because that specific velocity space volume representing trapped particles (i.e. with mirror points above the coronal exobase) in a collision-free treatment should be unpopulated. The rest of the upward velocity space would, however, be populated according to a Maxwellian at the exobase. In contrast that branch of the distribution function, describing electrons moving downwards towards the lower conjugated footpoint, evidently would be absent in just those velocity space volumes belonging either to trapped or to hyperbolic particles (i.e. particles with energies larger than the effective escape energy). These latter electrons have energies enabling them to overcome the remaining polarisation potential ramp $\Delta\Phi_\infty$ and escape irreversibly from the heliosphere. Hence such electrons cannot be expected to approach the corona from the top, if not especially generated by other sources further out in the heliosphere.

Collisionfree electrons moving upward from the corona are subject to gravitational forces, electric forces connected with the polarisation potential Φ and magnetic forces due to the magnetic field divergence and to the electron magnetic moment μ. This allows one to integrate the equation of particle motion yielding two invariants of the motion, namely the total energy H_0 and the invariant magnetic moment μ_0. These invariants lead to the following relations:

$$\left[\frac{1}{2}m_e v_\parallel^2 - e\Phi + \mu_0 B\right]_r = H_0, \tag{2}$$

and:

$$\left[\frac{\frac{1}{2}m_e v_\perp^2}{B}\right]_r = \mu_0. \tag{3}$$

Here $v_\parallel$ and $v_\perp$ are the electron velocity components parallel and perpendicular to the magnetic field $\boldsymbol{B}$, and $\Phi(r)$ is the effective polarisation potential. From these relations one derives that the pitch angle θ of an ascending electron (i.e $\theta > 0$) varies according to:

$$\tan^2(\theta) = \tan^2(\theta_0)\frac{B}{B_0}\left[1+\frac{2e\Delta\Phi}{m_e v_{\|0}^2} - \tan^2(\theta_0)\frac{\Delta B}{B_0}\right]^{-1}, \quad (4)$$

where “0” characterizes the respective quantities at the coronal exobase (i.e. at $r = r_0$), and where the definitions: $\Delta\Phi = \Phi - \Phi_0$ and $\Delta B = B - B_0$, have been used. At points r_∞ far above the exobase, where $\Delta B_\infty \simeq -B_0$, Eq. (4) yields:

$$\tan^2(\theta_\infty) \simeq \tan^2(\theta_0)\frac{B_\infty}{B_0}\left[1+\frac{2e\Delta\Phi_0}{m_e v_{\|0}^2} + \tan^2(\theta_0)\right]^{-1} \quad (5)$$

where $\Delta\Phi_0 = \Phi_\infty - \Phi_0$ denotes the potential difference between the coronal base and the asymptotic potential at infinity (i.e at large distances $r \to r_\infty$). With relations (2) to (5), astonishingly enough, one already determines the asymptotic solar wind electron flux $[n_e u_e]_\infty$ which in a quasi-neutral, stationary solar wind also fully determines the asymptotic solar wind mass flow $\Psi = m_i[n_e u_e]_\infty$

Assuming an upward emission of electrons from the exobase according to a Maxwellian one formally arrives at the following expression:

$$\Psi_\infty = m_i\left(\frac{r_0}{r_\infty}\right)^2 n_{e0}\left[\frac{kT_{e0}}{2\pi m_e}\right]^{\frac{1}{2}}(1+\Xi_0)\exp(-\Xi_0), \quad (6)$$

where $\Xi_0 = e\frac{\Delta\Phi_0}{kT_{e0}}$. Knowledge of the value of Ξ_0 would thus already completely determine the resulting solar wind mass flow Ψ_∞.

Some attempts have been made by Meyer-Vernet & Issautier (1998) and Meyer-Vernet (1999) to estimate Ξ_0 purely on the basis of solar coronal plasma properties, simply fulfilling requirements of quasineutrality and vanishing electrical currents, and have led them to the result:

$$5 \leq \Xi_0 \leq 28, \quad (7)$$

dependent on the value taken for κ if electrons at the coronal base $r = r_0$, are described instead by Maxwellians by so–called Kappa functions like:

$$f_{e0} = \frac{n_{e0}A_k(\kappa)}{2\pi(\kappa w^2)^{\frac{3}{2}}}\left[1+\frac{v^2}{\kappa w^2}\right]^{-(\kappa-1)}, \quad (8)$$

with:

$$A_k(\kappa) = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})\Gamma(\frac{3}{2})} \quad (9)$$

For the Maxwellian case, i.e. $\kappa \to \infty$, the value $\Xi_0 = 5$ is obtained, whereas for electron distributions with pronounced high–energy tails, i.e. with $\kappa = 3$, a value of $\Xi_0 = 28$ is obtained, leading to asymptotic solar wind velocities of 250 km s^{-1} ($\kappa = \infty$) and 700 km s^{-1} ($\kappa = 3$), respectively. Though this clearly reveals that the solar wind phenomenon is highly sensitive to the escape branch of the electron velocity distribution function in the upward hemisphere of velocity space, it nevertheless does not help to find consistent solar wind solutions.

As pointed out by Jockers (1970), the correct polarisation potential and thus the value of Ξ_0 can only be determined from a consistent knowledge of the solar wind dynamics which is closely connected with a consistent description of trapped electrons unfortunately not emanating from collision–free theories. Thus some relaxation processes of a wave–particle interaction type have to operate above the coronal base to explain observed solar wind quantities. In our view here these processes are due to quasilinear interactions of electrons with preexisting, convected whistler wave turbulences (see Secs. 6 through 8). Up to now trapped particles could not be described adequately by purely kinetic solar wind theories. On the other hand solar wind hydrodynamics can not account for truncated electron distributions. Hence we develop here a semi- hydrodynamic theory parametrizing the electron distribution in a kinetically motivated way permitting the calculation of all relevant velocity moments needed for a hydrodynamic view.

We assume that by wave–induced relaxation operating above the exobase the genuine Liouvillean velocity distribution is converted into one describing a macroscopic drift and the appearance of trapped particles on the basis of quasi–Maxwellians. We assume that the local electron distribution function $f_e(\boldsymbol{r},\boldsymbol{v})$ can be approximated by a truncated, shifted Maxwellian perhaps with a constant shift of U_0 with respect to the solar rest frame (SF). By some recalibration of the potential this drift can as well, however, also be set equal to $U_0 = 0$, since truncated Maxwellians define an inherent drift. Then with a density normalization $n(r)$ and a local thermal electron velocity spread C(r) the distribution is simply given by:

$$\begin{aligned} f_e(\boldsymbol{r},\boldsymbol{v})d^3v &= n(r)[\pi C^2(r)]^{-\frac{3}{2}}\{H(v_{\max}-v) \\ &+ H(\cos(\theta))H(v-v_{\max}(r))\} \\ &\times \exp\left[\frac{-v^2}{C^2(r)}\right]v^2 dv\sin(\theta)\,d\theta\,d\phi, \end{aligned} \quad (10)$$

Here polar velocity coordinates v, θ, ϕ have been used, the polar axis being identical with the magnetic field direction. Hence θ then automatically also represents the electron pitch-angle. The step–functions $H(X)$ have to take into account the appropriate local truncation with the effect of suppressing hyperbolic electrons in the sunward magnetic hemisphere of velocity space. The quantity $v_{\max}$ defines the local escape velocity of electrons, i.e. $v_{\max}^2 = \frac{2e\Delta\Phi(r)}{m_e}$.

It is evident that, due to this truncation, the functions $n(r)$, U_0 and $T(r) = \frac{m_e C^2(r)}{2k}$ in general are not strictly identical to the local density $n(r)$, bulk velocity $U(r)$ and temperature $T(r)$. They, in contrast, have to be obtained as moments of the above distribution function $f_e(r,v)$ by integration over velocity space and thus represent space–variable functions. First, the following relation must be valid:

$$\begin{aligned} &n_e(r) = n_i(r) \\ &= n(r)\int\int\int \quad [H(2e\Delta\Phi - m_e v'^2)+ \end{aligned} \quad (11)$$

$$H(\cos(\theta))H(m_e v'^2 - 2e\Delta\Phi(r))\big] \times \mathrm{Max}(v', r) d^3v'$$

where the potential difference, $\Delta\Phi = \Phi_\infty - \Phi$, has been introduced and where $H(X)$ are step functions (i.e. $H(X) = 1$ for positive arguments X, and $H(X) = 0$ for negative arguments X). $\mathrm{Max}(v, r)$ is the Maxwellian with a velocity dispersion $C(r)$. Individual electron velocities are denoted by v'. $\Phi(r)$ is the effective electron polarisation potential and $\Delta\Phi$ its difference with respect to that of infinity.

Eq. (11) can be evaluated yielding the following relation between $n(r)$ and $n_i(r)$:

$$n(r) = n_i(r)\left[\frac{4}{\sqrt{\pi}}\left(S2(0) - 0.5S2(\lambda)\right)\right]^{-1}. \tag{12}$$

Here the function $S2(x)$, for $j = 2$, is defined by the following integral function:

$$Sj(x) = \int_x^\infty c^j \exp(-c^2) dc, \tag{13}$$

The quantity $\lambda = \lambda(r)$ in Eq. (12) has the following definition:

$$\lambda = \left[\frac{e\Delta\Phi(r)}{m_e C^2(r)}\right]^{\frac{1}{2}} \tag{14}$$

Next we calculate the electron bulk velocity $\boldsymbol{U}(r)$. The truncated Maxwellian directly determines the radial solar electron flux or the solar wind proton flux in the form

$$\begin{aligned} n_e(r)U_{er}(r) &= n_i(r)U_{ir}(r) \\ &= n(r)\cos(\xi)\int\int\int\big[H(2e\Phi(r) - m_e v'^2) + H(\cos(\theta)) \\ &\quad \times H(m_e v'^2 - 2e\Phi(r))\big]\ \mathrm{Max}(v', r) v' \cos(\theta') d^3v' \\ &= n(r)\cos(\xi)\int\int\int\big[H^+ + H(\cos(\theta))H^-\big] \times \\ &\quad \mathrm{Max}(v', r) v' \cos(\theta') d^3v' \end{aligned} \tag{15}$$

where H^+ and H^- have been introduced as abbreviations with meanings evident by comparison with Eq. (10). The quantity $\cos(\xi)$ takes into account the local tilt by an angle ξ of the Archimedian spiral field with respect to the radial direction. Expression (15) can be evaluated to yield:

$$n_e(r)U_r(r) = \pi^{-\frac{1}{2}} C(r) n(r) \cos(\xi) S3(\lambda), \tag{16}$$

where $S3(x)$ is defined according to Eq. (13) for $j = 3$. It needs to be mentioned that the electron bulk is not moving in radial direction but into the direction of the local magnetic field with an electron bulk speed of $U_B = \pi^{-\frac{1}{2}} C(r) S3(\lambda) \frac{n(r)}{n_e(r)}$

Next we calculate the electron pressure and find accordingly:

$$P_e(r) = \frac{1}{6} m_e n(r) \int\int\int \big[H^+ + H(\cos(\theta))H^-\big] \mathrm{Max}'(v', r) w'^2 d^3v' \tag{17}$$

where w' is the electron velocity measured in the electron bulk flow frame (EBF) locally moving with the electron bulk velocity $U_B(r)\frac{\boldsymbol{B}}{B}$. Thus w' is related to the electron velocity v' by:

$$w'^2 = v'^2 - 2v' U_B \cos(\theta') + U_B^2. \tag{18}$$

The individual electron velocity has a tilt with respect to the radial direction given by: $\cos(\gamma') = \cos(\xi)\cos(\theta') + \sin(\xi)\sin(\theta')\cos(\phi')$.

Reminding oneself of the symmetry conditions of $f_e(r, v, \theta)$ the expression (17) evaluates to:

$$P_e(r) = \frac{2m_e C(r) n(r)}{3\sqrt{\pi}} \times \left[\left[S4(0) - 0.5S4(\lambda)\right] + \left(\frac{U_B(r)}{C(r)}\right)^2 \left[S2(0) - 0.5S2(\lambda)\right]\right] \tag{19}$$

Here again $S4(x)$ is calculated according to Eq. (13) for $j = 4$.

Of great interest for the thermodynamics and magnetohydrodynamics of the solar wind expansion is the electron heat conduction flow $\boldsymbol{q}_e$ which on the basis of the parametrized distribution function is represented by:

$$\boldsymbol{q}_e(r) = \frac{1}{2} m_e n(r) \int\int\int \big[H^+ + H(\cos(\theta))H^-\big] \times \mathrm{Max}(v', r)[\boldsymbol{v'} - \boldsymbol{U}](\boldsymbol{v'} - \boldsymbol{U})^2 d^3v' \tag{20}$$

and evidently is oriented purely parallel to the local magnetic field $\boldsymbol{B}$, i.e. $\boldsymbol{q}_e = \boldsymbol{q}_{eB}\frac{\boldsymbol{B}}{B}$. Again due to symmetry reasons expression (20) simplifies to yield the following modulus of heat conduction flow:

$$\begin{aligned} q_{eb}(r) &= \frac{1}{2} m_e n(r) \left(\pi C(r)\right)^{-\frac{3}{2}} \times \\ &\int\int\int \big[H^+ + H(\cos(\theta))H^-\big]\ \mathrm{Max}(v', r) \times \\ &\big[v'^2 - 2v' U_B \cos(\theta') + U^2\big]\left[v'\cos(\theta') - U_B\right] d^3v'. \end{aligned} \tag{21}$$

The latter expression can be evaluated and finally yields the following form:

$$\begin{aligned} q_{eb} &= \frac{1}{2} n_e m_e C^3(r) \left[\frac{S5(\lambda)}{4[S2(0) - 0.5S2(\lambda)]} + \right. \\ &\left. \frac{S3(\lambda)[S4(0) - 0.5S4(\lambda)]}{12[S2(0) - 0.5S2(\lambda)]^2} + \frac{S3^3(\lambda)}{64[S2(0) - 0.5S2(\lambda)]^3}\right] \\ &= \frac{1}{2} n_e m_e C^3(r)\Psi(r). \end{aligned} \tag{22}$$

By the use of truncated Maxwellians one is thus able to represent the heat conduction flow q_e as a functional of the lowest three velocity moments of this distribution, namely $n_e(r)$, $P_e(r)$ and $U_e(r)$, and thus reach a closed hydrodynamic system of governing differential equations.

3. Estimated effect of truncated distribution functions

Now we investigate a representation of solar wind electron pressures and heat conduction flows by means of truncated

Maxwellians and compare these expressions with observational results.

We study first the effect of the truncated Maxwellians on solar wind electron pressure. Adopting an effective electron polarisation potential $\Phi_e(r)$ we can easily calculate the resulting electron pressure associated with a truncated Maxwellian derived from Eq. (19) which leads to:

$$P_e(r) \; = \; \left(2m_e C^2(r)\frac{n(r)}{\sqrt{\pi}}\right) \times \qquad (23)$$
$$\left[[S4(0)-0.5S4(\lambda)] + \bar{U}^2\left[S2(0)-0.5S2(\lambda)\right]\right],$$

where λ was given in Eq. (11) and $\bar{U} = \frac{U_B}{C}$ was introduced. In view of the highly subsonic character of the solar wind electrons in regions inside 20 AU (i.e. $\bar{U} \ll 1$) the second term in the outer bracket is of second order in magnitude and estimate purposes for may be neglected here. Using the relation between $n(r)$ and $n_e(r)$ given by Eq. (9) one then obtains the following expression:

$$P_e(r) \; = \; \left[m_e C^2(r)\frac{n(r)}{2}\right] \times \qquad (24)$$
$$[S4(0)-0.5S4(\lambda)]\left[S2(0)-0.5S2(\lambda)\right]^{-1}$$
$$= \; \epsilon_e(r)P_e^0(r),$$

where the $\epsilon(r)$ describes the pressure reduction with respect to the classical pressure $P_e^0(r)$ resulting from an untruncated Maxwellian. It must be concluded that $P_e^0(r)$ is obtained from Eq. (23) for a potential barrier increased to infinite height, i.e. for $\Phi_e \to \infty$, or $\lambda \to \infty$. Realizing that $S2(x \to \infty) = S4(x \to \infty) = 0$, one thus arrives at the following expression for $\epsilon_e(r)$:

$$\epsilon_e(r) \; = \; [S4(0)-0.5S4(\lambda)]\left[S2(0)-0.5S2(\lambda)\right]^{-1}\frac{S2(0)}{S4(0)} \qquad (25)$$

In Figs. 3 and 4 of Fahr et al. (1997) it is demonstrated what effect a truncation of the Maxwellian has on the electron pressure. While in Fig. 3 of this paper the function $\epsilon_e(r)$ itself is shown, Fig. 4 displays the ratio $\Delta_e(r)$ of the pressure gradients dP_e/dr and dP_e^0/dr. In both cases it is evident that a physically motivated truncation of the Maxwellians not only reduces the effective electron pressure but also its gradient which represents an important force term in the equation of motion of the magnetohydrodynamic solar wind as already analyzed in quantitative terms by Fahr et al. (1990). Here as evident from the work of Meyer-Vernet & Issautier (1998) we again confirm the importance of the escape branch of the electron distribution function for the global solar wind dynamics.

4. Calculation of the electron heat conduction flow

Now, we test the effect of the newly formulated heat conduction flow as given in Eq. (19) on the distance-dependence of the electron temperature. With the expression (19) we have obtained:

$$q_{eB} = \frac{1}{2}n_e m_e C^3(r)\left[\frac{S5(\lambda)}{4[S2(0)-0.5S2(\lambda)]} + \right. \qquad (26)$$
$$\left. \frac{S3(\lambda)[S4(0)-0.5S4(\lambda)]}{12[S2(0)-0.5S2(\lambda)]^2} + \frac{S3^3(\lambda)}{64[S2(0)-0.5S2(\lambda)]^3}\right]$$

With Eqs. (9) and (13) we can now remove either the function $C(r)$ or the function λ from the above formula using the following relation:

$$C(r) = \frac{\pi U_r(r)\left[S2(0)-0.5S2(\lambda)\right]}{4\cos(\xi)S3(\lambda)} \qquad (27)$$

and finally then obtain the heat conduction flow as a function of the solar wind bulk velocity and the argument λ.

To further evaluate expression (26) and compare results with observational data we first derive an expression for λ evaluating the function $\Phi(r)$ or $\Delta\Phi(r)$, respectively, in as a consistent form as possible.

For that purpose, we start out from the generally accepted requirement that the hydrodynamical forces acting upon solar wind electrons, due to practical absence of inertial and gravitational forces, should cancel eachother leading in the CGL approximation for anisotropic pressure functions to the following expression (for a general derivation see Fahr et al. 1977):

$$0 = en_e\frac{d\Phi}{dz} - \frac{dP_{\parallel e}}{dz} - \frac{1}{B}\frac{dB}{dz}\left(P_{\perp e} - P_{\parallel e}\right) \qquad (28)$$

where z is the space coordinate parallel to the field $\boldsymbol{B}$, and where $P_{\perp e}$ and $P_{\parallel e}$ are the electron pressure tensor elements perpendicular and parallel to $\boldsymbol{B}$. For a purely radial field we can replace the space coordinate z by r and obtain:

$$\frac{d\Phi}{dr} = \frac{1}{en_e}\left[\frac{dP_{\parallel e}}{dr} + \frac{2}{r}(P_{\parallel e} - P_{\perp e})\right]. \qquad (29)$$

In this form this relation has also been used by Fichtner & Fahr (1991) or Meyer-Vernet & Issautier (1998). Here, however, we shall evaluate this expression in more detail making use of the parametrized form of the distribution function by a truncated Maxwellian given in Eq. (7). As evident from expression (23) for the scalar pressure P_e one can easily also derive analogously the following relations:

$$P_{\parallel e} = \frac{1}{3}P_e \qquad (30)$$

and:

$$P_{\perp e} = \frac{2}{3}P_e. \qquad (31)$$

With these relations we thus obtain from Eq. (29):

$$\frac{d\Phi}{dr} = \frac{1}{3en_e}\left[\frac{dP_e}{dr} - 2\frac{P_e}{r}\right] = \frac{1}{3en_e}\left[r^2\frac{d}{dr}\left(\frac{P_e}{r^2}\right)\right] \qquad (32)$$

Integrating this expression by parts then yields:

$$\frac{3e}{k}\Delta\Phi(r) = [T_e(r) - T_{e\infty}] - 4\int_r^{r_\infty} T_e\frac{dr}{r}. \qquad (33)$$

The outer border of the integration is hereby placed at a distance $r = r_\infty$ where the asymptotic level of the electric potential is achieved. Also the $(1/r^2)$–drop–off of the electron density at larger distances (i.e. $r \geq 1$ AU) has been used here.

Assuming, in addition, that the electron temperature drop–off can be represented in a satisfactorily accurate way by $T_e = T_e(r/r_0)^{-\alpha}$ (see e.g. observations by Scime et al. 1994) one finally obtains:

$$\lambda^2(r) = \frac{e\Delta\Phi}{kT_{er}} = \frac{1}{3}(1+\frac{4}{\alpha})\left[1-\frac{T_{e\infty}}{T_{er}}\right] = \frac{4+\alpha}{3\alpha}\left[1-\frac{T_{e\infty}}{T_{er}}\right] \tag{34}$$

To further evaluate Eq. (34) we have to define an adequate point r_∞. Hereby one should pay attention to the following: The above derivation because of the neglect of inertial forces can only be used in the region where solar wind electrons are still subsonic, i.e. inside a region where electron temperatures are larger than a critical value given by:

$$T_{ec} = \frac{m_e U_e^2}{\gamma_e} \tag{35}$$

with $\gamma_e = (f+2)/f$ being the ratio of electron heat capacities which for electrons bound to the magnetic field (i.e. $f = 1$) yields $\gamma_e = 3$. With $U_e = 450\,\mathrm{km\,s^{-1}}$ one thus finds $T_{ec} = 4.33\,10^3$ K connected with a critical definitions of λ by:

$$\lambda_c(r) = \left[\frac{4+\alpha}{3\alpha}\left[1-\frac{T_{ce}}{T_e}\right]\right]^{\frac{1}{2}}. \tag{36}$$

Using an adequate electron temperature profile taken from observations one now can evaluate the expression (26) for the heat conduction flow and compare it with observational data on $q_e(r)$.

At larger distances ($r \geq 0.7$ AU) $U = U_{er} = U_i$ can be taken as constant and thus the solar wind density drops off like $n_e(r) = n_{e0}(r_0/r)$. Solar wind electron temperatures $T_e(r)$ can for instance be obtained with the help of ULYSSES results published by Scime et al. (1994). These authors give temperatures separately for core (T_{ec}) and halo (T_{eh}) electrons in the following form:

$$T_{ec}(r) = 1.3\,10^5\left(\frac{r}{r_e}\right)^{-0.85}\ [\mathrm{K}] \tag{37}$$

and:

$$T_{eh}(r) = 9.2\,10^5\left(\frac{r}{r_e}\right)^{-0.38}\ [\mathrm{K}]. \tag{38}$$

Since the typical abundances of core and halo electrons were found to be (see Feldman et al. 1975):

$$A_c \simeq 0.96 \;\mathrm{and} A_h \simeq 0.04, \tag{39}$$

for our purposes here, due to the lack of any better information, one may thus reasonably well represent the effective electron temperature by the following combined expression:

$$T_e(r) = A_c T_{ec}(r) + A_h T_{eh}(r). \tag{40}$$

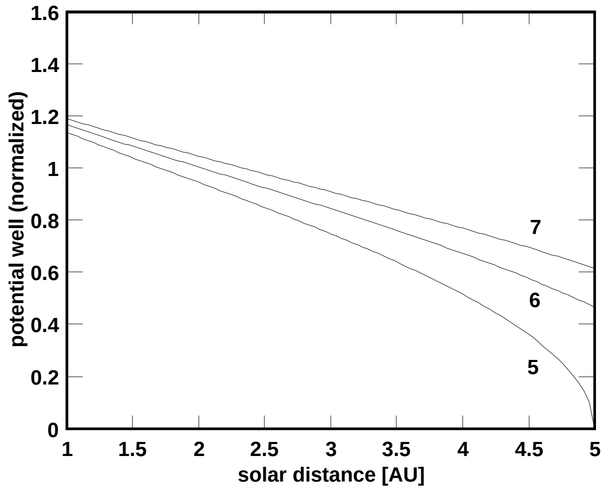

Fig. 1. Shown is the quantity $\lambda(r)$ (i.e. the electric potential energy difference to the asymptotic point r_∞ normalized by the thermal energy of the local electrons) for various values of r_∞ (i.e. 5, 6, 7 AU).

Adopting, however, this electron temperature profile and looking for the point where $T_e(r_\infty) = 4.3\,10^3$ Kelvin would be achieved, one would get the unreasonable result: $r \geq 1000$ AU.

Here we want to restrict ourselves to regions with measured electron temperatures, i.e. to 0.3 to 5.0 AU. Hence, we decide to finally define the quantity $\lambda(r)$ by:

$$\lambda(r) = \left[\frac{4+\alpha}{3\alpha}\left[1-\frac{T_e(5)}{T_e(r)}\right]\right]^{\frac{1}{2}}. \tag{41}$$

Assuming that the asymptotic level of the electric potential is already reached there.

In Fig. 1 we have displayed the quantity $\lambda(r)$ (i.e. the normalized potential step to the asymptotic plateau level as function of the solar distance r for various values of r_∞ = 5, 6, 7 AU. In addition in Fig. 2 we have shown, how the relevant integral functions $S2$, $S3$, $S4$, $S5$ needed in expression (26) to calculate the electron heatflow $q_{er}(r)$ vary with solar distance r, with $\lambda(r)$ defined by Eq. (41). In this figure the value $r_\infty = 5$ AU has been used.

5. The magnitude and radial gradient of the electron heat flow

Now we evaluate the expression (26) on the basis of the above expressions for T_e and λ given in Eqs. (38) and (39). Furthermore one may realize that $C(r)$ as measure of the velocity dispersion in our parametrized approach simply is a measure of the logarithmic slope of the electron distribution function, i.e. $-\frac{2}{m_e}C^2(r) \simeq \frac{dln(f)}{dE} = -\frac{1}{kT}$, in just the same way how the electron temperature is determined from the measured electron distribution function by Scime et al. (1994). This suggests clearly that this parameter function $C(r)$ can be set equal to:

$$C^2(r) \simeq \frac{2kT_e(r)}{m_e}. \tag{42}$$

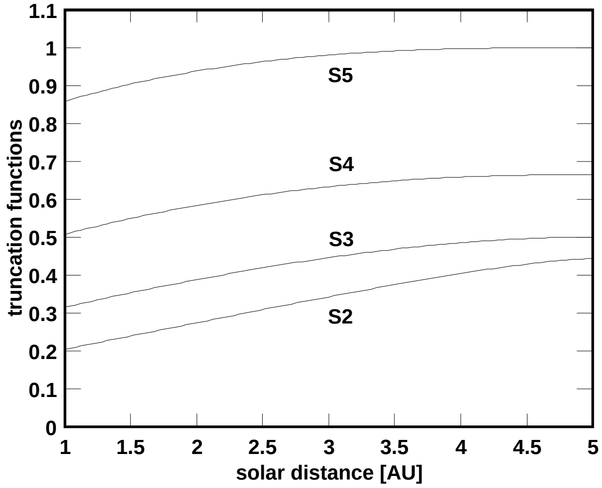

Fig. 2. Shown are the heatflow–relevant integral functions $S2(r)$, $S3(r)$, $S4(r)$ and $S5(r)$ as functions of the solar distance r for a boundary value of $r_\infty = 5$ AU.

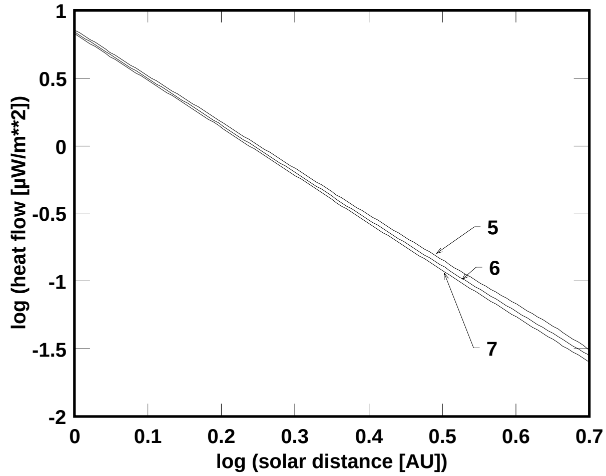

Fig. 3. Shown is the electron heat flow $q_e(r)$ in units of $[\frac{\mu W}{m^2}]$ as function of the solar distance r for various values of r_∞ (i.e 5, 6, 7 AU).

Thus, we evaluate the expression (26) for the heat conduction flow by:

$$\begin{aligned} q_{er}(r) &= q_{eb}(r)\cos(\xi) \\ &= \frac{1}{2} n_e m_e \cos(\xi) \left[\frac{2kT_e(r)}{m_e}\right]^{\frac{3}{2}} \Psi(r). \end{aligned} \tag{43}$$

With Eqs. (38) and (39) we can then numerically evaluate expression (43) and obtain with $n_e(r = 1\mathrm{AU}) = 8\mathrm{cm}^{-3}$ and $\cos(\xi) = 0.7$ at $r = 1$AU:

$$q_{er}(r = 1\mathrm{AU}) = 7.76 \frac{\mu W}{m^2} \tag{44}$$

which surprisingly enough is just the order of the heat conduction flow found by Scime et al. (1994) (i.e $8.8\frac{\mu W}{m^2}$). It is also consistent with values of between 5.0 to 8.0 $\frac{\mu W}{m^2}$ given by Feldman et al. (1975) and Pilipp et al. (1989).

In addition here we are interested in the study of the radial gradient of the electron heat flow which is also measured by Scime et al. (1994) with ULYSSES at its in-ecliptic itinerary to Jupiter. On this in–ecliptic itinerary mission ULYSSES predominantly was embedded in low speed solar wind (see Bame et al. 1993) with an average speed of $U = U_e = 400\,\mathrm{km\,s^{-1}}$ and average density of $n_e(r = 1\mathrm{AU}) = 8\,cm^{-3}$. Evaluating Eq. (38) for these above conditions and setting:

$$\cos(\xi) = \left(1 + \tan^2(\xi)\right)^{-\frac{1}{2}} = \left(1 + \left(\frac{r\Omega_0}{U_e}\right)^2\right)^{-\frac{1}{2}}, \tag{45}$$

we obtain the function $q_{er}(r)$ which is displayed as a function of the solar distance r in Fig. 3. As one can see from the linear curve appearing in the double–logarithmic plot of this figure the heat flow $q_{er}(r)$ behaves exactly like a power law in the radial coordinate r given by:

$$q_{er}(r) \simeq q_{er}(r = 1\ \mathrm{AU}) \left(\frac{r}{r_E}\right)^{-\gamma_e}, \tag{46}$$

where the exponent γ_e evaluates to $\gamma_e = 3.08$. This again, is a very nice result since it nearly exactly fits the result derived from ULYSSES solar wind electron observations (see Scime et al. 1994) yielding:

$$q_{er}(r) \simeq 8.8 \left(\frac{r}{r_E}\right)^{-3} \frac{\mu W}{m^2}. \tag{47}$$

As evident from the additional curves given in Fig. 3 it can be recognized that a variation of the value r_∞ plays a very inferior role for the result. Thus it seems as if, with our parametrized solar wind electron distribution function, we do solve two outstanding problems in the thermodynamic behaviour of solar wind electrons at larger distances.

1) The theoretically obtained magnitude of the electron heat flow is much smaller than that expected from the classical Spitzer–Härm theory (Spitzer & Härm 1953) on the basis of a so–called Fourier law with: $q_{er}(r) = -\kappa_{er}\frac{dT}{dr}$.

and:

2) The gradient of $q_{er}(r)$ obtained from the above theory is larger than expected for a normal collisionless expansion of solar wind electrons ($\gamma = 2$), but, interestingly enough, is exactly equal to the gradient found by ULYSSES observations (i.e. $\gamma = 3$).

This also means that in our parametrizing approach it is automatically arranged that free thermal solar wind electron energies are locally dissipated and thus represent a local energy source given by:

$$\frac{1}{r^2}\frac{d}{dr}(r^2 q_{er}(r)) = S_e(r) = \frac{\gamma_e - 2}{r_E} q_{er}(r) \left(\frac{r}{r_E}\right)^{-1} \tag{48}$$

This energy dissipation is enforced in our approach by the assumption of electron distributions which are truncated Maxwellians all over. In order to maintain such distribution functions in a collisionfree regime some relaxation process must be operative impeding the usual Liouville-Vlasov degeneration of the distribution function. Processes which we consider to be responsible for this relaxation are quasilinear whistler wave – electron interactions which we shall investigate in the next sections.

6. Dissipated wave energies and modulated electron distributions

It is evident that the truncated Maxwellians introduced in Sects. 2 and 3 can only be considered as appropriate to describe the effective kinetics of the solar wind electrons in parametrized form, if these functions can be physically motivated. Without collisional or "quasi-collisional" influences on the electrons at their evaporation from the lower corona by no means truncated Maxwellians could be good approximations since the hemispherical pitch angle isotropy would be violated. The classical "Liouville'an" distribution function resulting in case of collisionless evaporation is strongly pitch–angle dependent both in the antisunward and in the sunward part of the distribution with no particles populated in the elliptic branch of the velocity space (see Fahr & Shizgal 1983). To nevertheless explain the transport of electrons into these branches, and to better approach a truncated Maxwellian, either wave–induced pitch–angle diffusion and energy diffusion processes of electrons have to occur or the electron distribution functions have to be revealed as unstable with respect to driving waves by themselves.

The latter process has been discussed by Scime et al. (1994) and Gary et al. (1994). These authors point to the possibility of a heat flux instability with respect to whistler wave excitation. Representing the electron distribution function as given by two anisotropic Maxwellians (i.e. core and halo) with a relative drift Gary et al.(1994) can calculate positive whistler wave growth rates pointing to the fact that the electron heat flow may be instability–limited to a value of the order of $q_e \simeq \frac{3}{2} m_e c_{ec}^2 v_A$, with c_{ec} being the thermal velocity of the electron core and v_A being the local Alfven velocity. However, as already noticed by Dum et al. (1980) these growth rates are highly sensitive to specific features of the distribution function. Thus no clearcut result can be obtained with respect to the effectiveness of this wave growth with respect to reshaping the distribution function.

We therefore look into an alternative relaxation mechanism with explicit influences on the shaping of the electron distribution function. Here we think of quasilinear interactions of the electrons with preexisting whistler wave turbulence. Connected with such turbulences specific Fokker Planck diffusion coefficients can be evaluated which describe wave–induced electron diffusion processes in velocity space. The process operating with the highest rate, higher than the expansion rate, is pitch–angle scattering of electrons by resonant whistler waves (see e.g. Denskat et al. 1983). This process is appropriately described by the so-called pitch–angle diffusion coefficient $D_{\mu\mu}(v,\mu)$ given by (see Schlickeiser et al. 1991; Achatz et al. 1993):

$$D_{\mu\mu} = \frac{\pi}{2}\frac{\Omega_e^2}{B_0^2}(1-\mu^2)\left[\frac{I_+(\frac{\Omega_e}{v\mu})}{v\|\mu\|} + \frac{I_-(\frac{\Omega_e}{v\mu+2v_A})}{\|v\mu+2v_A\|},\right] \tag{49}$$

where Ω_e, v and μ are gyrofrequency, velocity and pitch–angle cosine of the electron, and where $I_+(k)$ and $I_-(k)$ are whistler waves with resonant wavenumbers k propagating parallel or antiparallel to $\boldsymbol{B}_0$. Both for negative and positive values of μ the pitch–angle diffusion process operates quite efficient and rapidly tends to isotropize the distribution function, whereas due to a resonance gap in the cyclotron interaction of electrons with whistler waves (e.g. see Dusenbery & Hollweg 1996; Schlickeiser et al. 1991) around pitch–angles with $\mu \simeq 0$ the pitch–angle diffusion between the two hemisphere $\mu \geq 0$ and $\mu \leq 0$ is strongly impeded. This quite naturally justifies the assumption of truncated Maxwellians since these are μ–isotropic in both hemispheres with the μ–anisotropy limited to $\mu \simeq 0$. Besides in the principles of this effect we are also interested in its quantitative strength which is connected with the level of whistler wave turbulence.

To clarify this point, we consider processes producing whistler turbulence. We consider the interaction of high frequency Alfvén and fast magnetosonic waves with electrons starting from the following assumptions:

1. The low frequency turbulence is described as a mixture of Alfvén (a) and fast magnetosonic (f) waves.
2. The source of the turbulent energy is due to pumping of wave energy from the largest to the smallest wavelengths (i.e from the lower to the higher frequencies, the whistler modes, where a part of the spectral energy flux is resonantly absorbed by solar wind electrons).
3. The initial power spectra are of the following form:

$$W_\omega^{a,f} = W_0^{a,f}\left(\frac{\omega}{\omega_0}\right)^{-\gamma_0} \quad \text{with} : \gamma_0 \leq 1 \tag{50}$$

where $W_0^{a,f}$ are reference powers at $\omega = \omega_0$, and where ω is the wave frequency measured in the solar rest frame.
4. The convective evolution of the power spectra with increasing solar distance is described by the following wave energy continuity equation:

$$\begin{aligned}&\operatorname{div}\left[\left(\frac{3}{2}U + U_g\right)W_\omega^{a,f}\right] - \frac{1}{2}(\boldsymbol{U}\circ\boldsymbol{\nabla})W_\omega^{a,f} = \\ &-\frac{\partial}{\partial\omega}[Q(W_\omega^a, W_\omega^f)]\end{aligned} \tag{51}$$

where U is the solar wind velocity and U_g is the group velocity of the waves in the solar wind reference frame. The source term on the right hand side describes the wavepower gain at frequency ω due to divergence in ω–space of the cascading wave energy flow in a saturated turbulence field. This term does not contain linear (L), but only nonlinear (NL) contributions and essentially allows to separate the frequency space into two regions using a critical frequency $\omega = \omega_c$ according to the following rule roughly valid here:

$$\begin{aligned} At:\ \omega < \omega_c \ &\Rightarrow\ L(W_\omega^{a,f}) = 0 \\ &\quad\text{(no gains from nonlinear terms)} \\ \omega > \omega_c \ &\Rightarrow\ NL(W_\omega^a, W_\omega^f) = 0 \\ &\quad\text{(no gains from linear terms)} \\ \omega = \omega_c \ &\Rightarrow\ L(W_\omega^{a,f}) \simeq NL(W_\omega^a, W_\omega^f) \\ &\quad\text{(equal gains from both terms)} \end{aligned} \tag{52}$$

Here the nonlinear terms are due to couplings between waves of the Alfvén (a) and of the fast magnetosonic (f) types. The above terms can be estimated by simple expressions if a radial symmetry of the problem with constant solar wind velocity U can be assumed, yielding:

$$L(W_\omega^{a,f}) \simeq \frac{U}{r} W_\omega^{a,f} \tag{53}$$

and:

$$NL(W_\omega^{a}, W_\omega^{f}) \simeq \Gamma_{nl}(W_\omega^{f,a}) W_\omega^{a,f}. \tag{54}$$

where Γ_{nl} is the nonlinear wavepower growth rate due to nonlinear wave couplings. For the critical frequency $\omega = \omega_c$ one thus obtains from Eqs. (53) and (54):

$$\Gamma_{nl}(W_{\omega_c}^{a,f}) \simeq \frac{U}{r}. \tag{55}$$

The nonlinear growth rate Γ_{nl} is expressed by Chashei and Shishov (1982a, 1982b) in the following form: a,f

$$\Gamma_{nl}(W_{\omega_c}^{a,f}) \simeq \epsilon^{a,f} \frac{W_\omega^{a,f}\omega_c}{\rho v_a^2} \frac{v_a}{U + v_a} \omega_c \tag{56}$$

where the numerical factors $\epsilon^{a,f}$ describing the efficiency of mode–couplings are shown to be of the order of 0.1. Using Eqs. (56) and (47) and approximating $(U+v_a)$ by U (i.e. super–Alfvénic solar wind flow), we then arrive with Eq. (55) at:

$$\epsilon^{a,f} \frac{W_0^{f,a}}{\rho v_a^2} \frac{v_a}{U} \left(\frac{\omega_c}{\omega_0}\right)^{-\gamma_0} \omega_c^2 \simeq \frac{U}{r} \tag{57}$$

yielding the critical frequency as:

$$\omega_c^{a,f} \simeq \omega_0 \left[\frac{U^2}{r v_a} \frac{\rho v_a^2}{\epsilon^{a,f} W_0^{f,a} \omega_0^2}\right]^{\frac{1}{2-\gamma_0}} \tag{58}$$

Associated with the approximate expression for the linear wave power sources one can derive the following expression for the total energy generation which cascades up to the nonlinear regime from the critical frequency $\omega = \omega_c$

$$Q_c^{a,f} \simeq \frac{U}{r} \int_0^{\omega_c} W_\omega^{a,f} d\omega \;=\; \frac{U}{r} W_0^{a,f} \left(\frac{\omega_c^{a,f}}{\omega_0}\right)^{-\gamma_c + 1} \times \frac{\omega_0}{(1-\gamma_0)} \tag{59}$$

Introducing ω_c from Eq. (54) one arrives at:

$$Q_c^{a,f} \simeq \frac{U}{r} \frac{W_0^{a,f}}{(1-\gamma_0)} \left[\frac{U^2}{r v_a} \frac{\rho v_A^2}{\epsilon^{a,f} W_0^{a,f} \omega_0^2}\right]^{\frac{(1-\gamma_0)}{(2-\gamma_0)}} = \frac{U}{r} W_0^{a,f} \tag{60}$$

The above expression for $Q_c^{a,f}$ denotes the total spectral energy flux in modes "a" and "f" respectively, integrated over the frequency range of the inertial range where $\gamma_0 = \frac{3}{2}$ is valid. Thus the active heating sources resulting from energy dissipation in the two modes are given by:

$$Q^{a,f} = Q_c^{a,f} \simeq \frac{U}{r} w_0^{a,f} \tag{61}$$

where the wave energy Q^f is dissipated to the electrons in the whistler frequency domain, mostly at the highest frequency end, i.e. at $\omega \simeq \Omega_e$, where Ω_e is the electron cyclotron frequency.

7. Radial dependence of the dissipated wave energy

For purposes of an estimation we may base our considerations on the background field given by Parker's Archimedian spiral and hence given by:

$$B_r = B_0 \left(\frac{r_0}{r}\right)^2;\; B_\theta = 0;\; B_\Phi = B_0 \frac{r_0^2 \Omega_S}{U r} \sin(\theta) \tag{62}$$

where θ is the ecliptic co–latitude and Ω_S is the solar rotation frequency. We now define for clarification a critical radius r_c where azimuthal and radial field components are just equal given by: $r_c = \frac{U}{(\Omega_S \sin(\theta))}$. One can then study the radial dependence of $Q^{a,f}$ for two distinct regions: i.e. for region I: $r \leq r_c$, and for region II: $r \geq r_c$.

Region I: $r \leq r_c$**.** In this region, with dominance of the radial field component, the following radial dependences can be assumed:

$$\rho \propto r^{-2};\; v_a \propto r^{-1};\; \rho v_a^2 \propto r^{-4};\; W_0 \propto r^{-3}; \quad (\mathrm{WKB-theory!}). \tag{63}$$

With these dependences one evaluates Eq. (58) into the following form:

$$Q^{a,f} \propto \frac{1}{r} r^{-3} \left[\frac{1}{r}\right]^{\frac{(1-\gamma_0)}{(2-\gamma_0)}} \propto \frac{1}{r} r^{-\frac{(7-4\gamma_0)}{(2-\gamma_0)}}. \tag{64}$$

Thus one finds that $W_0^{a,f}$ has the following r–dependence:

$$W_0^{a,f} \propto r^{-\frac{(7-4\gamma_0)}{(2-\gamma_0)}}. \tag{65}$$

For regions $r < 1$ AU HELIOS A/B data show that $W_0 \propto r^{-3.5}$ (Tu & Marsch 1995). Comparing this result with Eq. (57) allows one to conclude that $\gamma_0 = 0$, i.e. a nearly flat power spectrum. Thus one derives the following radial dependence of $Q^{a,f}$:

$$Q^{a,f} \simeq Q_c^{a,f} \left(\frac{r_c}{r}\right)^{4.5},\ \mathrm{with}\ : r \leq r_c. \tag{66}$$

Therefore one can conclude that the heat source connected with the dissipated wave energy in this region falls off like: $r^{-4.5}$.

Region 2: $r \geq r_c$. In this region the azimuthal field is dominant and thus the following radial dependences have to be considered:

$$\rho \propto r^{-2};\quad v_a \simeq \mathrm{const.};\quad \rho v_a^2 \propto r^{-2}; \quad W_0 \propto r^{-2};\quad (\mathrm{WKB-theory!}). \tag{67}$$

Evaluating again Eq. (61) we thus arrive at:

$$Q^{a,f} \propto \frac{1}{r} r^{-\frac{(5-3\gamma_0)}{(2-\gamma_0)}} \tag{68}$$

For a substantially flat spectrum with $\gamma_0 \simeq 0$ one therefore derives in this region the following radial dependence:

$$q^{a,f} \simeq Q_c^{a,f} \left(\frac{r_c}{r}\right)^{3.5},\ r \geq r_c. \tag{69}$$

8. Relevance of electron heating by waves

Now, we estimate the importance of the wave energy source given by Eq. (61) and evaluated for the region $r \leq r_c$ and $r \geq r_c$ by Eqs. (66) and (69). It is known that the level of magnetic field turbulences, essentially of Alfvénic type, at $r \simeq r_c \simeq 1$ AU is moderate (see Tu & Marsch 1995), implying that $W_0^\alpha = \frac{<\delta B^2>}{4\pi} \leq \frac{B^2}{4\pi}$ where B is the background interplanetary magnetic field and $<\delta B^2> = \sum <\delta B_{ii}^2>$ is the variance of the field fluctuations.

The level of fast magnetosonic waves (compressive MHD waves) responsible for the turbulent energy pumped into the dissipative whistler frequency domain can be estimated using relevant data of density fluctuations like those presented by Tu & Marsch (1995) yielding:

$$\frac{<\delta n^2>}{<n^2>} \simeq 0.1 \frac{<\delta B^2>}{B^2} \tag{70}$$

We may thus suppose that $W_0^f \simeq 0.1 W_0^\alpha$, also because in addition this estimate agrees with data given by Leamon et al. (1998). Using this result we finally obtain the following expression for the power Q^f transfered from fast magnetosonic waves to electrons in form of thermal energy:

$$Q^f \simeq Q_c^f \left(\frac{r_c}{r}\right)^\alpha, \tag{71}$$

where $\alpha = 4.5$ was obtained for region I: $r \leq r_c$, and $\alpha = 3.5$ was obtained for region II: $r \geq r_c$. Evaluation of Q_c in the form:

$$Q_c^f \simeq W_c^f \frac{U}{r_c} \simeq 0.1 \frac{B_c^2}{4\pi} \frac{U}{r_c}, \tag{72}$$

with $r \simeq r_c \simeq 1$ AU; $U = 400\,\mathrm{km\,s^{-1}}$; and $B_c = 5\,10^{-5}$ Gauss, yields:

$$q^w = Q_c^f r_c \simeq 1 \frac{\mu W}{m^2}. \tag{73}$$

A comparison of q^w given above with the mean value of the electron heat flow at 1 AU, i.e. $q_e(r = 1\mathrm{AU}) = 8.8 \frac{\mu W}{m^2}$, given by Scime et al. (1994) and also taken into account the radial dependence of this heat flow by $q_e \propto r^{-3}$, one can conclude that the direct heating of electrons by waves would not be sufficient to maintain the electron heatflow at regions $r \leq r_c \simeq 1$ AU where q^w according to Eq. (62) is given by $q^w \simeq r^{-3.5}$ and thus is falling off with distance faster than q_e. Wave heating may, however, become important at distances $r \geq r_c$, since here q^w according to Eq. (66) is given by: $q^w \simeq r^{-2.5}$, and thus drops off with r less rapidly than q_e.

On the other hand, in our concept of truncated Maxwellians we need to assume that quasilinear electron- whistler-wave interaction via pitch-angle scattering by waves is operating which has to be energetically effective enough so that the wave energy input into the whistler frequency domain is about equivalent to the energy losses due to the growing truncation with increasing distance. Inspection of Eq. (47) then requires that:

$$q_c^w \simeq r_c S_e(r_c) = (\gamma_e - 2) q_{er}(r_c) \frac{r_c}{r} \tag{74}$$

which with $r = r_c \simeq 1$ AU and Eqs. (39) and (40) simply requires that:

$$q_c^w \simeq 1 \frac{\mu W}{m^2} \simeq q_{er}(r = 1\ \mathrm{AU}) \tag{75}$$

$$= 1.08 \times 4.16 \frac{\mu W}{m^2} = 4.5 \frac{\mu W}{m^2} \tag{76}$$

This shows that the concept of truncated Maxwellians presented above regulating the solar wind electron heat flow connected with energy absorption from fast magnetosonic waves cascading up in frequency to the whistler frequency domain, appears to be feasible and reasonable.

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