Astron. Astrophys. 363, L13–L16 (2000)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Spatiotemporal fragmentation as a mechanism for different dynamical modes of behaviour in the solar convection zone

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Received 14 August 2000 / Accepted 17 October 2000

Abstract. Recent analyses of the helioseismic observations indicate that the previously observed surface torsional oscillations with periods of about 11 years extend significantly downwards into the solar convective zone. Furthermore, there are indications that the dynamical regimes at the base of the convection zone are different from those observed at the top, having either significantly shorter periods or non–periodic behaviour.

We propose that this behaviour can be explained by the occurrence of *spatiotemporal fragmentation*, a crucial feature of which is that such behaviour can be explained solely through nonlinear spatiotemporal dynamics, without requiring separate mechanisms with different time scales at different depths.

We find evidence for this mechanism in the context of a two dimensional axisymmetric mean field dynamo model operating in a spherical shell, with a semi–open outer boundary condition, in which the only nonlinearity is the action of the azimuthal component of the Lorentz force of the dynamo generated magnetic field on the solar angular velocity.

Key words: Sun: magnetic fields – Sun: oscillations – Sun: activity

1. Introduction

Recent analyses of the helioseismic data, from both the Michelson Doppler Imager (MDI) instrument on board the SOHO spacecraft (Howe et al. 2000a) and the Global Oscillation Network Group (GONG) project (Antia & Basu 2000) have provided strong evidence which indicates that the earlier observed time variations of the differential rotation on the solar surface, the so called 'torsional oscillations' with periods of about 11 years (e.g. Howard & LaBonte 1980; Snodgrass, Howard & Webster 1985; Kosovichev & Schou 1997; Schou et al. 1998), penetrate into the convection zone, to depths of at least 8 percent in radius.

Furthermore, these data have provided some evidence to suggest that variations in the differential rotation are also present around the tachocline at the bottom of the convection zone (Howe et al. 2000b). An important feature that distinguishes these variations from those observed at the upper parts of the convection zone is that they possess markedly different modes of behaviour: either possessing distinctly lower periods (of ~ 1.3 years), or being non–periodic (Antia & Basu 2000). Clearly, to firmly establish the precise nature of these variations, future observations are required. Whatever the outcome of such observations, however, both these sets of results point to the very interesting possibility that the variations in the differential rotation can have different periodicities/behaviours at different depths in the solar convection zone. It is therefore important to ask whether such different variations can in principle occur at different parts of the convection zone and, if so, what could be the possible mechanism(s) for their production.

The aim of this letter is to suggest that a natural mechanism for the production of such different dynamical modes of behaviour in the convection zone is through what we call *spatiotemporal fragmentation*, i.e. the occurrence of dynamical regimes at (given) values of the control parameters of the system, which possess different temporal behaviours at different spatial locations. This is to be contrasted with the usual temporal bifurcations, with identical temporal behaviour at each spatial point, which require changes in parameters to occur.

We find evidence for this mechanism in the context of a two dimensional axisymmetric mean field dynamo model in a spherical shell, with a semi–open outer boundary condition, in which the only nonlinearity is the action of the azimuthal component of the Lorentz force of the dynamo generated magnetic field on the solar angular velocity. The underlying angular velocity is chosen to be consistent with the most recent helioseismic data.

In addition to producing different dynamical variations in the differential rotation, including different periods, at the top and the bottom of the convection zone, this model is also ca-

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pable of producing butterfly diagrams which are in qualitative agreement with the observations as well as displaying torsional oscillations that penetrate into the convection zone, as recently observed by Howe et al. 2000a and Antia & Basu 2000 and studied by Covas et al. (2000).

2. The model

We shall assume that the gross features of the large scale solar magnetic field can be described by a mean field dynamo model, with the standard equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B} - \eta \nabla \times \mathbf{B}). \quad (1)$$

Here $\mathbf{u} = v\hat{\phi} - \frac{1}{2}\nabla\eta$, the term proportional to $\nabla\eta$ represents the effects of turbulent diamagnetism, and the velocity field is taken to be of the form $v = v_0 + v'$, where $v_0 = \Omega_0 r \sin\theta$, Ω_0 is a prescribed underlying rotation law and the component v' satisfies

$$\frac{\partial v'}{\partial t} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\mu_0 \rho r \sin\theta}.\hat{\phi} + \nu D^2 v', \quad (2)$$

where D^2 is the operator $\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}(\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) - \frac{1}{\sin\theta})$ and μ_0 is the induction constant. The source of the sole nonlinearity in the dynamo equation is the feedback of the azimuthal component of the Lorentz force (Eq. (2)), which modifies only slightly the underlying imposed rotation law, but thus limits the magnetic fields at finite amplitude. The assumption of axisymmetry allows the field $\mathbf{B}$ to be split simply into toroidal and poloidal parts, $\mathbf{B} = \mathbf{B}_T + \mathbf{B}_P = B\hat{\phi} + \nabla \times A\hat{\phi}$, and Eq. (1) then yields two scalar equations for A and B. Nondimensionalizing in terms of the solar radius R and time R^2/η_0, where η_0 is the maximum value of η, and putting $\Omega = \Omega^*\tilde{\Omega}$, $\alpha = \alpha_0\tilde{\alpha}$, $\eta = \eta_0\tilde{\eta}$, $\mathbf{B} = B_0\tilde{\mathbf{B}}$ and $v' = \Omega^* R\tilde{v}'$, results in a system of equations for A, B and v'. The dynamo parameters are the two magnetic Reynolds numbers $R_\alpha = \alpha_0 R/\eta_0$ and $R_\omega = \Omega^* R^2/\eta_0$, and the turbulent Prandtl number $P_r = \nu_0/\eta_0$. Ω^* is the solar surface equatorial angular velocity and $\tilde{\eta} = \eta/\eta_0$. Thus ν_0 and η_0 are the turbulent magnetic diffusivity and viscosity respectively, R_ω is fixed when η_0 is determined (see Sect. 3), but the value of R_α is more uncertain. The density ρ is assumed to be uniform.

When attempting to model astrophysical systems, boundary conditions are often rather ill–determined. We try to make physically motivated choices. For our inner boundary conditions we chose $B = 0$, ensuring angular momentum conservation, and an overshoot–type condition on $\mathbf{B}_P$ (cf. Moss & Brooke 2000). At the outer boundary, we used an open boundary condition $\partial B/\partial r = 0$ on B and vacuum boundary conditions for $\mathbf{B}_P$. The motivation for this is that the surface boundary condition is ill–defined, and there is some evidence that the more usual $B = 0$ condition may be inadequate. This issue has recently been discussed at length by Kitchatinov et al. (2000), who derive 'non–vacuum' boundary conditions on both B and $\mathbf{B}_P$.

Equations (1) and (2) were solved using the code described in Moss & Brooke (2000) (see also Covas et al. 2000) together with the above boundary conditions, over the range $r_0 \le r \le 1$, $0 \le \theta \le \pi$. We set $r_0 = 0.64$; with the solar convection zone proper being thought to occupy the region $r \gtrsim 0.7$, the region $r_0 \le r \lesssim 0.7$ can be thought of as an overshoot region/tachocline. In the following simulations we used a mesh resolution of 61×101 points, uniformly distributed in radius and latitude respectively.

In this investigation, we took Ω_0 to be given in $0.64 \le r \le 1$ by an interpolation on the MDI data obtained from 1996 to 1999 (Howe et al. 2000a). For α we took $\tilde{\alpha} = \alpha_r(r)f(\theta)$, where $f(\theta) = \sin^2\theta\cos\theta$ (cf. Rüdiger & Brandenburg 1995) and $\alpha_r = 1$ for $0.7 \le r \le 0.8$ with cubic interpolation to zero at $r = r_0$ and $r = 1$, with the convention that $\alpha_r > 0$ and $R_\alpha < 0$. Also, in order to take into account the likely decrease in the turbulent diffusion coefficient η in the overshoot region, we allowed a simple linear decrease from $\tilde{\eta} = 1$ at $r = 0.8$ to $\tilde{\eta} = 0.5$ in $r < 0.7$.

3. Results

We calibrated our model so that near marginal excitation the cycle period was about 22 years. This determined $R_\omega = 44000$, corresponding to $\eta_0 \approx 3.4 \times 10^{11}$ cm^2 sec^{-1}, given the known values of Ω^* and R. The first solutions to be excited in the linear theory are limit cycles with odd (dipolar) parity with respect to the equator, with marginal dynamo number $R_\alpha \approx -2.23$. The even parity (quadrupolar) solutions are also excited at a similar marginal dynamo number of $R_\alpha \approx -2.24$. It is plausible that the turbulent Prandtl number be of order unity, and we set $P_r = 1$. For the parameter range that we investigated, the even parity solutions are nonlinearly stable. Given that the Sun is observed to be close to an odd (dipolar) parity state, and that previous experience shows that small changes in the physical model can cause a change between odd and even parities in the stable nonlinear solution, we chose to impose dipolar parity on our solutions.

With these parameter values, we found that this model, with the underlying zero order angular velocity chosen to be consistent with the recent (MDI) helioseismic data, is capable of producing butterfly diagrams which are in qualitative agreement with the observations. An example is shown in Fig. 1. (The polar branch is a little too strong, but this feature can be weakened by adjusting the latitudinal dependence of α (see also Covas et al. 2000).) The model can also successfully produce torsional oscillations (see Fig. 2) that penetrate into the convection zone, similar to those deduced from recent helioseismic data (Howe et al. 2000a) and studied in Covas et al. (2000). We note, however, that an additional interesting feature of the present model is that the torsional oscillations have larger and more realistic amplitudes near the surface, of the order of 1 nHz, much larger than was found previously using the boundary condition $B = 0$ at the surface.

We found that the model is also capable of producing spatiotemporal fragmentation, near the base of the convection zone, hence resulting in oscillations in the differential rotation with, for example, half the basic period. To demonstrate this, we have

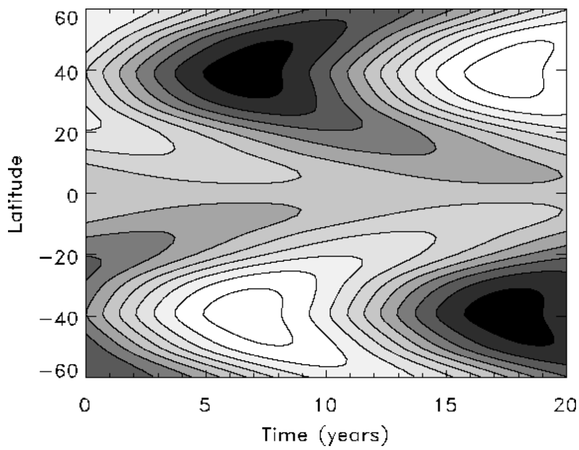

Fig. 1. Butterfly diagram of the toroidal component of the magnetic field $\boldsymbol{B}$ at fractional radius $r = 0.95$. Dark and light shades correspond to positive and negative values of B_ϕ respectively. Parameter values are $R_\alpha = -3.0$, $P_r = 1.0$ and $R_\omega = 44000$.

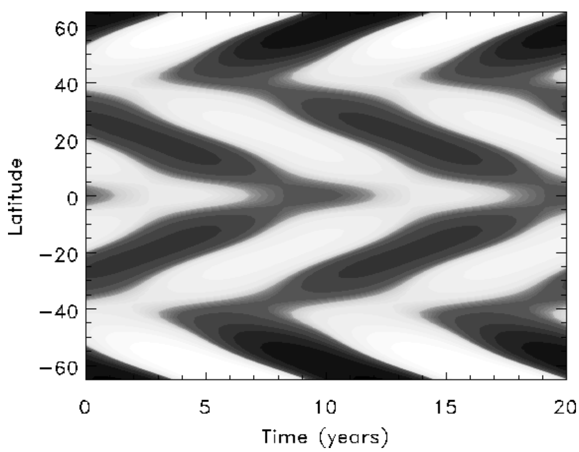

Fig. 2. Variation of rotation rate with latitude and time from which a temporal average has been subtracted to reveal the migrating banded zonal flows, taken at fractional radius $r = 0.99$. Darker and lighter regions represent positive and negative deviations from the time averaged background rotation rate. Parameter values are as in Fig. 1.

plotted in Figs. 3–5 the radial contours of the angular velocity residuals $\delta\Omega$ as a function of time for a cut at latitude $30°$, for several values of R_α. As can be seen, for smaller values of R_α (Fig. 3), we find torsional oscillations with the same period at the top and the bottom of the convection zone. As R_α is increased (Figs. 4 and 5), a spatiotemporal fragmentation occurs near the base of the convection zone, resulting in oscillations in the differential rotation with half the period of the oscillations near the top. For still higher values of R_α, the temporal variations in the differential rotation at the base of the convection zone start to become non-periodic, which might be of relevance if the failure of Antia & Basu (2000) to find shorter period oscillations near the bottom of the convection zone should turn out to be correct. We have also checked that the butterfly diagrams do not fragment and keep the same period independently of the depth and R_α value, continuing to resemble Fig. 1.

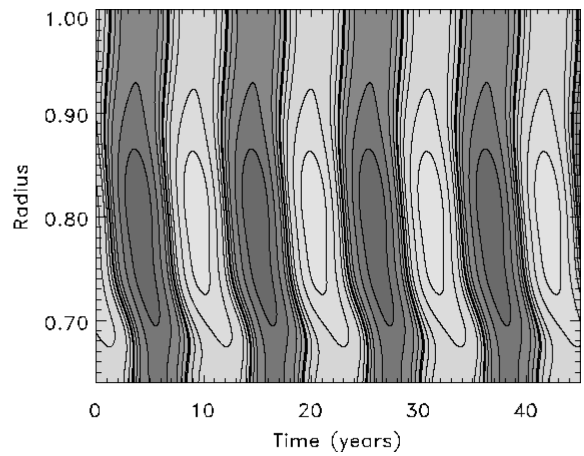

Fig. 3. Radial contours of the angular velocity residuals $\delta\Omega$ as a function of time for a cut at latitude $30°$. Parameter values are $R_\alpha = -3.0$, $P_r = 1.0$, $R_\omega = 44000$. Note how the torsional oscillations are very coherent from top to the base of the dynamo region showing that only one period is present.

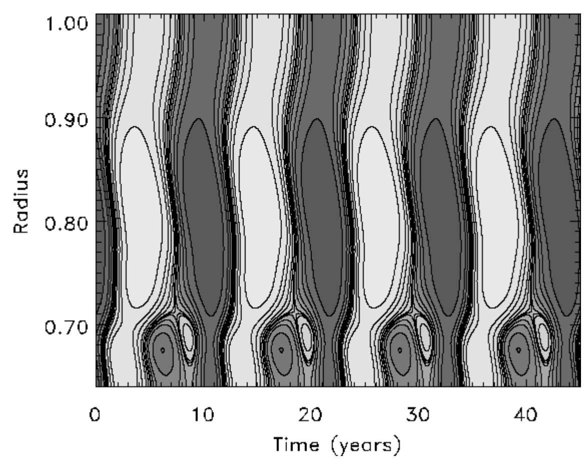

Fig. 4. Radial contours of the angular velocity residuals $\delta\Omega$ as in Fig. 3 for $R_\alpha = -7.0$. Note the emergence of spatiotemporal fragmentation towards the bottom of the convective zone, resulting in different periodicities there.

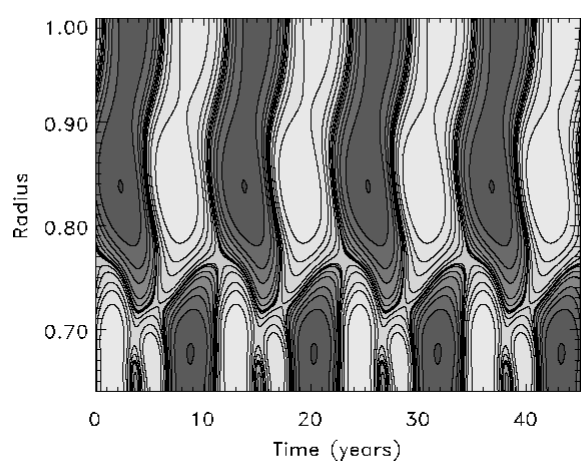

Fig. 5. Radial contours of the angular velocity residuals $\delta\Omega$ as in Fig. 3 for $R_\alpha = -10.0$.

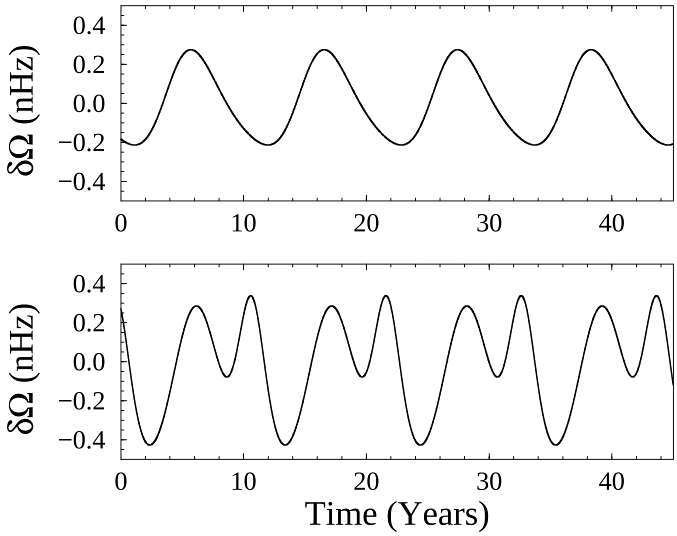

Fig. 6. 'Period halving' at $r = 0.68$ and latitude $30°$. The panels correspond, from left to right, to R_α values -3.0 and -7.0 respectively, and display increasing relative amplitudes of the secondary oscillations. Remaining parameters values are as in Fig. 3.

This fragmentation is made more transparent in Fig. 6 which shows the temporal oscillations in the angular velocity residuals $\delta\Omega$ at a fixed point, as R_α is increased, illustrating the presence of period halving.

4. Discussion

We have proposed spatiotemporal fragmentation as a natural mechanism for producing different types of variation in the differential rotation at the top and the bottom of the convection zone. To demonstrate the occurrence of such behaviour, we have studied a solar dynamo model, with a semi–open outer boundary condition, calibrated to have the correct cycle period, with a mean rotation law given by recent helioseismic observations. We note in passing that in a few simulations performed with the boundary condition $B = 0$ at the surface, we have not so far found this phenomenon, although we cannot yet make a definitive statement on this point. In addition to producing butterfly diagrams in qualitative agreement with those that are observed, as well as torsional oscillations that penetrate into the convection zone, we have shown that this model can also produce spatiotemporal fragmentation, resulting in different oscillatory modes of behaviour near the top and the bottom of the convection zone.

We emphasize that the main aim of this letter is to propose a mechanism that can be expected to operate in general nonlinear dynamo settings, and which is capable of producing multiple periods and/or non-periodic oscillations in parts of the convective zone. The specific results given here, such as the single period halving, are based on a particular dynamo model which inevitably includes many simplifying assumptions, not least of which is that the density is uniform. (It is unclear how the inclusion of a radial dependence $\rho(r)$ would affect our results — we note that current solar dynamo models commonly take a uniform density.) We expect that the mechanism is of quite general applicability, and so it is plausible that a more sophisticated model might exhibit further bifurcations, thus producing different reduced periods and oscillatory regimes. It may also be useful to bear in mind in this connection that three period halvings would result in 11 years$/2^3 \sim 1.3$ years! We shall return to a more detailed study of the underlying dynamics as well as a quantitative study of different dynamo models elsewhere. We have chosen $P_r = 1$ in order to obtain larger amplitude torsional oscillations near the surface. We have checked that fragmentation still occurs at smaller values of P_r.

Inevitably the uncertainties associated with the inversion of the helioseismic data so deep in the convection zone are quite large. Thus we believe that the mechanism discussed here may, by demonstrating what modes of dynamical behaviour are theoretically possible, act as a conceptual aid in interpreting current and further observations.

Acknowledgements. We would like to thank R. Howe and M. Thompson for many useful discussions regarding the helioseismic observations and for providing us with the time average of the solar rotation. EC is supported by a PPARC fellowship. RT would like to thank R. Brandenberger and R. Unruh for their hospitality during a visit to the University of British Columbia, where some of this work was done.

References

Antia H. M., Basu S., 2000, ApJ, 541, 442
Covas E., Tavakol R., Moss D., Tworkowski A., 2000, A&A Lett., 360, 21
Howard R., LaBonte B. J., 1980, ApJ Lett., 239, 33
Howe R., et al., 2000a, ApJ Lett., 533, 163
Howe R., et al., 2000b, Science, 287, 2456
Kitchatinov L. L., Mazur M. V., Jardine M., 2000, A&A, 359, 531
Kosovichev A. G., Schou J. 1997, ApJ, 482, 207
Moss D., Brooke J., 2000, MNRAS, 315, 521
Rüdiger R., Brandenburg A., 1995, A&A, 296, 557
Schou J., Antia H. M., Basu S. et al., 1998, ApJ, 505, 390
Snodgrass H. B., Howard R. F., Webster L., 1985, Sol. Phys., 95, 221