

# Nonlinear model pulsations for long-period Cepheids

## I. Galactic Cepheids

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**Abstract.** Nonlinear pulsation models for long-period Cepheids in Galaxy were constructed and their theoretical light and velocity curves were compared with observations. Two different mass-luminosity ( $M - L$ ) relations were assumed, one for canonical evolutionary models and the other for models with overshooting. A model sequence was constructed by varying the masses and correspondingly the luminosities for both relations. The values of the effective temperatures  $T_e$  of the models were assumed to be 200 K smaller than those of theoretical blue edge for the same masses and luminosities. Each sequence consisted of about 50 models with pulsation periods from about 10 to 100 days. Nonlinear hydrodynamic simulations were performed to get limit cycles. When nonlinear pulsation settled into limit cycles, light and velocity curves were Fourier decomposed and compared with observational results. It is concluded that the models with the overshooting-type are globally in better agreement with observations than those with the standard relation, while there are discrepancies for higher order Fourier components in both cases. Two additional model sequences were constructed by changing the value of artificial viscosity coefficients and of  $T_e$ . Decreasing the artificial viscosity can produce a slightly better agreement between models and observations for the higher order Fourier components, while the effects of different  $T_e$  on the same components are small.

The stability of the limit cycles are briefly discussed along with the lack of indications of modal resonance phenomena and the possible importance of the degree of nonadiabaticity.

**Key words:** stars: variables: Cepheids – stars: variables: general – stars: oscillations – hydrodynamics

### 1. Introduction

Features of observed light and velocity curves of pulsating stars must contain information about the stellar physical status, and this motivates the widespread application of Fourier decomposition techniques to regular variable stars. Simon & Lee (1981) performed the Fourier decomposition of the Cepheid

light curves, and found a sharp rise of the relative phase (or phase difference) and a remarkable dip of the ratio of amplitudes of the low order Fourier components in the vicinity of the period  $P \sim 10$  days. Velocity curves then were analyzed with the same technique by Simon & Teays (1983), and the lower order Fourier components showed a similar behavior.

Many theoretical nonlinear models were calculated with various physical assumptions and different hydrodynamic codes to explain this feature, which is also called Hertzsprung progression (Hertzsprung, 1926). A modal resonance between the second overtone and the fundamental mode, first proposed by Simon & Schmidt (1976), was identified as responsible for it. However the mass of canonical evolutionary models, such as those obeying the mass–luminosity relation  $M-L$  obtained by Becker et al. (1977), had to be reduced to about 60% to realize this resonance in the observed period range, which is called bump mass discrepancy. Non-linear calculations have succeeded in reproducing the main features of light and velocity curves (see e.g. Buchler et al. 1990), and the introduction of the OPAL opacity confirmed these positive results, and also considerably reduced the bump mass discrepancy (Moskalik et al. 1992).

While the mass-discrepancy of double-mode Cepheids was resolved with the OPAL opacity (Moskalik et al. 1992; Kanbur & Simon 1994), there are still some unsolved problems of *non-linear* pulsations for shorter period Cepheids. The double-mode behavior seems to be now reproducible thanks to the introduction of turbulent convection (e.g. Buchler & Kollath 1999), and this should be in part also the case of the resonance in the first overtone pulsators between the first and the fourth overtone modes (for this problem, see Antonello & Aikawa 1993, 1995; Schaller & Buchler 1993). Feuchtinger et al. (2000) constructed convective models for the first overtone pulsators and succeeded in reproducing the observed feature of the resonance. But the positive results are limited to Cepheids in Galaxy, and presently it is not possible to reproduce the features observed in Cepheids of low metallicity galaxies such as the Magellanic Clouds (e.g. Buchler & Kollath 1999).

There have not been so many calculations of nonlinear pulsations for longer period Cepheids. Carson & Stothers (1984) pointed out that the amplitude variation with pulsation periods

**Table 1.** The characteristics of model sequences

Sequences	$M/M_{\odot}$		$\log(L/L_{\odot})$		$\log(Te)$		$P$ (d)		number of models
	min	max	min	max	min	max	min	max	
(a)	8.0	13.5	3.78	4.62	3.702	3.805	14.1	69.2	56
(b)	5.6	10.7	3.54	4.53	3.687	3.726	11.1	78.4	53

in their nonlinear models based on the canonical  $M-L$  relation did not fit the observations. On the other hand, Davis et al. (1981) claimed that the light curve features in X Cygni ( $P = 16.4$  d) could be explained only by a model based on this relation. Simon & Kanbur (1995) made a large number of nonlinear models and tried to compare them with observations using the Fourier parameters.

The comparison of model output with observations seems to be more difficult for longer period Cepheids than for shorter period ones, because the modal couplings are a crucial mechanism for explaining many features in shorter period Cepheids, but we do not know what are the mechanisms in longer period Cepheids. We propose that oscillations with considerably strong non-adiabaticity is one of the possible keys to characterize the pulsation in longer period Cepheids.

In this paper, we shall investigate features of light and velocity variations in nonlinear pulsation models of longer period Cepheids systematically, and compare them with observations. Antonello & Morelli (1996) published a large set of Fourier components of observed light curves. Kovács et al. (1990) published Fourier components of observed velocity curves also for longer period Cepheids, and we shall supplement it with other available data for two stars.

In Sect. 2 we describe our nonlinear pulsation modelling, we compare our results with observations in Sect. 3, we briefly discuss several interesting features in nonlinear models in Sect. 4 and make conclusions in Sect. 5.

## 2. Nonlinear Cepheid modeling

Nonlinear models were made with the code TGRID (Simon & Akiawa, 1983). This code has a facility of rezoning adapted to temperature changes in the vicinity of the hydrogen ionization region, and gives smooth light curves. The adopted chemical composition was  $X=0.70$  and  $Z=0.02$ , and we used the opacity table (s92 380) supplied by OP project (Seaton et al. 1994) and OPTFIT code (Seaton 1993) for fitting and smoothing of the table. We ignored entirely the effects of convection in the envelope, and the diffusion approximation was used for radiative transfer treatment.

We were interested in two problems: one was to find out any effects of modal resonances, and the other was to see the differences in the features of light and velocity curves due to different  $M-L$  relations.

As a canonical  $M-L$  relation we adopted the one from Becker et al. (1977):

$$\log L/L_{\odot} = 3.68 \log M/M_{\odot} + 0.46 \quad (1)$$

The second relation was a full overshooting-type one given by Chiosi (1990) and modified according to more recent evolutionary models (see e.g. Antonello et al. 1997):

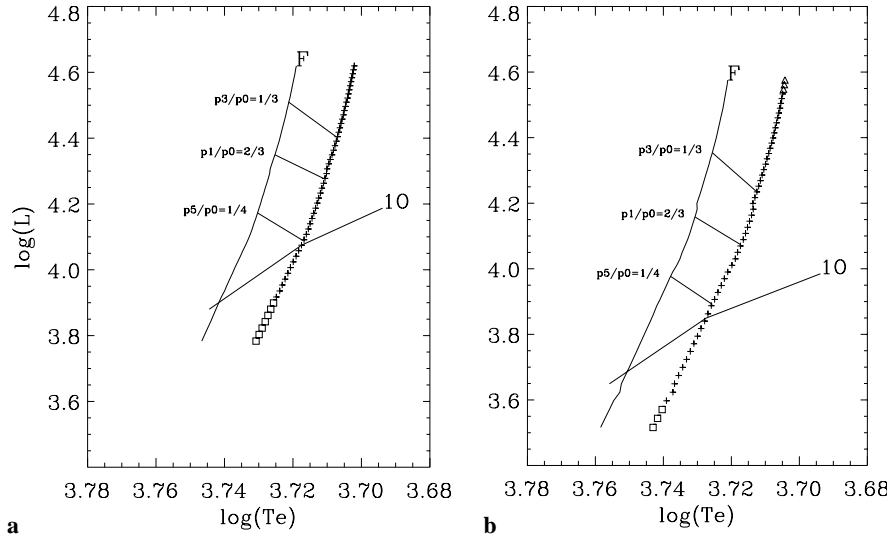
$$\log L/L_{\odot} = 3.52 \log M/M_{\odot} + 0.91 \quad (2)$$

A model sequence for each  $M-L$  relation was constructed in the following way. The theoretical blue edge was calculated with LNA analysis using Castor's type code for a given mass and the corresponding luminosity; then a lower  $T_e$  by 200 K than the corresponding blue edge was chosen for the same mass. The sequence was obtained by changing the mass of models by small steps. We shall call the model sequences obtained with Eqs. (1) and (2) as sequences (a) and (b), respectively. Fig. 1 shows the location of the sequences in the HR diagram along with the blue edges of the fundamental and first overtone modes and some lines identifying the expected resonances from linear period ratio. Sequence (a) has a mass range  $8.0M_{\odot} \leq M \leq 13.5M_{\odot}$ , mass step of  $0.1M_{\odot}$  and covers pulsation periods from 13.5 days to 69.2 days. Similarly, sequence (b) has mass range  $5.5M_{\odot} \leq M \leq 11.0M_{\odot}$  and pulsation periods from 9.3 days to 67.5 days. The characteristics of the model sequences are summarized in Table 1.

The observed structure and width of Cepheid instability strip for longer period Cepheids is not clear. Fernie (1990) claimed that the observed red edge might not be parallel to the blue edge, and moreover the width in  $\log Te$  could be as large as 0.11. There are several theoretical estimates of the width such as from 0.034 to 0.072 by Deupree (1980) and an upper limit at 0.057 by Buchler et al. (1990); Bono et al. (1999a) discuss in detail new estimates (see also Buchler & Kolláth 1999). Our sequences are located anyway near the blue side of the instability strip. We examined the effects of different locations in the instability strip on the features of light and velocity curves by making other models similar to those of sequence (b) except for the  $T_e$  value, which was 400 K smaller than the blue edge.

In nonlinear simulations, we used artificial viscosity of the von-Neumann-Richtmeyer type with a cutoff parameter to stabilize numerical instabilities (see Eq. (1) in Kovács & Buchler, 1993, for the expression of the artificial viscosity). In this expression there are two parameters,  $C_Q$  and  $\alpha$ , whose proper values are not known.  $C_Q$  controls the efficiency of the artificial viscosity, and  $\alpha$  is used to suppress unnecessary effects produced in the inner part of stellar envelope; we adopted  $C_Q = 4.0$ ,  $\alpha = 0.05$ .

Nonlinear simulations were carried out with  $8 \times 10^5$  time-steps as a first run, covering a time interval from 900 to 2000 pulsation cycles. Initially static models were perturbed with the velocity profile of the eigen function of the fundamental mode



**Fig. 1a and b.** The location of model sequences in the HR diagram. **a** standard models, **b** overshooting type models. The blue edges of the fundamental (F) and first overtone (10) modes are shown, along with some lines indicating modal resonances (linear period ratios). Crosses: models with limit cycles; triangles: models with irregular oscillations; squares: mode switching models.

with an amplitude of  $-10 \text{ km s}^{-1}$ . Pulsations settled into limit cycles within this first run, except for a few cases of shorter period models which required a second run with the same time-steps.

### 3. Theory versus observation

Bolometric corrections taken from Flower (1977; supergiant stars) were applied to convert the bolometric magnitudes of model output to visual magnitudes, and the last 300 days data of light and velocity variations at the photosphere for each model were then Fourier-decomposed.

The light curves as well as velocity curves were fitted with the formula:

$$A_0 + \sum_n A_n \cos(2\pi n f(t - T_0)) + \phi_n \quad (3)$$

where  $T_0$  is an arbitrary epoch, and  $f$  is the frequency,  $f = 1/P_{nl}$ , where  $P_{nl}$  is the period of the limit cycle, which was obtained with the least-square power spectrum method (Vanicek, 1971). The maximum order of the fit was 20. Since Antonello & Morelli (1996) suggested the presence of resonance features in observed higher order Fourier components, in the present work we have considered the components up to the 6th order. The weighted phases (or phase differences)  $\phi_{n1} = \phi_n - n\phi_1$  and the relative amplitude ratios,  $R_{n1} = A_n/A_1$  were then derived.

Fig. 2 shows the weighted phases and amplitude ratios of theoretical light curves for various model sequences, while Fig. 3 shows the same quantities of theoretical velocity curves. The values of the weighted phases have an ambiguity of  $2k\pi$ , where  $k$  is a positive or negative integer number. The proper choice of the arbitrary constant is not easy when a large change of the weighted phase occurs in a narrow period range, such as at  $P \sim 10\text{--}15$  days, in particular for the higher order components.

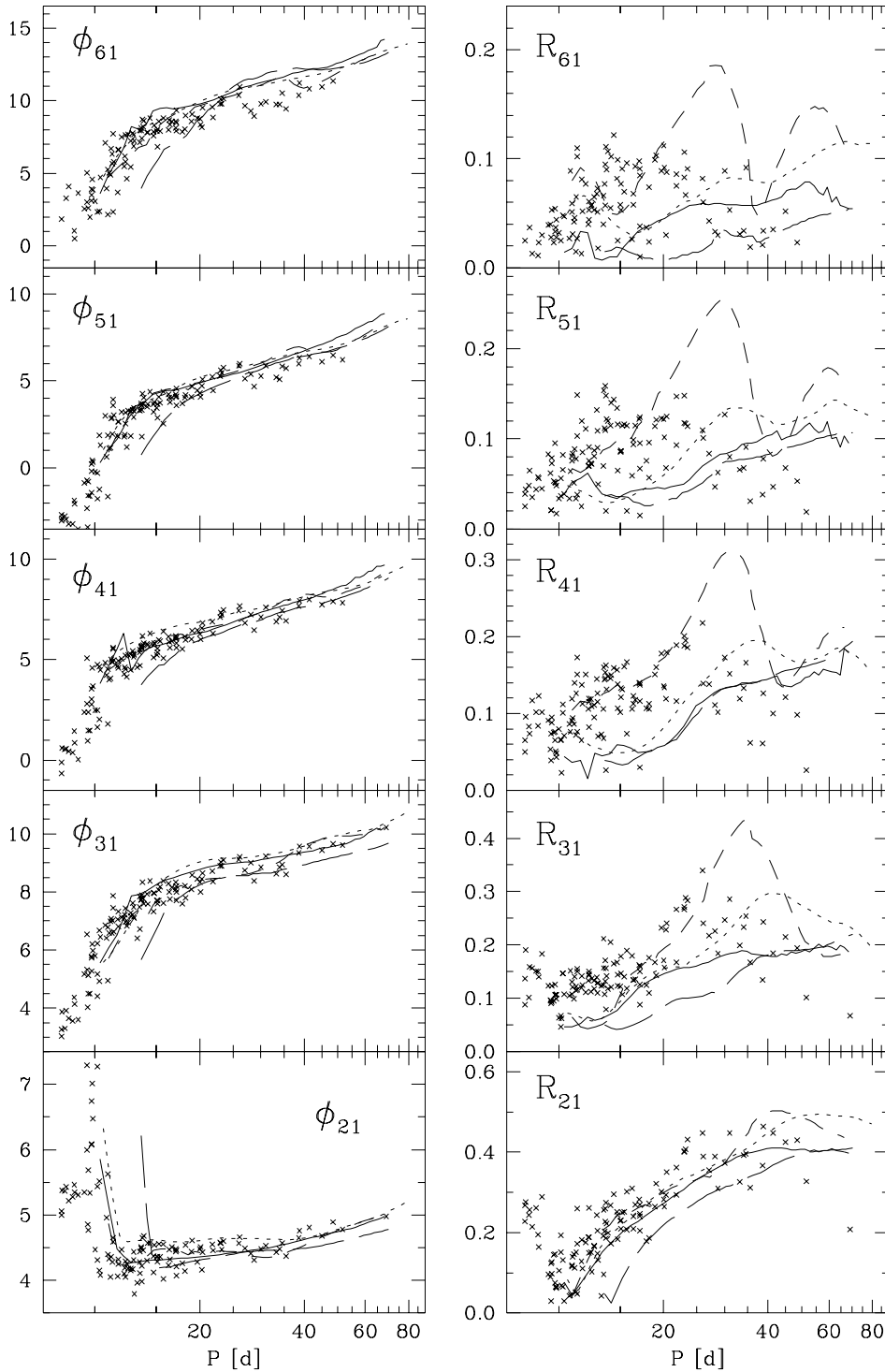
#### 3.1. Light curves

The model sequence for the canonical  $M\text{--}L$  relation (long-dashed line in Fig. 2) is not in agreement with observations, particularly for lower order amplitude ratios, such as  $R_{21}$  and  $R_{31}$ . We can see that the theoretical parameters are systematically shifted with respect to the observations, which is originated from the fact that the model sequence has the  $P_0/P_2$  resonance at longer period than 10 days. This problem has been discussed as the ‘‘bump’’ mass problem (e.g. Moskalik et al. 1992).

Observed values of the lower order amplitude ratios have a minimum at 10 d, a peak around 30 d and then tend to decrease with increasing period. The model sequence fails to reproduce such trends. This failure is related to the difficulty of reproducing the Bailey diagram, first pointed out by Carson & Stothers (1984). As regards the weighted phases, a discrepancy of the model sequence can be seen in the shorter period range.

The models with overshooting type  $M\text{--}L$  relation (continuous line in Fig. 2) are in better agreement with observations, particularly the low order amplitude ratios and weighted phases. The sequence follows well the observations at shorter periods. It also has a maximum for  $R_{21}$  around 40 days, while we notice only a plateau for  $R_{31}$  and  $R_{41}$ .

What is the physical mechanisms of this decreasing of the amplitude ratios and of the amplitudes themselves in nonlinear pulsation of long-period Cepheids? Non-adiabaticity, such as shock dissipation, could be responsible for this trend. Empirically, the degree of nonadiabaticity of the pulsation in a model is evaluated with  $\xi = L/M^{1.6}$  (Christy 1975). We expect therefore that the nonadiabatic effects become stronger along with pulsation periods. This difference in the degree of nonadiabaticity along with pulsation periods could cause the amplitude changes with period. Moreover, the pulsations of models of sequence (b) for a given period have stronger nonadiabatic effects than those of sequence (a). By this reasoning, a slightly more overluminous  $M\text{--}L$  relation than the one of sequence (b) should give a better agreement with observed low order amplitude ratios.



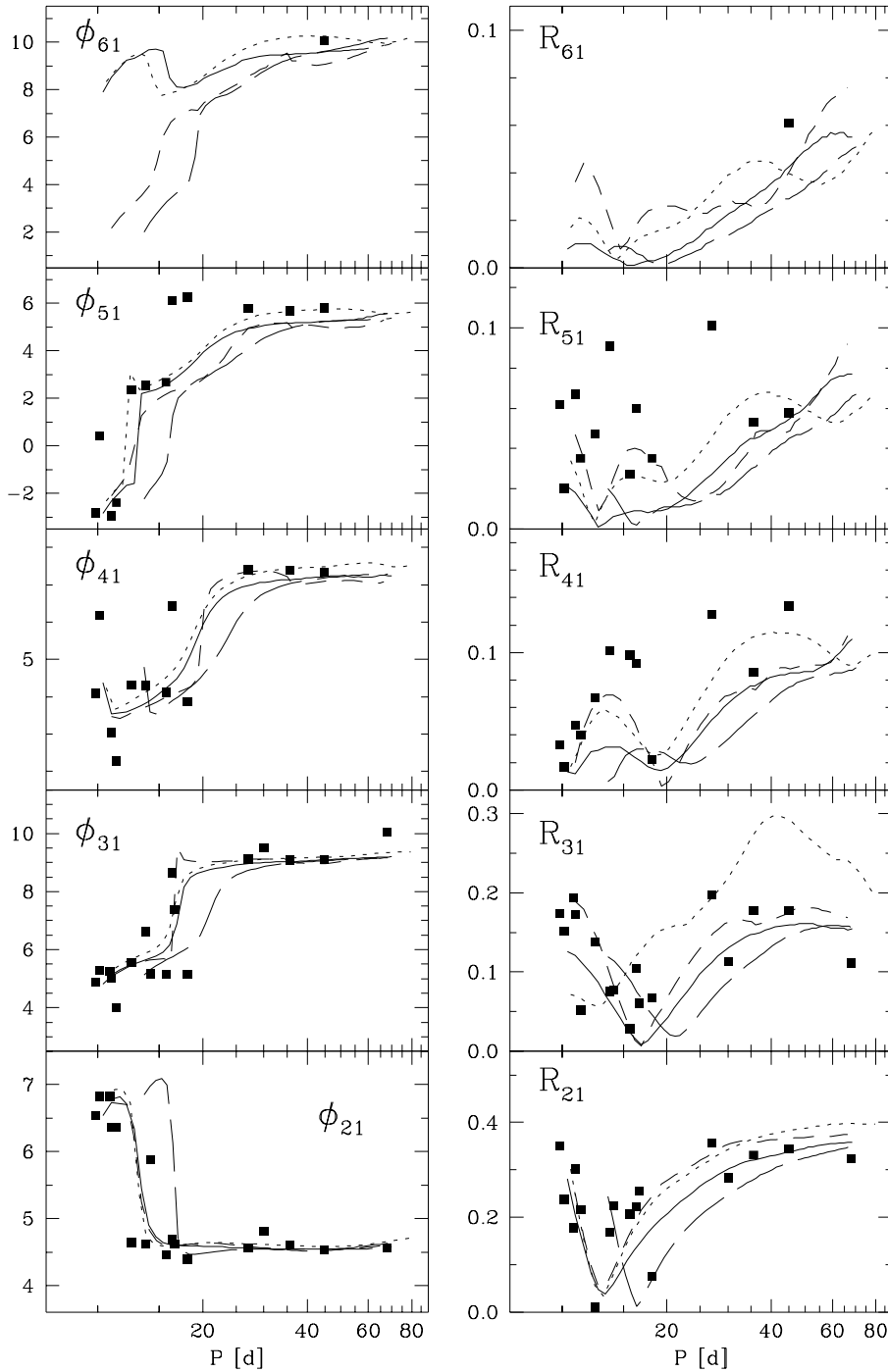
**Fig. 2.** Weighted phases (or phase differences),  $\phi_{n1} = \phi_n - n\phi_1$  and the relative amplitude ratios,  $R_{n1} = A_n/A_1$  of theoretical light curves compared with observations. *Long-dashed line*: model sequence with the canonical  $M-L$  relation (a); *continuous line*: model sequence with overshooting-type relation (b); *short-dashed line*: model sequence (b) with low artificial viscosity; *dotted line*: cooler model sequence (b)

### 3.2. Velocity curves

Fig. 3 shows the weighted phases and the amplitude ratios of theoretical velocity curves compared with observations. The velocity at the photosphere of theoretical models was converted according to observational convention before performing the Fourier decomposition. The Fourier parameters obtained by Kovács et al. (1990), converted owing to the different fitting formula adopted by us (see e.g. Antonello & Aikawa 1995), are

also plotted along with the parameters of two more stars, SV Vul and S Vul, obtained from the analysis of observational data of Gorynya et al. (1992, 1996). Table 2 includes the period of the two stars, obtained with a least squares best fit to the data, the number of data points, the standard deviation of the fit and the amplitude.

We note that the models of sequence (b) follow well the observations in the low order Fourier parameters case. The ob-



**Fig. 3.** Weighted phases,  $\phi_{n1} = \phi_n - n\phi_1$  and the relative amplitude ratios,  $R_{n1} = A_n/A_1$  of theoretical velocity curves compared with observations. Symbols as in Fig. 2

servational data, however, have some scatter due to the different quality and small number of data points. The models of sequence (a) have a systematic shift for the longer periods. Both sequence (a) and (b) models disagree with observations as regards the high order amplitude ratios.

### 3.3. Sensitivities

Observed high order amplitude ratios such as  $R_{51}$  and  $R_{61}$  have a maximum around 15 days. This maximum is caused by a dip

just after the light minimum, a sharp increase of the ascending branch and a narrow peak at maximum of the light curves. Good examples are BN Pup and CY Cas. Both our model sequences (a) and (b) are in poor agreement with these features. Since we suspected that probably a too large artificial viscosity  $C_Q$  value might suppress the short wavelength features, we made another model sequence for overshooting-type  $M-L$  relation with  $C_Q = 1$  instead of  $C_Q = 4$ . The results are shown in Figs. 2 and 3 (short-dashed lines). As expected, we have some improvements; unfortunately, there are also strong enhancements

**Table 2.** Fourier parameters of velocity curves of two long period Cepheids

Stars	$P$	$N$	$\sigma$	$R_{21}$	$\phi_{21}$	$R_{31}$	$\phi_{31}$	$R_{41}$	$\phi_{41}$	$R_{51}$	$\phi_{51}$	$R_{61}$	$\phi_{61}$
SV Vul	44.8659	87	0.80	0.358	4.43	0.177	2.59	0.120	0.98	0.066	4.04	0.022	2.38
S Vul	68.362	68	1.65	0.338	4.64	0.161	3.37	—	—	—	—	—	—

of “artificial dip” in nonlinear models with periods longer than 30 days.

We also examined the effects of different  $T_e$  values. For this purpose, another model sequence was constructed with the same stellar parameters as sequence (b) except for  $T_e$  which was 400K cooler than the corresponding theoretical blue edge. The results are shown in Fig. 2 and Fig. 3 (dotted lines). Compared with the original sequence (b), there are differences in some period ranges which could explain some observational scatter; there is, however, an enhancement of the amplitude ratios at longer periods. As regards the velocity curves, in this case there is a significant difference in the  $R_{31}$  values.

### 3.4. Modal resonances

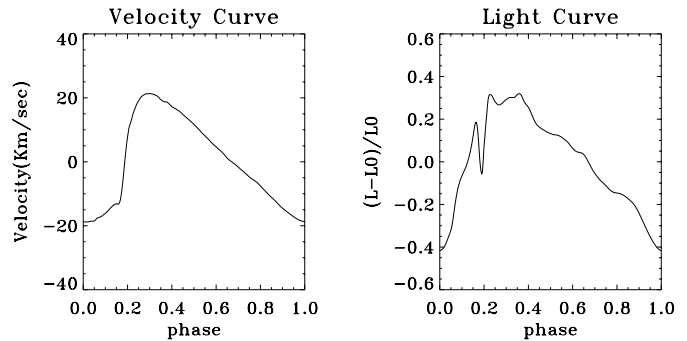
We expect that modal couplings in nonlinear pulsation, particularly modal resonances, may cause abrupt changes in the theoretical Fourier parameter distribution, even if we change the model input parameters smoothly. We know at least two examples of how the Fourier components behave at modal resonances. One is the resonance of the fundamental mode and the second overtone mode at about 10 d. Observationally, in the light curve case,  $\phi_{21}$  has a peak at 10 d, while  $R_{21}$  has a considerably deep dip. These features are reproduced well by hydrodynamic calculations (see e.g. Moskalik et al. 1992), and may be explained even by analytical models (Aikawa 1987). Antonello & Poretti (1986) pointed out that there is evidence of a resonance at a period of about 3 days in s-Cepheids, which is supposed to be caused by modal resonance in first overtone pulsators between first and fourth overtone mode. The feature of  $\phi_{21}$  at this resonance is slightly different from the one at 10 days, and a dip of  $R_{21}$  is less remarkable. Probably, the difference between these two features comes from the fact that the overtone pulsators have less modulated light curves and lower amplitudes than fundamental mode Cepheids. On the other hand,  $\phi_{31}$  both in fundamental and first overtone mode Cepheids shows a progressive change of  $2\pi$  near the observed resonance periods.

Table 3 summarizes pulsation periods of possible resonances for our model sequences. From the previous two examples we expect that, if a resonance is effective, the weighted phases and the amplitude ratios should have considerable changes in a narrow range of pulsation periods. We notice possible features only due to the resonance of  $P_5/P_0 = 1/4$  at about 25 days in sequence (a) and 18 days in sequence (b) in higher order components.

Subharmonic resonances like as  $P_1/P_0 = 2/3$  should be effective only when both the modes involved are pulsationally unstable. From Fig. 1, we expect that some models with very

**Table 3.** Periods of modal resonances in longer period Cepheid models

Resonances	sequence(a)	sequence(b)
$P_5/P_0 = 1/4$	24.7	19.3
$P_1/P_0 = 2/3$	35.7	27.8
$P_3/P_0 = 1/3$	45.5	37.9



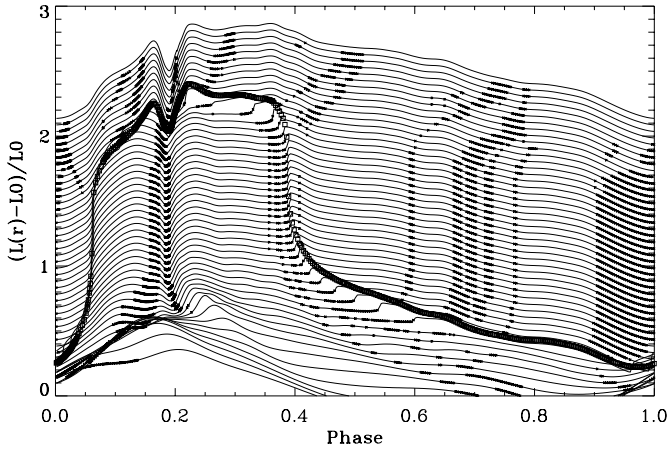
**Fig. 4.** The velocity and light curves of one of the longer period Cepheid model. The luminosity is normalized with respect to the luminosity of static equilibrium,  $L_0$ .

low effective temperatures may satisfy this condition. Indeed, Moskalik & Buchler (1991) found this subharmonic resonance in models at about 800 K to the right of the blue edge.

There is no evidence of the resonance  $P_3/P_0 = 1/3$ , which was suggested by Antonello & Morelli (1996). Probably in this long period region, nonadiabatic effects become very strong and resonances are no more effective in nonlinear pulsation. However, in Magellanic Clouds there is a narrow dip of Fourier parameters and Antonello (1998) has suggested the resonance  $P_0/P_1 = 2$  at  $P \sim 120$  d. This issue will be treated further on in our next paper (Aikawa & Antonello 2000) along with possible explanations of the dip.

## 4. Discussion

It is known that S Vul, the Cepheid with the longest period in the Galaxy, has a flat-topped light curve (Fernie 1970). This feature is common among the long period Cepheids, and determines the characteristics of the lower order Fourier components. We suppose that strong nonadiabaticity, particularly shock dissipation, must be responsible for this. Fig. 4 shows the light and velocity curves of one of the long period models ( $M = 10.7M_\odot$ ,  $\log L/L_\odot = 4.53$ ,  $T_e = 5069$  K) of sequence (b). Fig. 5 displays the luminosity profile in the outer part of the envelope of this model. In these figures, the time of the maximum velocity is



**Fig. 5.** Luminosity profile in the outer part of envelope in the longest period model of sequence (b). The luminosity is normalized with that of static model,  $L_0$ , and the luminosity curve of each zone is shifted vertically. The times and the zones where the artificial viscosities have non-zero values are indicated with the symbol  $x$ . The locations of the front of hydrogen ionization and recombination are marked with a square

in contraction is defined as phase 0. In Fig. 4 we can see a secondary peak in the descending branch of light curve. In Fig. 5, the regions of the envelope and phases where the artificial viscosities have non-zero values are patched with a symbol,  $x$ , and the location of the hydrogen ionization is also marked.

We can see a strong shock propagation at phase 0 generated at the hydrogen ionization front, which is located deep in the envelope. We expect that strong disturbances will come up in the atmosphere at this phase. In fact, according to observations (Benz & Mayor, 1982), the turbulent velocity observed in long period Cepheids is strongest at this phase during pulsation. They also show that the turbulence velocities are stronger in longer period than in shorter period Cepheids. Ishida & Takeuti (1991) made a similar diagram to Fig. 5 for a classical Cepheid model of intermediate mass. Compared with their results, long-period Cepheids have an additional feature. There is a strong shock wave propagation just after light maximum, detached at the phase 0.25 from the ionization front. This shock phenomenon produces the plateau at light maximum (see the plateau from 0.2 to 0.4 phase in Fig. 5). Davis (1972) first pointed out the same phenomenon in his W Virginis model.

Strong nonadiabaticity in longer period Cepheids may produce similar dynamical behaviors to those of less-massive supergiant stars but in much milder manner. Buchler & Kovács (1987) and Aikawa (1987) showed bifurcation processes and chaos in less-massive supergiant stars. These bifurcation processes are not related to modal resonances. Aikawa (2000) confirmed that Cepheid models with  $P \sim 100$  d show the period doubling bifurcation due to strong nonadiabaticity. Van Genderen (1983) reported likely fluctuations in light curves of long-period Cepheids in LMC (for instance, HV 883 with  $P = 133.9$  d). Some of the nonlinear models of long period Cepheids in our sequences (marked by a triangle in Fig. 1) show quite violent pulsations and some difficulties in numerical simulations.

These models probably show irregular pulsations rather than limit cycles.

Some shorter period models of sequence (a) and (b) indicate instability of fundamental limit cycles (the models are indicated with a square in Fig. 1), and we confirm that nonlinear pulsations are going to settle into the first overtone mode in some cases. They are located in a region of the instability strip where the fundamental mode limit cycle is unstable, and the first overtone has a stable limit cycle. Spangenberg (1975) and other authors tried to classify the regions of the instability strip into several categories on the basis of the stability of the limit cycles. They used old Los Alamos opacity, but the global feature should not change with the new opacity. According to these studies, the fundamental limit cycles are unstable in a bluer region of the instability strip. Goupil et al. (1997) found that a sequence of Pop. I Cepheid models, which are located at 100 K to the right of the blue edge, show this instability at shorter periods. Our calculations suggest that this instability region must be extended to about 200 K to the right of the blue edge for  $P \sim 10$  d. On the other hand, the results of our model sequence located at 400 K to the right of the blue edge and the models by Moskalik et al. (1992) show that the instability region does not extend to further cooler regions. We, therefore, expect that the width of the instability strip must be quite narrow in this period region, if the instability of the fundamental mode limit cycle is responsible for a gap in the distribution of Cepheids at about 10 d, as suggested by Goupil et al. (1997).

Davis et al. (1981) compared theoretical light curves and observations of X Cygni, and claimed that only a nonlinear pulsation model with the canonical  $M-L$  relation can explain the observed light curve features. According to observational data in Fig. 2, there is a large scatter of higher component amplitude ratio values at  $P \sim 15$  d. This means that Cepheids with this period have many short wavelength varieties, such as dips and bumps, for instance in RW Cas case. We therefore conclude that it is questionable to compare model output with observed features of “short wavelength” phenomena in single Cepheid to derive general conclusions about the physical parameters of stars.

## 5. Conclusions

Sequences of nonlinear pulsation models for long period Cepheids in Galaxy have been constructed for comparison purposes with observations, in terms of weighted phases and amplitude ratios of Fourier components of light and radial velocity curves. The sequence of more luminous models are in better agreement with observations, and we pointed out that a slightly overluminous  $M-L$  relation than the overshooting-type one is necessary to explain a considerable drop of low order amplitude ratios at the end of long-period Cepheid sequence. Simon (1995) suggested that the  $M-L$  relation for bump Cepheids should be intermediate between the canonical and overshooting-type one; however we suggest that the degree of overshooting may be a function of pulsation periods, i.e. it depends on the stellar mass.

The characteristics of nonlinear pulsations in longer period Cepheids are intermediate between the slightly nonadiabatic behavior of short-period Cepheids and the strong nonadiabaticity in less-massive supergiant stars. Antonello & Aikawa (1998) suggested that a lower degree of nonadiabaticity may favor modal resonant phenomena and permanent double-mode pulsation. Modal resonant phenomena may be ineffective in long-period Cepheids due to strong nonadiabaticity; however the question is whether a different metallicity could affect this behavior.

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