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Nonlinear model pulsations for long-period Cepheids

II. Magellanic Cloud Cepheids

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Abstract. Nonlinear, radiative models for long–period Cepheids in the Magellanic Clouds were constructed and the theoretical light and velocity curves were compared with observations. We assumed a metal content $Z = 0.005$, and two mass–luminosity M–L relations: one was obtained from stellar evolution calculation with convective overshooting models, and the other, which gives an even larger L for the same M, was suggested by the study of a bump Cepheid in the Magellanic Clouds. Both relations produce results which are in rough agreement with the observed light and radial velocity curves of Cepheids with period $P \gtrsim 30$ d. The latter relation, however, gives better results when compared with observations for $P \sim 10$ d.

The longest P of stable limit cycle is about 130 d; in a previous paper we obtained, for $Z = 0.020$, a maximum P of about 70 d. This result compares reasonably well with Cepheids observed in Galaxy and Magellanic Clouds; in other words, the longest P of Cepheids should depend on the average metallicity of the parent galaxy.

The nonlinear P of more massive models with low metallicity tends to lengthen significantly for a very small increase of M, and this could explain the flattening of the PL relation for $P \gtrsim 100$ d, which is observed in metal poor galaxies.

A discussion of the shortcomings of the models for a period as short as about 10 d and some comments on resonance effects are also reported.

Key words: stars: variables: Cepheids – stars: variables: general – stars: oscillations – hydrodynamics – galaxies: Magellanic Clouds

1. Introduction

As a by-product of microlensing surveys (MACHO, EROS and OGLE projects), huge amounts of data on light curves of Magellanic Cloud Cepheids with short period P are now available, which allows significant tests of nonlinear model predictions. In a comparison between Galactic and Magellanic Cepheid light curves, Buchler et al. (1996) pointed out a serious problem for stars in a period range around 10 days, while the advent of the OPAL opacity reduces the beat Cepheid problems (e.g. Morgan & Welch 1997). The theoretical models are not able to reproduce the observed features when going from the metal content of Galaxy ($Z \sim 0.02$) to that of LMC ($Z \sim 0.01$) and SMC (~ 0.005; the metal content for the Magellanic Cloud Cepheids has been estimated spectroscopically e.g. by Luck et al. 1998). A similar conclusion to that of Buchler et al. (1996) was obtained by Wood et al. (1997) after the comparison of the observed light curve of the Cepheid HV 905 with nonlinear pulsation models. The authors claimed that in order to get a good agreement with the observed light curve, a quite 'luminous' mass–luminosity M–L relation should be supposed. Owing to this, the overshooting which would be required in the evolution calculations is roughly the double of the currently used values. In the present paper, we report about the results of a study of nonlinear models for a wide range of pulsation periods, obtained for two M–L relations: one is derived from stellar evolution calculations with fully convective overshooting (e.g. Chiosi 1990), and the other is that suggested by Wood et al. (1997). The light and velocity curves of the limit cycles are compared with observations in terms of the Fourier components; the available data taken from the literature were analyzed by Antonello (1998). Carson & Stothers (1984) pointed out that the models of very long period Cepheids do not follow the usual linear period–luminosity PL relation, and this result was used to explain the observed behavior of the relation for $P \gtrsim 100$ d. In the present paper we will further detail this behavior; moreover, we will also briefly discuss some nonlinear effects related to resonances.

2. Nonlinear Cepheid models

The overall strategy of nonlinear model construction was the same as that discussed in Aikawa & Antonello (2000; hereinafter Paper I). The adopted chemical composition was $X = 0.7$, $Y = 0.295$ and $Z = 0.005$, and the opacity (s92 364) was that supplied by the OP project (Seaton et al. 1994), with OPTFIT code (Seaton 1993) for fitting and smoothing of the opacity tables. We assumed the M–L relation suggested by Chiosi (1990) for full convective overshooting evolutionary models, modified according to more recent models (see Paper I).

$$\log L/L_\odot = 3.52 \log M/M_\odot + 0.91. \qquad (1)$$

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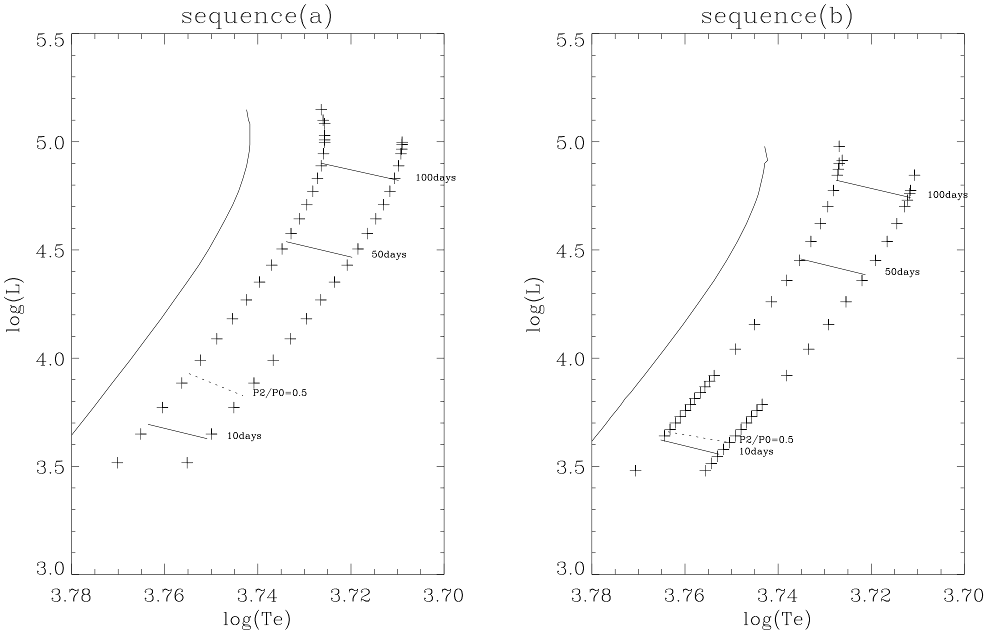

Fig. 1. The location of model sequences in the HR diagram; sequence(a) was obtained with the M–L relation (1), sequence(b) with the relation (2). The sequences, which are indicated by plus signs, were obtained for T_e which was 200 K and 400 K lower than the corresponding blue edge. The resonance line $P_2/P_0 = 0.5$ (linear period ratio; dotted line), some periods of the models and the blue edge of the F-mode (continuous line) are also indicated.

Another relation was assumed for comparison purposes:

$$\log L/L_\odot = 3.52 \log M/M_\odot + 1.18. \qquad (2)$$

It was obtained simply by shifting the zero point in relation (1) in the way required by Wood et al. (1997) to explain the light curve of HV 905 in LMC. The theoretical blue edge was calculated with linear nonadiabatic LNA analysis using Castor's type code for a given mass and the corresponding luminosity, and the values of the effective temperature T_e of the model sequences were chosen 200 K and 400 K smaller than the corresponding blue edge. We will call the model sequence obtained with Eqs. (1) and (2) as sequence (a) and (b), respectively. Fig. 1 shows the location of the sequences in the logT_e–log$L$ diagram along with the blue edges of the fundamental mode and the resonance line $P_2/P_0 = 0.5$ that we expect from linear period ratios. Sequence (a) has a mass range $5.5M_\odot \leq M \leq 16.0M_\odot$ and covers pulsation periods from about 7 to 155 days. Similarly, the mass range of sequence (b) is $4.5M_\odot \leq M \leq 12.0M_\odot$, and it covers almost the same range of pulsation periods as sequence (a). For other features of nonlinear modeling, see Paper I.

3. Theory versus observation

We used the same definitions of the weighted phases (or phase differences) $\phi_{n1} = \phi_n - n\phi_1$ and the relative amplitude ratios, $R_{n1} = A_n/A_1$, and the same procedures of the Fourier fitting as in Paper I.

Fig. 2 shows the weighted phases and relative amplitude ratios of theoretical light and velocity curves for various model sequences, compared with observations. The observed parameters which are presently available concern LMC and SMC Cepheids with $P \gtrsim 30$ d (Antonello 1998). Since for shorter period Cepheids only light curve data would be available, we decided to take into account the short period Cepheids in Galaxy as a reference. The comparison with the theoretical models will be instructive in any case, since it is known that for $P \sim 10$ d the ϕ_{21} and R_{21} values are roughly similar in Galaxy, LMC and SMC. On the other hand, nothing can be said for the present about the higher order Fourier parameters of LMC and SMC Cepheids.

We summarize here the main results of the comparison.

- For both M–L relations, the theoretical models reproduce the global features of the trends of observed Fourier components, in particular in the radial velocity case. On the whole, the comparison between models and observations of Cepheids with $P \gtrsim 30$ d in Magellanic Clouds indicates that it is not possible to choose reliably between sequence (a) and sequence (b) models as the best ones. In this period range, R_{31} values of radial velocity curves appear to be probably sensitive to T_e. Some discrepancies occur in the light curve case for ϕ_{31} at P near 100 d, since the observations suggest some structure which is not reproduced by models.
- The main differences between the Fourier components of the two model sequences are the locations of the effects related to the resonance $P_2/P_0 = 0.5$ (e.g. dips of R_{n1} values), which are shifted towards longer periods for sequence (a) models. Since there are no significant differences among the light curve parameters of Galaxy and Magellanic Cloud Cepheids for $P \sim 10$ d, the almost flat distribution of ϕ_{21} values of theoretical light curves is openly against the observational results (see e.g. Buchler 1998).
- Theoretical velocity curves have a sharper response at the 10 days resonance than light curves. Taking into account also the results reported in Paper I, we can conclude that the theoretical radial velocities are less sensitive to different metallicities than theoretical light curves; it would be interesting to check this result with radial velocity observations of LMC and SMC Cepheids with period near 10 d.

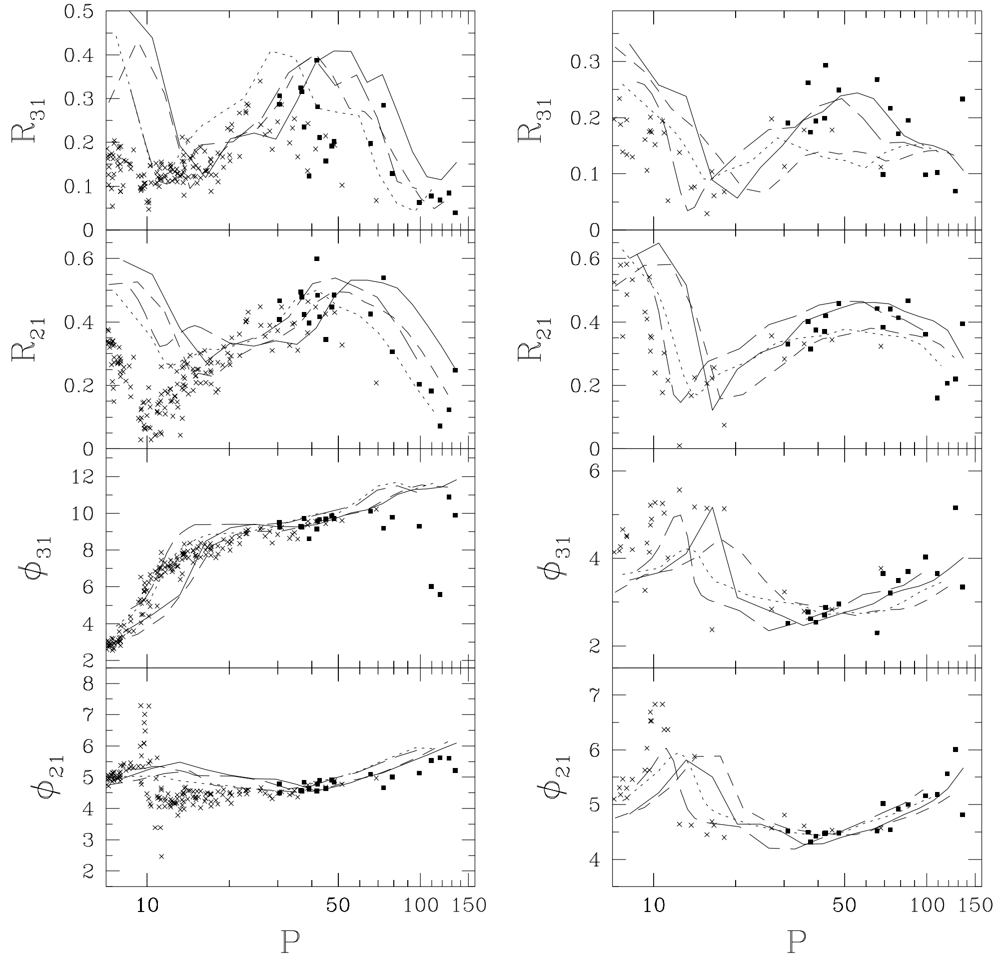

Fig. 2. Weighted phases, $\phi_{n1} = \phi_n - n\phi_1$ and relative amplitude ratios, $R_{n1} = A_n/A_1$ of theoretical light curves (left panels) and velocity curves (right panels) against period, compared with observations of Cepheids in Galaxy (crosses) and Magellanic Clouds (filled squares). Short dashed line: sequence (a), -200 K; continuous line: sequence (a), -400 K; dotted line: sequence (b), -200 K; long dashed line: sequence (b), -400 K

- Antonello (1998) found a new progression of the light and radial velocity curves of longer P Cepheids, and suggested the resonance $P_1/P_0 = 2$ at $P_0 \sim 130$ d as the responsible mechanism for this effect. The present models offer probably half of the required theoretical support to this interpretation. The radial velocity phase differences for $P \gtrsim 100$ d increases in a way which recalls what occurs at $P \sim 10$ d, and similarly also the amplitude ratios; unfortunately, it is not possible to get limit cycles for longer P and to verify the complete similarity. As regards the light curves, there is an interesting hint that R_{31} reaches a minimum value in this P range, which is expected in case of a resonance; on the other hand the light curve phase differences are structureless, but there are similar problems also for the 10 d resonance. This is an interesting problem theoretically on resonance phenomena under strong non-adiabatic pulsation, if the feature comes from the mode resonance.

4. Nonlinear effects

4.1. Existence of a limit cycle

For most of the models we started the simulation with small amplitude, which grew until the pulsation settled into a limit cycle oscillation. Some of the models, however, did not have limit cycles in nonlinear regime. For the most luminous models in Fig. 1 the amplitude at the photosphere became so large that it was not possible to continue the simulation. There are similar cases in the less-massive supergiant stars and most of them show chaotic pulsations as steady states of nonlinear pulsation. Although we cannot confirm a steady state of nonlinear pulsation of the present models, we suspect these stars do not have limit cycles. Thus, we expect that observed counterparts of these stars, namely those with P longer than about 150 d (Madore 1985) are not regular pulsators. It is interesting to compare this result with that obtained in Paper I for galactic Cepheids. The models with $Z = 0.020$ studied in Paper I have no limit cycles for $P \gtrsim 70$ d, which indicates a dependence of the maximum P on the metallicity, in the sense that such a P is longer for lower metallicities. The observed stars in Galaxy and Magellanic Clouds confirm qualitatively this theoretical result.

4.2. Nonlinear periods

It is well known that the differences among periods in linear adiabatic, linear nonadiabatic and limit cycle pulsation calculations are quite small in short period Cepheids. For longer period Cepheids, however, the differences become significant. There is a strong coupling between acoustic waves and thermal waves, and this makes the nonadiabatic periods quite longer than adi-

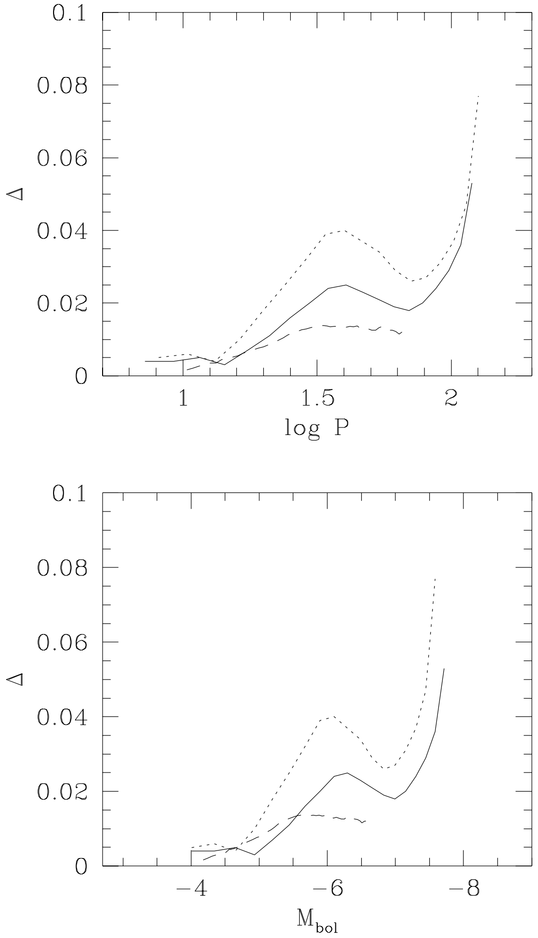

Fig. 3. The relative difference of nonlinear and LNA P as a function of LNA P (upper panel) and bolometric magnitude (lower panel) of sequence (a) models (continuous line; -400 K; dotted line: -200 K), compared with models of Cepheids in Galaxy (dashed line). One should note that nonlinear periods are much longer than those calculated from LNA models of long P Cepheids

abatic ones in luminous cepheids (Aikawa 1985). We show in Fig. 3 that the pulsation periods at limit cycles are even longer than the linear nondiabatic ones, and the relative difference Δ between linear nonadiabatic (P_{lna}) and nonlinear (P_{nl}) periods,

$$\Delta = (P_{nl} - P_{lna})/P_{lna}, \tag{3}$$

increases remarkably for small changes of the luminosity L. The result is that the slope of the theoretical PL relation for longer P is quite smaller than for shorter P.

Carson & Stothers (1984) tried to explain the observed flattening of the PL relation of Cepheids in Magellanic Clouds for $P \gtrsim 100$ d using nonlinear pulsation models, and ascribed this behavior to the low T_e values. However we note that if it was just a matter of T_e, the period-luminosity-color PLC relation should not show a ‘flattening’. Laney & Stobie (1986) remark that Magellanic Cloud stars with $P > 100$ d are subluminous both in the PL *and* PLC relations, and by about the same amount, and are not unusually red for their periods. We conclude that the reason for such an observed effect is not just the low T_e, but it is the lengthening of the nonlinear P displayed in Fig. 3. Actually the models can explain quantitatively only a small part of the observed effect.

It is possible to conclude from Fig. 3 that the lengthening is typical of low metallicity galaxies, because, on the one hand, it should not be possible to find very long P Cepheids in metal rich galaxies, and on the other hand for larger Z values the lengthening appears to be generally smaller.

5. Conclusions

Nonlinear pulsation models for long period Cepheids in Magellanic Clouds have been compared with observations in terms of weighted phases and amplitude ratios of Fourier components of light and radial velocity curves. While the longer P ($\gtrsim 30$ d) Cepheids appear unsensitive to the adopted M–L relation, for shorter P Cepheids it seems that an overluminous relation gives better results than a full overshooting–type relation. The results indicate also that regular limit cycles of more massive models depend on the metal content, in the sense that more massive models can have a stable limit cycle for lower metal content. The nonlinear P of such metal poor models tends to be much longer than the LNA one, and this can explain the flattening of the PL relation for $P \gtrsim 100$ d, which is observed in metal poor galaxies. Finally, the difficulties of radiative models in reproducing the observed features of resonance effects on the light curves are confirmed.

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